Four-generation model based on an S_4 permutation symmetry

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We consider an S_4 family-symmetry model with four generations of leptons and quarks. A mass relation of the form $m_L/m_\tau = m_v/m_b = m_a/m_t$ is obtained. The predicted values of individual 4×4 Kobayashi-Maskawa matrix elements are within recent experimental data. If the charged-lepton mass of the fourth generation is at 30 GeV and the t-quark mass is at 35 GeV, the masses of the d- and u-type quarks of the fourth generation are predicted to be 73 ± 2 GeV and 523 ± 3 GeV, respectively, taking account of the renormalization-group effect.

I. INTRODUCTION

One of the problems in the standard model[1] is the so-called generation problem: why leptons and quarks appear repeatedly in the same manner. To answer this question some ideas have been proposed: leptons and quarks have substructures which realize the family replication; there is a symmetry between generations in addition to the $SU(2)\times U(1)$ symmetry. The first idea is called a composite model[2] and the second idea is divided mainly into two types of models: one is a horizontal-gauged-symmetry model[3] and the other is a discrete-symmetry model.[4–8] The composite model needs new fundamental constituents and the horizontal-gauged-symmetry model needs extra gauge bosons to avoid the appearance of Nambu-Goldstone bosons after the spontaneous breaking of the family symmetry.

On the other hand the discrete-symmetry model does not need any new fields that are not contained in the standard model. If one does not want to introduce any new fields, the discrete model seems to be a minimal extension of the original unified theory of weak and electromagnetic interactions to settle the generation problem. For this reason the discrete model has been studied by many authors. However, in general, we are obliged to proliferate Higgs sectors if we introduce such a discrete symmetry. As a natural consequence, the flavor-changing neutral current (FCNC) will arise from neutral-Higgs-boson couplings. This is the price to be paid for adopting the discrete symmetry. If we want to forbid the appearance of the FCNC at any cost, it is possible to do that in a discrete model,[5] but it is impossible to get the fermion mixings to be in agreement with experimental data. On the other hand if we admit the appearance of the FCNC by neutral Higgs bosons,[7,8] the fermion mixings which do not contradict with experimental data can be obtained. Even if we adopt such a model, we do not want the FCNC to appear in the low-energy region because the processes $K_L \to \bar{\mu}\mu$, μe, etc., are known from experiment to have rates too small. So we usually assume heavy neutral Higgs bosons to suppress such rare processes.

Thus we can construct two kinds of discrete models: one forbids the FCNC and the other admits the FCNC. If we attach importance to correct prediction of the fermion mixings rather than the appearance of the FCNC, we should adopt the latter discrete model rather than the former one. Some years ago the latter discrete model was proposed by Pakvasa and Sugawara.[7] They constructed a three-generation model of leptons and quarks based on the permutation symmetry S_4, which is a discrete group of degree 4. At that time they concentrated on a problem of fermion masses and predicted the t-quark mass m_t to lie in 26 GeV $\le m_t \le$ 35 GeV. A few years later, Yamanaka, Sugawara, and Pakvasa[8] (YSP) determined the Kobayashi-Maskawa (KM) matrix[9] in their model. It is remarkable that their prediction for the KM matrix was in agreement with experimental data well. Thus YSP's model is one of the hopeful candidates to settle the generation problem.

In these family-symmetry models, there still remains an important question how many generations there are in nature. Unfortunately, not only YSP's model but also family-symmetry models have not yet answered the question. Therefore if we want to get some information about the number of generations N, we are forced to rely on another theoretical reasoning or experimental results at present. There are two kinds of theoretical constraint on the number of the generations. The one is $N \ge 3$ which comes from the CP-violation mechanism suggested by Kobayashi and Maskawa.[9] The other is $N < \frac{33}{2}$ which comes from asymptotic freedom[10] in QCD. On the other hand, there is an astrophysical constraint[11] that suggests the existence of a fourth neutrino being almost on the verge of exclusion. In the near future the precise measurement of $e^+e^- \to \nu\bar{\nu}\gamma$ (Ref. 12) and the decay width of Z^0 will provide us the number of neutrinos. Since the number of neutrinos will be equal to the number of generations, they are the most promising experiments to determine the number of generations of leptons and quarks. Additional information about the number of generations may be obtained from the large B_d^0-$\bar{B}_d^0$ mixing found recently by the ARGUS Collaboration,[13] which may suggest the existence of fourth-generation quarks.[14] We have generally believed that the number of generations is 3, but, as stated above, there is no reason that we must not suppose more than three generations. Then it might be interesting to extend YSP's three-generation model to the four-generation one.

In this paper we construct the four-generation model based on an S_4 permutation symmetry. Before dealing with the S_4 permutation symmetry, we discuss the general permutation symmetry S_n in the next section. A more specific model based on an S_4 symmetry for four generations of leptons and quarks is analyzed in detail in Sec. III. In Sec. IV an additional reflection symmetry is introduced. It is remarkable that the fermions of the first and second generations do not acquire masses by the symmetry. The resulting mass formula for the fermions of the third and fourth generations enables us to estimate the quark masses of the fourth generation. We estimate the masses taking account of the renormalization-group effect. In Sec. V we show that the reflection symmetry leads to the symmetric KM matrix. We also show the values of individual matrix elements. We need to give masses to the fermions of the first and second generations since those fermions are forced to be massless by the reflection symmetry. In Sec. VI we give them masses by breaking the reflection symmetry. It is shown that the KM matrix elements are modified slightly as the fermions of the first and second generations acquire masses. In the final section we summarize the prediction of the model. Throughout this paper, (a,v)[15] denotes the fourth-generation quarks with electric charges $(\frac{2}{3},-\frac{1}{3})e$ and (ν_L,L) denotes the fourth-generation leptons with $(0,-1)e$.

II. THE S_n FAMILY SYMMETRY

In this section we discuss the assignment of the fermions to the representation of S_n. We shall denote the left-handed (LH) doublets, right-handed (RH) singlets, and Higgs scalars doublets by

$$\begin{bmatrix} \nu^i \\ l^i \end{bmatrix}_L, \; l_R^j, \; \begin{bmatrix} u^i \\ d^i \end{bmatrix}_L, \; u_R^j, \; d_R^j, \; \phi_k = \begin{bmatrix} \phi_k^+ \\ \phi_k^0 \end{bmatrix},$$

where $i,j=1-N$ for N generations and $k=1-K$ for K Higgs doublets. We then have the Yukawa couplings of the general form

$$L_Y = \sum_{i,j,k} f_{ijk}\overline{(\nu^i l^i)_L}\phi_k l_R^j + \sum_{i,j,k} g_{ijk}\overline{(u^i d^i)_L}\phi_k d_R^j + \sum_{i,j,k} h_{ijk}\overline{(u^i d^i)_L}\tilde{\phi}_k u_R^j + \text{H.c.} , \tag{2.1}$$

where $\tilde{\phi}_k = i\tau_2\phi_k^*$. The coupling constants in each charge sector are complex numbers in general.[16]

Let us impose the S_n permutation symmetry on the Yukawa couplings (2.1). LH fermions l_L, u_L, and d_L are assigned to the representations $D(l_L)$, $D(u_L)$, and $D(d_L)$, respectively; RH fermions l_R, u_R, and d_R are assigned to the representations $D(l_R)$, $D(u_R)$, and $D(d_R)$, respectively. The representation D may be irreducible or reducible at the present stage. Since LH quarks form doublets of SU(2), we must have $D(u_L)=D(d_L)$. Then, assuming the symmetry between u- and d-type quarks, we should also take $D(u_R)=D(d_R)$. In addition, assuming the symmetry between leptons and quarks, we should also take $D(l_L)=D(u_L)=D(d_L)$, and $D(l_R)=D(u_R)=D(d_R)$. Such an assignment gives a proportional relation between f_{ijk}, g_{ijk}, and h_{ijk} in each element. Therefore the fermion mass matrices of the charged lepton and the u- and d-type quarks form the same structure.

Let us show that the representation D should not be irreducible, but reducible. If each field is assigned to one irreducible representation, each charge sector has only one coupling constant, so the fermion mass matrices are proportional to one another:

$$M_l \propto M_- \propto M_+^* , \tag{2.2}$$

where M_l, M_-, and M_+ represent charged lepton, down-quark, and up-quark mass matrices. Note that the up-quark matrix has an asterisk because u-type quarks couple to $\tilde{\phi}^k$. The proportional relation (2.2) gives the mass relation

$$m_e:m_\mu:m_\tau:m_L:\cdots = m_d:m_s:m_b:m_v:\cdots = m_u:m_c:m_t:m_a:\cdots . \tag{2.3}$$

This relation is, however, in contradiction with the observed ratio, i.e.,

$$m_e:m_\mu \neq m_d:m_s .$$

Therefore each of the LH and RH fermions should not be assigned to one irreducible representation of S_n; at least either the LH or RH fermion should be assigned to some reducible representation of S_n. If we take such an assignment, we may get a more realistic mass relation. We will show an example with a reducible representation in the next section.

