Low-energy lepton violation from supersymmetric flipped SU(5)

David E. Brahm and Lawrence J. Hall
Physics Department, University of California, Berkeley, California 94720
and Theoretical Physics Group, Lawrence Berkeley Laboratory, 1 Cyclotron Road, Berkeley, California 94720
(Received 11 May 1989)

We construct a supersymmetric flipped SU(5)⊗U(1) model which violates R parity and electron number at low energies, through a superpotential term $\frac{1}{2}C^{ijk}L_iL_jE_k^c$. Rotation of the electron and Higgs superfields makes this term also responsible for charged-lepton masses. The model employs a missing-partners mechanism for the Higgs fields and a seesaw mechanism for the neutrinos. It correctly predicts the approximate electron mass and several mass relations, as well as numerical values for the grand unification scale and the C^{ijk} coefficients. The electron-neutrino Majorana mass is close to experimental limits, and provides constraints. Interesting Z^0 decays are predicted: e.g., $Z^0 \to e^-\mu^+e^+\mu^-$ with invariant mass peaks in the (e,μ) channels.

I. INTRODUCTION AND MOTIVATION

It is often said that the most important experimental signature of supersymmetry is missing energy. In fact this signature only occurs in those supersymmetric models which are also R_P invariant and have a neutral lightest superpartner (LSP). It has recently been stressed that it is worthwhile searching for supersymmetric signatures in models without R_P invariance;[1] such signatures are exotic and typically easily identified.[2,3] It is trivial to write down R_P-violating SU(3)⊗SU(2)⊗U(1) models, even with the minimal field content of Q, U^c, D^c, L, E^c, h, and $\bar{h}$. The $\Delta L \neq 0$ model contains LLE^c, QLD^c, and $\mu L\bar{h}$, while the $\Delta B \neq 0$ model contains $U^cD^cD^c$ in the superpotential. At the SU(3)⊗SU(2)⊗U(1) level there is little reason to choose the usual R_P-invariant model (conserving both B and L) over models which violate either B or L. In each model the effective theory at the TeV scale contains a global U(1) symmetry (R_P, B or L). Clearly, experimental searches should be made for all three. However, this leaves open an important theoretical question: which version is most likely to be the remnant of symmetry breaking at a higher-energy scale?

The very simplest unified schemes do tend to give the standard R_P-conserving model. For example, R_P can result in SO(10) models from the requirement that all interactions have an even number of spinor representations. In SU(5) theories, LLE^c, QLD^c, and $U^cD^cD^c$ all come from the same operator, which must therefore be absent to avoid proton decay (see Fig. 1). On the other hand, there is absolutely no reason that these simplest of all ground unified theories (GUT's) are the ones chosen by nature. We find it interesting that only mild additions are required to obtain an R_P-violating low-energy theory. Even in SU(5), lepton-number violation can occur at the renormalizable level in the low-energy effective theory.[2] This is because Higgs and lepton doublets have the same gauge quantum numbers and can have mass mixing. This case is particularly interesting because the flavor dependence of the lepton-number violation is highly constrained. Another unusual possibility is that the extra low-energy global symmetry is a discrete Z_N symmetry ($N > 2$), as might arise from compactification in superstring-inspired models.[4] In this case the L or B violation which causes LSP decay occurs via higher-dimension operators.

In this paper we consider a new way of obtaining the R_P-violating $\Delta L \neq 0$ model. Our model is based on the flipped SU(5)⊗U(1) gauge group[5-7] [$\overline{\mathrm{SU}(5)}$ for short], where electric charge is embedded partly in each simple factor. Unlike conventional SU(5) (Ref. 8), this group allows simple operators which yield LLE^c, without giving $U^cD^cD^c$. In subsequent sections we describe the model and its experimental consequences in some detail. Certainly the model is not perfect: it loses the two good predictions of conventional SU(5), namely, $\sin^2\theta_W$ (Ref. 9) and m_b/m_τ (Ref. 10); it is not even a true GUT in that the group is not semisimple. Nevertheless, we find some elegant and unusual features, together with some constrained predictions, which we list here.

(i) This model has the fewest superfields of any supersymmetric SU(5) model known to us, all in low-dimensional representations (5's and 10's).

(ii) Decuplets (10, $\overline{10}$) of SU(5) break SU(5)⊗U(1) to SU(3)⊗SU(2)⊗U(1) and leave light Higgs doublets in a very elegant missing-partners mechanism.[11,7]

(iii) The grand unified scale is generated dynamically by renormalization-group scaling of supersymmetry-

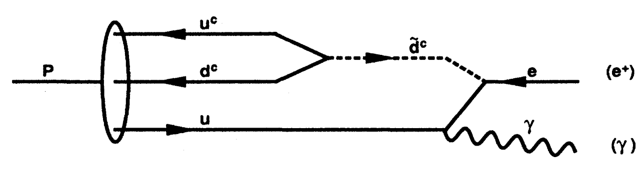

FIG. 1. Proton decay from $U^cD^cD^c$ and QLD^c.

breaking scalar masses.

(iv) The Higgs mixing term $mh\bar{h}$, with m at the weak scale, is generated by the same symmetry that suppresses low-energy B violation.

(v) The incorrect mass relation of conventional SU(5), $m_d/m_s = m_e/m_\mu$, is absent.

(vi) The charged-lepton masses do not arise from Yukawa couplings of the unified theory. The μ and τ masses arise from the same higher-dimension operators which are responsible for the L violation, which is therefore highly constrained.

(vii) The electron mass occurs at even higher dimension and is consequently small.

(viii) A seesaw mechanism makes neutrinos light, with ν_τ a candidate for dark matter.

(ix) The electron neutrino has a Majorana mass close to its present limit from neutrinoless double beta decay.

(x) $Z^0 \to e^-\mu^+e^+\mu^-$, where each (e,μ) pair has invariant mass equal to $m(\tilde{\nu}_\mu)$, occurs with a large branching ratio if $m_{\tilde{\nu}} < M_Z/2$. If sneutrinos are heavier, $Z^0 \to e^-\tau^+e^+\tau^-$ is the dominant signal, but is too rare to be seen at CERN LEP.

