Corrections to Higgs-boson-mass sum rules from the sfermion sector of a supersymmetric model

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We consider the minimal supersymmetric extension of the standard model, and compute the corrections to Higgs-boson-mass sum rules that are possibly sensitive to the large mass of the sfermion sector and/or to large trilinear Higgs-boson–sfermion–sfermion Yukawa couplings. We demonstrate that when the parameters are chosen so that corrections to the electroweak ρ are within experimental limits, large corrections to the mass sum rules arise only in certain extreme regions of parameter space, corresponding to very large ratios for squark masses.

In two previous papers we have explored corrections to Higgs-boson-mass sum rules of the minimal supersymmetric extension of the standard model[1,2] (MSSM). In the first paper, we demonstrated that the sum rules predicted at the tree level are screened against large one-loop corrections that derive from the Higgsino-gaugino sector and which grow quadratically with the possibly large mass scale of this sector. The leading dependence is logarithmic, generally implying quite small corrections to the mass sum rules from this source. In the second paper, we gave a general proof that this screening will take place for one-loop corrections from any heavy sector, so long as there are no large trilinear Yukawa couplings. However, we noted there that the sfermion sector gives rise to Higgs-boson–sfermion–sfermion trilinear couplings that could, in principle, be large, thereby leading to possibly large corrections to the natural relations (such as the Higgs-boson-mass sum rules) predicted by supersymmetry (SUSY). However, at the same time we noted that large trilinear couplings of this type will also impact other natural predictions of the standard model (SM), in particular leading to corrections to the tree-level prediction of

$$\rho=\frac{m_W^2}{m_Z^2\cos^2\theta_W}=1 \ . \tag{1}$$

In particular, it is possible that the very strong experimental limit[3]

$$\frac{\delta\rho}{\rho}\lesssim 0.01 \tag{2}$$

restricts the magnitude of the trilinear Higgs-boson–sfermion–sfermion couplings to such an extent that large corrections to the Higgs-boson-mass sum rules are not possible. In this paper, we explore this issue quantitatively.

We have computed the corrections to the simplest of the mass sum rules, namely,

$$m_W^2=m_{H^+}^2-m_{A^0}^2 \ , \tag{3}$$

using the formalism and approach described in Ref. 1. The sfermion sector is incorporated using the MSSM formalism developed in Ref. 4. We review a portion of the notation used there, using the squark sector as an example. The masses and interactions of the squarks are determined by the F- and D-term superpotential contributions and by the most general soft-supersymmetry-breaking potential involving the squark fields:

$$\begin{aligned}V_{\text{soft}}=&\widetilde{M}_Q^2\widetilde{Q}^{i*}\widetilde{Q}^i+\widetilde{M}_U^2\widetilde{U}^*\widetilde{U}+\widetilde{M}_D^2\widetilde{D}^*\widetilde{D}\\&+\frac{g}{\sqrt{2}m_W}\left[\epsilon^{ij}\frac{A_d^2}{\cos\beta}H_1^i\widetilde{Q}^j\widetilde{D}\right.\\&\left.-\epsilon^{ij}\frac{A_u^2}{\sin\beta}H_2^i\widetilde{Q}^j\widetilde{U}+\text{H.c.}\right] ,\end{aligned} \tag{4}$$

where the first terms appearing in Eq. (4) are the bilinear mass-squared terms, and the second terms are the crucial trilinear terms describing the interaction of the two Higgs fields with the squark fields. The $\widetilde{Q}$ field is the spin-0 partner of the usual left-handed doublet field, while the $\widetilde{U}$ and $\widetilde{D}$ are the fields of the spin-0 partners of the right-handed quarks. In more conventional notation,

$$\widetilde{Q}^i=\begin{bmatrix}\widetilde{u}_L\\ \widetilde{d}_L\end{bmatrix},\quad \widetilde{U}^*=\widetilde{u}_R,\quad \widetilde{D}^*=\widetilde{d}_R \ . \tag{5}$$

(See Ref. 4 for details; note, however, that we use here a slightly different notation for the trilinear coupling—we have replaced $m_6m_qA_q$ by A_q^2.) An important point to note regarding the squark sector mass eigenstates is that the trilinear A terms cause mixing between the left-handed and right-handed eigenstates according to mass matrices of the form

$$\mathcal{M}_{U,D}^2=\begin{bmatrix}\widetilde{M}_Q^2 & A_{u,d}^2\\ A_{u,d}^2 & \widetilde{M}_{U,D}^2\end{bmatrix} . \tag{6}$$

In Eq. (6) we neglect all terms proportional to quark masses and other SUSY parameters (in particular we drop terms involving the μ parameters of SUSY, see Ref. 4). Note, in particular, that $A_{u,d}$ cannot increase without bound when $\widetilde{M}_Q$, $\widetilde{M}_U$, and $\widetilde{M}_D$ are fixed, without violating the requirement of a positive-definite determinant for these mass-squared matrices.

