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Constraints on unstable neutrinos from cosmology

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We present a detailed numerical calculation of the cosmological limits on the lifetime of Dirac and Majorana neutrinos decaying into invisible modes.

I. INTRODUCTION

The possibility that neutrinos account for the missing mass in the Universe has been considered seriously during the last two decades.[1,2] Stable neutrinos would be of cosmological relevance if their mass is either in the range 40–100 eV or 1–10 GeV. Stated in other words, the limits on stable neutrino masses that follow from cosmology imply that a neutrino must weigh less than 40–100 eV or its mass must be bigger than 1–10 GeV. (This last limit is known as the Lee-Weinberg bound.[2]) A massive neutrino, however, can decay. If it decays, the limits on the mass will obviously depend on the neutrino lifetime. This more general case of neutrinos decaying into invisible modes has been discussed in Ref. 3. The evaluation of the limits in Ref. 3 was, however, done using some simplifying assumptions. Because of the importance of these limits, we have carried a detailed calculation of the limits on the neutrino lifetime as a function of its mass, which we shall present in this paper. Our results are somewhat different than those of Ref. 3. The origin of the discrepancy is that we have dropped some of their simplifying assumptions. We discuss this in detail at the end of the paper. A similar analysis corresponding to the case of a heavy stable neutrino has been performed recently in Ref. 4.

We shall consider a massive sequential neutrino—a neutral lepton of a $SU(2)_L \times U(1)_Y$ doublet—and investigate its contribution to the Universe energy density. The contribution contains a piece coming from the neutrino itself and pieces related to the decay products. Our limits follow from the requirement that this contribution does not exceed the total energy density of the Universe. We will restrict ourselves to the case that the neutrino can decay only into invisible modes, e.g., three lighter neutrinos, a light neutrino plus a stable boson, etc. If it decays into photons and/or charged particles, other constraints (as the data on the microwave background) would apply and, usually, this leads to more strict restrictions. We will consider the case that the neutral lepton is a Dirac neutrino as well as the case that it is a Majorana neutrino.

The paper is organized as follows. In Sec. II we study the case of heavy neutrinos and in Sec. III we complete the discussion with the consideration of light neutrinos. In Sec. IV we present our results.

II. EVOLUTION AND DECOUPLING OF HEAVY NEUTRINOS

We consider first the case of a heavy neutrino, which by definition is nonrelativistic by the time of decoupling. We shall use the rate equation[2]

$$\frac{dn}{dt} = \langle \sigma |v| \rangle (n^2 - n_0^2) - 3Hn \ , \tag{1}$$

which can be deduced from the Boltzmann equation[5] provided some hypotheses are made.[6] In Eq. (1), n is the neutrino number density, n_0 is the corresponding equilibrium value,

$$n_0(T) = \frac{2}{(2\pi)^3} \int \frac{d^3p}{\exp(\sqrt{m^2+p^2}/T)+1} \ , \tag{2}$$

and $\langle \sigma |v| \rangle$ is the average value of the annihilation cross section times the modulus of the relative initial velocity (the average is defined precisely in Ref. 5). The Hubble parameter H in Eq. (1) is the Universe expansion rate at time t and is given by

$$H = \left[\frac{8\pi\rho G}{3} \right]^{1/2} \ , \tag{3}$$

where, in a radiation-dominated Universe with photon temperature T_γ, the energy density can be written as

$$\rho = \frac{\pi^2}{30} g_* T_\gamma^4 \ . \tag{4}$$

Here g_* is the total number of effective degrees of freedom. At a given temperature of the radiation-dominated Universe, the main contribution to g_* is due to relativistic particles with $m < T_\gamma$. The values of g_* would then be approximated by a series of step functions. The contribution of nonrelativistic particles to g_* is suppressed by the Boltzmann factor, but the existence of a large number of such particles, e.g., strong resonances, makes it non-negligible. In our analysis we shall borrow the realistic evaluation of g_* performed in Ref. 7, where relativistic as well as nonrelativistic particles are considered.