To avoid processes such as $\mu \to e\gamma$ and $K_L \to \mu e$, we cannot allow strong violation of several lepton family numbers.[1] Of the nine terms $\frac{1}{2}C^{ijk}L_iL_jE_k^c$ (by antisymmetry of the generation indices i and j), each of which violates exactly one or three family numbers, our theory must allow only those that violate a single family number. Our model primarily violates electron number. The strictest limits on our coefficients C^{212} and C^{313} then come from the electron-neutrino Majorana mass.

II. A $\overline{\text{SU(5)}}$ MODEL WITH ELECTRON-NUMBER VIOLATION

Our model employs a discrete symmetry called η_P (replacing R_P or the $H \to -H$ symmetry of Ref. 7), which has been chosen to allow the operator which contains LLE^c but forbid B violation. Our left-handed chiral superfields have the following SU(5) structure, U(1) charge, and η_P charge:[12]

$$\begin{aligned} &F_i = (\mathbf{10},\ 1,\ -1),\quad \bar{f}_i = (\bar{\mathbf{5}},\ -3,\ 1)\ ,\\ &E_i^c = (\mathbf{1},\ 5,\ -1),\quad H = (\mathbf{10},\ 1,\ 3)\ ,\\ &\bar{H} = (\overline{\mathbf{10}}, -1,\ 4)\ ,\\ &h = (\mathbf{5},\ -2,\ -2),\quad \bar{h} = (\bar{\mathbf{5}},\ 2,\ 4)\ . \end{aligned} \tag{1}$$

Here i,j,k are generation indices (1 to 3). The matter multiplets contain

$$F = \begin{bmatrix} 0 & d_3^c & -d_2^c & d_1 & u_1 \\ & 0 & d_1^c & d_2 & u_2 \\ & & 0 & d_3 & u_3 \\ & & & 0 & \nu^c \\ & & & & 0 \end{bmatrix},\quad \bar{f} = \begin{bmatrix} u_1^c \\ u_2^c \\ u_3^c \\ \nu \\ e \end{bmatrix}\ . \tag{2}$$

H and $\bar{H}$ take GUT-scale vacuum expectation values (VEV's) $\langle H_{45}\rangle = \langle \bar{H}_{45}\rangle = V$ (we will explore the dynamical generation of the GUT scale in Sec. III), while the remaining fields in H and $\bar{H}$ acquire GUT-scale masses from the missing-partners mechanism[11,7] (along with the triplet parts of h and $\bar{h}$) and the super-Higgs effect. The doublet parts of h and $\bar{h}$ are the low-energy Higgs fields, taking VEV's in the h_5 and $\bar{h}_5$ directions; these become the isospin $+\frac{1}{2}$ piece of the h doublet (hypercharge $-\frac{1}{2}$) and the isospin $-\frac{1}{2}$ piece of the $\bar{h}$ doublet (hypercharge $+\frac{1}{2}$).

Under η_P, $\theta \to i\theta$ and superfields transform as $e^{2\pi i\eta_P/8}$, so for a superpotential term to be invariant the sum of the η_P charges must equal 4 (mod 8). To zeroth order in $1/M_P$ our superpotential is simply

$$W^{(0)} = \lambda_1^{ij}F_iF_jh + \lambda_2^{ij}F_i\bar{f}_j\bar{h} + \lambda_4 HHh + \lambda_5 \bar{H}\bar{H}\bar{h}\ . \tag{3}$$

This generates down-quark masses, up-quark masses, and the missing-partners mechanism.

The lepton-number-violating term arises at first order in $\epsilon \equiv (M_G/M_P)$ (where M_G is the GUT scale):

$$W^{(1)} = \lambda_A M_P^{-1} H\bar{f}\bar{f}E^c \to \tfrac{1}{2}\bar{C}^{ijk}L_iL_jE_k^c \tag{4}$$

as H takes its VEV, so we expect $\bar{C}^{ijk} \le \epsilon$. In formulating η_P, we saw immediately that no symmetry could allow down masses (FFh), charged-lepton masses ($h\bar{f}E^c$), and the L-violating term of (4) ($H\bar{f}\bar{f}E^c$), without also allowing the B- and L-violating term $HFF\bar{f}$. We could have chosen to retain $h\bar{f}E^c$ but not $H\bar{f}\bar{f}E^c$, violating L through a term $(H\bar{H})^nH\bar{f}h \to \mu L\bar{h}$. Then a rotation of L and h would generate $\frac{1}{2}C^{ijk}LLE^c$ with C^{313} and/or C^{323} dominant, as in Hall and Suzuki's SU(5) model.[2] Instead, we chose (by our choice of η_P) to disallow the $h\bar{f}E^c$ term and generate lepton masses (for μ and τ only) from $H\bar{f}\bar{f}E^c$, by the rotation of L_1 and h.

This rotation is due to the superpotential terms

$$\begin{aligned} W^{(2)} &= \lambda_B M_P^{-8}(H\bar{H})^4 H\bar{f}h + \lambda_C M_P^{-11}(H\bar{H})^6 h\bar{h} \\ &\to \mu^i L_i\bar{h} + mh\ \ \bar{h}(\mu^i = \epsilon^9\lambda_B M_P,\ m = \epsilon^{12}\lambda_C M_P)\ . \end{aligned} \tag{5}$$

The combination of fields which couples to $\bar{h}$ will be the one we call the low-energy Higgs field; this coupling is responsible for breaking Peccei-Quinn (PQ) symmetry and allowing both Higgs fields to take VEV's,[13] so that our theory correctly breaks $SU(2)_L \otimes U(1)_Y \to U(1)_{EM}$. Let us first rotate the L_i (which are still degenerate, being massless) so that the linear combination of these which couples to $\bar{h}$ is now called L_1, then rotate this L_1 with h such that the linear combination which couples with $\bar{h}$ is now called h':

$$\begin{aligned} &h' = c_1h + s_1L_1,\quad L_1' = c_1L_1 - s_1h\ ,\\ &L_2' = L_2,\quad L_3' = L_3\ . \end{aligned} \tag{6}$$

The rotation angle is given by $s_1/c_1 = \mu/m$.

