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$\langle \bar{q}q \rangle$ component of the quark self-energy in the light-cone gauge

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The operator-product expansion is used to evaluate the lowest-order, quark-condensate component of the quark self-energy in the light-cone gauge. The on-shell value of the self-energy is found to agree with that obtained in covariant gauges.

A portion of the nonperturbative content of QCD may be probed by augmenting the perturbative theory with the operator-product expansion (OPE) of those nonperturbative vacuum expectation values (NPVEV's) already known to contribute to QCD sum-rule phenomenology.[1,2] These NPVEV's incorporate gauge-invariant condensates into the Green's functions of QCD, leading to power-law dependencies that differ from those anticipated by purely perturbative methods. In covariant gauges, such OPE methods have already been employed to determine the $O(\alpha_s)$ contribution of the chiral-symmetry-violating quark condensate ($\langle \bar{q}q \rangle$) to the quark self-energy, and the pole position of the $\langle \bar{q}q \rangle$-corrected quark propagator has been demonstrated to be gauge-parameter independent.[3,4] Such insensitivity to the choice of gauge is a property well known to characterize purely perturbative QCD self-energies when evaluated on shell.[5] Moreover, an effective mass of ~ 300 MeV for u and d quarks is obtained from the pole position of the $\langle \bar{q}q \rangle$-corrected propagator, a scale suggestive of a constituent mass.[3,6] An important test of these results is to evaluate the $\langle \bar{q}q \rangle$ component of the quark self-energy in a *noncovariant* gauge, since an effective mass devolving from a (gauge-dependent) self-energy must be gauge independent in order to be of physical interest.

In covariant gauges, only the leading-order and next-to-leading-order terms in the NPVEV $\langle 0|{:}\bar{\psi}(x)\psi(y){:}|0\rangle$ contribute to the quark self-energy.[7,8] This fortuitous decoupling of higher-order OPE contributions from the quark self-energy is a direct consequence of the $g^{\mu\nu}/p^2$ and $p^\mu p^\nu/p^4$ dependence characterizing the gluon propagator in covariant gauges. The differing behavior of the gluon propagator in noncovariant gauges [e.g., the $(n^\mu p^\nu + p^\mu n^\nu)/(n\cdot p)^2$ structure in the light-cone gauge] necessarily couples *all* orders of the OPE into the quark self-energy. The sum of all such OPE terms (involving progressively steep power-law dependencies) must eventually lead to the same effective mass obtained in covariant gauges from leading-order and next-to-leading order OPE terms, if that mass is to have any physical significance. We therefore test the covariant-gauge results of Refs. 3 and 4 (as well as the methodological consistency of augmenting perturbative QCD in any gauge with NPVEV's) by evaluating the quark-condensate component of the quark self-energy in a noncovariant gauge, the light-cone gauge.[9]

The OPE for the quark propagator is

$$iS_F(p) = \int d^4x\, e^{ip\cdot x}\langle 0|T[\psi(x)\bar{\psi}(0)]|0\rangle$$
$$= C_I(p) + C_{\bar{q}q}(p)\langle \bar{q}q\rangle + C_{GG}(p)\langle G^{\mu\nu}G^{\mu\nu}\rangle$$
$$+\text{higher-dimension condensates}\ . \qquad (1)$$

The first term in (1) is purely perturbative, whereas the second term is the nonperturbative contribution of interest. The coefficient $C_{\bar{q}q}(p)$ in this term can be evaluated perturbatively, as the OPE factors the long-distance behavior of QCD into the condensates, provided such condensates are of mass dimension less than 10 (Ref. 1). To evaluate this coefficient, we permit $\langle \bar{q}q\rangle$ to enter the perturbation series directly via the nonlocal NPVEV $\langle {:}\bar{\psi}(z)\psi(y){:}\rangle$, a residual normal-ordered contribution to the Wick expansion of[2]

$$\left\langle 0 \left| T\left[\psi(x)\int d^4y \int d^4z\, L^I_{\rm QCD}(y) L^I_{\rm QCD}(z)\bar{\psi}(0)\right]\right| 0\right\rangle .$$