III. THE S_4 FAMILY SYMMETRY FOR FOUR GENERATIONS

In the preceding section we set up a general scheme for N generations of leptons and quarks based on the permutation symmetry S_n. We shall here apply this scheme to the case of four generations: $N=4$. Although there is uncertainty about the degree of the permutation group, we take here the permutation group of degree 4, S_4. The permutation group S_4 has irreducible representations of dimensions **3**, **3'**, **2**, **1**, and **1'** (the prime means antisymmetric representation). Since the group S_4 does not have an irreducible representation of dimension **4**, we cannot assign four-generation fermions to one irreducible representation. The Higgs doublets also must be assigned to more than one irreducible representation. Although there are some assignments, we shall here deal with one of them. The LH doublets and the RH singlets of fermions, and Higgs doublets are assigned as follows:[17]

$$\underset{\mathbf{1}}{\begin{pmatrix}\nu_1\\ l_1\end{pmatrix}_L},\quad \underset{\mathbf{3}}{\left\{\begin{pmatrix}\nu_2\\ l_2\end{pmatrix}_L,\ \begin{pmatrix}\nu_3\\ l_3\end{pmatrix}_L,\ \begin{pmatrix}\nu_4\\ l_4\end{pmatrix}_L\right\}};\quad \underset{\mathbf{1}}{l_{1R}},\quad \underset{\mathbf{1}}{l_{2R}},\quad \underset{\mathbf{2}}{\{l_{3R},l_{4R}\}};$$

$$\underset{\mathbf{1}}{\begin{pmatrix}u_1\\ d_1\end{pmatrix}_L},\quad \underset{\mathbf{3}}{\left\{\begin{pmatrix}u_2\\ d_2\end{pmatrix}_L,\ \begin{pmatrix}u_3\\ d_3\end{pmatrix}_L,\ \begin{pmatrix}u_4\\ d_4\end{pmatrix}_L\right\}};\quad \begin{matrix} d_{1R}, & d_{2R}, & \{d_{3R},d_{4R}\} \\ u_{1R}, & u_{2R}, & \{u_{3R},u_{4R}\} \\ \mathbf{1} & \mathbf{1} & \mathbf{2}\end{matrix};\quad \underset{\mathbf{1}}{\chi},\quad \underset{\mathbf{3}}{\{\phi_0,\phi_1,\phi_2\}}\ . \tag{3.1}$$

When we assign the LH doublets and RH singlets as $\mathbf{1}\oplus\mathbf{3}$, $\mathbf{1}\oplus\mathbf{1}\oplus\mathbf{2}$, respectively, there are some assignments of Higgs doublets to get S_4-singlet Yukawa couplings. Only when Higgs doublets are assigned to $\mathbf{1}\oplus\mathbf{3}$ do all fermions acquire masses. This is why we introduce four Higgs doublets. The general Higgs potential invariant under $SU(2)\times U(1)\times S_4$ by this assignment is[18]

$$\begin{aligned} V=&\mu_0^2(\bar\phi_0\phi_0+\bar\phi_1\phi_1+\bar\phi_2\phi_2)+\alpha(\bar\phi_0\phi_0+\bar\phi_1\phi_1+\bar\phi_2\phi_2)^2+\beta[\tfrac{1}{2}(\bar\phi_0\phi_1+\bar\phi_1\phi_0)^2+\tfrac{1}{6}(\bar\phi_0\phi_0+\bar\phi_1\phi_1-2\bar\phi_2\phi_2)^2]\\ &+\gamma[\tfrac{1}{2}(\bar\phi_0\phi_2+\bar\phi_2\phi_0)^2+\tfrac{1}{2}(\bar\phi_1\phi_2+\bar\phi_2\phi_1)^2+\tfrac{1}{2}(\bar\phi_0\phi_0-\bar\phi_1\phi_1)^2]\\ &+\delta[\tfrac{1}{2}(\bar\phi_0\phi_{1}-\bar\phi_1\phi_0)^2+\tfrac{1}{2}(\bar\phi_1\phi_2-\bar\phi_2\phi_1)^2+\tfrac{1}{2}(\bar\phi_2\phi_0-\bar\phi_0\phi_2)^2]+\mu_\chi^2\bar\chi\chi+a\bar\chi\chi\sum_{i=0}^{2}(\bar\phi_i\phi_i)+b(\bar\chi\chi)^2\ . \end{aligned} \tag{3.2}$$

As shown in Appendix A, the vacuum expectation values (VEV's) of neutral members of the four Higgs doublets are given by

$$\langle\chi^0\rangle=v,\quad \langle\phi_0^0\rangle=\frac{\xi}{\sqrt{2}}e^{i\phi},\quad \langle\phi_1^0\rangle=\frac{\xi}{\sqrt{2}}e^{-i\phi},\quad \langle\phi_2^0\rangle=\xi_2\ , \tag{3.3}$$

the potential is minimized when

$$\cos 2\phi=-\frac{\gamma+\delta}{\beta+\delta}\left[\frac{\xi_2}{\xi}\right]^2 . \tag{3.4}$$

The minimum is stable if

$$b>0,\quad |\gamma+\delta|>|\beta+\delta|,\quad |\xi_2|<|\xi|\ . \tag{3.5}$$

The Higgs-boson Yukawa coupling invariant under $SU(2)\times U(1)\times S_4$ is

$$\begin{aligned} L_Y=&A\overline{(\nu_1 l_1)}_L\chi l_{1R}+B\overline{(\nu_1 l_L)}_L\chi l_{2R}\\ &+C[\overline{(\nu_2 l_2)}_L\phi_0+\overline{(\nu_3 l_3)}_L\phi_1+\overline{(\nu_4 l_4)}_L\phi_2]l_{1R}+D[\overline{(\nu_2 l_2)}_L\phi_0+\overline{(\nu_3 l_3)}_L\phi_1+\overline{(\nu_4 l_4)}_L\phi_2]l_{2R}\\ &+E\left[\frac{1}{\sqrt{2}}[\overline{(\nu_2 l_2)}_L\phi_1+\overline{(\nu_3 l_3)}_L\phi_2]l_{3R}+\frac{1}{\sqrt{6}}[\overline{(\nu_2 l_2)}_L\phi_0+\overline{(\nu_3 l_3)}_L\phi_1-2\overline{(\nu_4 l_4)}_L\phi_2]l_{4R}\right]\\ &+\left[\text{similar terms for quarks with}\begin{Bmatrix}A,B\\ C,D,E\end{Bmatrix}\to\begin{Bmatrix}A_+,B_- & A_-,B_-\\ C_+,D_+,E_+; & C_-,D_-,E_-\end{Bmatrix};\right.\\ &\qquad\left. l_{iL}\to q_{iL},\quad l_{iR}\to(u_{iR},d_{iR});\quad (\chi,\phi_i)\to(\tilde\chi,\tilde\phi_i;\chi,\phi_i)\right]+\text{H.c.}\ , \end{aligned} \tag{3.6}$$

where

$$\tilde\chi=\begin{bmatrix}\bar\chi^0\\ -\chi^-\end{bmatrix},\quad \tilde\phi_i=\begin{bmatrix}\bar\phi_i^0\\ -\phi_i^-\end{bmatrix}.$$

At the tree level the mass matrices of leptons and quarks are obtained from (3.3) and (3.6):

$$\overline{(l_1 l_2 l_3 l_4)}_L M_l\begin{bmatrix}l_1\\ l_2\\ l_3\\ l_4\end{bmatrix}_R+\overline{(d_1 d_2 d_3 d_4)}_L M_-\begin{bmatrix}d_1\\ d_2\\ d_2\\ d_4\end{bmatrix}_R+\overline{(u_1 u_2 u_3 u_4)}_L M_+\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{bmatrix}_R+\text{H.c.}\ ,$$

where

$$M_l=\frac{\xi E}{2}\begin{bmatrix} \epsilon_{1l}\omega & \epsilon_{2l}\omega & 0 & 0 \\ \epsilon_{3l}e^{i\phi} & \epsilon_{4l}e^{i\phi} & e^{-i\phi} & \frac{1}{\sqrt{3}}e^{i\phi} \\ \epsilon_{3l}e^{-i\phi} & \epsilon_{4l}e^{-i\phi} & e^{i\phi} & \frac{1}{\sqrt{3}}e^{-i\phi} \\ \epsilon_{3l}\kappa & \epsilon_{4l}\kappa & 0 & \frac{-2}{\sqrt{3}}\kappa \end{bmatrix} . \tag{3.7}$$

Here $\kappa=\sqrt{2}\xi_2/\xi$, $\omega=\sqrt{2}v/\xi$, $\epsilon_{1l}=\sqrt{2}A/E$, $\epsilon_{2l}=\sqrt{2}B/E$, $\epsilon_{3l}=\sqrt{2}C/E$, and $\epsilon_{4l}=\sqrt{2}D/E$. $M_{\mp}$ have the same form as M_l with $E_{\mp}$ replacing E, and $\phi\to\pm\phi$. We find explicitly that the relation $M_l\propto M_-\propto M_+^*$ is broken. The Lagrangian (3.6) and the corresponding mass matrix (3.7) are the general form for the assignment (3.1). However, there are too many coupling constants in (3.6) to predict the fermion masses and the fermion mixings. Therefore, as a first approximation, we impose on the Lagrangian a reflection symmetry which reduces the number of parameters and increases the predictability. In addition, it explains naturally why the fermions of the first and second generations are much lighter than those of the third and fourth ones. In Sec. VI, we will reintroduce those Yukawa couplings dropped out by the reflection symmetry as a perturbation to give masses to the fermions of the first and second generations.

IV. THE MASSES OF THE FOURTH-GENERATION QUARKS

In this section we study the model with a reflection symmetry (R) and predict the masses of the fourth-generation quarks.