In our calculations of the corrections to the squared masses entering into Eq. (3), we have retained only the leading corrections from each type of diagram. By "leading" we mean those terms which grow most rapidly with the A and $\tilde{M}$ parameters. (This is the same approximation used above in the squark mass matrix.) In the case of the Higgs-boson mass-squared corrections, the two basic diagram types are (a) diagrams with two trilinear Higgs-boson–squark–squark vertices, yielding leading corrections proportional to A^4/m_W^2 and (b) diagrams with a quartic Higgs-boson–Higgs-boson–squark–squark vertex, resulting in leading terms proportional to $\tilde{M}^2$. For the W mass squared both types of diagram yield leading corrections proportional to $\tilde{M}^2$. We have examined other terms and found that they are always suppressed compared to those we keep when $A^2 \gg m_W^2$ and $\tilde{M}^2 \gg m_W^2$. In considering the corrections to the Higgs-boson-mass sum rule of Eq. (3), it is useful to define the ratio

$$R \equiv \frac{m_{H^+}^2 - m_{A^0}^2}{m_W^2} \ . \tag{7}$$

At the tree level $R=1$. To the extent that one-loop corrections are small, we can write the one-loop correction to R in the form

$$\Delta R = \frac{\Delta m_{H^+}^2 - \Delta m_{A^0}^2 - \Delta m_W^2}{m_W^2} \ . \tag{8}$$

The expressions for $\Delta m_{H^+}^2$, $\Delta m_{A^0}^2$, and Δm_W^2 appear in the Appendix. We have employed dimensional regularization. For the leading correction terms that we focus on, all terms in $1/\epsilon$ cancel in the mass sum rule. Indeed, the leading correction to Δm_W^2 is finite, while the $1/\epsilon$ terms in the leading corrections to $\Delta m_{H^+}^2$ and $\Delta m_{A^0}^2$ are the same. Thus, even though it is most convenient to express the results using the renormalization scale μ_0, the correction to the sum rule does not have any dependence on μ_0. Of course, in Veltman's renormalization scheme[5] the expression for Δm_W^2 leads immediately to a result for $\delta\rho$:

$$\delta\rho = \frac{\Delta m_W^2}{m_W^2} \ , \tag{9}$$

since $\Delta m_Z^2 = 0$ for the graphs we consider. We have checked that our expression for $\delta\rho$ in the Appendix agrees in the unmixed eigenstate limit with the expressions in the literature. (See, for example, Ref. 6.)

In order to gain some intuition regarding the behavior of the corrections to ρ and to the Higgs-boson mass-squared sum rules, as a function of the A and $\tilde{M}$ parameters, we give results in the special case where we take

$$\tilde{M}_U = \tilde{M}_D = \tilde{M}_Q \equiv M, \quad A_u = A_d \equiv A \ . \tag{10}$$

[Note that in this special case the contributions from the class (b) quartic vertex diagrams are precisely zero in our leading approximation.] We consider the limit where M and A are large compared to m_W. It is convenient to first define the coefficient

$$\eta \equiv \frac{3\alpha}{16\pi \sin^2\theta_W} \simeq 1.9\times 10^{-3} \ , \tag{11}$$

where we have used $\alpha = \frac{1}{137}$ and $\sin^2\theta_W = 0.225$. The factor of 3 in η is the color factor appropriate for a single family of squarks. In general, one must sum over all squarks and sleptons. For $\delta\rho$ we find the expression

$$\delta\rho = \frac{\eta}{m_W^2}\left[M^2 - \frac{M^4 - A^4}{2A^2} \ln\frac{M^2+A^2}{M^2-A^2} \right] \tag{12}$$

and for ΔR we find

$$\Delta R = -\frac{\eta}{m_W^4}\left[(\cot\beta + \tan\beta)^2 A^4 \times \left[1 - \frac{1}{2}\frac{M^2}{A^2}\ln\frac{M^2+A^2}{M^2-A^2} \right] \right] - \delta\rho \ ; \tag{13}$$