The lowest-order contribution to $C_{\bar{q}q}(p)$, as represented by Fig. 1, is thus given by[4]

$$\Sigma(p) = \frac{-4g^2}{3}\int d^4(y-z)e^{ip\cdot(y-z)}$$
$$\times\gamma^\mu\langle 0|{:}\bar{\psi}(z)\psi(y){:}|0\rangle\gamma^\nu$$
$$\times\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(y-z)}D_{\mu\nu}(k)\ . \qquad (2)$$

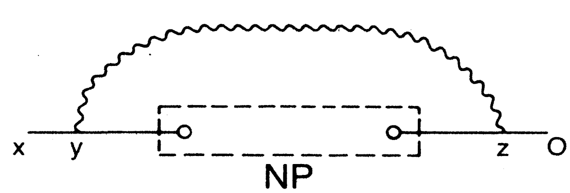

FIG. 1. Configuration-space Feynman-diagrammatic representation of the $O(\alpha_s)$ contribution of the quark condensate to the quark self-energy. The box encloses a nonperturbative vacuum expectation value of quark fields.

In the light-cone gauge, the gluon propagator $D_{\mu\nu}(k)$ in (2) is given by

$$\langle T[A_\mu^a(x)A_\nu^b(y)]\rangle \equiv i\delta^{ab}\int \frac{d^4p}{(2\pi)^4}D_{\mu\nu}(p)\ , \tag{3a}$$

$$D_{\mu\nu}(p)=\frac{-g^{\mu\nu}}{p^2}+\frac{n^\mu p^\nu+n^\nu p^\mu}{(n\cdot p)p^2},\quad n^2=0\ . \tag{3b}$$

The nonperturbative content of (2) resides in the NPVEV $\langle 0|{:}\bar\psi(z)\psi(y){:}|0\rangle$. The quark-condensate projection of this NPVEV is given by[10]

$$\langle 0|{:}\bar\psi(z)\psi(y){:}|0\rangle=\frac{\langle\bar q q\rangle}{3}\sum_{N=0}^{\infty}\frac{(-im)^{2N}(y-z)^{2N}}{N!(N+1)!4^{N+1}}+\frac{\langle\bar q q\rangle}{3}\sum_{N=0}^{\infty}\frac{(-im)^{2N+1}\gamma\cdot(y-z)(y-z)^{2N}}{2(N+2)!N!4^{N+1}}$$
$$+\text{contributions from higher-dimension condensates}\ , \tag{4}$$

where $\langle\bar q q\rangle\equiv\langle 0|{:}\bar\psi(0)\psi(0){:}|0\rangle$ with color and Dirac indices contracted. This expression is obtained by performing a two-variable Taylor expansion about $y=z=0$ and then extracting the gauge-invariant components through use of the fixed-point gauge.[11] One then obtains (4) from the covariantized Taylor series by eliminating covariant derivatives through use of the $\not{D}\psi=-im\psi$ equation of motion.[2,12]

By substituting the quark-condensate component of (4) into the self-energy of (2) and then performing the integration over $(y-z)$, one obtains the following expression for $\Sigma(p)$:

$$\Sigma(p)=\frac{-4g^2}{9}\langle\bar q q\rangle\sum_{N=0}^{\infty}\frac{m^{2N}}{N!(N+1)!4^{N+1}}\left[\frac{\partial^2}{\partial p^2}\right]^N\gamma^\mu\gamma^\nu\left[\frac{-g^{\mu\nu}}{p^2}+\frac{p^\mu n^\nu+n^\mu p^\nu}{p^2(n\cdot p)}\right]$$
$$+\frac{2g^2}{9}\langle\bar q q\rangle\sum_{N=0}^{\infty}\frac{m^{2N+1}}{N!(N+2)!4^{N+1}}\frac{\partial}{\partial p^\lambda}\left[\frac{\partial^2}{\partial p^2}\right]^N\gamma^\mu\gamma^\lambda\gamma^\nu\left[\frac{-g^{\mu\nu}}{p^2}+\frac{p^\mu n^\nu+n^\nu p^\mu}{p^2(n\cdot p)}\right]\ . \tag{5}$$

