Some comments on flipped SU(5)×U(1) and flipped unification in general

S. M. Barr
Bartol Research Institute, University of Delaware, Newark, Delaware 19716
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A general group-theoretical discussion of flipped embeddings is given. In addition to the well-known flipped SU(5) and flipped SO(10), the existence of flipped E_6 and E_7 is shown, as well as several families and special cases of flipped embeddings. A possible physical reason, essentially based on the group theory of flipped embeddings, why nature prefers the low-energy group $SU(3)\times SU(2)\times U(1)$ to alternatives such as $SU(4)\times U(1)$ and SU(5) is pointed out.

There has been much recent interest in the so-called "flipped SU(5)" as a unification group in the context of superstring theory. The existence of this group structure was first mentioned in the literature in a paper of De Rújula, Georgi, and Glashow.[1] It was independently rediscovered in Ref. 2, where it was given the name presently attached to it and where it was first explicitly studied in detail. In Refs. 2 and 3 the group and Higgs structure of flipped SU(5) unification were worked out as well as the implications for $\sin^2\theta_w$, the proton-decay lifetime, and proton-decay branching ratios. The natural generalization to "flipped SO(10)" was first made in the literature by Kephart and Nakagawa[4] (although already known[5] to the authors of Ref. 1). In the context of ordinary, that is, nonsupersymmetric, grand unified theories there was no reason to regard flipped SU(5) or flipped SO(10) as having any superiority over other unified schemes. However in the context of subsequent developments certain features of flipped unification have been found to be advantageous. The most significant breakthrough in that regard was the observation of Ref. 6 that the "missing-partner mechanism" can be implemented much less artificially in flipped than in standard SU(5). This mechanism for explaining the splitting of the fundamental Higgs multiplet into a superheavy color triplet and a light weak doublet requires introducing a **75** and $(\mathbf{50}+\overline{\mathbf{50}})$ of Higgs field (in addition to the $\mathbf{5}+\overline{\mathbf{5}}$) in ordinary SU(5), whereas in flipped SU(5) only a $\mathbf{10}+\overline{\mathbf{10}}$ is needed. A second key advantage only arises in the context of superstring unification. The breaking of $SU(5)\times U(1)$ down to the standard model in the flipped way requires (again) only an SU(5) **10** of Higgs fields[2,3,6] whereas to break in the standard way requires an adjoint (**24**) of Higgs fields (or a larger representation) which would *not* be present in the matter content of a superstring model.[7] It has also been found that "flipping" allows one to overcome some of the difficulties associated with giving neutrinos acceptable masses in superstring-inspired models.[7]

In this paper we would like to make two points—one group theoretical and one physical. The group-theoretical point is that the phenomenon of flipped embeddings generalizes to many groups. For example, we shall show that flipped E_6 and flipped E_7 also exist. Moreover, we point out that a flipped embedding sequence $SO(2N)\to SU(N)\times U(1)\to SU(N-p)\times SU(p)\times U(1)$ exists[8] for all N and p. This is interesting because it means that in flipped SU(5) models one can end up with a low-energy group of SU(5) or $SU(4)\times U(1)$ as well as $SU(3)\times SU(2)\times U(1)$ just as in standard SU(5). This will bring us to our physical point which is that flipped unification may provide a key to understanding why the low-energy group $SU(3)\times SU(2)\times U(1)$ is preferred. We say "may" provide such a key because the ideas involved are quite speculative and involve several assumptions.

The idea of a flipped embedding arises in the following group-theoretical context. Consider the sequence of subgroups

$$G\supset H\times U(1)_X\supset[K\times U(1)_Z]\times U(1)_X\ ,$$

where G is a simple group, H and K are simple or semisimple, and (rank K)=(rank H)-1=(rank G)-2. The generators of $U(1)_X$ and $U(1)_Z$ are X and Z, respectively. What we will call the *standard* breaking chain is

$$G\to H\to K\times U(1)_Z\ .$$

What we call the *flipped* breaking chain is

$$G\to H\times U(1)_X\to K\times U(1)_{\tilde Z}\ ,$$

where the generator $\tilde Z$ of $U(1)_{\tilde Z}$ is given by

$$\tilde Z=\alpha Z+\beta X,\quad \beta\neq 0 \tag{1}$$

and where the representations of G decompose under $K\times U(1)_{\tilde Z}$ into the same representations with the same charges as under $K\times U(1)_Z$.