the first term comes from the difference of the Higgs-boson mass-squared corrections. It is useful to note that for $A \simeq M$ we have

$$\delta\rho \simeq \eta M^2/m_W^2 \ , \tag{14}$$

whereas ΔR clearly becomes logarithmically infinite in this limit. From Eqs. (11) and (14) we note that once $M \gtrsim 2.5 m_W$ in the $A \simeq M$ case, $\delta\rho$ exceeds 0.01 in clear disagreement with the experimental bound of Eq. (2). Thus, for large M, an expansion in the limit of $A \ll M$ is most relevant, for which we find

$$\delta\rho \simeq \eta \frac{2A^4}{3m_W^2 M^2}, \quad \Delta R \simeq -\eta \frac{A^8}{3M^4 m_W^4}(\tan\beta + \cot\beta)^2 - \delta\rho \ . \tag{15}$$

In this case, we may rewrite the result for ΔR using the expression for $\delta\rho$. From Eq. (15) we find, for $A \ll M$,

$$\begin{aligned}\Delta R &\simeq -\frac{16\pi \sin^2\theta_W}{4\alpha}(\tan\beta+\cot\beta)^2 \delta\rho^2 - \delta\rho \\ &\simeq -387(\tan\beta + \cot\beta)^2\delta\rho^2 - \delta\rho \ .\end{aligned} \tag{16}$$

(Note that corrections to R are generally negative.) From this equation, we see that if $\delta\rho \simeq 0.01$ then the first term is dominant. For example, at $\tan\beta = 1.2$, we find $\Delta R \simeq -0.17$. The small-A expansion makes clear an important general feature. Requiring a small value for $\delta\rho$, for example, $\delta\rho \simeq 0.01$, amounts to fixing a value for A at fixed M:

$$\frac{A}{M} \simeq 1.67 \left[\frac{m_W}{M} \right]^{1/2} \ . \tag{17}$$

Then, M scales as A^2, and A becomes $\lesssim 0.5M$ once $M \gtrsim 10 m_W$ ($A \gtrsim 5 m_W$). We also note that in the limit of $A \ll M$, the squark mass-squared matrix of Eq. (6) makes it clear that the squark eigenstates are approximately pure L and R, with squared masses increasing as M^2 (and, thus, as A^4 at fixed $\delta\rho$). More generally, we shall find that these same scaling systematics are preserved, but that by allowing for substantial deviations from the parameter equalities of our special case, Eq. (10), we can find corners of parameter space where restricting $\delta\rho$ to lie below 1% does not so strongly limit ΔR.

With these features in mind, we can turn to a numerical examination of the size of $\delta\rho$ and of ΔR, as a function of the five parameters A_u, A_d, $\tilde{M}_Q$, $\tilde{M}_U$, and $\tilde{M}_D$. We begin by graphically illustrating the above special case of equal parameters, Eq. (10). In Fig. 1 we plot contours of constant $\delta\rho$ and constant ΔR in the two-dimensional A and M parameter space. One sees immediately that restricting $\delta\rho$ to lie below some value limits the size of ΔR corrections. For instance, the $\delta\rho=1\%$ contour clearly always falls between the 10% and 30% contours of ΔR.

We next illustrate the behavior of $\delta\rho$ for more general parameter choices. In Fig. 2 we illustrate results for the parameter configuration of $\tilde{M}_U=\tilde{M}_D=\tilde{M}_Q=1$ TeV as a function of A_u and A_d. Note that corrections to both $\delta\rho$ and ΔR grow rapidly as the general magnitude of A_u and A_d increases. This is as expected since these A parame-