The rate equation (1) can be cast in a more convenient form by defining

$$Y = \frac{n}{s}, \quad x = \frac{m}{T}, \quad \alpha = \frac{T}{T_\gamma} \ , \tag{5}$$

with m and T the mass and temperature of the neutrino species.[8] The entropy density

$$s=\frac{2\pi^2}{45}g_{*s}T_\gamma^3 \qquad (6)$$

is written in terms of g_{*s}, defined as the effective degrees of freedom of all relativistic bosons (B) and fermions (F) which would have the same entropy at $T=T_\gamma$:

$$g_{*s}=\sum_B g_B\left[\frac{T_i}{T_\gamma}\right]^3+\frac{7}{8}\sum_F g_F\left[\frac{T_i}{T_\gamma}\right]^3 . \qquad (7)$$

Using the definitions of Eq. (5) in Eq. (1) we get

$$\frac{dY}{dx}=-K\langle\sigma|v|\rangle(Y^2-Y_0^2) , \qquad (8)$$

where $Y_0=n_0/s$ (equilibrium value) and

$$K=\frac{mM_P g_{*s}}{x^2\alpha}\left[\frac{\pi}{45g_*}\right]^{1/2} , \qquad (9)$$

with $M_P=1.22\times10^{19}$ GeV the Planck mass.

The precise form of the cross section σ of the annihilation process $N\bar N\to f\bar f$ into fermions, depends on the nature of the neutrino N. As we have stated in the Introduction we shall assume that N is the neutral member of a leptonic $SU(2)_L\times U(1)_Y$ doublet, with the corresponding charged lepton being heavier than N.

For ease of presentation we shall only display the cross sections corresponding to the case that N is nonrelativistic. (In Sec. III use is made of the full cross sections.) When N is a Dirac neutrino we obtain

$$\sigma|v|=\frac{G_F^2m^2}{2\pi}\sum_i(1-z_i^2)^{1/2}\left\{\left[1+\left[\frac{8-5z_i^2}{6(1-z_i^2)}-\frac{z_i^2}{3}\right]\beta^2\right](C_V^2+C_A^2)+\left[\tfrac{1}{2}z_i^2\left[1+\frac{2-z_i^2}{2(1-z_i^2)}\beta^2\right]\right](C_V^2-C_A^2)\right\} . \qquad (10)$$

The cross section when N is a Majorana neutrino is given by

$$\sigma|v|=\frac{G_F^2m^2}{\pi}\sum_i(1-z_i^2)^{1/2}\left\{\left[\frac{4-2z_i^2-2z_i^4}{3(1-z_i^2)}\right]\beta^2C_V^2+\left[z_i^2+\left[\frac{4-11z_i^2+17z_i^4/3}{3(1-z_i^2)}\right]\beta^2\right]C_A^2\right\} . \qquad (11)$$

In Eqs. (10) and (11), β is the neutrino velocity in the center-of-mass coordinate system, $z_i=m_i/m$ with m_i the mass of the final-state fermion and

$$C_V=J_3-2q\sin^2\theta_W,\quad C_A=J_3 . \qquad (12)$$

Here, J_3 is the weak-isospin component[9] and q the charge of the fermion. The fermions included in our calculations are e, μ, and τ and the corresponding neutrinos, together with the quarks u, d, s, c, and b. We have taken $m_u=m_d=350$ MeV, $m_s=550$ MeV, $m_c=1.8$ GeV, and $m_b=5$ GeV. We do not include the top quark since it is too heavy to be relevant in our analysis.

