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Quark-number susceptibility in quenched quantum chromodynamics

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Using staggered fermions, we measure the quark-number susceptibility of quenched quantum chromodynamics in a Monte Carlo simulation on an $8^3\times4$ lattice. At low temperatures it is consistent with zero and it rises suddenly to nonzero values across the known deconfinement phase transition. Implications for the interpretation of this susceptibility as an indicator of the presence of light baryonic modes in the plasma are discussed.

Numerical simulations of lattice quantum chromodynamics (QCD) have led us to an exciting prediction of a new and hitherto unseen state of matter called the quark-gluon plasma (QGP). While the QGP could have existed a few microseconds after the big bang, the heavy-ion-collision experiments at CERN and BNL may provide us with a glimpse of it under laboratory conditions. At present, it seems that the best way to gain more theoretical insight into the nature of the QGP is through numerical simulations, especially since there are strong indications[1] that close to the critical temperature T_c the QGP is definitely not an ideal gas of quarks and gluons. Of course, at sufficiently large temperatures, it should become an ideal gas due to the asymptotic nature of QCD. However, such large temperatures may not be accessible in the proposed heavy-ion experiments.

A particularly interesting probe to study the dominant modes of QGP was suggested recently by Gottlieb *et al.*[2] and McLerran.[3] On an $8^3\times4$ lattice Gottlieb *et al.* simulated QCD with two light flavors and measured the quark-number susceptibility at zero chemical potential as a function of temperature. They found it to be vanishingly small in the low-T phase. At T_c, it rose suddenly to a nonzero value and behaved like T^2 thereafter. They interpreted this to be due to the presence of dominant low-mass dynamical modes of the QGP which are baryonic in nature.

The quark-number susceptibility is analogous to the familiar magnetic susceptibility. If μ denotes the chemical potential for baryons then the baryon- or quark-number density and the corresponding susceptibility are, respectively, given by

$$n(\mu,T)=\frac{T}{V}\frac{\partial \ln Z(\mu,T)}{\partial\mu}\,, \tag{1}$$

and

$$\chi(\mu,T)=\frac{\partial n(\mu,T)}{\partial\mu}\,. \tag{2}$$

Here V is the spatial volume and T is the temperature of the system. For sufficiently large V, the susceptibility at zero chemical potential $\chi(0,T)$ is clearly proportional to T^2 on dimensional grounds and the information about the dynamics of the system is contained in a dimensionless function of T, $f(T)$, multiplying it. In a simplistic picture of a noninteracting gas of particles of mass m, $f(T)\sim\exp(-m/T)\sim0$, if $m\gg T$ and $f(T)\simeq1$, if $m\ll T$. This is very similar to the behavior observed in Ref. 2. If there is more than one quark flavor ($N_f>1$), then μ becomes a linear combination of the corresponding chemical potentials and χ/N_f replaces χ.

It might be emphasized, however, that the above picture is too simple minded, especially close to T_c. As mentioned earlier, QGP is far from being an ideal gas near T_c and may, in fact, be nonperturbative in nature. Of course, one could argue that the susceptibility is expected to be sensitive only to low-mass dynamical modes which are baryonic; heavy or nonbaryonic modes are not counted by it and the former may exhibit an ideal-gas behavior. It seems interesting to test this simplistic picture by taking the limit $N_f\to0$, which suppresses the dynamical creation and annihilation of light-quark pairs (an obvious source of light plasma modes). One can then calculate Eq. (2) again and compare with the results of full QCD. Naively, if the light quarks are the direct source of the baryon number in the plasma, one might expect the two computations to produce susceptibilities which behave quite differently with respect to temperature (for example, suppressed pair creation could lead to a susceptibility which is flatter across the transition). Assuming that the limit $N_f\to0$ leads smoothly to the quenched approximation $N_f=0$, we have measured the susceptibility in quenched QCD as a function of temperature and found qualitatively the same behavior as in full QCD. Thus, our work necessitates a deeper look into the interpretation of Ref. 2.