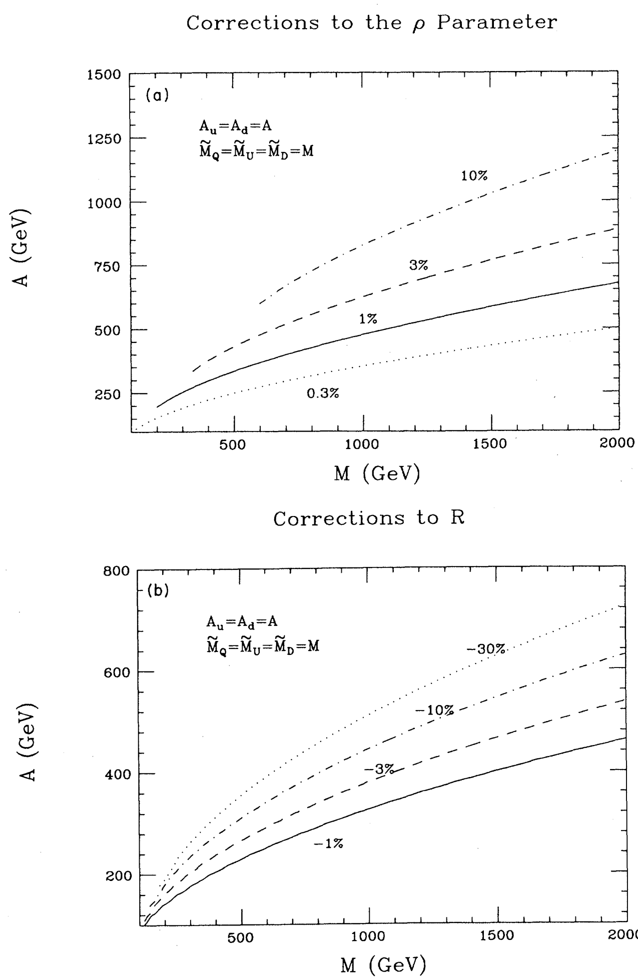

FIG. 1. We plot contours of (a) constant $\delta\rho$ and (b) constant ΔR, as a function of A and M for the special case of Eq. (10). Percentage values of corrections corresponding to the various curves are indicated.

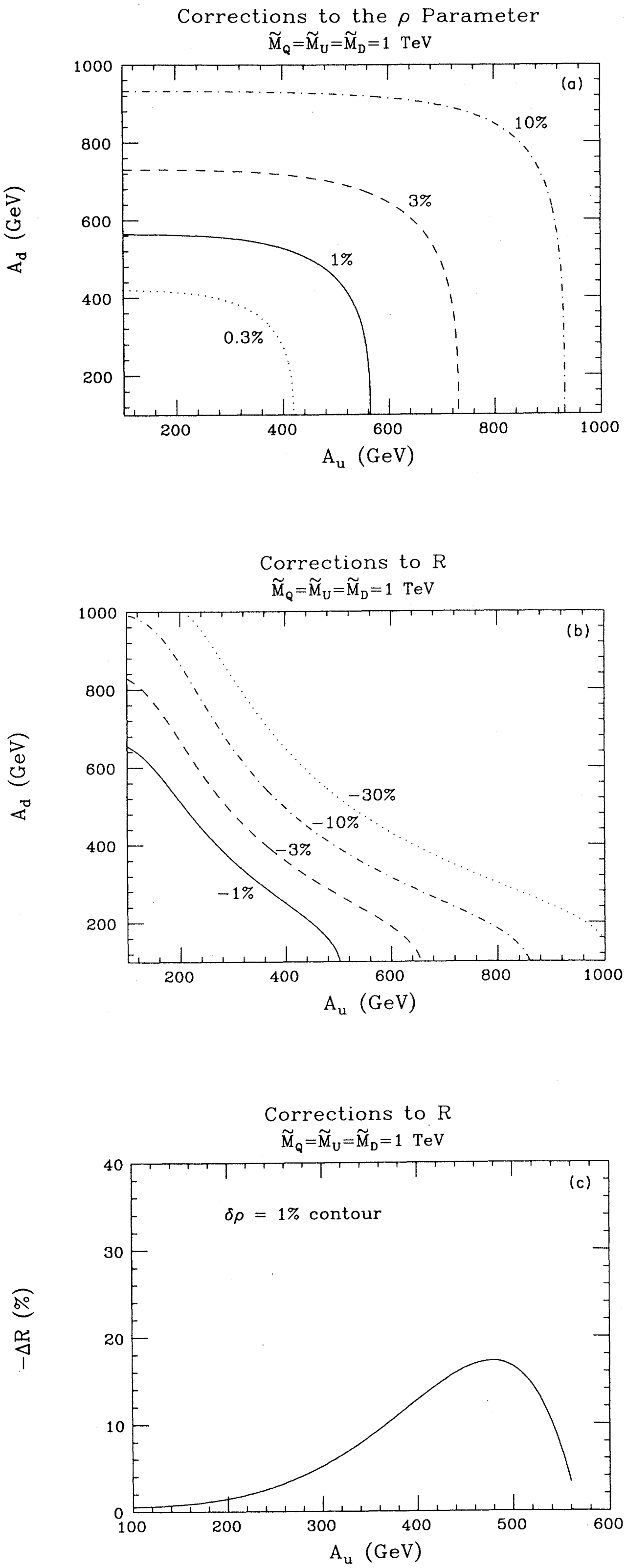

FIG. 2. We consider the parameter configuration of fixed $\tilde{M}_Q=\tilde{M}_U=\tilde{M}_D=1$ TeV and show contours as a function of A_u and A_d. In (a) we plot contours of fixed $\delta\rho$; in (b) we plot contours of fixed ΔR; and in (c) we plot ΔR as we move along the 1% contour of $\delta\rho$ in the parameter A_u.