We have solved the rate equation (8) taking as initial condition at $x\to0$ (or $T\gg m$) that the heavy-neutrino species is in thermal and chemical equilibrium

$$Y=Y_0 . \qquad (13)$$

In Fig. 1 we display the behavior of Y as a function of x, corresponding to the results for a neutrino with $m=100$ MeV and 1 GeV. The characteristic features of this figure are easy to understand. For high temperatures Y tracks its equilibrium value Y_0. When T is on the order of the neutrino mass ($x\simeq1$), Y starts decreasing due to the Boltzmann factor. As a consequence, the rate at which the neutrinos interact cannot keep the chemical equilibrium in the expanding Universe, and Y departs from equilibrium. After decoupling, the neutrino number density dilutes as T^3 and thus Y evolves again smoothly. In the case that the neutrino is stable, it is (the value in) the asymptotic limit ($x\to\infty$) which gives the present population of neutrinos in the Universe. However, we are concerned with the more general case where the neutrino is unstable, and thus the neutrino population will decrease. This happens in fact for typical values of x much above the range shown in Fig. 1.

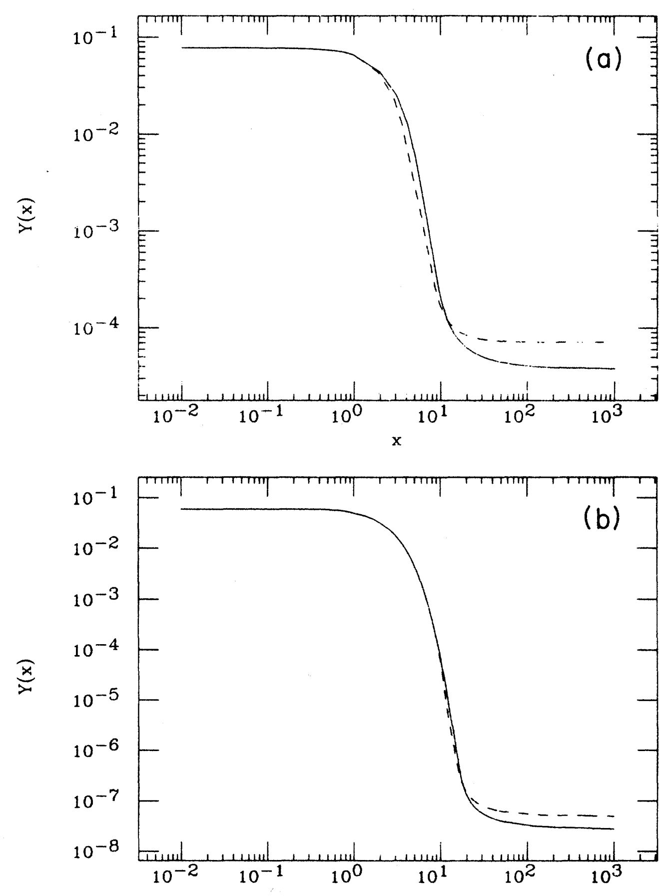

FIG. 1. Evolution of the specific neutrino density Y as a function of $x=m/T$, for (a) $m=100$ MeV and (b) $m=1$ GeV. Solid curves are the results for a Dirac neutrino and dashed curves correspond to a Majorana neutrino.

We are interested in the case that the neutrino energy density is an important fraction of the total energy density of the Universe. After the decoupling of the massive neutrinos, the Universe is still radiation dominated. The energy density of radiation dilutes as T^4, compared to the neutrino energy density which goes as T^3. It follows that, as the Universe cools, there will be a temperature T_3 at which the contribution of the neutrinos to the energy density equals the radiation contribution. However, the neutrinos will decay into invisible modes, and the decay products will dominate the energy density. Thus, the Universe will be again radiation dominated for temperatures less than a certain T_4. In our numerical analysis, the consideration of these different epochs in the Universe evolution will be important.

III. THE CASE OF LIGHT NEUTRINOS

For heavy neutrinos, we have used the rate equation (8). If the neutrino is not heavy, this equation is no longer valid since now the neutrinos are relativistic at decoupling[6] and we have to rely on other methods.