The lepton mass matrix is now generated from Eq. (4):

$$W^{(1)} \to s_1\bar{C}^{i1k}L_i'h'E_k^c + c_1\bar{C}^{i1k}L_i'L_1'E_k^c + \bar{C}^{23k}L_2'L_3'E_k^c\ . \tag{7}$$

Since $\overline{C}^{ijk}$ is antisymmetric in (i,j) the first term gives masses to the muon and tau only, smaller than the quark masses by $O(\epsilon)$, while the electron remains massless. A term

$$W^{(3)}=\lambda_E M_P^{-4}(H\overline{H})^2\overline{f}E^c h \tag{8}$$

gives a small electron mass of $\epsilon^4\langle h'\rangle$; we will find from M_W/M_P that $\epsilon=0.04$ in our model, so $m_e\approx$MeV. The electron is light because it is the combination of L_i that rotated with h. The second and third terms of $W^{(1)}$ violate lepton number as desired. The rotation also affects the down-quark mass term:

$$\lambda_1^{ij}Q_i h D_j^c \to c_1\lambda_1^{ij}Q_i h' D_j^c - s_1\lambda_1^{ij}Q_i L_1' D_j^c \; . \tag{9}$$

The second term is electron-number violating, diagonal in the quark flavors, and dominated by the third quark generation: $D^{313}Q_3L_1D_3^c$. From (7) and (9) we see

$$C^{212}=\frac{c_1 m_\mu}{s_1\langle h'\rangle},\quad C^{313}=\frac{c_1 m_\tau}{s_1\langle h'\rangle}\;,$$
$$D^{111}=\frac{-s_1 m_d}{c_1\langle h'\rangle}\;, \tag{10}$$
$$D^{212}=\frac{-s_1 m_s}{c_1\langle h'\rangle},\quad D^{313}=\frac{-s_1 m_b}{c_1\langle h'\rangle}\;.$$

With the diagonalization of the lepton and quark mass matrices, all other lepton-number-violating coefficients vanish except for the three terms terms C^{23k}, which must be taken small by fiat. Our theory violates only electron number.

The electron-neutrino Majorana mass diagrams of Fig. 2 constrain the rotation angle. Too much rotation $(\mu \gg m)$ makes D^{313} large and Fig. 2(a) dominates, with

$$m_M(\nu_e)=\frac{3A_b}{16\pi^2}\left[\frac{s_1}{c_1}\right]^2\frac{m_b^4 M_{3/2}}{\langle h'\rangle^2 m_{\tilde{b}^c}^2}<2\text{ eV}\;, \tag{11a}$$

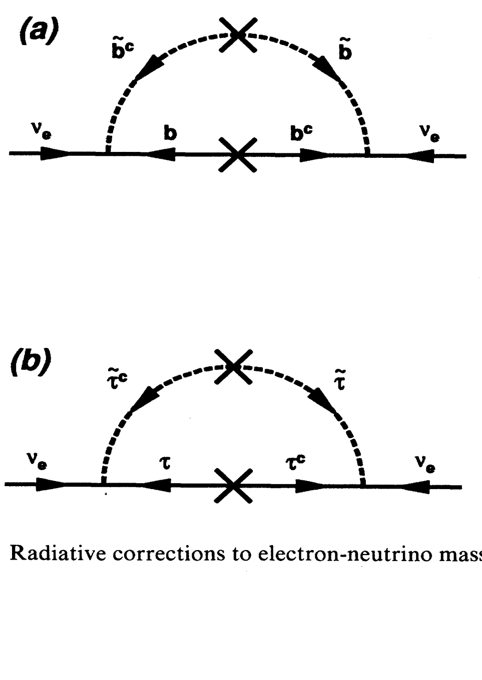

FIG. 2. Radiative corrections to electron-neutrino mass.

where the limit is obtained from neutrinoless double-beta-decay experiments. Too little rotation $(\mu\le m)$ makes C^{313} large and Fig. 2(b) dominates, with

$$m_M(\nu_e)=\frac{A_\tau}{16\pi^2}\left[\frac{c_1}{s_1}\right]^2\frac{m_\tau^4 M_{3/2}}{\langle h'\rangle^2 m_{\tilde{\tau}}^2}<2\text{ eV}\;. \tag{11b}$$

Reconciling (11a) and (11b) forces us to make the rotation angle about 20° and the A parameters (renormalized down to $M_{3/2}$) small, about $\frac{1}{10}$. Small values for the A's are plausible from the renormalization equations if $A\approx -4M_0$ at the Planck scale; Eq. (A11) of Ref. 13 predicts $A_b, A_\tau(M_{3/2})=A+4M_0$ [another possibility is to take A_b and A_τ of opposite sign, so (11a) and (11b) tend to cancel]. With[13]

$$M_{3/2}=1\text{ TeV},\quad \langle h'\rangle=130\text{ GeV}\;,$$
$$m_{\tilde{b}^c}^2=7.6M_{3/2}^2,\quad m_{\tilde{\tau}}^2=1.5M_{3/2}^2\;, \tag{12}$$

we calculate a barely acceptable mass for $A\approx 0.1$ and $s_1\approx 0.38$. This allows us to predict (in the sense that the given values require the least fine-tuning of A) the C^{ijk}'s and D^{ijk}'s from (10).

To achieve this rotation angle we must make $\lambda_B/\lambda_C=2\times10^{-4}$. Then to get the weak scale right in Eq. (5) (with $\lambda_C\approx 1$), we find $\epsilon=0.04$.