ters characterize the strength of the Higgs-boson$-\tilde{q}-\tilde{q}$ trilinear couplings that evade the screening theorem. However, from Fig. 2(c) we see that when $\delta\rho=0.01$ the corrections to ΔR are limited to $\lesssim 17\%$, and are even smaller over a substantial range of A_u.

In Fig. 3 we show results for the parameter configuration of fixed $A_u=A_d=0.5$ TeV and $\tilde{M}_Q=1$ TeV, varying $\tilde{M}_U$ and $\tilde{M}_D$. From Figs. 3(a) and 3(b) we observe that the $\delta\rho$ and ΔR corrections decrease as the scale of $\tilde{M}_U$ and $\tilde{M}_D$ increases. This, of course, agrees with the screening theorem of Ref. 2. Again we may examine the behavior of ΔR as we follow along the $\delta\rho=0.01$ contour. We see that ΔR has a maximum at $\tilde{M}_U \sim \tilde{M}_D$ which is roughly $\delta\rho \lesssim 17\%$.

There are two common features of the configurations discussed in Figs. 2 and 3 that are noteworthy. First, we point out that the maximum $\delta\rho$ is achieved at the symmetric point corresponding to our special case limit of Eq. (10). Second, in moving along the $\delta\rho=1\%$ contour, the ratios of squark masses never become large. This, it turns out, is why ΔR is never particularly large. We shall see that the largest ΔR values are obtained at fixed $\delta\rho$ for configurations where large squark mass ratios emerge.

To illustrate a case in which very large ΔR values are possible, we show in Fig. 4 results for the parameter configuration of fixed $A_u=A_d=0.5$ TeV as a function of $\tilde{M}_Q$ and $\tilde{M}_U=\tilde{M}_D$. As far as we have been able to determine, this is the parameter configuration leading to the largest ΔR values while keeping $\delta\rho$ at 1%. From Figs. 4(a) and 4(b) we observe a pattern similar to that of Figs. 3(a) and 3(b). We see that for fixed $A_u=A_d$, the $\delta\rho$ and ΔR corrections decrease as the scale of $\tilde{M}_Q$ and $\tilde{M}_U=\tilde{M}_D$ increases, in agreement with the screening theorem. As we follow along the $\delta\rho=0.01$ contour, ΔR varies substantially, lying below 20% in absolute magnitude when $\tilde{M}_Q$ and $\tilde{M}_U=\tilde{M}_D$ are similar in magnitude, but becoming quite large when $\tilde{M}_Q$ is either small [implying that $\tilde{M}_U=\tilde{M}_D$ is large, see Fig. 4(a)] or very large (implying that $\tilde{M}_U=\tilde{M}_D$ is small). However, we emphasize that it is actually the size relative to $A_u^2(A_d^2)/m_W$ that matters [just as in Eq. (15), for the special case] and not the absolute magnitude; the entire behavior scales as a function of the common value of A_u and A_d.

The scaling features are illustrated in Fig. 5. There we compare results for ΔR and squark masses for (i) $A_u=A_d=1$ TeV with the results for (ii) $A_u=A_d=0.5$ TeV, as we move along the $\delta\rho=1\%$ contour keeping $\tilde{M}_U=\tilde{M}_D$ and varying $\tilde{M}_Q$, as in Fig. 4(c). In this comparison between cases (i) and (ii), we have plotted case (ii) results after scaling $\tilde{M}_Q$ by a factor of 4 and [in Fig. 5(b)] squark masses by a factor of 4. In these plots we have required that squark masses be heavier than 800 GeV [200 GeV] in cases (i) [(ii)], respectively, so that we are clearly in the domain of validity of the approximation in which we retain only the "leading" terms in $\delta\rho$ and ΔR described earlier. Regarding ΔR, it is apparent that when we double the value of $A_u=A_d$, the curves nearly match if we quadruple $\tilde{M}_Q$. At corresponding points on the ΔR curve, we see from Fig. 5(b) that the required squark masses are also quadrupled (as is $\tilde{M}_U=\tilde{M}_D$). Thus, we