For staggered fermions the partition function Z, obtained by integrating out the fermionic degrees of freedom, is given by

$$Z=\int\prod_{x,\tau}dU_x^\tau\exp[-S_{\rm eff}(\{U_x^\tau\},\beta,m_ia,\mu_ia)]\,,$$

where the effective action is

$$S_{\rm eff}=S_g+\tfrac{1}{4}\sum_{i=1}^{N_f}{\rm Tr}\ln M^i\,.$$

The gluonic action S_g and the inverse quark propagator M^i are given by

$$S_g=\beta\sum_{x,\tau>\nu=0}^{3}(1-\tfrac{1}{3}\,{\rm Re\,tr}U_x^\tau U_{x+\hat\tau}^{\nu\dagger}U_{x+\hat\nu}^\tau U_x^{\nu\dagger})$$

and

$$M_{x,y}^i=m_ia\delta_{x,y}+\tfrac{1}{2}\sum_{\nu=1,2,3}(-1)^{x_0+\cdots+x_{\nu-1}}(U_x^\nu\delta_{y,x+\hat\nu}-U_y^{\nu\dagger}\delta_{y,x-\hat\nu})+\tfrac{1}{2}[f(\mu_ia)U_x^0\delta_{y,x+\hat0}-g(\mu_ia)U_y^{0\dagger}\delta_{y,x-\hat0}]\,. \quad (3)$$

Our notation is fairly standard. The above formulas are written for N_f flavors of quarks of mass m_ia and the corresponding chemical potentials are μ_ia on the lattice. The functions f and g have to satisfy certain constraints[4] to ensure that the energy density obtained from Z above remains finite in the continuum limit of $a\to 0$. They suffice to obtain a finite number density and susceptibility as well.[2] Two different choices for them have been proposed in the literature,[5] which do satisfy these constraints. However, as we shall now show, we do not need to worry about the details of the prescription.

In the general case above, one can obtain N_f different conserved densities and $N_f(N_f+1)/2$ susceptibilities, including those corresponding to the variations of number densities n_i with the chemical potential μ_j. It is obvious from Eqs. (1) and (2) that they will, in general, involve first and second derivatives of the functions f and g. Using the constraints on these functions, it can be easily shown that $f'(0)=-g'(0)=1$ and $f''(0)=g''(0)=1$. Thus the number densities and the susceptibilities are independent of the prescription to introduce the chemical potentials on the lattice, given by the functions f and g, if one evaluates them either at zero chemical potential or in the continuum limit where $\mu a\to 0$. While we will be concerned with zero chemical potentials here, we note that corrections due to nonzero a, being prescription dependent, can potentially cause non-negligible changes in the critical parameters, particularly if they are obtained in the strong-coupling region.

A physically meaningful way of constructing susceptibilities for the baryon number or the third component of the isospin is to define appropriate linear combinations of $\partial n_i/\partial\mu_j$, as done in Ref. 2:

$$\chi_0(\mu,T)=\frac{1}{N_f}\sum_{i,j}\frac{\partial n_i}{\partial\mu_j}\,,$$
$$\chi_3(\mu,T)=\frac{1}{N_f}\left(\frac{\partial}{\partial\mu_1}-\frac{\partial}{\partial\mu_2}\right)(n_1-n_2)\,. \quad (4)$$

If all m_i (and the corresponding μ_i) are equal, then these equations simplify to

$$\chi_0(T)=\chi_3(T)+\frac{N_fT}{4V}\left[\left\langle{\rm Tr}\frac{1}{M}\tilde M_0{\rm Tr}\frac{1}{M}\tilde M_0\right\rangle-\left\langle{\rm Tr}\frac{1}{M}\tilde M_0\right\rangle^2\right],\quad \chi_3(T)=\frac{4T}{V}\left\langle{\rm Tr}\frac{1}{M}M_0-{\rm Tr}\frac{1}{M}M_0\frac{1}{M}M_0\right\rangle, \quad (5)$$

where all μ_i have been set to zero and the matrix M_0 ($\tilde M_0$) is the time component of the inverse quark propagator in Eq. (3) (with opposite sign).