The neutrino seesaw mechanism is driven by the term

$$W^{(4)}=\lambda_N M_P^{-5}(H\overline{H})^2(F\overline{H})^2\;. \tag{13}$$

ν^c gets a Majorana mass of order $\epsilon^6 M_P\approx 4\times10^{10}$ GeV; then the λ_2 term couples ν^c to ν, giving ν a seesaw mass:

$$m_{\nu_L}\approx\frac{m_u^2}{4\times10^{10}\text{ GeV}}$$
$$=3\times10^{-6}\text{ eV}(e),\; 0.1\text{ eV}(\mu),\; 100\text{ eV}(\tau)\;, \tag{14}$$

with $m_t=70$ GeV. The ν_τ is cosmologically stable and is a constituent of the dark matter.

Undesirable superpotential terms are neatly suppressed by η_P:

$$W^{(5)}=\lambda_F M_P^{-13}(H\overline{H})^6HFF\overline{f}+\lambda_G M_P^{-13}(H\overline{H})^7F\overline{H}$$
$$+\lambda_H M_P^{-5}(H\overline{H})^4\;. \tag{15}$$

The first violates B and L, while the second and third threaten the flatness of the $H=\overline{H}=V$ direction. Actually we need $\lambda_H\le 10^{-4}$ to avoid this pitfall. Other highly suppressed or unimportant terms are collected for completeness's sake in $W^{(6)}$:

$$W^{(6)}=M_P^{-3}(H\overline{H})\overline{H}\overline{h}\overline{h}\overline{f}+M_P^{-5}(H\overline{H})^2HHF\overline{f}$$
$$+M_P^{-8}(H\overline{H})^4HFh+M_P^{-12}(H\overline{H})^5\overline{H}\overline{H}\overline{H}hE^c$$
$$+M_P^{-13}(H\overline{H})^6HHH\overline{f}\;. \tag{16}$$

We do not have exact mass relation predictions, but up to Yukawa couplings we expect

$$m(\mu)/m(c)\approx m(\tau)/m(t)\approx\epsilon=0.04 \text{ (experimentally}\approx 0.08,\ 0.03)\ , \tag{17}$$

$$m(s)/m(c)\approx m(b)/m(t)\approx c_1=0.9 \text{ (experimentally}\approx 0.15,\ 0.07)\ , \tag{18}$$

$$m(e)\approx\epsilon^4\langle h'\rangle\approx \text{ MeV (experimentally}=511\text{ keV})\ . \tag{19}$$

Other numerical predictions of our model are

$$M_G/M_P=0.04\ , \tag{20}$$

$$m(\nu)\approx 2\text{ eV}(e),\ 0.1\text{ eV}(\mu),\ 100\text{ eV}(\tau)\ , \tag{21}$$

$$\begin{aligned} &C^{212}=0.002,\quad C^{313}=0.034\ ,\\ &D^{212}=-0.0006,\quad D^{313}=-0.015\ , \end{aligned} \tag{22}$$

with other C^{ijk}'s and D^{ijk}'s small or zero. Note the largest C^{ijk} is indeed $O(\epsilon)$ as expected.

III. RENORMALIZATION SCALING BEHAVIOR

In this section we will find $g_1=g_5$ at M_G, we will derive the gaugino mass relation necessary for our model to break down correctly to the standard model, and we will explore the origin of the GUT scale.

g_1 is the coupling constant associated with the normalized U(1) charge $q=\sqrt{1/40}\,Q$, Q being the charge given in Eq. (1). From renormalization scaling of known low-energy couplings,[14] $g_5(M_G)=0.724$ (from α_3 and α_2) and $g_y(M_G)=0.703$ ($y=\sqrt{3/5}\,Y$, and taking $\sin^2\theta_W=0.228$). Since we know

$$y=\sqrt{24/25}\,q+\sqrt{1/25}\,T_{24} \tag{23}$$

we can define an angle θ_X analogous to the Weinberg angle:

$$\begin{aligned} &g_y=g_1\sqrt{25/24}\cos\theta_X=g_5\sqrt{25}\sin\theta_X\ ,\\ &\tan\theta_X=\sqrt{1/24}\frac{g_1}{g_5}\ . \end{aligned} \tag{24}$$

The known values give us $\sin^2\theta_X=0.0377$ and $g_1/g_5=0.97$ at M_G. An SO(10) embedding would predict $\sin^2\theta_X=0.04$ and $g_1/g_5=1$. Without committing to SO(10), we will hereafter take $g_1=g_5\equiv g$.

We must be sure the vacuum breaks correctly, since there are three flat directions in our model: $\langle H\rangle=\langle\overline{H}\rangle=V$, $\langle F_i\rangle=\langle\overline{H}\rangle=\hat{V}$ (for one value of i), and $\langle h\rangle=\langle\overline{h}\rangle=v$. The direction chosen by the theory to break the $\overline{\text{SU}(5)}$ symmetry and define the GUT scale depends on the renormalization behavior of the scalar masses. The supergravity supersymmetry-breaking Lagrangian is[15]

$$\begin{aligned} \mathcal{L}_{\text{SX}}=&-m_A^2|A_i|^2-[AW_3(A_i)+BW_2(A_i)+\text{H.c.}]\\ &-(\tfrac{1}{2}M_0\lambda^a\lambda^a+\text{H.c.})\ . \end{aligned} \tag{25}$$

We take all scalar masses m_A equal, all trilinear soft breaking coefficients A equal, all bilinear coefficients B equal, and all gaugino masses M_0 equal, all $O(M_{3/2})$, at M_P. We must ask which sum goes negative first as we go down in energy: $(m_H^2+m_{\overline{H}}^2)$, $(m_F^2+m_{\overline{H}}^2)$, or $(m_h^2+m_{\overline{h}}^2)$. It is easy to make m_F^2 scale less quickly than m_H^2 by making λ_1 and λ_2 small compared to λ_4. Making the h masses scale more slowly than the H masses is more difficult. It turns out that the HHh term contributes equally to the renormalization scaling of m_h^2 and m_H^2, so the correct breaking depends on gaugino masses only. We start with Ref. 16, Eqs. (A20)–(A23), then eliminate all terms involving ϕ, λ_1, or λ_2; set $\lambda_4=\lambda_5\equiv\lambda$; and define $\hat{m}_H^2\equiv(m_H^2+m_{\overline{H}}^2)/2$, etc.; to get

$$\frac{d}{dt}(\hat{m}_H^2)=\frac{1}{16\pi^2}\left[6\lambda^2(2\hat{m}_H^2+\hat{m}_h^2+\hat{A}^2)-\frac{g^2}{5}(144M_5^2+M_1^2)\right]\ , \tag{26a}$$

$$\frac{d}{dt}(\hat{m}_h^2)=\frac{1}{16\pi^2}\left[6\lambda^2(2\hat{m}_H^2+\hat{m}_h^2+\hat{A}^2)-\frac{g^2}{5}(96M_5^2+4M_1^2)\right]\ . \tag{26b}$$