When the sequence is $SO(10)\supset SU(5)\times(1)_X\supset SU(3)\times SU(2)\times U(1)_Z\times U(1)_X$ that flipped breaking chain is what has been called[2] "flipped SU(5)." In a way this is a misnomer since the flipped is distinguished from the standard embedding at the point when $H\times U(1)$ breaks to $K\times U(1)$. In a way, a better term would be "flipped $SU(3)\times SU(2)\times U(1)$." Nevertheless, the name flipped SU(5) has stuck and is justified to the extent that one can get the correct "flipped" SU(5) multiplets from the "standard" SU(5) multiplets by interchanging or flipping $\nu\leftrightarrow e$ and $u\leftrightarrow d$. For this flipped sequence $\alpha=-\frac{1}{5}$. (This is of course enough to determine β once the relative normalization of the generators Z and X is defined.) The other flipped embedding that is well known is $G=\mathrm{E}_6$, $H=\mathrm{SO}(10)$, $K=\mathrm{SU}(5)$ which is called usually "flipped SO(10)." For this $\alpha=-\frac{1}{4}$. Both of these are instances of the general case $G=E_N$, $H=E_{N-1}$, and $K=E_{N-2}$ for which both standard and flipped breaking always exist. [$\mathrm{E}_5\equiv\mathrm{SO}(10)$, $\mathrm{E}_4\equiv\mathrm{SU}(5)$, $\mathrm{E}_3\equiv\mathrm{SU}(3)\times\mathrm{SU}(2)$, $\mathrm{E}_2\equiv\mathrm{SU}(2)\times\mathrm{SU}(2)$, $\mathrm{E}_1\equiv\mathrm{SU}(2)$. These identifications are based on isomorphisms of the Lie algebras.[9]] For these $\alpha(N)$ is given by $\alpha(8)=-\frac{1}{2}$, $\alpha(7)=-\frac{1}{3}$, $\alpha(6)=-\frac{1}{4}$, $\alpha(5)=-\frac{1}{5}$, $\alpha(4)=\frac{2}{3}$, $\alpha(3)=\frac{1}{2}$. To illustrate let us consider $N=8$. Under the subgroups $\mathrm{E}_8\supset\mathrm{E}_7\times\mathrm{U}(1)_X\supset[\mathrm{E}_6\times\mathrm{U}(1)_Z]\times\mathrm{U}(1)_X$ the fundamental (adjoint) decomposes as (in obvious notation)

$$\begin{array}{ccccccccccccc}
\mathbf{248}\to & \mathbf{133}^0 & + & \mathbf{56}^1 & + & \mathbf{56}^{-1} & + & \mathbf{1}^2 & + & \mathbf{1}^0 & + & \mathbf{1}^{-2}\,. \\
 & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 & \mathbf{78}^{0,0} & & \mathbf{27}^{1,1} & & \mathbf{27}^{1,-1} & & \mathbf{1}^{0,2} & & \mathbf{1}^{0,0} & & \mathbf{1}^{0,-2} \\
 & + & & + & & + & & & & & & \\
 & \mathbf{27}^{-2,0} & & \overline{\mathbf{27}}^{\,-1,1} & & \overline{\mathbf{27}}^{\,-1,-1} & & & & & & \\
 & + & & + & & + & & & & & & \\
 & \overline{\mathbf{27}}^{\,2,0} & & \mathbf{1}^{3,1} & & \mathbf{1}^{+3,-1} & & & & & & \\
 & + & & + & & + & & & & & & \\
 & \mathbf{1}^{0,0} & & \mathbf{1}^{-3,1} & & \mathbf{1}^{-3,-1} & & & & & &
\end{array}$$