We require the Lagrangian (3.6) to be invariant under the sign changes of S_4-singlet fields.[19] The remaining couplings are then

$$L_Y=E\left[\frac{1}{\sqrt{2}}[\overline{(\nu_2 l_2)}_L\phi_1+\overline{(\nu_3 l_3)}_L\phi_0]l_{3R}+\frac{1}{\sqrt{6}}[\overline{(\nu_2 l_2)}_L\phi_0+\overline{(\nu_3 l_3)}_L\phi_1-2\overline{(\nu_4 l_4)}_L\phi_2]l_{4R}\right]$$
$$+[\text{similar terms for quarks with } E\to(E_+,E_-),\ \ l_{iL}\to q_{iL},\ \ l_{iR}\to(u_{iR},d_{iR}),\ \ \phi_i\to(\tilde{\phi}_i,\phi_i)]+\text{H.c.} \tag{4.1}$$

Of course the reduced Lagrangian also preserves $SU(2)\times U(1)\times S_4$ invariance. The corresponding charged-lepton mass matrix is then

$$M_l=\frac{\xi E}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-i\phi} & \frac{1}{\sqrt{3}}e^{i\phi} \\ 0 & 0 & e^{i\phi} & \frac{1}{\sqrt{3}}e^{-i\phi} \\ 0 & 0 & 0 & \frac{-2}{\sqrt{3}}\kappa \end{bmatrix} . \tag{4.2}$$

The quark mass matrices $M_{\mp}$ are gotten by the replacement of parameters as explained in Sec. III. We will find that the matrices are proportional to one another:

$$M_l\propto M_-\propto M_+^* . \tag{4.3}$$

The relation (4.3) is in agreement with (2.2) in Sec. II, but it should be noticed that the mass relation (2.3) is not obtained in this case because all the fermions are not assigned to one irreducible representation. When these mass matrices are diagonalized by biunitary transformations,

$$U_l M_l V_l^\dagger=\text{diag}(m_e,m_\mu,m_\tau,m_L) ,$$
$$U_- M_- V_-^\dagger=\text{diag}(m_d,m_s,m_b,m_v) , \tag{4.4}$$
$$U_+ M_+ V_+^\dagger=\text{diag}(m_u,m_c,m_t,m_a) ,$$

the fermion mass eigenvalues are given by

$$m_e=0,\quad m_\mu=0,\quad m_\tau=\mu_l\sqrt{\lambda_3},\quad m_L=\mu_l\sqrt{\lambda_4} ,$$
$$m_d=0,\quad m_s=0,\quad m_b=\mu_-\sqrt{\lambda_3},\quad m_v=\mu_-\sqrt{\lambda_4} , \tag{4.5}$$
$$m_u=0,\quad m_c=0,\quad m_t=\mu_+\sqrt{\lambda_3},\quad m_a=\mu_+\sqrt{\lambda_4} ,$$

where

$$\mu_l=\frac{\xi|E|}{2},\quad \mu_-=\frac{\xi|E_-|}{2},\quad \mu_+=\frac{\xi|E_+|}{2} ,$$

and

$$\lambda_4^3=\tfrac{2}{3}\{(2+\kappa^2)\mp[(2+\kappa^2)^2-3(2\kappa^2+\sin^2 2\phi)]^{1/2}\} .$$

It should be noticed that two fermions are massless, and two fermions are massive in each charge sector. We regard the massless fermions as those of the first and

TABLE I. The v-quark mass with respect to the mass of the fourth-generation charged lepton. Inputs in the calculations are $m_\tau=1.7842$ GeV, $2M_b=M(\Upsilon(4S))=10.575$ GeV, and $\alpha_s(Q^2)$ at $Q^2=34^2$ GeV^2; see Ref. 23.

$\alpha_s(Q^2=34^2\ \mathrm{GeV}^2)$	$\Lambda_{\overline{\mathrm{MS}}}$ (GeV)	m_L (GeV)	M_v (GeV)
0.151	0.36	30	74.3
		40	97.5
		50	120
0.156	0.42	30	73.7
		40	96.6
		50	119
0.161	0.48	30	73.2
		40	95.9
		50	118

second generations, the massive ones as those of the third and fourth generations. We remove the degeneracy of the mass for the first and second generations and give them masses in Sec. VI.

Thus we get a mass relation between the fermions of the third and fourth generations:

$$\frac{m_\tau}{m_L}=\frac{m_b}{m_v}=\frac{m_t}{m_a} . \tag{4.6}$$

The mass relation (4.6) is somewhat different from the result that was obtained by YSP. They got a mass relation between the masses of the second- and third-generation fermions and predicted the t-quark mass. However, in our model with R, the quark masses of the third generations are not related to those of the second generations, so the t-quark mass remains an unknown parameter, as well as the masses of the v and a quarks.

We shall estimate the masses of the fourth-generation quarks using (4.6). The mass relation (4.6) will hold at some high-energy scale $\mu_0 \gg 10^2$ GeV. Regarding (4.6) as the relation between effective masses, the naive prediction of quark masses will be reduced by the QCD renormalization-group effect. We give a detail of this subject below. To interpret the quark mass for a confined quark it seems that the most reasonable definition is due to Georgi and Politzer.[20] However, their prescription is

TABLE II. The a-quark mass with respect to the mass of the fourth-generation charged lepton. Inputs in the calculations are $M_t=35, 40, 50, 100, 180$ GeV, $m_\tau=1.7842$ GeV, and $\alpha_s(Q^2)$ at $Q^2=34^2$ GeV^2; see Ref. 23.

$\alpha_s(Q^2=34^2\ \mathrm{GeV}^2)$	$\Lambda_{\overline{\mathrm{MS}}}$ (GeV)	m_L (GeV)	M_a (GeV)
	$M_t=35$ GeV		
0.151	0.36	30	525
		40	690
		50	853
0.156	0.42	30	523
		40	686
		50	848
0.161	0.48	30	520
		40	683
		50	845
	$M_t=40$ GeV		
0.151	0.36	30	603
		40	792
		50	979
0.156	0.42	30	600
		40	788
		50	974
0.161	0.48	30	598
		40	785
		50	970
	$M_t=50$ GeV		
0.151	0.36	30	758
		40	996
		50	1232
0.156	0.42	30	755
		40	992
		50	1226
0.161	0.48	30	752
		40	988
		50	1221
	$M_t=100$ GeV		
0.151	0.36	30	1542
		40	2029
		50	2510
0.156	0.42	30	1536
		40	2021
		50	2501
0.161	0.48	30	1532
		40	2014
		50	2492
	$M_t=180$ GeV		
0.151	0.36	30	2810
		40	3700
		50	4580
0.156	0.42	30	2801
		40	2388
		50	4565
0.161	0.48	30	2794
		40	3677
		50	4551

gauge dependent, so it renders the result ambiguous. To remove this gauge dependence Kanaya, Sugawara, Pakvasa, and Tuan[21] (KSPT) introduced a gauge-independent quark propagator, and with the aid of resulting mass anomalous dimension, they discussed the threshold effects in a gauge-independent manner. After we review their prescription briefly, we shall apply KSPT's method to predict the masses of the v and a quarks.

First of all, the confined mass is defined at the threshold for pair production in accordance with Georgi and Politzer. The mass anomalous dimension γ_m derived from KSPT is given by

$$\gamma_m=\frac{\mu}{m}\frac{\partial m}{\partial\mu}=\frac{g^2}{3\pi^2}\left[-\tfrac{3}{2}+3\eta-3\eta^2\ln\left[1+\frac{1}{\eta}\right]\right], \tag{4.7}$$

where g is the renormalized gauge coupling constant and $\eta=(m/\mu)^2$; μ is a renormalization point and m is a running mass of a quark. Since g also depends on μ, one needs to know how g behaves and the value of g at some point. The μ dependence is given by a β function (which is gauge independent to this order[20]):

$$\beta=\mu\frac{\partial g}{\partial\mu}=-\frac{g^3}{16\pi^2}\left[11-\tfrac{2}{3}\sum_i\left[1-6\eta_i-\frac{16\eta_i^2}{\zeta_i}\ln\left|\frac{\zeta_i-1}{\zeta_i+1}\right|\right]\right], \tag{4.8}$$

where $\zeta_i=(1+4\eta_i)^{1/2}$ and the summation is over quark flavors. We may solve these equations for $m(\mu)$ and $g(\mu)$ using γ_m and β. Now we are ready to apply this procedure to the mass formula given by Eq. (4.6):

$$m_v(\mu)=\frac{m_L}{m_\tau}m_b(\mu),\quad m_a(\mu)=\frac{m_L}{m_\tau}m_t(\mu)\ . \tag{4.9}$$

Starting from the threshold $2M_b$, $2M_t$ we look for M_v, M_a at the threshold such that $m_v(2M_v)=(m_L/m_\tau)m_b(2M_v)=M_v$, $m_a(2M_a)=(m_L/m_\tau)m_a(2M_a)=M_a$ are satisfied. The results for the quark masses are shown in Tables I and II for different values of m_L. We have used[22,23] $m_\tau=1.7842$ GeV, $2M_b=10.575$ GeV and values of $\alpha(Q^2=34^2\ \mathrm{GeV}^2)=0.156\pm0.005$, $\Lambda_{\overline{\mathrm{MS}}}=0.42\pm0.06$ GeV, where $\overline{\mathrm{MS}}$ denotes the modified minimal subtraction scheme.

V. NUMERICAL ESTIMATION FOR THE 4×4 KM MATRIX

We will now calculate numerically all the KM matrix elements corresponding to the matrix of the form (4.2) as a first approximation. When we diagonalize the quark mass matrices, the following matrices U_- and V_- are obtained after absorbing some phases into quark fields:

$$U_-^\dagger=\begin{bmatrix} -\frac{e^{i(\phi+\varphi)}}{N_1}Wc_\theta-i\frac{e^{i\varphi}}{N_2}s_\theta & \frac{e^{i(\phi+\varphi)}}{N_1}Ws_\theta-i\frac{e^{i\varphi}}{N_2}c_\theta & 0 & 0 \\ \frac{\kappa^2}{N_1}c_\theta-\frac{ie^{-i\phi}}{N_2}s_\theta & -\frac{\kappa^2}{N_1}s_\theta-\frac{ie^{-i\phi}}{N_2}c_\theta & -\frac{X_3^*}{N_3} & -\frac{X_4^*}{N_4} \\ -\frac{\kappa^2e^{2i\phi}}{N_1}c_\theta+\frac{ie^{i\phi}}{N_2}s_\theta & \frac{\kappa^2e^{2i\phi}}{N_1}s_\theta+\frac{ie^{i\phi}}{N_2}c_\theta & -\frac{X_3}{N_3} & -\frac{X_4}{N_4} \\ -i\frac{\kappa s}{N_1}e^{i\phi}c_\theta-\frac{s}{\kappa N_2}s_\theta & i\frac{\kappa s}{N_1}e^{i\phi}s_\theta-\frac{s}{\kappa N_2}c_\theta & \frac{2\kappa}{N_3}(\lambda_3-2) & \frac{2\kappa}{N_4}(\lambda_4-2) \end{bmatrix}, \tag{5.1}$$

$$V_-^\dagger=\begin{bmatrix} c_\theta & -s_\theta & 0 & 0 \\ s_\theta & c_\theta & 0 & 0 \\ 0 & 0 & -\frac{2c}{K_3} & -\frac{2c}{K_4} \\ 0 & 0 & \frac{\sqrt{3}(2-\lambda_3)}{K_3} & \frac{\sqrt{3}(2-\lambda_4)}{K_4} \end{bmatrix}, \tag{5.2}$$

where

$$W=2\kappa^2+\sin^2 2\phi,\quad c_\theta=\cos\theta,\quad s_\theta=\sin\theta,\quad c=\cos 2\phi,\quad s=\sin 2\phi\ ,$$

$$N_1^2=(2\kappa^2+\sin^2 2\phi)(3\kappa^2+\sin^2 2\phi),\quad N_2^2=3+\frac{\sin^2 2\phi}{\kappa^2}\ ,$$

$$N_j^2=\tfrac{8}{3}\{[(2+\kappa^2)\lambda_j-(2\kappa^2+\sin^2 2\phi)](1+2\kappa^2)-3(\lambda_j-1)(2\kappa^2+\sin^2 2\phi)\}\ ,$$

$$K_j^2=4\cos^2 2\phi+3(2-\lambda_j)^2,\quad X_j=e^{i\phi}[e^{2i\phi}+(\lambda_j-1)e^{-2i\phi}]\quad (j=3,4)\ .$$