To have the right-hand side of (26a) greater than that of (26b), we must have, at M_G,

$$M_1(M_G)\geq 4M_5(M_G)\ , \tag{27}$$

discouraging any thoughts of a nearby superunification into SO(10). For a comparable analysis in the $\overline{\text{SU}(5)}$ model of Antoniadis *et al.*, see Ref. 14.

Equation (26a) determines the GUT scale, from

$$\hat{m}_H^2(t=0)=m_A^2,\quad \hat{m}_H^2(t=\ln\epsilon)=0\ . \tag{28}$$

Let

$$\alpha=\frac{18\lambda^2}{16\pi^2},\quad \beta=\frac{6\lambda^2A^2}{16\pi^2}-\frac{29g^2M_0^2}{16\pi^2}\ . \tag{29}$$

Then with $\hat{m}_h^2\approx\hat{m}_H^2$ and [from Ref. 16, (A11) and (A12)] $\hat{A}=Ae^{\alpha t}$, but ignoring scaling of g and λ, the solution is

$$\epsilon=\left[1-\frac{\alpha m_A^2}{\beta}\right]^{1/\alpha}\ . \tag{30}$$

For $\epsilon=0.04$, one choice of parameters obeying (30) is

$$g=0.724,\quad \lambda=0.86,\quad m_A=M_0,\quad A=-4M_0\ . \tag{31}$$

We have taken $g=0.724$ as calculated at the beginning of this section, and $A=-4M_0$ (at M_P) as discussed following Eq. (11b). M_0 is undetermined.

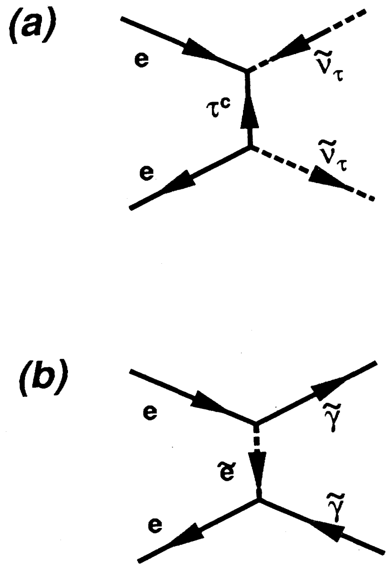

FIG. 3. t-channel-exchange diagrams.

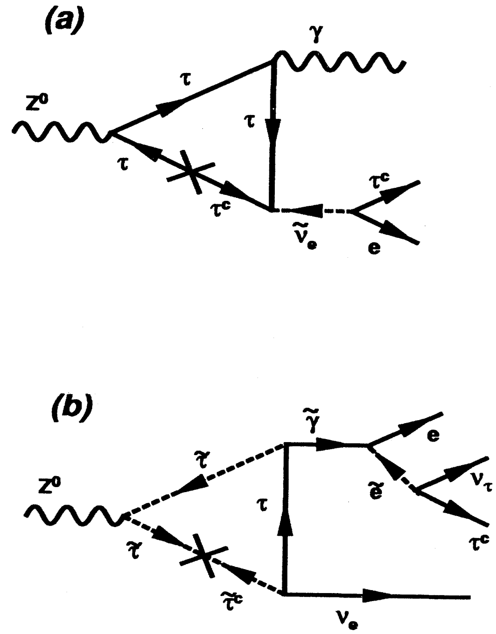

FIG. 5. (a) $Z^0 \to \gamma \bar{\tilde{\nu}}_e$, (b) $Z^0 \to \bar{\nu}_\tau \tilde{\gamma}$.

IV. SIGNATURES OF LEPTON-NUMBER VIOLATION IN RARE Z^0 DECAYS

The signatures of a model violating L differ greatly from the missing-energy signatures of R_P-invariant models. Superpartners can be pair produced, as in the e^+e^- collisions of Fig. 3, and will decay into ordinary matter through processes such as those in Fig. 4. One-loop diagrams such as those in Fig. 5 show how a Z^0 could produce a single superpartner and one ordinary particle of fixed energy, a truly spectacular decay which unfortunately has too small a branching ratio to be seen at LEP. Prominent Bhabha-scattering resonances occur at the $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$ masses, as seen in Fig. 6; though note these interactions are absent in our model. (They do occur in the μ version of our model; see the Appendix.) These signatures are discussed further in Refs. 1–3.

We will concentrate on rare Z^0 decays which occur in our model with branching ratios of 10^{-6} or larger, so as to be seen at LEP. From Eq. (22), we see our largest L-violating coefficient is $C^{313}=0.034$, which provides the interactions of Fig. 7.