To evaluate the number density of light neutrinos after decoupling we follow a standard procedure.[3,10] We calculate the decoupling temperature T_D by equating the rate of expansion of the Universe with the rate of interaction of neutrinos:[11]

$$H=\frac{\langle \sigma|v|nn\rangle}{\langle n\rangle} . \tag{14}$$

The knowledge of T_D (Ref. 12) allows us to get the number density at decoupling $n(T_D)$ and the corresponding value of Y, as defined in Eq. (5).

The method of obtaining T_D using Eq. (14) might seem inaccurate. To gain confidence in it we have performed the following check on the method. We calculate the decoupling temperature of *heavy* neutrinos using Eq. (14). Independently, we also evaluate this temperature using Eq. (8), defining T_D as the temperature at which Y departs from Y_0. The two results are very similar and we find this encouraging.

IV. RESULTS AND DISCUSSION

In Secs. II and III we have shown our strategy to calculate Y after decoupling and to evolve it until the present Y(today). If the neutrino were stable the present-day energy density would simply be given by $\rho=mY_Ts$, where Y_T is Y(today) times the degrees of freedom of the neutrinos species (this has to include the antineutrino contribution when the neutral lepton is of the Dirac type), and s is the present entropy density.

The present-day contribution of unstable neutrinos to the energy density contains two pieces. First, neutrinos that have not decayed contribute as

$$\rho_{ND}=mY_Tse^{-(t_0-t_D)/\tau} , \tag{15}$$

where t_0 is the Universe lifetime, τ the neutrino lifetime, and t_D the decoupling time. The second contribution is due to the decay products of the neutrino disintegration. This can be written as

$$\rho_D=mY_Ts\int_{t_D}^{t_0} e^{-(t-t_D)/\tau}\frac{T_0}{T}\frac{dt}{\tau} , \tag{16}$$

where T_0 is today's neutrino temperature.

Our limits follow from the requirement

$$\rho_{ND}+\rho_D\le\rho_{\max} , \tag{17}$$

with $\rho_{\max}$ the upper bound on the energy density of the Universe, which we discuss below. For each neutrino mass, Eq. (17) determines the upper bound on the lifetime τ. We describe now the way we get this upper bound.

To use Eq. (17) we have to evaluate the integral in Eq. (16). A complication is due to the fact that to calculate that integral we need to know for each temperature whether we are in a radiation- or matter-dominated universe. This depends on the values of the transition temperatures T_3 and T_4. However, these temperatures depend in turn on the precise value of τ. To extract our limits we have solved three coupled equations. One of them is Eq. (17), while the other two are the equations that define T_3 and T_4.

Our results are presented in Fig. 2. The limits depend on T_γ(today). We take T_γ(today)=2.7 K. We also need