Here θ is a mixing parameter between two degenerate mass eigenstates belonging to the same mass eigenvalue of zero, and φ is a relative phase between eigenstates of S_4-singlet and -triplet LH fermions. It should be noticed that φ in (5.1) is not eliminated by the redefinition of quark phases. As shown in Appendix B, the KM matrix U_{KM} is symmetric and each element of the 4×4 KM matrix consists of four dimensionless independent real parameters. The full expressions of our KM matrix elements are given as follows with the help of (B4) and (5.1):

$$\begin{aligned}
U_{11}&=\frac{\cos^2\theta}{N_1^2}f_1(\phi,\varphi,\kappa)+\frac{2\cos\theta\sin\theta}{N_1N_2}g_1(\phi,\varphi,\kappa)-\frac{\sin^2\theta}{\kappa^2N_2^2}h_1(\phi,\varphi,\kappa)\\
&\quad+i\left[\frac{\cos^2\theta}{N_1^2}f_2(\phi,\varphi,\kappa)+\frac{2\cos\theta\sin\theta}{N_1N_2}g_2(\phi,\varphi,\kappa)-\frac{\sin^2\theta}{\kappa^2N_2^2}\sin 2\varphi\right]\ ,\\
U_{12}&=-\frac{\cos\theta\sin\theta}{N_1}f_1(\phi,\varphi,\kappa)+\frac{\cos^2\theta-\sin^2\theta}{N_1N_2}g_1(\phi,\varphi,\kappa)-\frac{\cos\theta\sin\theta}{\kappa^2N_2^2}h_1(\phi,\varphi,\kappa)\\
&\quad+i\left[-\frac{\cos\theta\sin\theta}{N_1^2}f_2(\phi,\varphi,\kappa)+\frac{\cos^2\theta-\sin^2\theta}{N_1N_2}g_2(\phi,\varphi,\kappa)-\frac{\cos\theta\sin\theta}{\kappa^2N_2^2}\sin 2\varphi\right]\ ,\\
U_{1j}&=-\frac{1}{N_1N_j}2\kappa^2(2\cos 2\phi+2-\lambda_j)\sin 2\phi\sin\phi\cos\theta+\frac{2}{N_2N_j}(2\cos 2\phi+2-\lambda_j)\sin 2\phi\sin\theta\\
&\quad+\frac{i}{N_1N_j}2\kappa^2(2\cos 2\phi+2-\lambda_j)\sin 2\phi\cos\phi\cos\theta\ ,\\
U_{22}&=\frac{\sin^2\theta}{N_1^2}f_1(\phi,\varphi,\kappa)-\frac{2\cos\theta\sin\theta}{N_1N_2}g_1(\phi,\varphi,\kappa)-\frac{\cos^2\theta}{\kappa^2N_2^2}h_1(\phi,\varphi,\kappa)\\
&\quad+i\left[\frac{\sin^2\theta}{N_1^2}f_2(\phi,\varphi,\kappa)-\frac{2\cos\theta\sin\theta}{N_1N_2}g_2(\phi,\varphi,\kappa)-\frac{\cos^2\theta}{\kappa^2N_2^2}\sin 2\varphi\right]\ ,\\
U_{2j}&=\frac{1}{N_1N_j}2\kappa^2(2\cos 2\phi+2-\lambda_j)\sin 2\phi\sin\phi\sin\theta+\frac{2}{N_2N_j}(2\cos 2\phi+2-\lambda_j)\sin 2\phi\cos\theta\\
&\quad-\frac{i}{N_1N_j}2\kappa^2(2\cos 2\phi+2-\lambda_j)\sin 2\phi\cos\phi\sin\theta\ ,\\
U_{34}&=\frac{1}{N_3N_4}\frac{16}{3}[(\kappa^2-\sin^2 2\phi)\cos 2\phi-\kappa^2\cos^2 2\phi]\ ,\\
U_{jj}&=\frac{1}{N_j^2}[(2\cos 2\phi+4\kappa^2)\lambda_j^2-16\kappa^2\lambda_j+8(2\kappa^2-\sin^2 2\phi\cos 2\phi)]\ ,
\end{aligned}\tag{5.3}$$

where $j=3,4$, and

$$f_1(\phi,\varphi,\kappa)=(2\kappa^2+\sin^2 2\phi)^2\cos 2(\phi+\varphi)+2\kappa^4\cos^2 2\phi-\kappa^2\sin^2 2\phi\cos 2\phi\ ,$$

$$f_2(\phi,\varphi,\kappa)=(2\kappa^2+\sin^2 2\phi)^2\sin 2(\phi+\varphi)+2\kappa^4\sin 2\phi\cos 2\phi-\kappa^2\sin^3 2\phi\ ,$$

$$g_1(\phi,\varphi,\kappa)=-(2\kappa^2+\sin^2 2\phi)\sin(\phi+2\varphi)+(2\kappa^2\cos 2\phi-\sin^2 2\phi)\sin\phi\ ,$$

$$g_2(\phi,\varphi,\kappa)=(2\kappa^2+\sin^2 2\phi)\cos(\phi+2\varphi)-(2\kappa^2\cos 2\phi-\sin^2 2\phi)\cos\phi\ ,$$

$$h_1(\phi,\varphi,\kappa)=2\kappa^2\cos 2\phi-\sin^2 2\phi+\kappa^2\cos 2\varphi\ .$$

Now we need to give the suitable values for the four independent real parameters to determine the magnitude of the KM matrix elements. We should remember that ϕ is related to the ratio of the fermion masses of the third and fourth generations. Taking account of this fact, it is more convenient to introduce the following mass ratio P defined by

$$P=\frac{4}{3}\frac{m_\tau^2 m_L^2}{(m_\tau^2+m_L^2)^2} \tag{5.4}$$

instead of ϕ. Of course it is also possible to parametrize P with the masses of the third- and fourth-generation quarks in the same charge sectors instead of with those of leptons. We will here give the suitable values of m_L to get the first parameter P.

We will notice that the second parameter θ is constrained by a ratio of the KM matrix elements $|U_{ub}|$ and $|U_{cb}|$. According to the recent experimental results from the ARGUS and CLEO Collaborations,[24]

$$0.07<|U_{ub}/U_{cb}|<0.19\ , \tag{5.5}$$

the corresponding upper and lower bounds on $\cos\theta$ are 0.998 and 0.983, respectively. In Fig. 1 the correlation of $|U_{ub}|$ and $|U_{cb}|$ is shown for different choices of the relevant parameter θ. It was found that the ratio $|U_{ub}/U_{cb}|$ has little effect on the quark masses of the fourth generation by numerical estimation.

Finally we determine the residual two parameters κ and φ so that the magnitude of the KM matrix element U_{ud} equals 0.974 20. Thus we obtain all the KM matrix elements numerically under the suitable choices of values for P, θ, κ, and φ. The magnitude of the individual KM matrix elements for $\cos\theta=0.99$ associated with $|U_{ub}/U_{cb}|\sim 0.14$ is shown in Table III for different values of m_L. The results do not contradict experimental data.[22]

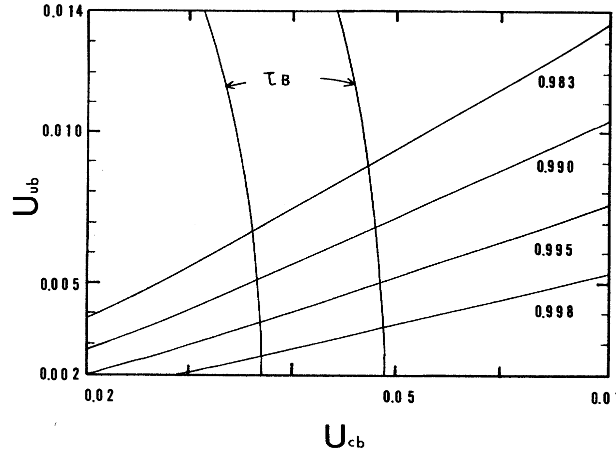

FIG. 1. The correlation of $|U_{ub}|$ and $|U_{cb}|$ for different values of $\cos\theta$ at 0.983, 0.990, 0.995, 0.998, and constraints set by B-hadron lifetime τ_B measurement, which is related to $|U_{ub}|^2+0.48|U_{cb}|^2=(0.90\pm0.24)\times10^{-3}$; see Ref. 23.