Equation (5) is simplified by performing the Dirac algebra and explicitly performing the differentiation with respect to p^λ in the second term:

$$\Sigma(p)=\frac{-4g^2}{9}\langle\bar q q\rangle\sum_{N=0}^{\infty}\frac{m^{2N}}{N!(N+1)!4^{N+1}}\left[\frac{\partial^2}{\partial p^2}\right]^N\left[\frac{-2}{p^2}\right]$$
$$+\frac{2g^2}{9}\langle\bar q q\rangle\sum_{N=0}^{\infty}\frac{m^{2N+1}}{N!(N+2)!4^{N+1}}\left[\frac{\partial^2}{\partial p^2}\right]^N\left[\frac{-4\not{p}}{p^4}+\frac{4\not{n}}{p^2(n\cdot p)}\right]\ . \tag{6}$$

The n-independent terms in (6) have previously been evaluated in the covariant-gauge calculation, and are found to truncate for $N\geq 1$ (Refs. 7 and 12). Thus the quark-condensate component of the quark self-energy becomes

$$\Sigma(p)=\frac{2g^2}{9}\langle\bar q q\rangle\left[\frac{1}{p^2}-\frac{m\not{p}}{2p^4}\right]+\frac{8g^2}{9}\langle\bar q q\rangle\sum_{N=0}^{\infty}\frac{m^{2N+1}\not{n}}{N!(N+2)!4^{N+1}}\left[\frac{\partial^2}{\partial p^2}\right]^N\left[\frac{1}{p^2(n\cdot p)}\right]\ . \tag{7}$$

We thus see from (7) that all orders of the OPE contribute to the self-energy in the light-cone gauge, in contrast with covariant gauges where no terms in Σ are more than linear in m. The derivatives appearing in (7) are evaluated by observing that since n is a null vector, the inverse power of $n\cdot p$ cannot be increased by differentiation without introducing factors of $n^2(=0)$ into the numerator:

$$\left[\frac{\partial^2}{\partial p^2}\right]^N\frac{1}{p^2(n\cdot p)}=\frac{4^N N!N!}{p^{2N+2}(n\cdot p)}\ . \tag{8}$$

Substitution of (8) into (7) leads to the following series representation for $\Sigma(p)$:

$$\Sigma(p)=\frac{2g^2}{9}\langle\bar q q\rangle\left[\frac{1}{p^2}-\frac{m\not{p}}{2p^4}+\frac{\not{n}}{m(n\cdot p)}\sum_{N=0}^{\infty}\frac{1}{(N+1)(N+2)}\left[\frac{m^2}{p^2}\right]^{N+1}\right]\ . \tag{9}$$

This series converges for $|p^2|\geq m^2$, and can be evaluated through utilization of the series[13]

$$\sum_{k=1}^{\infty}\frac{x^k}{k(k+1)}=1+\frac{1-x}{x}\ln(1-x) \tag{10}$$

$(|x|\leq 1)$ in order to obtain the following expression for the $\langle\bar q q\rangle$ component of the quark self-energy for $m^2\leq|p|^2$:

$$\Sigma(p)=\frac{2g^2}{9}\langle\bar q q\rangle\left\{\frac{1}{p^2}-\frac{m\not{p}}{2p^4}+\frac{\not{n}}{m(n\cdot p)}\left[1+\left[\frac{p^2}{m^2}-1\right]\ln\left[1-\frac{m^2}{p^2}\right]\right]\right\}\ . \tag{11}$$

To evaluate the on-shell value of $\Sigma(p)$ it is useful to recall that

$$\bar{u}\gamma^\mu u = \frac{p^\mu}{m}\bar{u}u \ , \tag{12}$$

where $\bar{u}, u$ are the external, on-shell fermion wave functions. Using (12) and taking the mass-shell limit in (11), we obtain the following on-shell value for $\Sigma(p)$ in the light-cone gauge:

$$\Sigma(m) = \frac{g^2\langle \bar{q}q \rangle}{3m^2} \ . \tag{13}$$

This value is identical to the on-shell covariant-gauge value, which is derived from the covariant gauge self-energies:[4,12]

$$\Sigma(p) = \frac{g^2\langle \bar{q}q \rangle}{9p^2}\left[(3+a) - \frac{am\not{p}}{p^2}\right] , \tag{14}$$

on the $\not{p} = m$ mass shell.

In summary, the lowest-order quark-condensate component of the quark self-energy has been evaluated in the light-cone gauge to all orders in the mass parameter m. The value of this self-energy on the quark mass shell is identical with that obtained in covariant gauges. This result provides further field-theoretical support for the gauge independence of the dynamical quark mass generated by the quark condensate.

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