Under both $\mathrm{E}_8\to\mathrm{E}_6\times\mathrm{U}(1)_Z$ and $\mathrm{E}_8\to\mathrm{E}_6\times\mathrm{U}(1)_{\tilde Z}$ where $\tilde Z=-\frac{1}{2}Z-\frac{3}{2}X$ one gets the same set of representations. In this case the pairs that flip are $[\mathbf{27}^{-2,0},\mathbf{27}^{1,1}]$, $[\mathbf{1}^{3,1},\mathbf{1}^{-3,-1}]$, $[\mathbf{1}^{-3,1},\mathbf{1}^{0,2}]$, and $[\mathbf{1}^{3,-1},\mathbf{1}^{0,-2}]$. There is also an equivalent flipping $\tilde Z=-\frac{1}{2}Z+\frac{3}{2}X$. [It should be noted that in this sequence $\mathrm{E}_7\times\mathrm{U}(1)_X$ is not a maximal subgroup of E_8. Rather $\mathrm{E}_8\supset\mathrm{E}_7\times\mathrm{SU}(2)\supset\mathrm{E}_7\times\mathrm{U}(1)_X$.] To take a second example, consider the same exceptional sequence with $N=7$. Under $\mathrm{E}_7\supset\mathrm{E}_6\times\mathrm{U}(1)_X\supset[\mathrm{SO}(10)\times\mathrm{U}(1)_Z]\times\mathrm{U}(1)_X$ the fundamental representation of E_7 decomposes as follows:

$$\begin{array}{ccccccccc}
\mathbf{56}\to & \mathbf{1}^3 & + & \mathbf{1}^{-3} & + & \mathbf{27}^1 & + & \overline{\mathbf{27}}^{\,-1}\,. \\
 & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 & \mathbf{1}^{0,3} & & \mathbf{1}^{0,-3} & & \mathbf{16}^{1,1} & & \overline{\mathbf{16}}^{\,-1,-1} \\
 & & & & & + & & + \\
 & & & & & \mathbf{10}^{-2,1} & & \mathbf{10}^{+2,-1} \\
 & & & & & + & & + \\
 & & & & & \mathbf{1}^{4,1} & & \mathbf{1}^{-4,-1}
\end{array}$$

$\mathrm{E}_7\to\mathrm{SO}(10)\times\mathrm{U}(1)_Z$ and $\mathrm{E}_7\to\mathrm{SO}(10)\times\mathrm{U}(1)_{\tilde Z}$ with $\tilde Z=-\frac{1}{3}Z+\frac{4}{3}X$ give the same representations with the following pairs being flipped: $[\mathbf{10}^{-2,1},\mathbf{10}^{+2,-1}]$, $[\mathbf{1}^{0,3},\mathbf{1}^{4,1}]$, $[\mathbf{1}^{0,-3},\mathbf{1}^{-4,-1}]$.

A second generalization is to the orthogonal series.[8] Consider the sequence of subgroups $\mathrm{SO}(2N)\supset\mathrm{SU}(N)\times\mathrm{U}(1)_X\supset\mathrm{SU}(N-p)\times\mathrm{SU}(p)\times\mathrm{U}(1)_Z\times\mathrm{U}(1)_X$. Here for all N (odd and even) and all $p=0,\ldots,N$ there are both standard and flipped embeddings. [Here SU(0) and SU(1) are defined to be trivial.] For these cases $\alpha=\pm(2p/N-1)$. The spinors of SO(2N) are 2^{N-1} dimensional. For N odd we have conjugate complex spinors 2^{N-1} and $\overline{2^{N-1}}$ with

$$2^{N-1}\to\sum_{k=0}^{(N-1)/2}[2k]^{(N-4k)}$$

under $\mathrm{SO}(2N)\to\mathrm{SU}(N)\times\mathrm{U}(1)_X$. (Here $[p]$ means a tensor representation with p antisymmetric indices. $[p]^q$ refers to such a representation with charge q under $\mathrm{U}(1)_X$.) If one component of a Higgs field in the $[p]^{(N-2p)}$ representation develops a superlarge expectation value then the group $\mathrm{SU}(N)\times\mathrm{U}(1)_X$ will break to the group $\mathrm{SU}(N-p)\times\mathrm{SU}(p)\times\mathrm{U}(1)_{\tilde Z}$ in the *flipped* way. To get the *standard* breaking to $\mathrm{SU}(N-p)\times\mathrm{SU}(p)\times\mathrm{U}(1)_Z$ one needs an adjoint Higgs field (or larger). Since all $[p]$ (or their conjugates $[N-p]=[\bar p]$) are contained in the spinor 2^{N-1}, in a grand-unified-theory (GUT) model all such flipped breakings are achievable in an equally simple way. For SO(2N) with N even the same situation obtains.