VI. THE LIGHT FERMION MASSES AND MIXINGS

Our work from Sec. III onwards has all been concerned with R in addition to the $SU(2)\times U(1)\times S_4$ symmetry. Since the fermions of the first and second generations became massless by R, the reason why the fermions of the third and fourth generations are much heavier than those of the first and second ones was naturally explained. Therefore R is a good symmetry as a first approximation. As a second approximation, we must give those massless fermions masses keeping the mass hierarchy $m_1,m_2\ll m_3,m_4$. We should break R in order to carry it out. R is broken if the Yukawa couplings, which were dropped out by the symmetry, were reintroduced. Since we want to preserve the mass hierarchy derived

TABLE III. The 4×4 Kobayashi-Maskawa matrix (no-perturbation calculation) for different values of the mass of the fourth-generation charged lepton. Inputs in the calculations are $m_\tau=1.7842$ GeV, $\cos\theta=0.99$, $\mathrm{sgn}(\cos2\phi)=\mathrm{sgn}(\sin\phi)=1$, and $\mathrm{sgn}(\sin2\phi)=\mathrm{sgn}(\cos\phi)=-1$.

m_L (GeV)	κ	$\cos2\varphi$	$\lvert U_{KM}\rvert$			
			0.974 20	0.225 58	0.006 76	0.000 23
30	2.3310×10^{-3}	−0.40	0.225 58	0.973 06	0.047 53	0.001 64
			0.006 76	0.047 53	0.996 46	0.068 87
			0.000 23	0.001 64	0.068 87	0.997 62
			0.974 20	0.225 60	0.005 88	0.000 15
40	1.5193×10^{-3}	−0.40	0.225 60	0.973 34	0.041 30	0.001 06
			0.005 88	0.041 30	0.997 79	0.051 58
			0.000 15	0.001 06	0.051 58	0.998 66
			0.974 20	0.225 60	0.005 99	0.000 12
50	1.2270×10^{-3}	0.00	0.225 60	0.973 32	0.041 68	0.000 86
			0.005 99	0.041 68	0.998 26	0.041 23
			0.000 12	0.000 86	0.041 23	0.991 48

from R, we assume that the couplings are sufficiently small to break R slightly. Then it is possible to deal with the couplings as the perturbation and to give masses to the fermions of the first and second generations.

We have already shown in Sec. III that the original Yukawa couplings make the mass matrices M_l, M_-, and M_+^* of the form (3.7). Let us form $M_lM_l^\dagger$ by the charged-lepton mass matrix[25] M_l:

$$M_lM_l^\dagger = M_l^2 + \delta M_l^2 , \tag{6.1}$$

where

$$M_l^2 = \frac{\xi^2 E^2}{4}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{4}{3} & \frac{1}{3}e^{2i\phi}+e^{-2i\phi} & -\frac{2}{3}\kappa e^{i\phi} \\ 0 & \frac{1}{3}e^{-2i\phi}+e^{2i\phi} & \frac{4}{3} & -\frac{2}{3}\kappa e^{-i\phi} \\ 0 & -\frac{2}{3}\kappa e^{-i\phi} & -\frac{2}{3}\kappa e^{i\phi} & \frac{4}{3}\kappa^2 \end{bmatrix},$$

and

$$\delta M_l^2 = \frac{\xi^2 E^2}{4}\begin{bmatrix} \tau_{1l}\omega^2 & \tau_{2l}\omega e^{i\phi} & \tau_{2l}\omega e^{i\phi} & \tau_{2l}\kappa\omega \\ \tau_{2l}\omega e^{i\phi} & \tau_{3l} & \tau_{3l}e^{2i\phi} & \tau_{3l}\kappa e^{i\phi} \\ \tau_{2l}\omega e^{-i\phi} & \tau_{3l}e^{-2i\phi} & \tau_{3l} & \tau_{3l}\kappa e^{-i\phi} \\ \tau_{2l}\kappa\omega & \tau_{3l}\kappa e^{-i\phi} & \tau_{3l}\kappa e^{-i\phi} & \tau_{3l}\kappa^2 \end{bmatrix}.$$

Here $\omega=\sqrt{2}v/\xi$, $\tau_{1l}=\epsilon_{1l}^2+\epsilon_{2l}^2$, $\tau_{2l}=\epsilon_{1l}\epsilon_{3l}+\epsilon_{2l}\epsilon_{4l}$, and $\tau_{3l}=\epsilon_{3l}^2+\epsilon_{4l}^2$. $M_\mp M_\mp^\dagger$ have the same form as $M_lM_l^\dagger$ with $E_\mp$ replacing E, $\tau_{i\mp}$ replacing τ_l, and $\phi\to\pm\phi$. We regard the first term as the unperturbed matrix and the second one as a perturbation on the first term. Each unperturbed matrix has two eigenvalues of zero as shown in Sec. III. The two eigenstates, for M_l^2 and M_-^2, belonging to the eigenvalue of zero, are

$$\left[-\frac{e^{i(\phi+\varphi)}}{N_1}W \;\; \frac{\kappa^2}{N_1} \;\; -\frac{\kappa^2 e^{2i\phi}}{N_1} \;\; -i\frac{\kappa s}{N_1}\right]^T ,$$

$$\left[-i\frac{e^{i\varphi}}{N_2} \;\; -i\frac{e^{-i\phi}}{N_2} \;\; \frac{ie^{i\phi}}{N_2} \;\; -\frac{s}{\kappa N_2}\right]^T ,$$

where a superscript T means taking the transpose of the matrix. For M_+^2 the eigenstates have the form $\phi\to-\phi$. We denote these eigenstates by $|1l\rangle$ and $|2l\rangle$, $|1-\rangle$ and $|2-\rangle$, and $|1+\rangle$ and $|2+\rangle$, respectively. The perturbations δM_l^2, δM_-^2, and δM_+^2 remove the degeneracy in the first-order calculation and make the massless fermion massive. We get the masses of the fermions as the solutions of a quadratic equation:

$$\begin{aligned} &W_l^2-(\langle 1l|\delta M_l^2|1l\rangle+\langle 1l|\delta M_l^2|2l\rangle)W_l+\langle 1l|\delta M_l^2|1l\rangle\langle 2l|\delta M_l^2|2l\rangle-|\langle 1l|\delta M_l^2|2l\rangle|^2=0 ,\\ &W_-^2-(\langle 1-|\delta M_-^2|1-\rangle+\langle 2-|\delta M_-^2|2-\rangle)W_-+\langle 1-|\delta M_-^2|1-\rangle\langle 2-|\delta M_-^2|2-\rangle-|\langle 1-|\delta M_-^2|2-\rangle|^2=0 ,\\ &W_+^2-(\langle 1+|\delta M_+^2|1+\rangle+\langle 2+|\delta M_+^2|2+\rangle)W_++\langle 1+|\delta M_+^2|1+\rangle\langle 2+|\delta M_+^2|2+\rangle-|\langle 1+|\delta M_+^2|2+\rangle|^2=0 . \end{aligned} \tag{6.2}$$

We shall identify these solutions as the squared fermion masses of the first and second generations:

$$\begin{aligned} m_\mu^{e2} &= \frac{\mu_l^2}{2}\left\{\frac{F_{1l}}{N_1^2}+\frac{F_{2l}}{N_2^2}\mp\left[\left[\frac{F_{1l}}{N_1^2}-\frac{F_{2l}}{N_2^2}\right]^2+\frac{4|F_{3l}|^2}{N_1^2N_2^2}\right]^{1/2}\right\} ,\\ m_s^{d2} &= \frac{\mu_-^2}{2}\left\{\frac{F_{1-}}{N_1^2}+\frac{F_{2-}}{N_2^2}\mp\left[\left[\frac{F_{1-}}{N_1^2}-\frac{F_{2-}}{N_2^2}\right]^2+\frac{4|F_{3-}|^2}{N_1^2N_2^2}\right]^{1/2}\right\} ,\\ m_c^{u2} &= \frac{\mu_+^2}{2}\left\{\frac{F_{1+}}{N_1^2}+\frac{F_{2+}}{N_2^2}\mp\left[\left[\frac{F_{1+}}{N_1^2}-\frac{F_{2+}}{N_2^2}\right]^2+\frac{4|F_{3+}|^2}{N_1^2N_2^2}\right]^{1/2}\right\} , \end{aligned} \tag{6.3}$$

where

$$\begin{aligned} F_{1j} &= \tau_{1j}\omega(2\kappa^2+\sin^2 2\phi)^2+6\tau_{2j}\omega\kappa^2(2\kappa^2+\sin^2 2\phi)\sin 2\phi\,\sin\varphi+9\tau_{3j}\kappa^4\sin^2 2\phi ,\\ F_{2j} &= \tau_{1j}\omega^2-6\tau_{2j}\omega\,\sin 2\phi\,\sin\varphi+9\tau_{3j}\sin^2 2\phi ,\\ F_{3j} &= i\tau_{1j}\omega^2(2\kappa^2+\sin^2 2\phi)e^{-i\phi}+3\tau_{2j}\omega[(2\kappa^2+\sin^2 2\phi)e^{-i\varphi}+\kappa^2\sin 2\varphi e^{i\varphi}]e^{-i\phi}\\ &\quad -i\kappa^2(2+\sin^2\phi)e^{-i\phi}-\kappa^2(2i\,\sin^2 2\phi-2\,\sin 4\phi)e^{-i\phi} . \end{aligned}$$

The corresponding eigenstates are given by

$$\begin{aligned} &|e\rangle=\cos\theta|1l\rangle+\sin\theta|2l\rangle, \quad |\mu\rangle=-\sin\theta|1l\rangle+\cos\theta|2l\rangle ,\\ &|d\rangle=\cos\theta_-|1-\rangle+\sin\theta_-|2-\rangle, \quad |s\rangle=-\sin\theta_-|1-\rangle+\cos\theta_-|2-\rangle ,\\ &|u\rangle=\cos\theta_+|1+\rangle+\sin\theta_+|2+\rangle, \quad |c\rangle=-\sin\theta_+|1+\rangle+\cos\theta_+|2+\rangle , \end{aligned}$$

where

$$\cos\theta_j=\frac{\sqrt{2}\mu_j^2}{m_{2j}^2-m_{1j}^2}Q(m_{2j}^2,m_{1j}^2,F_{1j},F_{2j}),\quad Q(m_{2j}^2,m_{1j}^2,F_{1j},F_{2j})=\frac{\dfrac{F_{1j}}{N_1^2}-\dfrac{F_{2j}}{N_{2j}}+\left[\dfrac{m_{2j}^2-m_{1j}^2}{\mu_j^2}\right]^{1/2}}{\dfrac{m_{2j}^2-m_{1j}^2}{\mu_j^2}-\left[\dfrac{F_{1j}}{N_1^2}-\dfrac{F_{2j}}{N_2^2}\right]}\ . \tag{6.4}$$

Here $j=1,-,+$; $m_{1l}^2=m_e^2$, $m_{2l}^2=m_\mu^2$; $m_{1-}^2=m_d^2$, $m_{2-}^2=m_s^2$; $m_{1+}^2=m_u^2$, $m_{2+}^2=m_c^2$.