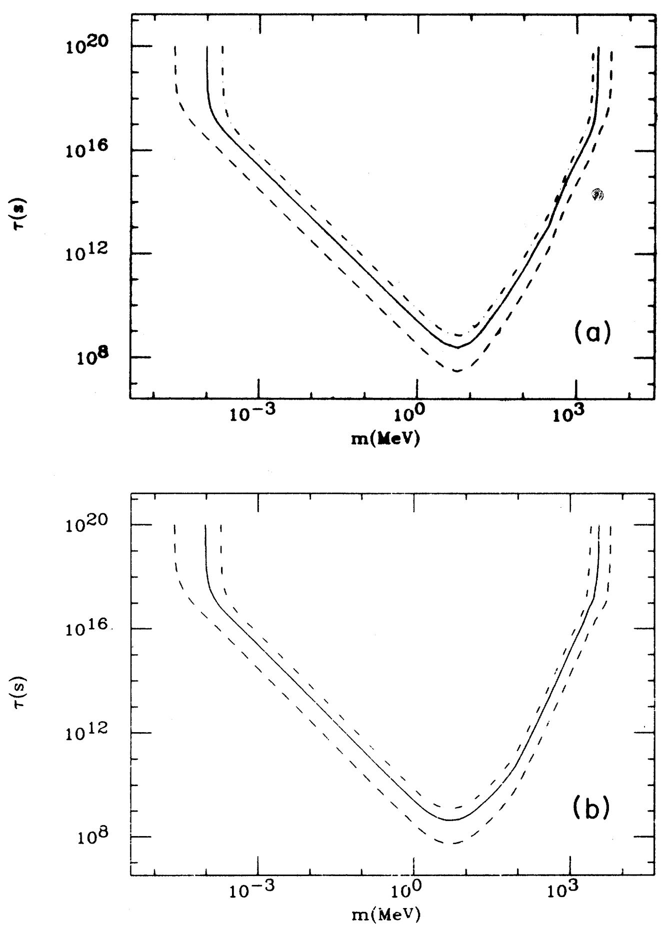

FIG. 2. Limits on the neutrino lifetime as a function of the neutrino mass, for the (a) Dirac and (b) Majorana case. We show the limits corresponding to $\Omega h^2=2$ (dashed-dotted curve), $\Omega h^2=1$ (solid curve), and $\Omega h^2=\frac{1}{4}$ (dashed curve).

to specify a value for ρ_{max}. We have set, as usual,[10]

$$\rho_{max}=1.05\times10^{-2}\Omega h^2\ \text{MeV/cm}^3\ , \tag{18}$$

where

$$\Omega=\frac{\rho_0}{\rho_c},\quad h=\frac{H_0}{100\ \text{km}\,\text{s}^{-1}\text{Mpc}^{-1}}\ . \tag{19}$$

Here, ρ_0 and H_0 are the present-day values of the energy density and Hubble parameter and ρ_c is the critical density.

The observational limits on Ω and h are[13]

$$0.5\leq h\leq 1,\quad \Omega\leq 2\ . \tag{20}$$

It follows that a conservative limit on the neutrino lifetime is obtained by using $\Omega h^2=2$. Given the large uncertainties in Ω and h, we also display in Fig. 2 the bound obtained assuming $\Omega h^2=1$ and $\frac{1}{4}$.

Dicus, Kolb, and Teplitz[3] were the first to evaluate the contribution of unstable neutrinos to the Universe energy density. We have introduced the following improvements in our work: (i) we make use of the precise calculation of g_* in Ref. 7 (Dicus, Kolb, and Teplitz use a constant g_*); (ii) we use the Boltzmann equation when treating heavy neutrinos, and this allows us to take into account quantitatively that freeze-out is not instantaneous; (iii) we use the exact cross sections and consider all open channels (Dicus, Kolb, and Teplitz consider only the $\nu_e\bar{\nu}_e$ channel); (iv) we consider the different eras of the Universe that arise in the presence of neutrinos and take this into account in our calculations, solving the three coupled equations we mention above. All these points introduce numerical differences in the final results. For example, the minimum lifetime obtained in Dicus, Kolb, and Teplitz[3] is $\tau\leq2.4\times10^7$ s, using $t_u=3.15\times10^{17}$ s ($\Omega h^2\simeq\frac{1}{2}$). This has to be compared to our results $\tau\leq2.5\times10^8$ s (for $\Omega h^2=1$) and $\tau\leq3.3\times10^7$ s (for $\Omega h^2=\frac{1}{4}$). We see that there is almost 1 order of magnitude of difference among the lifetimes. This minimum lifetime corresponds to neutrino masses of a few MeV. For greater masses, we notice that the differences are even greater. When $m=1$ GeV, the bound on the neutrino lifetime in Dicus, Kolb, and Teplitz[3] is $\tau\leq6.7\times10^{13}$ s while our limits are $\tau\leq4.3\times10^{15}$ s ($\Omega h^2=1$) and $\tau\leq5.5\times10^{14}$ s ($\Omega h^2=\frac{1}{4}$).