Not all sequences give flipped embeddings. In Table I we present certain families of cases $G\supset H\times\mathrm{U}(1)_X\supset K\times\mathrm{U}(1)_Z\times\mathrm{U}(1)_X$ some of which have flipped embeddings and some of which do not. These comprise all the *families* of cases involving the phenomenologically interesting U, SO, and E series. Note that SO(2N) always has two subgroups of the form $H\times\mathrm{U}(1)$ with rank $H=N-1$: $\mathrm{SO}(2N)\to\mathrm{SO}(2N-2)\times\mathrm{O}(2)$ and $\mathrm{SO}(2N)\to\mathrm{SU}(N)\times\mathrm{U}(1)$. Similarly SU($N$) has the subgroups $\mathrm{SU}(N-1)\times\mathrm{U}(1)$ and $\mathrm{SU}(N-p)\times\mathrm{SU}(p)\times\mathrm{U}(1)$. E_N always contains $\mathrm{E}_{N-1}\times\mathrm{U}(1)$. Putting these together in all possible ways gives the cases in Table I.

Certain groups can give a U(1) factor in a different way. For example, E_6 contains the maximal subgroups $\mathrm{SU}(6)\times\mathrm{SU}(2)$ and $\mathrm{SU}(3)\times\mathrm{SU}(3)\times\mathrm{SU}(3)$ from which one can consider various cases like $G=\mathrm{E}_6$, $H=\mathrm{SU}(5)\times\mathrm{SU}(2)$, $K=\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{SU}(2)$. In Table II we present a selection of various special cases of this sort that do not fall into families. Again, some of these approaches have flipped embeddings and some do not.

At first glance it appears very remarkable, almost miraculous, that flipped embeddings exist at all. One is solving a set of simultaneous linear equations of the form $\tilde Z_i=\alpha Z_i+\beta X_i$ where i refers to a multiplet of $K\times\mathrm{U}(1)_Z\times\mathrm{U}(1)_X$. There is only one unknown (once we normalize X and Z, β is determined up to a sign by α), but as many equations as there are multiplets. The problem is very overdetermined and one would expect only the trivial (i.e., standard) solution $\alpha=1,\beta=0$.

At second glance it appears almost trivial that flipped embeddings exist since they correspond to symmetries of the Dynkin diagrams. Consider the Dynkin diagram for SO(10). It contains the Dynkin diagram of SU(5) in two ways. If we call these two SU(5) subgroups $\mathrm{SU}(5)_1$, and $\mathrm{SU}(5)_2$ then SO(10) contains the subgroups $\mathrm{SO}(10)\supset\mathrm{SU}(5)_1\supset\mathrm{SU}(3)_1\times\mathrm{SU}(2)_1\times\mathrm{U}(1)_1$ and $\mathrm{SO}(10)\supset\mathrm{SU}(5)_2\supset\mathrm{SU}(3)_2\times\mathrm{SU}(2)_2\times\mathrm{U}(1)_2$. A representation of SO(10) is guaranteed to give the same set of charges when decomposed under $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ since these are automorphic to each other. Thus it seems obvious that SO(10) will contain a standard embedding $\mathrm{SU}(3)_1\times\mathrm{SU}(2)_1\times\mathrm{U}(1)_1$ and an inequivalent flipped embedding $\mathrm{SU}(3)_1\times\mathrm{SU}(2)_1$

TABLE I. Families of flipped embeddings. α defines the embedding through $\tilde{Z}=\alpha Z+\beta X$; see Eq. (1) of text. An × in last column means that no flipped embedding exists for that sequence. [Note that "flipped SU(5)" corresponds both to the first family with $N=5$, and to the third with $N=5,p=2$.]

G	H	K	α
E_N	E_{N-1}	E_{N-2}	$\alpha(8)=-\frac{1}{2}$ $\alpha(7)=-\frac{1}{3}$ $\alpha(6)=-\frac{1}{4}$ $\alpha(5)=-\frac{1}{5}$ $\alpha(4)=\frac{2}{3}$ $\alpha(3)=\frac{1}{2}$
SU(N)	SU($N-1$)	SU($N-2$)	$\alpha=1/(N-1)$
SO($2N$)	SU(N)	SU($N-p$)×SU(p)	$\alpha=\pm\left\lvert\frac{2p}{N}-1\right\rvert$
SO($2N+1$)	SU(N)	SU($N-p$)×SU(p)	Reduces trivially to previous case
SO($2N$)	SO($2N-2$)	SO($2N-4$)	$\alpha=0$
SU(N)	SU($N-p$)×SU(p)	SU($N-p$)×SU(q)×SU($p-q$)	×
SO($2N$)	SO($2N-2$)	SU($N-1$)	×

×U(1)$_2$. However, this argument is a bit too superficial. A key question is whether U(1)$_2$ commutes with SU(3)$_1$×SU(2)$_1$. If so, as indeed is the case, then the argument for the existence of a flipped embedding is valid. In all of the cases in Table I where there is a flipped embedding, it can be attributed to such a symmetry of the Dynkin diagram. The cases where this argument fails are those where U(1)$_2$ and K_1 do *not* commute. Thus the existence of flipped embeddings, while not a miracle or fluke in any sense, is not a complete triviality either.