If there were not δM^2's, the diagonalizing matrices of the d- and u-type quark mass matrices would be related to each other: $U_-^T=U_+^\dagger$, and $V_-^T=V_+^\dagger$, from which it follows that θ_- is equal to θ_+. The perturbation, however, causes the equality to break. For $\theta_-\neq\theta_+$, most of the KM matrix elements are modified as follows:

$$\begin{aligned}
U_{11}=&\frac{\cos\theta_+\cos\theta_-}{N_1^2}f_1(\phi,\varphi,\kappa)+\frac{\cos\theta_+\sin\theta_-+\cos\theta_-\sin\theta_+}{N_1N_2}g_1(\phi,\varphi,\kappa)-\frac{\sin\theta_+\sin\theta_-}{\kappa^2N_2^2}h_1(\phi,\varphi,\kappa)\\
&+i\left[\frac{\cos\theta_+\cos\theta_-}{N_1^2}f_2(\phi,\varphi,\kappa)+\frac{\cos\theta_+\sin\theta_-+\cos\theta_-\sin\theta_+}{N_1N_2}g_2(\phi,\varphi,\kappa)-\frac{\sin\theta_+\sin\theta_-}{\kappa^2N_2^2}\sin2\varphi\right],\\
U_{12}=&-\frac{\cos\theta_+\sin\theta_-}{N_1^2}f_1(\phi,\varphi,\kappa)+\frac{\cos\theta_+\cos\theta_--\sin\theta_+\sin\theta_-}{N_1N_2}g_1(\phi,\varphi,\kappa)-\frac{\cos\theta_-\sin\theta_+}{\kappa^2N_2^2}h_1(\phi,\varphi,\kappa)\\
&+i\left[-\frac{\cos\theta_+\sin\theta_-}{N_1^2}f_2(\phi,\varphi,\kappa)+\frac{\cos\theta_+\cos\theta_--\sin\theta_+\sin\theta_-}{N_1N_2}g_2(\phi,\varphi,\kappa)-\frac{\cos\theta_-\sin\theta_+}{\kappa^2N_2^2}\sin2\varphi\right],\\
U_{21}=&-\frac{\cos\theta_-\sin\theta_+}{N_1^2}f_1(\phi,\varphi,\kappa)+\frac{\cos\theta_-\cos\theta_+-\sin\theta_-\sin\theta_+}{N_1N_2}g_1(\phi,\varphi,\kappa)-\frac{\cos\theta_+\sin\theta_-}{\kappa^2N_2^2}h_1(\phi,\varphi,\kappa)\\
&+i\left[-\frac{\cos\theta_-\sin\theta_+}{N_1^2}f_2(\phi,\varphi,\kappa)+\frac{\cos\theta_-\cos\theta_+-\sin\theta_-\sin\theta_+}{N_1N_2}g_2(\phi,\varphi,\kappa)-\frac{\cos\theta_+\sin\theta_-}{\kappa^2N_2^2}\sin2\varphi\right],\\
U_{1j}=&-\frac{1}{N_1N_j}2\kappa^2(2\cos2\phi+2-\lambda_j)\sin2\phi\sin\phi\cos\theta_++\frac{2}{N_2N_j}(2\cos2\phi+2-\lambda_j)\sin2\phi\sin\theta_+\\
&+\frac{i}{N_1N_j}2\kappa^2(2\cos2\phi+2-\lambda_j)\sin2\phi\cos\phi\cos\theta_+\ ,\\
U_{j1}=&-\frac{1}{N_1N_j}2\kappa^2(2\cos2\phi+2-\lambda_j)\sin2\phi\sin\phi\cos\theta_-+\frac{2}{N_2N_j}(2\cos2\phi+2-\lambda_j)\sin2\phi\sin\theta_-\\
&+\frac{i}{N_1N_j}2\kappa^2(2\cos2\phi+2-\lambda_j)\sin2\phi\cos\phi\cos\theta_-\ ,\\
U_{22}=&\frac{\sin\theta_+\sin\theta_-}{N_1^2}f_1(\phi,\varphi,\kappa)-\frac{\cos\theta_+\sin\theta_--\cos\theta_-\sin\theta_+}{N_1N_2}g_1(\phi,\varphi,\kappa)-\frac{\cos\theta_+\cos\theta_-}{\kappa^2N_2^2}h_1(\phi,\varphi,\kappa)\\
&+i\left[\frac{\sin\theta_+\sin\theta_-}{N_1^2}f_2(\phi,\varphi,\kappa)-\frac{\cos\theta_+\sin\theta_--\cos\theta_-\sin\theta_+}{N_1N_2}g_2(\phi,\varphi,\kappa)-\frac{\cos\theta_+\cos\theta_-}{\kappa^2N_2^2}\sin2\varphi\right],\\
U_{2j}=&\frac{1}{N_1N_j}2\kappa^2(2\cos2\phi+2-\lambda_j)\sin2\phi\sin\phi\sin\theta_++\frac{2}{N_2N_j}(2\cos2\phi+2-\lambda_j)\sin2\phi\cos\theta_+\\
&-\frac{i}{N_1N_j}2\kappa^2(2\cos2\phi+2-\lambda_j)\sin2\phi\cos\phi\sin\theta_+\ ,\\
U_{j2}=&\frac{1}{N_1N_j}2\kappa^2(2\cos2\phi+2-\lambda_j)\sin2\phi\sin\phi\sin\theta_-+\frac{2}{N_2N_j}(2\cos2\phi+2-\lambda_j)\sin2\phi\cos\theta_-\\
&-\frac{i}{N_1N_j}2\kappa^2(2\cos2\phi+2-\lambda_j)\sin2\phi\cos\phi\sin\theta_-\ .
\end{aligned}\tag{6.5}$$

Only the elements U_{34} and $U_{jj}(j=3,4)$ are not changed by the perturbation. An example of the magnitudes of the KM matrix elements based on Eqs. (6.4) is given in Table IV. In the calculation the parameters θ_+ $(=\theta)$, ϕ, φ, and κ were taken to be equal to the values used in Sec. V, but θ_- was so determined that U_{11} would be equal to 0.9760 which is the maximum value given by experiments assuming more than four generations.[22] We will notice

TABLE IV. The 4×4 Kobayashi-Maskawa matrix (perturbation calculation) for different values of the mass of the fourth-generation charged lepton. Inputs in the calculations are the same for the parameter in Table III, except for θ_-.

m_L (GeV)	$\cos\theta_-$	$\cos2\varphi$	$\lvert U_{KM}\rvert$			
30	0.9913	-0.40	0.976 00	0.217 65	0.006 76	0.000 23
			0.217 66	0.974 86	0.047 53	0.001 64
			0.006 31	0.047 60	0.996 46	0.068 87
			0.000 21	0.001 64	0.068 87	0.997 62
40	0.9913	-0.40	0.976 00	0.217 70	0.005 88	0.000 15
			0.217 71	0.975 13	0.041 30	0.001 06
			0.005 49	0.041 36	0.997 79	0.051 58
			0.000 14	0.001 06	0.051 58	0.998 66
50	0.9915	0.00	0.976 00	0.217 65	0.005 99	0.000 12
			0.217 66	0.973 51	0.041 68	0.000 86
			0.005 52	0.041 75	0.998 26	0.041 23
			0.000 11	0.000 86	0.041 23	0.991 48

that $\cos\theta_-$ and $\cos\theta_+$ are nearly equal to 0.99 in the case without perturbation. It is easy to see that the KM matrix becomes symmetric and the numerical values of the elements shown in Table III are unchanged only when $\theta_+=\theta_-$.

Note that the fermion masses of the first and second generations contain ten unknown parameters τ_{ij} ($i=l,-,+$; $j=1,2,3$), and ω as shown in Eq. (6.3). If the value of ω is given, each parameter τ_{ij} is bounded from both above and below. Especially the allowed region of τ_{i2} is usually much narrower than those of τ_{i1} and τ_{i3}. So, as a typical value, we fix τ_{i2} at the value where $|\tau_{i2}|$ becomes maximum. Then there remain six undetermined parameters. They can be determined by three quadratic equations (6.2) for fermion masses of the first and second generations and three equations (6.4) for mixing angles. In Tables V–VII we have listed examples of the values τ_{ij}. In the derivation of those values we gave the fermion masses of the first and second generations, $\cos\theta_l=\cos\theta_-=\cos\theta_+=0.990$, and $\omega=0.5$ as input parameters.