We have computed these susceptibilities along with $\langle\bar\psi\psi\rangle$ and the average Polyakov loop $\langle L\rangle$ in a simulation of QCD in the quenched approximation. In this approximation, $S_{\rm eff}=S_g$ and the dynamics of the theory is completely governed by gluons. Fermionic operators measured in such a background field of gluons can be seen as representing the properties of static quarks interacting via gluonic forces. To define such operators consistently by means of the same formulas as are used in full QCD, one must assume that the quenched approximation can be reached smoothly as the $N_f\to 0$ limit of full QCD. This assumption underlies all calculations performed in the quenched approximation; its validity can be checked by comparing quenched calculations to full QCD calculations performed with very small N_f (see Ref. 6). In particular, we have used Eq. (5) to compute $\chi_3(T)$ [which becomes equal to $\chi_0(T)$ in the quenched approximation] in the sense of the limit $N_f\to 0$. A direct comparison of the results of Gottlieb *et al.*[1] with those of quenched QCD should be very helpful in understanding the origin of the baryonic excitations these authors have seen in the QGP.

We employed an $8^3\times 4$ lattice to simulate the theory at several values of the couplings which included the deconfinement transition: $5.50\le\beta\le 5.75$. χ was computed with the method of random vectors[2] using a conjugate-gradient algorithm to calculate M^{-1}. Up to 3 (5) such vectors were used in the hadron (plasma) phase for bare-quark masses $ma=0.1$, 0.05, and 0.025 (and 0.01). The inverse was obtained by demanding that an average element of the residue vector be less than $\sim 10^{-3}$. At each β 10000 interactions have been typically per-

TABLE I. The susceptibility for quenched QCD on an $8^3\times4$ lattice as a function of the bare-quark mass ma and the coupling β. The last column has been obtained by a linear extrapolation of all the data at each coupling.

β \ ma	0.10	0.05	0.025	0.01	0.00
5.75	0.0760(10)	0.0880(10)	0.0925(10)	0.0945(10)	0.0974(8)
5.73	0.0765(40)	0.0905(25)	0.0995(35)	0.0975(35)	0.1007(29)
5.70	0.0640(15)	0.0805(15)	0.0850(15)	0.0915(20)	0.0938(14)
5.68	0.0625(30)	0.0745(30)	0.0780(35)	0.0885(30)	0.0885(26)
5.67	0.0400(50)	0.0440(60)	0.0520(70)	0.0575(80)	0.0565(59)
5.65	0.0185(25)	0.0235(30)	0.0295(30)	...	0.0319(36)
5.60	0.0080(20)	0.0095(30)	0.0130(30)	...	0.0136(35)
5.50	−0.0010(20)	0.0045(25)	−0.0065(40)	...	0.0008(39)

formed and $\langle\bar\psi\psi\rangle$ and χ were calculated every 100 iterations.

Table I contains our results for the susceptibility at all quark masses that we studied. One sees that χ increases monotonically as the quark mass is decreased, which is consistent with its expected behavior. The results for $ma=0$ are obtained by linear extrapolations from the data at all the masses (extrapolating from the data at $ma\leq0.05$ gives very similar results). Figure 1 plots these extrapolated values for χ along with similarly extrapolated values for $\langle\bar\psi\psi\rangle$ and with the deconfinement order parameter $\langle L\rangle$. We see that χ jumps abruptly at the critical point of the previously known[7] phase transition where $\langle\bar\psi\psi\rangle$ and $\langle L\rangle$ also jump. The qualitative similarity of these results with those of Ref. 2 is remarkable.

Figure 2 compares our results for χ/N_fT^2 for $m/T=0.2$ as a function of T/T_c, to the corresponding results[2] for χ_3 in the full theory. This direct comparison assumes asymptotic scaling for both data sets. We expect that its violations should not be very significant since we are using dimensionless ratios for which a weaker assumption of scaling may be valid, and since the known scaling violations in both the theories are similar in magnitude.[8,9] While remaining aware of these caveats about scaling, one sees that our results are indeed similar to those of Ref. 2. For both $N_f=0$ and 2, the jump coincides with the onset of the plasma phase, with the difference that for $N_f=0$ the phase transition is expected to have more to do with deconfinement, while for $N_f=2$ it has to do with chiral symmetry. One further notices that the susceptibility for the quenched theory is lower than that for the full theory at a given T/T_c.