Our model predicts the decay $Z^0 \to e^-\tau^+e^+\tau^-$, through the three diagrams of Fig. 8. If $m(\tilde{\nu})<M_Z/2$, this process proceeds primarily through the production of two on-shell $\tilde{\nu}_\tau$'s [Fig. 8(a)], and the branching ratio is large ($>10^{-4}$). In this case, the $(e^-\tau^+)$ pair and the $(e^+\tau^-)$ pair would each have an invariant mass equal to the $\tilde{\nu}_\tau$ mass. In fact, sneutrino pair production would lead to equally large branching ratios for all three of these reactions:

$$\begin{aligned} &Z^0 \to e^-\tau^+e^+\tau^-, \quad \sqrt{s_{e^-\tau^+}}=\sqrt{s_{e^+\tau^-}}=m(\tilde{\nu}_\tau) , \\ &Z^0 \to e^-\mu^+e^+\mu^-, \quad \sqrt{s_{e^-\mu^+}}=\sqrt{s_{e^+\mu^-}}=m(\tilde{\nu}_\mu) , \qquad (32) \\ &Z^0 \to \tau^-\tau^+\tau^+\tau^-, \quad \sqrt{s_{\tau^-\tau^+}}=\sqrt{s_{\tau^+\tau^-}}=m(\tilde{\nu}_e) . \end{aligned}$$

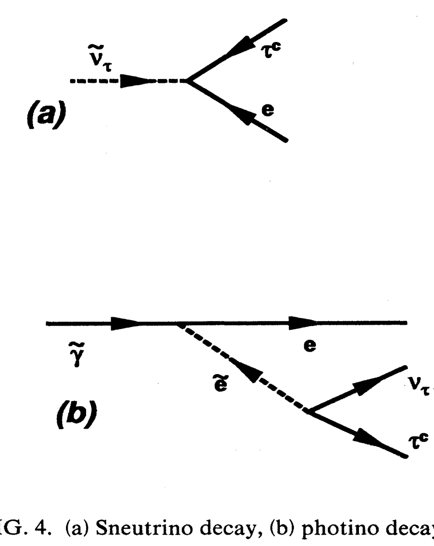

FIG. 4. (a) Sneutrino decay, (b) photino decay.

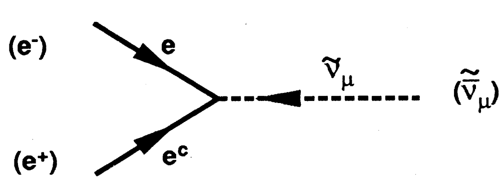

FIG. 6. Sneutrino resonance in the μ version.

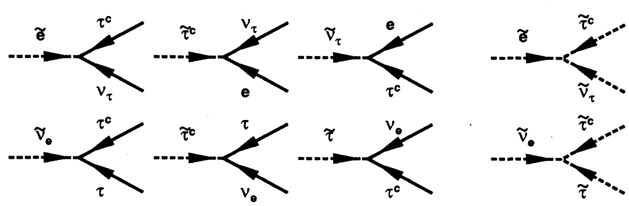

FIG. 7. R_P-violating interactions from C^{313}.

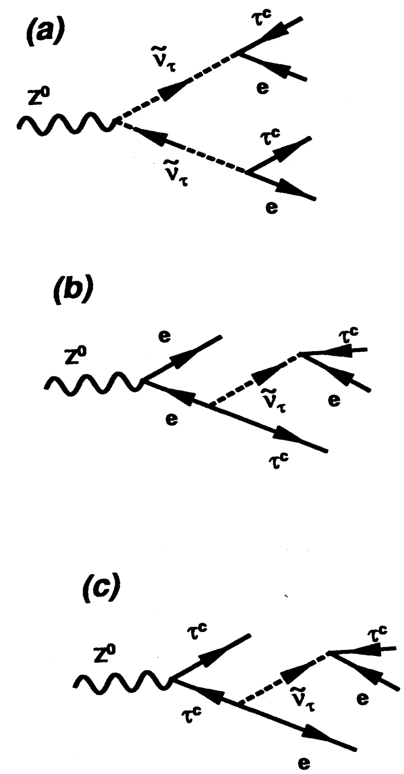

FIG. 8. $Z^0 \rightarrow e^- \tau^+ e^+ \tau^-$.

If $m(\tilde{\nu}) \geq M_Z/2$, all three diagrams of Fig. 8 contribute to $Z^0 \rightarrow e^- \tau^+ e^+ \tau^-$, with Figs. 8(b) and 8(c) dominating at higher sneutrino masses. In this case only one $(e\tau)$ pair comes out with invariant mass equal to $m(\tilde{\nu}_\tau)$. Similar diagrams lead to

$$\begin{aligned}
&Z^0 \rightarrow e^- \tau^+ e^+ \tau^- \propto (C^{313})^2 , \\
&Z^0 \rightarrow e^- \mu^+ e^+ \mu^- \propto (C^{212})^2 , \\
&Z^0 \rightarrow \tau^- \tau^+ \tau^+ \tau^- \propto (C^{313})^2 , \\
&Z^0 \rightarrow \mu^- \mu^+ \tau^- \tau^+ \propto (C^{212})^2 , \\
&Z^0 \rightarrow b\bar{b}\tau^- \tau^+ \propto (D^{313})^2 , \\
&Z^0 \rightarrow e^- \bar{b} e^+ b \propto (D^{313})^2 .
\end{aligned} \tag{33}$$

Figure 9 shows total calculated branching ratios for $Z^0 \rightarrow e^- \tau^+ e^+ \tau^-$ (solid line), plotted against $m(\tilde{\nu}_\tau)$, for $C^{313}=1$ (a), and for $C^{313}=0.034$ (b) as in our model. The patterned lines show the contributions from each of the three diagrams of Fig. 8. For $C^{313}=0.034$, we see that a signal would only be seen at LEP if $m(\tilde{\nu}) < M_Z/2$, but for $C^{313}=1$ a signal would be seen for sneutrinos as heavy as 70 GeV.