VII. CONCLUSION

We have presented a four-generation model based on the $SU(2)\times U(1)\times S_4$ symmetry. The assignment of left-handed fermions, right-handed fermions, and Higgs scalar doublets to representations of S_4 are $1\oplus3$, $1\oplus1\oplus2$, and $1\oplus3$, respectively. There were too many Yukawa couplings in this assignment, so we were not able to predict the fermion masses and mixings exactly. Therefore, as a first approximation, we imposed on the $SU(2)\times U(1)\times S_4$ symmetry an additional reflection symmetry (R) which means an invariance of the Yukawa couplings between fermions and Higgs bosons under the sign changes of S_4-singlet fields. In this case the fermions of the first and second generations became massless; those of the third and fourth ones became massive. Thus the reason why the fermions of the third and fourth generations are much heavier than those of the first and second ones is naturally explained by R. The mass relation for the fermions of the third and fourth generations,

$$\frac{m_L}{m_\tau}=\frac{m_v}{m_b}=\frac{m_a}{m_t}\ ,$$

is particular in our assignment with reflection symmetry. By taking account of KSPT's renormalization-group effect, the estimation of M_v and M_a (defined at threshold for pair production) has been done for suitable choice of m_L and M_t. If m_L and M_t are in the energy range of

TABLE V. The perturbation in the charged-lepton mass matrix. Inputs in the calculation are $m_e=5.11\times10^{-4}$ GeV, $m_\mu=0.1056$ GeV, $\omega=0.5$, and $\cos\theta=0.990$.

m_L (GeV)	τ_{1l}	τ_{2l}	τ_{3l}
30	1.19×10^{-4}	3.37×10^{-7}	1.29×10^{-2}
40	6.75×10^{-5}	5.68×10^{-7}	1.73×10^{-2}
50	4.33×10^{-5}	-1.94×10^{-7}	1.70×10^{-2}

TABLE VI. The perturbation in the d-type quark mass matrix. Inputs in the calculation are $m_d=0.01$ GeV, $m_s=0.1$ GeV, $\omega=0.5$, and $\cos\theta_-=0.990$.

m_L (GeV)	M_v (GeV)	τ_{1-}	τ_{2-}	τ_{3-}
30	74.3	1.74×10^{-5}	2.53×10^{-6}	2.82×10^{-3}
40	97.5	1.01×10^{-5}	3.73×10^{-6}	3.89×10^{-3}
50	120	6.74×10^{-6}	-1.24×10^{-6}	3.97×10^{-3}
30	73.7	1.77×10^{-5}	2.49×10^{-6}	2.87×10^{-3}
40	96.6	1.03×10^{-5}	3.66×10^{-6}	3.96×10^{-3}
50	119	6.86×10^{-6}	-1.21×10^{-6}	4.03×10^{-3}
30	73.2	1.79×10^{-5}	2.46×10^{-6}	2.91×10^{-3}
40	95.9	1.05×10^{-5}	3.61×10^{-6}	4.02×10^{-3}
50	118	6.97×10^{-6}	-1.19×10^{-6}	4.10×10^{-3}

KEK TRISTAN, for example, $m_L = 30$ GeV (Ref. 26) and $M_t = 35$ GeV (Ref. 27), then the masses of the v and a quarks are found to be $M_v \sim 73 \pm 2$ GeV, $M_a \sim 523 \pm 3$ GeV. The former gives an expectation that the v quark can be produced in the decay such as $W^- \to v\bar{c}, v\bar{u}$. The latter gives expectation that some mesons containing the a quark, for example, $a\bar{u}$, $a\bar{c}$, $\bar{a}d$, $\bar{a}s$, and $\bar{a}b$, etc., can be detected at the Superconducting Super Collider (SSC).

As a second approximation, we broke R slightly to make massless fermions of the first and second generations massive. We assumed the couplings dropped out by R to be sufficiently small since we needed to keep the mass hierarchy $m_1, m_2 \ll m_3, m_4$ derived from R. As a result, we were able to give masses to the fermions of the first and second generations keeping the mass hierarchy.

We predicted the values of individual 4×4 KM matrix elements. Good agreement with recent experimental data has been found. The hierarchy of the fourth column in the 4×4 KM matrix $(U_{14}, U_{24}, U_{34}, U_{44})$ is obtained in the form $(\lambda^4, \lambda^3, \lambda^2, 1)$, where λ is the Cabibbo-angle parameter $\lambda = U_{12}$. The hierarchy is naturally caused by the structure of quark mass matrices M_- and M_+ of the form (3.7). Recently some quark mass matrices (Fritzsch type,[28] Stech type,[29] and Gronau-Jonson-Schechter type[30]) with four generations of quarks have been investigated. However, many examples have not yet determined the hierarchy of the KM matrix. Although our quark mass matrices do not belong to any examples above, they are also hopeful candidates to provide the phenomenological KM matrix.[31] There remain the important problems of B_d^0-$\bar{B}_d^0$ mixing, B_s^0-$\bar{B}_s^0$ mixing, and CP violation in the four-generation case. These subjects will be discussed in separate papers in detail.

This model has two serious problems. One is flavor changing couplings of neutral Higgs bosons, which also arose in YSP's model. YSP have avoided this difficulty

TABLE VII. The perturbation in the u-type quark mass matrix. Inputs in the calculation are $m_u = 7.5\times10^{-3}$ GeV, $m_c = 1.5$ GeV, $\omega = 0.5$, and $\cos\theta_+ = 0.990$.

m_L (GeV)	M_a (GeV)	τ_{1+}	τ_{2+}	τ_{3+}
		$M_t = 35$ GeV		
30	525	7.84×10^{-5}	5.67×10^{-7}	8.43×10^{-3}
40	690	4.57×10^{-5}	8.39×10^{-7}	1.16×10^{-2}
50	853	3.00×10^{-5}	-2.81×10^{-7}	1.17×10^{-2}
30	523	7.90×10^{-5}	5.63×10^{-7}	8.49×10^{-3}
40	686	4.62×10^{-5}	8.29×10^{-7}	1.17×10^{-2}
50	848	3.07×10^{-5}	-2.74×10^{-7}	1.19×10^{-2}
30	520	7.99×10^{-5}	5.57×10^{-7}	8.59×10^{-3}
40	683	4.66×10^{-5}	8.22×10^{-7}	1.18×10^{-2}
50	845	3.06×10^{-5}	-2.76×10^{-7}	1.19×10^{-2}
		$M_t = 40$ GeV		
30	603	5.94×10^{-5}	7.49×10^{-7}	6.39×10^{-3}
40	792	3.47×10^{-5}	1.10×10^{-6}	8.81×10^{-3}
50	979	2.28×10^{-5}	-3.70×10^{-7}	8.87×10^{-3}
30	600	6.00×10^{-5}	7.41×10^{-7}	6.45×10^{-3}
40	788	3.50×10^{-5}	1.09×10^{-6}	8.89×10^{-3}
50	974	2.30×10^{-5}	-3.66×10^{-7}	8.97×10^{-3}
30	598	6.04×10^{-5}	7.36×10^{-7}	6.50×10^{-3}
40	785	3.53×10^{-5}	1.08×10^{-6}	8.96×10^{-3}
50	970	2.32×10^{-5}	-3.63×10^{-7}	9.04×10^{-3}
		$M_t = 50$ GeV		
30	758	3.76×10^{-5}	1.18×10^{-6}	4.04×10^{-3}
40	996	2.19×10^{-5}	1.74×10^{-6}	5.56×10^{-3}
50	1232	1.44×10^{-5}	-5.86×10^{-7}	5.60×10^{-3}
30	755	3.79×10^{-5}	1.17×10^{-6}	4.07×10^{-3}
40	992	2.21×10^{-5}	1.73×10^{-6}	5.61×10^{-3}
50	1226	1.45×10^{-5}	-5.80×10^{-7}	5.66×10^{-3}
30	752	3.82×10^{-5}	1.16×10^{-6}	4.10×10^{-3}
40	988	2.23×10^{-4}	1.72×10^{-6}	5.65×10^{-3}
50	1221	1.46×10^{-5}	-5.76×10^{-7}	5.71×10^{-3}
		$M_t = 100$ GeV		
30	1542	9.09×10^{-6}	4.90×10^{-6}	9.70×10^{-4}
40	2029	5.28×10^{-6}	7.25×10^{-6}	1.33×10^{-3}
50	2510	3.46×10^{-6}	-2.43×10^{-6}	1.36×10^{-3}
30	1536	9.16×10^{-6}	4.86×10^{-6}	9.80×10^{-4}
40	2021	5.33×10^{-6}	7.20×10^{-6}	1.34×10^{-3}
50	2501	3.49×10^{-6}	-2.41×10^{-6}	1.37×10^{-3}
30	1532	9.21×10^{-6}	4.83×10^{-6}	9.90×10^{-4}
40	2014	5.36×10^{-6}	7.15×10^{-6}	1.35×10^{-3}
50	2492	3.52×10^{-5}	-2.40×10^{-6}	1.38×10^{-3}
		$M_t = 180$ GeV		
30	2810	2.74×10^{-6}	1.62×10^{-5}	2.61×10^{-4}
40	3700	1.59×10^{-6}	2.41×10^{-5}	3.37×10^{-4}
50	4580	1.04×10^{-6}	-8.10×10^{-6}	4.30×10^{-4}
30	2801	2.76×10^{-6}	1.61×10^{-5}	2.63×10^{-3}
40	3688	1.60×10^{-6}	2.39×10^{-5}	3.40×10^{-4}
50	4565	1.04×10^{-6}	-8.05×10^{-6}	4.33×10^{-4}
30	2794	2.77×10^{-6}	1.60×10^{-5}	2.65×10^{-4}
40	3677	1.61×10^{-6}	2.38×10^{-5}	3.43×10^{-4}
50	4551	1.05×10^{-6}	-8.00×10^{-6}	4.53×10^{-4}

under an assumption that the neutral members have superheavy masses more than a few TeV (Refs. 8 and 18), but it will require further investigation because the tree-level unitarity violation[32] may arise in such a high-energy region. The other is the large mass difference of M_a and M_v which leads to a large deviation of the ρ parameter [33] of the standard model from $\rho=1$. If the t-quark mass M_t were lighter than 25 GeV, the a-quark mass M_a could be predicted to be lighter than 370±3 GeV. Unfortunately recent experimental data on the lower bound of M_t (Ref. 27) exclude that region, so we cannot remedy the disturbing fact. From this point, this model is yet regarded as a toy model. A possible way out would be to construct a model with another assignment of the representation of S_4. This subject will also be discussed in a forthcoming paper.