It could turn out eventually that any of the flipped embeddings in Tables I and II could have relevance to the real world. But if we restrict ourselves to the superstring framework then probably only "flipped SU(5)" has any likelihood of being relevant. Suppose we consider the possibility that the world in four dimensions *is* described by a supersymmetric SU(5)×U(1) unified model with the following characteristics. We assume the matter multiplets are in SU(5)×U(1) representations that would come from the decomposition of a **16**, $\overline{\mathbf{16}}$, **10**, or **1** of SO(10) that in turn would come from the decomposition of a **27** or $\overline{\mathbf{27}}$ of E$_6$. Suppose further that supersymmetry is broken softly at low energies as in hidden sector models. We can imagine all this as emerging from superstring theory or not. Can we hope to understand why the low-energy theory is SU(3)×SU(2)×U(1) rather than SU(4)×U(1) or SU(5) which are also rank 4? We will now present a speculative answer to that question which involves several assumptions, of course, and that uses in an essential way the group-theoretical features of flipped unification. Among the possible Higgs fields available to do symmetry breaking in the situation we describe are $(\mathbf{10}^1+\overline{\mathbf{10}}^{-1})$, $(\overline{\mathbf{5}}^{-3}+\mathbf{5}^{+3})$, and $(\mathbf{1}^5+\mathbf{1}^{-5})$ representations of SU(5)×U(1). Not all of these representations of SU(5)×U(1) are necessarily present in the sub-Planck-scale world, but *a priori* there is no reason to expect the $\mathbf{5}^{+3}$ to be less likely to be present than the $\overline{\mathbf{10}}^{-1}$, say. [If we did have some argument why this would be so, it would be a simpler reason for SU(3)×SU(2)×U(1) rather than SU(4)×U(1) being the low-energy group. But we are not aware of any such argument at the present time.] Let us say then that Higgs fields in all these representations are present in the four-dimensional theory. Then we have three possibilities for a rank-4 low-energy group:

TABLE II. A sample of special cases that do not fall into the families of Table I. × in last column means that no flipped embedding exists, or that only a trivial one with $\alpha=-1$ exists.

G	H	K	α
E$_6$	SU(6)	SU(3)×SU(3)	$\alpha=-\frac{1}{2}$
E$_6$	SU(5)×SU(2)	SU(3)×SU(2)×SU(2)	$\alpha=\frac{4}{5}$
E$_6$	SU(5)×SU(2)	SU(4)×SU(2)	$\alpha=\frac{1}{5}$
E$_6$	SU(3)×SU(3)×SU(2)	SU(3)×SU(2)×SU(2)	$\alpha=0$
E$_6$	SU(3)×SU(3)×SU(2)	SU(3)×SU(3)	$\alpha=\frac{1}{2}$
E$_7$	SO(12)	SU(6)	$\alpha=-\frac{1}{2}$
E$_8$	E$_7$	SO(12)	$\alpha=0$
E$_7$	SO(12)	SO(10)	×
E$_6$	SU(6)	SU(5)	×
E$_6$	SU(6)	SU(4)×SU(2)	×
E$_6$	SU(5)×SU(2)	SU(5)	×
E$_6$	[SU(4)×SU(2)]×SU(2)	[SU(4)]×SU(2)	×
E$_6$	[SU(4)×SU(2)]×SU(2)	[SU(4)×SU(2)]	×
E$_7$	E$_6$	SU(6)	×

if $\langle 10^{-1}+\overline{10}^{-1}\rangle\neq 0$, then SU(3)×SU(2)×U(1), if $\langle \overline{5}^{-3}+5^{3}\rangle\neq 0$, then SU(4)×U(1), and if $\langle 1^{5}+1^{-5}\rangle\neq 0$ then SU(5). These are all flipped breakings in the sequence $SO(2N)\supset SU(N)\times U(1)\supset SU(N-p)\times SU(p)\times U(1)$ of Table I. Which is to be preferred?