Binétruy, Girardi, and Salati[3] calculated the contribution of unstable neutrinos to the Universe energy density taking into account that the freezing phenomenon is not instantaneous. Compared to this work, the improvements we have introduced are essentially points (i), (iii), and (iv) above. These authors present, in fact, two limits, corresponding to a radiation- and to a matter-dominated universe. As we discuss in point (iv) above, we consider the evolution of the Universe in the presence of neutrinos and end up with only one limit. Numerical differences are also important. For $m=1$ GeV, the bound on the neutrino lifetime in Binétruy, Girardi, and Salati,[3] for $h=\frac{1}{2}$, is $\tau\leq1.7\times10^{13}$ s (radiation-dominated universe) and $\tau\leq2.7\times10^{14}$ s (matter-dominated universe).

We have compared the T_D obtained with the Boltzmann equation to T_D coming from the method used by Dicus, Kolb, and Teplitz, i.e., by equating the interaction time with the expanding time of the Universe. We have shown they agree.

In the present paper, the cosmological limits on the lifetime of a Majorana neutrino are also presented. The differences among the two types of neutrinos are more significant for heavy neutrinos, as it should be. The minimum lifetime, corresponding to neutrino masses of a few MeV, is $\tau\leq2.5\times10^8$ s for Dirac neutrinos and $\tau\leq4.3\times10^8$ s for Majorana neutrinos. For $m=1$ GeV and $\Omega h^2=1$, we get the limits $\tau\leq4.3\times10^{15}$ s (Dirac) and $\tau\leq1.4\times10^{15}$ s (Majorana). The asymptotic mass limit for a heavy stable neutrino is $m\geq2.5$ GeV (Dirac) and $m\geq3.5$ GeV (Majorana) for $\Omega h^2=1$. The limit on heavy stable neutrino masses was considered in Ref. 4. Besides the fact that our work considers stable and unstable neutrinos and that smooth transition from unstable to stable is shown in our plots, we also have some differences with Ref. 4. Indeed, we distinguish among T_ν and T_γ and among g_* and g_{*s} in the Boltzmann equation.

In conclusion, we have performed a very detailed calculation of unstable neutrinos to the Universe energy density. Compared to previous work on this subject, we have made several improvements and have obtained somewhat different results.

ACKNOWLEDGMENTS

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[6] The assumptions needed to go from the Boltzmann equation to the rate equation (1) are specified in Ref. 5. They are (a) the fermion pairs resulting from the neutrino annihilation are in thermal equilibrium, (b) that at decoupling the neutrino is nonrelativistic, and (c) the initial chemical potential of the neutrino species is very small.

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[8] The precise value of T depends on the decoupling temperature T_D, defined in Sec. III. If $T_D\leq m_\pi$, then $T^3=\frac{4}{11}T_\gamma^3$. This is

no longer true for higher decoupling temperatures. For example, for $T_D \geq m_\mu$ one has $T^3 = \frac{43}{53} T_\gamma^3$.

[9]Our normalization is such that $J_3(\text{neutrino}) = \frac{1}{2}$.

[10]See, for instance, M. Turner, in *Architecture of Fundamental Interactions at Short Distances,* proceedings of the Les Houches Summer School, Les Houches, France, 1985, edited by P. Ramond and R. Stora (Les Houches Summer School Proceedings Vol. 44) (North-Holland, Amsterdam, 1987), p. 513.

[11]In Eqs. (10) and (11) we have displayed the cross sections in the nonrelativistic limit. To analyze the case of light neutrinos we have used the full cross sections.

[12]There is a well-known limitation of the standard method of equating the expansion rate with the neutrino interacting rate: namely, that the method singles out a unique temperature. However, decoupling of neutrinos is not an instantaneous effect.

[13]G. A. Tammann, A. Sandage, and A. Yahil, in *Physical Cosmology,* Les Houches Summer School in Theoretical Physics, Les Houches, France, 1979, edited by R. Balian, J. Audouze, and D. N. Schramm (North-Holland, Amsterdam, 1980), p. 53.