V. CONCLUSION

Our model shows how L violation can occur in a supersymmetric GUT model, and how rotation of one lepton family with the Higgs field singles out that family to be light and violated. Since the electron is light, we conclude electron number is violated. The rotation angle is constrained from both sides by limits on the electron-neutrino Majorana mass, so from the lepton and quark masses we can predict our largest L-violating terms are $C^{313} L_3 L_1 E_3^c$ ($C^{313}=0.034$), $C^{212} L_2 L_1 E_2^c$, ($C^{212}=0.002$), and $D^{313} Q_3 L_1 D_3^c$ ($D^{313}=-0.015$). We can then calculate branching ratios for Z^0 decays which depend only on the unknown sneutrino masses. If $m(\tilde{\nu}) < M_Z/2$, the

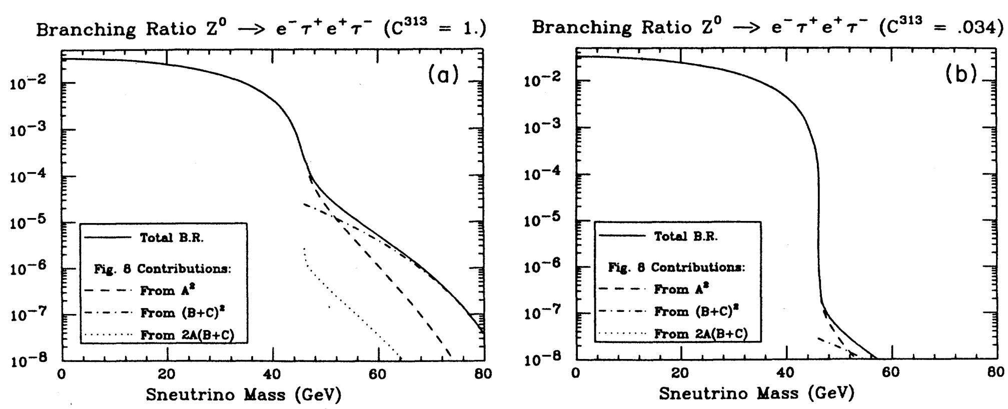

FIG. 9. Branching ratios for (a) $C^{313}=1$, (b) $C^{313}=0.034$.

clearest signal would come from $Z^0 \to e^- \mu^+ e^+ \mu^-$, where each (e,μ) pair has invariant mass equal to $m(\tilde{\nu}_\mu)$. If $m(\tilde{\nu}) > M_Z/2$, then branching ratios are proportional to $(C^{ijk})^2$ [or $(D^{ijk})^2$], so for our model $Z^0 \to e^- \tau^+ e^+ \tau^-$ dominates, though with $C^{313} = 0.034$ this branching ratio is still rather small. These rare Z^0 decays replace the usual missing-energy signatures, since the LSP is unstable.

ACKNOWLEDGMENTS

We wish to thank M. Suzuki, S. Hsu, and J. Burton for helpful conversations. L.J.H. acknowledges support from the Sloan Foundation and the Presidential Young Investigators Program. D.E.B. acknowledges support from the National Science Foundation. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and in part by the National Science Foundation under Grant No. PHY85-15857.

APPENDIX: A μ VERSION OF OUR MODEL

To avoid the neutrino mass constraint, a second version of our model makes the rotated lepton be the muon instead. Now our left-handed chiral superfields have the following SU(5) structure, U(1) charge, and ζ_P charge:

$$\begin{aligned} &F_i = (\mathbf{10},1,2), \quad \bar{f}_i = (\bar{\mathbf{5}},-3,-2) , \\ &E_i^c = (1,5,2), \quad H = (\mathbf{10},1,-7) , \\ &\bar{H} = (\overline{\mathbf{10}},-1,0) , \\ &h = (\mathbf{5},-2,5), \quad \bar{h} = (\bar{\mathbf{5}},2,9) . \end{aligned} \tag{A1}$$

Superfields transform as $e^{2\pi i \zeta_P/18}$, so the sum of the ζ_P charges in an allowed term must equal 9 (mod 18). The same mechanisms occur as in the η_P model; only the numbers are different, and the rotated lepton superfield is L_2. Our superpotential is

$$W^{(0)} = \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{f}_j \bar{h} + \lambda_4 HHh + \lambda_5 \bar{H}\bar{H}\bar{h} , \tag{A2}$$

$$W^{(1)} = \lambda_A M_P^{-1} H\bar{f}\bar{f}E^c \to \tfrac{1}{2} C^{ijk} L_i L_j E_k^c , \tag{A3}$$

$$W^{(2)} = \lambda_B M_P^{-18} (H\bar{H})^9 H\bar{f}\bar{h} + \lambda_C M_P^{-21} (H\bar{H})^{11} h\bar{h} , \tag{A4}$$

$$W^{(3)} = \lambda_E M_P^{-4} (H\bar{H})^2 \bar{f} E^c h , \tag{A5}$$

$$W^{(4)} = \lambda_N M_P^{-15} (H\bar{H})^7 (F\bar{H})^2 , \tag{A6}$$

$$\begin{aligned} W^{(5)} = {} & \lambda_F M_P^{-33} (H\bar{H})^{16} HFF\bar{f} + \lambda_G M_P^{-33} (H\bar{H})^{17} F\bar{H} \\ & + \lambda_H M_P^{-15} (H\bar{H})^9 , \end{aligned} \tag{A7}$$

$$\begin{aligned} W^{(6)} = {} & M_P^{-3} (H\bar{H}) \bar{H}h\bar{h}\bar{f} + M_P^{-15} (H\bar{H})^7 HHF\bar{f} \\ & + M_P^{-18} (H\bar{H})^9 HFh + M_P^{-22} (H\bar{H})^{10} \bar{H}\bar{H}\bar{H} h E^c \\ & + M_P^{-33} (H\bar{H})^{16} HHH\bar{f} . \end{aligned} \tag{A8}$$

By setting $m = 50$ GeV (with $\lambda_C = 0.5$) and $\mu = 1$ TeV (with $\lambda_B = 0.04$), we get $\epsilon = 0.17$ and $c_2 = 0.05$. The ν^c gets a Majorana mass of order $\epsilon^{16} M_P \approx 5 \times 10^6$ GeV, so that neutrino masses become 0.02 eV (e), 700 eV (μ), 500 keV (τ). When L_2 rotates with h, the muon neutrino picks up a mass from the diagrams of Fig. 2, but experimental limits on the ν_μ mass are much more forgiving.