The $(10^1+\overline{10}^{-1})$, $(\overline{5}^{-3}+5^3)$, (1^5+1^{-5}) all have D-flat directions. We suppose that these are also F-flat, with the tree-level potential arising from the soft supersymmetry- (SUSY-)breaking terms $m_{10}^2|H_{10}|^2+m_{\overline{10}}^2|H_{\overline{10}}|^2+m_5^2|H_5|^2+m_{\overline{5}}^2|H_{\overline{5}}|^2+m_1^2|H_1|^2+\cdots$. Following the scenario of Ref. 7 we imagine that the effective potential turns over at very large values of the classical field due to radiative effects. The one-loop contribution to $V_{\rm eff}$ is

$$V_{1\ \rm loop}=\frac{1}{64\pi^2}{\rm Str}[M^4(H_i)\ln M^2(H_i)/Q^2]\ ,$$

where Q^2 is a renormalization scale. This supertrace must be taken over all species of particles and to evaluate it we must know, among other things, the full superpotential. We do not know this, of course, indeed we do not even know how many of each type of multiplet there are. The most well-known part of the model is the gauge sector, where we know the particle content exactly to be a massive vector supermultiplet in the adjoint of SU(5)×U(1). The masses of these gauge particles will depend on four parameters: g_5 and g_1 (the gauge couplings), and μ_5 and μ_1 (the SUSY-breaking gaugino masses), where the subscripts refer to SU(5) and U(1). We will take $g_5=g_1\equiv g$ and $\mu_5=\mu_1\equiv\mu$ for reasons of simplicity. Actually, as will be apparent, the one-loop potential is much less sensitive to the values of g_1 and μ_1 than to g_5 and μ_5, so our qualitative conclusion does not depend much on this simplifying assumption. (We expect $g_5\approx g_1$ at the Planck scale anyway.) A much more serious simplifying assumption we will make is to ignore completely the contributions from the other sectors, about which we know less. One could justify this if the dominant source of SUSY breaking were gaugino masses as assumed in Ref. 7. Moreover those of the Yukawa couplings that we do know experimentally *are* quite small compared to the gauge couplings. If that is assumed to be a general feature of the couplings in the superpotential then the matter contributions to $V_{1\ \rm loop}$ (which go as the square of these couplings) will be much smaller than the gauge contributions (which go as g^2). What we find if we make this assumption is that the one-loop effective potential is more negative in the SU(3)×SU(2)×U(1) direction than the SU(4)×U(1) direction, and more negative in the SU(4)×U(1) direction than in the SU(5) direction.

The SU(3)×SU(2)×U(1) direction corresponds to $(10^1+\overline{10}^{-1})$ getting a vacuum expectation value (VEV) (which we will call V_{10}) and $\langle\overline{5}^{-3}\rangle=\langle 5^3\rangle=\langle 1^5\rangle=\langle 1^{-5}\rangle=0$. Thirteen gauge bosons get mass, twelve of them a mass of $g_5|V_{10}|=g|V_{10}|$, and one a mass of

$$\frac{1}{\sqrt{10}}(24g_5^2+g_1^2)^{1/2}|V_{10}|\equiv\sqrt{5/2}\,g|V_{10}|\ .$$

(Note the weak dependence on g_1 we mentioned above.) The scalar partners of these get mass

$$\left[g^2V_{10}^2+\frac{m_{10}^2+m_{\overline{10}}^2}{2}\right]^{1/2}$$

and

$$\left[\tfrac{5}{2}g^2V_{10}^2+\frac{m_{10}^2+m_{\overline{10}}^2}{2}\right]^{1/2},$$

respectively. And the gauginos get mass

$$\left[g^2V_{10}^2+\frac{\mu^2}{2}\pm g\mu|V_{10}|\right]^{1/2}$$

and

$$\left[\tfrac{5}{2}g^2V_{10}^2+\frac{\mu^2}{2}\pm\sqrt{5/2}\,g\mu|V_{10}|\right]^{1/2},$$

respectively. A little arithmetic gives

$$V_{1\ \rm loop}(V_{10})=-12f(g^2,m_{10}^2+m_{\overline{10}}^2,\mu^2,V_{10}^2)-f(\tfrac{5}{2}g^2,m_{10}^2+m_{\overline{10}}^2,\mu^2,V_{10}^2)\ , \tag{2a}$$

$$f(g^2,m^2,\mu^2,V^2)\equiv\frac{1}{64\pi^2}g^2V^2\left[\left[-\tfrac{1}{2}-\ln\frac{g^2V^2}{Q^2}\right]m^2+\left[8+8\ln\frac{g^2V^2}{Q^2}\right]\mu^2\right]. \tag{2b}$$

Note that the dominant contribution which is the first term in Eq. (2a) is just proportional to the number of broken generators of SU(5), i.e., the dimension of SU(5)/SU(3)×SU(2)×U(1), which is twelve.