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APPENDIX A: THE MINIMUM OF THE HIGGS POTENTIAL

We denote the VEV's of the Higgs potential by

$$\langle \phi_0^0 \rangle = x e^{-i\vartheta_0}, \quad \langle \phi_1^0 \rangle = y e^{-i\vartheta_1}, \quad \langle \phi_2^0 \rangle = z e^{-i\vartheta_2}, \quad \langle \chi^0 \rangle = v e^{-i\vartheta_3} . \tag{A1}$$

In terms of the VEV's we have

$$\begin{aligned} V = {} & \mu_0^2(x^2+y^2+z^2) + \alpha(x^2+y^2+z^2)^2 + \beta[2x^2y^2\cos^2(\vartheta_0-\vartheta_1) + \tfrac{1}{6}(x^2+y^2-2z^2)^2] \\ & + \gamma[2x^2z^2\cos^2(\vartheta_0-\vartheta_2) + 2y^2z^2\cos^2(\vartheta_1-\vartheta_2) + \tfrac{1}{2}(x^2-y^2)^2] \\ & - \delta[2y^2z^2\sin^2(\vartheta_1-\vartheta_2) + 2z^2x^2\sin^2(\vartheta_2-\vartheta_0) + 2x^2y^2\sin^2(\vartheta_0-\vartheta_1)] + \mu_\chi^2 v^2 + a v^2(x^2+y^2+z^2) . \end{aligned} \tag{A2}$$

We can immediately obtain the minimization conditions

$$\begin{aligned} \frac{\partial V}{\partial x} = {} & (4\alpha + \tfrac{2}{3}\beta + 2\gamma)x^3 + \{2\mu_0^2 + [4\alpha + 4\beta\cos^2(\vartheta_0-\vartheta_1) + \tfrac{2}{3}\beta - 2\gamma - 4\delta\sin^2(\vartheta_0-\vartheta_1)]y^2 \\ & + [4\alpha - \tfrac{4}{3}\beta + 4\gamma\cos^2(\vartheta_0-\vartheta_2) - 4\delta\sin^2(\vartheta_2-\vartheta_0)]z^2 + 2av^2\}x = 0 , \end{aligned} \tag{A3a}$$

$$\begin{aligned} \frac{\partial V}{\partial y} = {} & (4\alpha + \tfrac{2}{3}\beta + 2\gamma)y^3 + \{2\mu_0^2 + [4\alpha + 4\beta\cos^2(\vartheta_0-\vartheta_1) + \tfrac{2}{3}\beta - 2\gamma - 4\delta\sin^2(\vartheta_0-\vartheta_1)]x^2 \\ & + [4\alpha - \tfrac{4}{3}\beta + 4\gamma\cos^2(\vartheta_1-\vartheta_2) - 4\delta\sin^2(\vartheta_1-\vartheta_2)]z^2 + 2av^2\}y = 0 , \end{aligned} \tag{A3b}$$

$$\begin{aligned} \frac{\partial V}{\partial z} = {} & (4\alpha + \tfrac{8}{3}\beta)z^3 + \{2\mu_0^2 + [4\alpha - \tfrac{4}{3}\beta + 4\gamma\cos^2(\vartheta_0-\vartheta_2) - 4\delta\sin^2(\vartheta_2-\vartheta_0)\}x^2 \\ & + [4\alpha - \tfrac{4}{3}\beta + 4\gamma\cos^2(\vartheta_1-\vartheta_2) - 4\delta\sin^2(\vartheta_1-\vartheta_2)]y^2 + 2av^2\}x = 0 , \end{aligned} \tag{A3c}$$

$$\frac{\partial V}{\partial v} = 2\mu_\chi^2 + 2a(x^2+y^2+z^2)v = 0 , \tag{A3d}$$

$$\frac{\partial V}{\partial \vartheta_0} = -2\beta x^2y^2\sin2(\vartheta_0-\vartheta_1) - 2\gamma x^2z^2\sin2(\vartheta_0-\vartheta_2) + 2\delta[x^2z^2\sin2(\vartheta_2-\vartheta_0) - x^2y^2\sin2(\vartheta_0-\vartheta_1)] = 0 , \tag{A3e}$$

$$\frac{\partial V}{\partial \vartheta_1} = -2\beta x^2y^2\sin2(\vartheta_1-\vartheta_0) - 2\gamma y^2z^2\sin2(\vartheta_1-\vartheta_2) + 2\delta[-y^2z^2\sin2(\vartheta_1-\vartheta_2) - x^2y^2\sin2(\vartheta_1-\vartheta_0)] = 0 , \tag{A3f}$$

$$\frac{\partial V}{\partial \vartheta_2} = 2\gamma[x^2z^2\sin2(\vartheta_0-\vartheta_2) + y^2z^2\sin2(\vartheta_1-\vartheta_2)] + 2\delta[y^2z^2\sin2(\vartheta_1-\vartheta_2) - z^2x^2\sin2(\vartheta_2-\vartheta_0)] = 0 , \tag{A3g}$$

$$\frac{\partial V}{\partial \vartheta_3} = 0 . \tag{A3h}$$

The Eq. (A3h) shows that ϑ_3 is an arbitrary phase, so we may set $\vartheta_3=0$. The Eq. (A3g) is not independent of (A3e) and (A3f); this reflects the fact that V depends only on two angles; we can set $\vartheta_2=0$. When we substitute Eqs. (A3e)–(A3g) into Eqs. (A3a)–(A3c), we get

$$(4\alpha-\tfrac{4}{3}\beta+2\gamma-2\delta)x^2+(4\alpha+\tfrac{8}{3}\beta-2\gamma-2\delta)y^2 +(4\alpha-\tfrac{4}{3}\beta+2\gamma-2\delta)z^2+2\mu_0^2+2av^2=0\ , \quad \text{(A4a)}$$

$$(4\alpha+\tfrac{8}{3}\beta-2\gamma-2\delta)x^2+(4\alpha-\tfrac{4}{3}\beta+2\gamma-2\delta)y^2 +(4\alpha-\tfrac{4}{3}\beta+2\gamma-2\delta)z^2+2\mu_0^2+2av^2=0\ , \quad \text{(A4b)}$$

$$(4\alpha-\tfrac{4}{3}\beta+2\gamma-2\delta)x^2+(4\alpha-\tfrac{4}{3}\beta+2\gamma-2\delta)y^2 +\left[4\alpha+\tfrac{8}{3}\beta-2\frac{(\gamma+\delta)^2}{\beta+\delta}\right]z^2+2\mu_0^2+2av^2=0\ , \quad \text{(A4c)}$$

which implies $x^2=y^2$. It should be noticed that $x^2=y^2=z^2$ is not necessarily satisfied. This relation immediately leads to $\sin2\vartheta_0+\sin2\vartheta_1=0$ or $\vartheta_1=-\vartheta_0+2m\pi$ ($m=0,\pm1,\pm2,\ldots$) with the help of Eq. (A3g). Thus there remains only one phase ϑ_0. Substituting $x=y\equiv(1/\sqrt{2})\xi$, $z\equiv\xi_2$, $\vartheta_0\equiv-\phi$, and $\vartheta_2=\vartheta_3=0$ into (A1), the VEV's of the Higgs potential are represented as (3.3). The minimization condition (3.4) can also be obtained if we rewrite Eqs. (A3e) and (A3f) as

$$\sin4\phi=-2\frac{\gamma+\delta}{\beta+\delta}\left[\frac{\xi_2}{\xi}\right]^2\sin2\phi\ , \quad \text{(A5)}$$

and divide it by $\sin2\phi\neq0$.

APPENDIX B: THE SYMMETRIC KM MATRIX

Let us prove that the $N\times N$ KM matrix becomes symmetric in a model which has only one Yukawa coupling constant in each charge sector. We then have quark mass matrices M_- and M_+ which are related to each other:

$$M_-\propto M_+^* \quad \text{(B1)}$$

They can be diagonalized by the biunitary transformation:

$$U_-M_-V_-^\dagger=D_-\ , \quad \text{(B2)}$$

$$U_+M_+V_+^\dagger=D_+\ , \quad \text{(B3)}$$

where D_- and D_+ means diagonal matrices whose diagonal elements are real mass eigenvalues. Taking the complex conjugate of (B3),

$$U_+^*M_+^*V_+^T=D_+^*=D_+\ ,$$

with the help of (B1) and (B2), we get

$$U_+^*M_+^*V_+^T\propto U_-M_-V_-^\dagger\ .$$

The following relation is induced immediately:

$$U_+=U_-^*\ , \quad \text{(B4)}$$

$$V_+=V_-^*\ . \quad \text{(B5)}$$

On the other hand the KM matrix is defined by the matrices U_- and U_+:

$$U_{\rm KM}=U_+U_-^\dagger\ . \quad \text{(B6)}$$

On substituting (B4) in (B6) we obtain

$$U_{\rm KM}=U_{\rm KM}^T\ . \quad \text{(B7)}$$

Here we show that the $N\times N$-symmetric unitary matrix has N dimensionless independent real parameters. The proof is the following: Let U_{jk} be an element of unitary matrix. This element consists of a real and an imaginary part:

$$U_{jk}=x_{jk}+iy_{jk}\quad (j,k=1\text{–}N)\ . \quad \text{(B8)}$$

When the unitary matrix is symmetric, there are $N(N+1)/2$ elements for x's and y's, respectively. On the other hand, the unitary condition reads

$$\sum_{k=1}^{N}[x_{jk}x_{lk}+y_{jk}y_{lk}+i(y_{jk}x_{lk}-x_{jk}y_{lk})]=\delta_{jl}\ . \quad \text{(B9)}$$

For $j=l$ only the real part has N conditions, and for $j\neq l$ each real and imaginary part has $N(N-1)/2$ conditions in Eq. (B9). Therefore we arrive at the number of independent real parameters

$$2\times N(N+1)/2-N-2\times N(N-1)/2=N\ .$$

We thus find that if the mass matrices satisfying (B1) in any model with a family symmetry, one can always get a symmetric KM matrix that has N independent real parameters. Such a proportional relation (B1) will not be derived when we assign u-type quarks to the representation different from that of d-type quarks, then the KM matrix does not become symmetric.

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