Combining the lepton mass and LLE^c terms [see Eqs. (7) and (8)] now gives (with $i \neq 2$)

$$\begin{aligned} W^{(1)} + W^{(3)} \to {} & \lambda^{2k} \langle h' \rangle E_2' E_k^c + (c_2 \lambda^{ik} + s_2 C^{i2k}) \langle h' \rangle E_i' E_k^c \\ & + C^{13k} L_1' L_3' E_k^c + (c_2 C^{i2k} - s_2 \lambda^{ik}) L_i' L_2' E_k^c . \end{aligned} \tag{A9}$$

Here $\lambda \equiv \epsilon^4 \lambda_E$. A rotation is assumed to diagonalize the mass matrices. The muon mass is $\epsilon^4 \langle h' \rangle \approx 100$ MeV. The tau (and electron) masses are smaller than the quark masses by $O(\epsilon)$. Six of the lepton-number-violating terms are suppressed (since c_2 and λ are both small). The Yukawa couplings must conspire to make only one of the three remaining C^{13k} large, and only one of the two remaining charged-lepton masses (e,τ) large. From Eq. (10) we see that in the quark sector we have a large $Q_3 L_2 D_3^c$ term, with $D^{323} = -0.72$. Up to Yukawa couplings we expect

$$m(e)/m(u) \approx m(\tau)/m(t) \approx \epsilon = 0.17 \text{ (experimentally } \approx 0.09,\ 0.03) , \tag{A10}$$

$$m(s)/m(c) \approx m(b)/m(t) \approx c_2 = 0.05 \text{ (experimentally } \approx 0.15,\ 0.07) , \tag{A11}$$

$$m(\mu) \approx \epsilon^4 \langle h' \rangle \approx 100 \text{ MeV (experimentally } = 106 \text{ MeV)} , \tag{A12}$$

$$m(\nu) \approx 0.02 \text{ eV}(e),\ 700 \text{ eV}(\mu),\ 500 \text{ keV}(\tau) . \tag{A13}$$

This model has the advantage that it explains why down-type heavy quarks are lighter than their up-type partners (by c_2), and it avoids the strict mass limits on ν_e. Muon number (rather than electron number) is violated. $\tilde{\nu}_\mu$'s would be produced copiously at e^+e^- colliders through the diagram of Fig. 6. However, this model fails to explain the lightness of the electron.

[1]S. Dimopoulos and L. J. Hall, Phys. Lett. B **207**, 210 (1987).
[2]L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).
[3]C. Aulakh and R. Mohapatra, Phys. Lett. **119B**, 136 (1983); I. H. Lee, Nucl. Phys. **B246**, 120 (1984); G. G. Ross and J. W. F. Valle, Phys. Lett. **151B**, 375 (1985); J. Ellis, G. Gelmini, C. Jarlskog, G. G. Ross, and J. W. F. Valle, *ibid.* **150B**, 142 (1985); S. Dawson, Nucl. Phys. **B261**, 297 (1985); F. Zwirner, Phys. Lett. **132B**, 103 (1983); R. Barbieri and A. Masiero,

Nucl. Phys. **B267**, 679 (1986); S. Dimopoulos, R. Esmailzadeh, L. J. Hall, and G. D. Starkman, Report No. SLAC-PUB-4797, 1988 (unpublished).

[4]M. C. Bento, L. J. Hall, and G. G. Ross, Nucl. Phys. **B292**, 400 (1987).

[5]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **45**, 413 (1980).

[6]S. M. Barr, Phys. Lett. **112B**, 219 (1982).

[7]I. Antoniadis, J. Ellis, J. S. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **194**, 231 (1987). Antoniadis *et al.* discuss a $\overline{\mathrm{SU}(5)}$ model with only dimension four (trilinear) terms in the superpotential, and impose a discrete symmetry ($H \to -H$) to eliminate R_P-violating terms at this level. However, non- (and super-) renormalizable terms obeying this symmetry would violate B and L badly [e.g., $M_P^{-3}(H\overline{H})HFF\overline{f}$] as well as destroying the flat directions of the theory [e.g., $M_P F\overline{H}$ or $M_P^{-1}(H\overline{H})^2$]. Their model also requires four singlet fields ϕ_m, three driving the neutrino seesaw mechanism by marrying the ν_i^c, and one taking a weak-scale VEV to mix h and $\overline{h}$. We achieve these goals through the additional nonrenormalizable terms in our superpotential.

[8]H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[9]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[10]A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B135**, 66 (1978).

[11]S. Dimopoulos and F. Wilczek, in *The Unity of the Fundamental Interactions,* proceedings of the 19th International School of Subnuclear Physics, Erice, Italy, 1981, edited by A. Zichichi (Subnuclear Series, Vol. 19) (Plenum, New York, 1983), p. 237.

[12]The η_P charges are just one of a family of equivalent assignments, differing by a multiple of the U(1) charge: $F(x-1)$, $\overline{f}(-3x+1)$, $E^c(5x-1)$, $H(x+3)$, $\overline{H}(-x+4)$, $h(-2x-2)$, $\overline{h}(2x+4)$, all mod(8).

[13]M. Claudson, L. J. Hall, and I. Hinchliffe, Nucl. Phys. **B228**, 501 (1983), especially Fig. 5.

[14]M. Drees, Phys. Lett. B **206**, 265 (1988).

[15]M. B. Wise, in *Proceedings of the Fourth Theoretical Advanced Study Institute in Elementary Particle Physics,* Santa Fe, New Mexico, 1987, edited by R. Slansky and G. West (World Scientific, Singapore, 1988), p. 787; A. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara, and C. Savoy, Phys. Lett. **119B**, 343 (1982); L. Ibanez, *ibid.* **118B**, 73 (1982); H. P. Nilles, *ibid.* **115B**, 1973 (1982); L. J. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D **27**, 2359 (1983).

[16]J. Ellis, J. S. Hagelin, S. Kelley, and D. V. Nanopoulos, Nucl. Phys. **B311**, 1 (1988).