Now consider the SU(4)×U(1) direction which corresponds to $(\overline{5}^{-3}+5^3)$ getting a VEV which we denote V_5. We find

$$V_{1\ \rm loop}(V_5)=-8f(g^2,(m_5^2+m_{\overline{5}}^2),\mu^2,V_5^2)-f(\tfrac{5}{8}g^2,(m_5^2+m_{\overline{5}}^2),\mu^2,V_5^2)\ , \tag{3}$$

where here the 8 is just the dimension of SU(5)/SU(4)×U(1), i.e., again the number of broken SU(5) generators. Finally, for a Higgs-singlet VEV, V_1, which breaks SU(5)×U(1) to SU(5) we find

$$V_{1\ \rm loop}(V_1)=-f(\tfrac{5}{8}g^2,m_1^2+m_{\overline{1}}^2,\mu^2,V_1^2)\ . \tag{4}$$

If $\mu^2>m_i^2$ as assumed in Ref. 7, or if $m_i^2+m_{\overline{i}}^2$ are approximately equal for $i=10,5,1$, then we see that the potential is steepest in the SU(3)×SU(2)×U(1) direction and least steep in the SU(5) direction, and that this effect comes from counting the number of broken generators of SU(5). Now, in fact, the one-loop effective potential is unbounded below. One presumes that higher-order or nonperturbative effects will cure this, but unless one can compute these effects one cannot make any assertions about the direction in which the absolute minimum lies. However, in the spirit of MAC (most attractive channel) arguments, it seems reasonable to take the one-loop result as indicative. In any event we can say that there is a physical effect which favors the SU(3)×SU(2)×U(1) channel but that other higher-order effects conceivably could counteract this tendency.

This mechanism makes essential use of flipped breaking. Suppose instead we had considered an adjoint Higgs field of SU(5) (which would *not* be allowed in the framework of superstrings). The directions that break to SU(3)×SU(2)×U(1) and SU(4)×U(1) are, respectively,

$$\frac{1}{\sqrt{60}}\mathrm{diag}(2\,2\,2\,-3\,-3)$$

and

$$\frac{1}{\sqrt{40}}\mathrm{diag}(1\,1\,1\,1\,-4)\ .$$

The coefficient in front of the gauge contribution to the one-loop effective potential will be proportional to $12\{[2-(-3)]/\sqrt{60}\}^2$ for the SU(3)×SU(2)×U(1) direction and $8\{[1-(-4)]/\sqrt{40}\}^2$ for the SU(4)×U(1) direction. [The first factor being in each case the *number* of broken generators, and the second coming from normalizing the SU(5) generators consistently.] Thus the one-loop potential does not prefer either direction. It is easy to see that this pattern remains true for $\mathrm{SU}(N)\to\mathrm{SU}(N-p)\times\mathrm{SU}(p)\times\mathrm{U}(1)$. The normalized generator that commutes with the unbroken group is

$$\frac{1}{\sqrt{2Np(N-p)}}\left[\underbrace{p\ \cdots\ p}_{N-p},\underbrace{(N-p)\cdots\ -(N-p)}_{p}\right]$$

and the number of broken generators is $2p(N-p)$. Thus the gauge contribution to $V_{1\,\mathrm{loop}}$(gauge) is proportional to $2p(N-p)[N/\sqrt{2Np(N-p)}]^2$=independent of p. On the other hand, for the flipped case $\mathrm{SU}(N)\times\mathrm{U}(1)\to\mathrm{SU}(N-p)\times\mathrm{SU}(p)\times\mathrm{U}(1)$ the SU(N) part of the gauge contribution to $V_{1\,\mathrm{loop}}$ is just proportional to $-2p(N-p)$ and so will be deepest for $p=[N/2]$.

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