According to the simple picture described in the Introduction, the quark-number susceptibility should act as a counter of light baryonic modes. Might one then interpret the similarity of our results to those of Ref. 2 as indicating the presence of light baryonic modes in the plasma even in the quenched approximation? Such modes are not expected from a classical point of view, governed by the purely gluonic action. Unlike in full QCD, the relation between the partition function and any fermionic observable is not fully clear in the quenched approximation. Nevertheless,

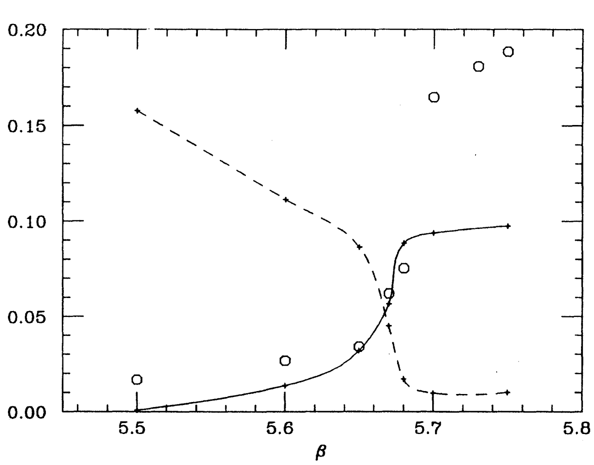

FIG. 1. The deconfinement order parameter $\langle L\rangle$ (circles), the $m=0$ chiral condensate $\langle\bar\psi\psi\rangle$ (dashed curve), and the $m=0$ susceptibility χ (solid curve) in lattice units as functions of the coupling β on an $8^3\times4$ lattice.

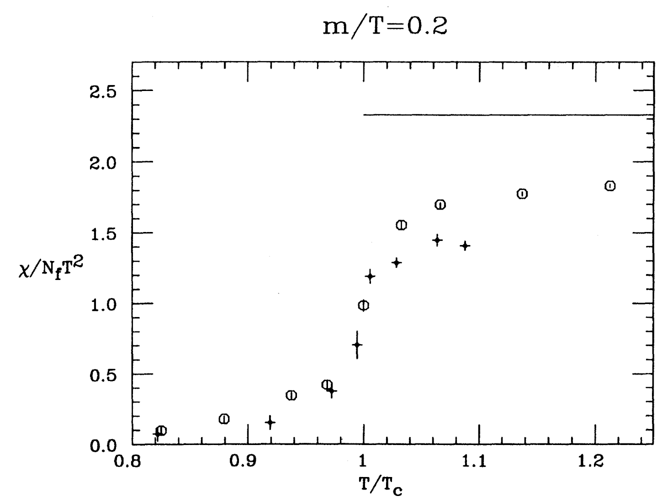

FIG. 2. Susceptibility in units of N_fT^2 as a function of T/T_c for quark mass $m/T=0.2$ on an $8^3\times4$ lattice. Daggers are for the quenched theory and circles are for full QCD (after Ref. 2). The horizontal line indicates the value of the susceptibility for a massless ideal gas on the same size lattice.

if we accept that observables measured in the quenched approximation are equal to the $N_f \rightarrow 0$ limits of the corresponding observables of the full theory, Fig. 2 tends to suggest that the light dynamical quarks do not contribute substantially to the observables in Eq. (5). These would thus be insensitive to the intrinsic nature of the plasma with respect to its light baryonic or quark modes. In the limiting sense of $N_f \rightarrow 0$, our results seem to point to alternate sources of baryon number in the plasma which could include heavy (constituent?) quarks or bound states in sufficiently large numbers.

Another way to understand our results is to recognize that what one sees in Fig. 2, for both the full theory and the quenched theory, is a reflection of the response of the operator M^{-1} to a sudden change of dynamics of the gauge fields, which appears to be qualitatively similar in the two theories. Indeed, using the usual renormalization-group arguments which are employed to explain the similarity of the hadron spectrum in these two theories, one can conclude from Fig. 2 that the physical coupling of the external field to the medium is similar in the two cases, if the necessary change of scale Λ is properly taken care of. In such a picture, it is not meaningful to speak of the quark modes as separate from the gluonic ones in a baryonless plasma due to the intrinsic renormalization scale dependence of these modes. One can also argue in this picture that the essential physics behind the phase transitions seen in the two theories is the same, again apart from a scale change. Furthermore, one can understand in the same way why the energy density[1,10] or the hadronic screening lengths[11] in the full theory and the quenched theory are also similar in nature. Of course, one does expect physical differences in the two theories which cannot be scaled away. In our case, the difference of the susceptibilities above the phase transition may be such a quantity. However, to establish its physical significance one would need to show that it survives the continuum limit.

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