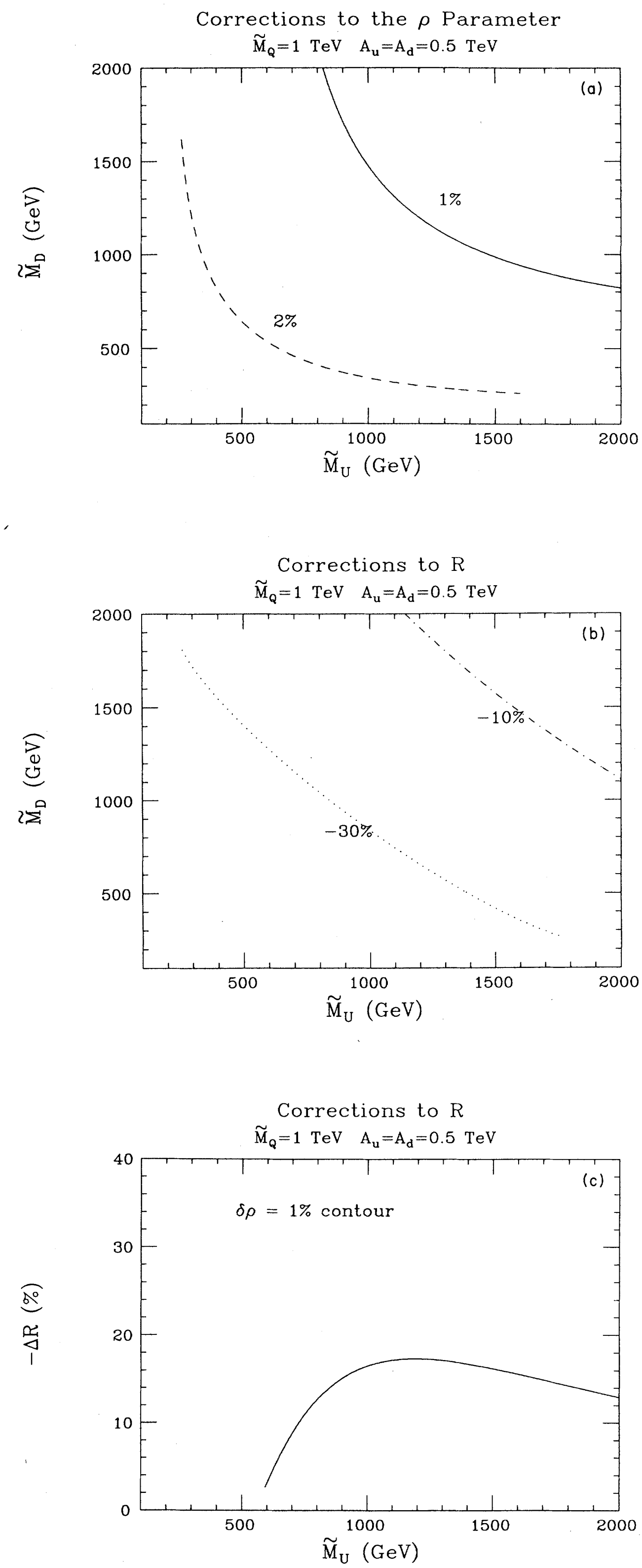

FIG. 3. We consider the parameter configuration of fixed $A_u=A_d=0.5$ TeV and $\tilde{M}_Q=1$ TeV, and show contours as a function of $\tilde{M}_U$ and $\tilde{M}_D$. In (a) we plot contours of fixed $\delta\rho$; in (b) we plot contours of fixed ΔR; and in (c) we plot ΔR as we move along the 1% contour of $\delta\rho$ in the parameter $\tilde{M}_U$.

have the same type of scaling behavior found in the "small-A" limit of the $A_u=A_d=A$ and $\tilde{M}_Q=\tilde{M}_U=\tilde{M}_D=M$ case discussed analytically earlier, see Eqs. (15)–(17). The reason that we get such large values for ΔR in the present asymmetric case is that a large ratio between $\tilde{M}_Q$ and $\tilde{M}_U=\tilde{M}_D$ leads to large ratios for the physical squark masses, as is evident from Fig. 5(b). ΔR is much more sensitive to such large ratios than is $\delta\rho$. This is because $\delta\rho$ is primarily sensitive to large mass splitting between up- and down-squark mass eigenstates, whereas ΔR is sensitive also to large mass splitting be-

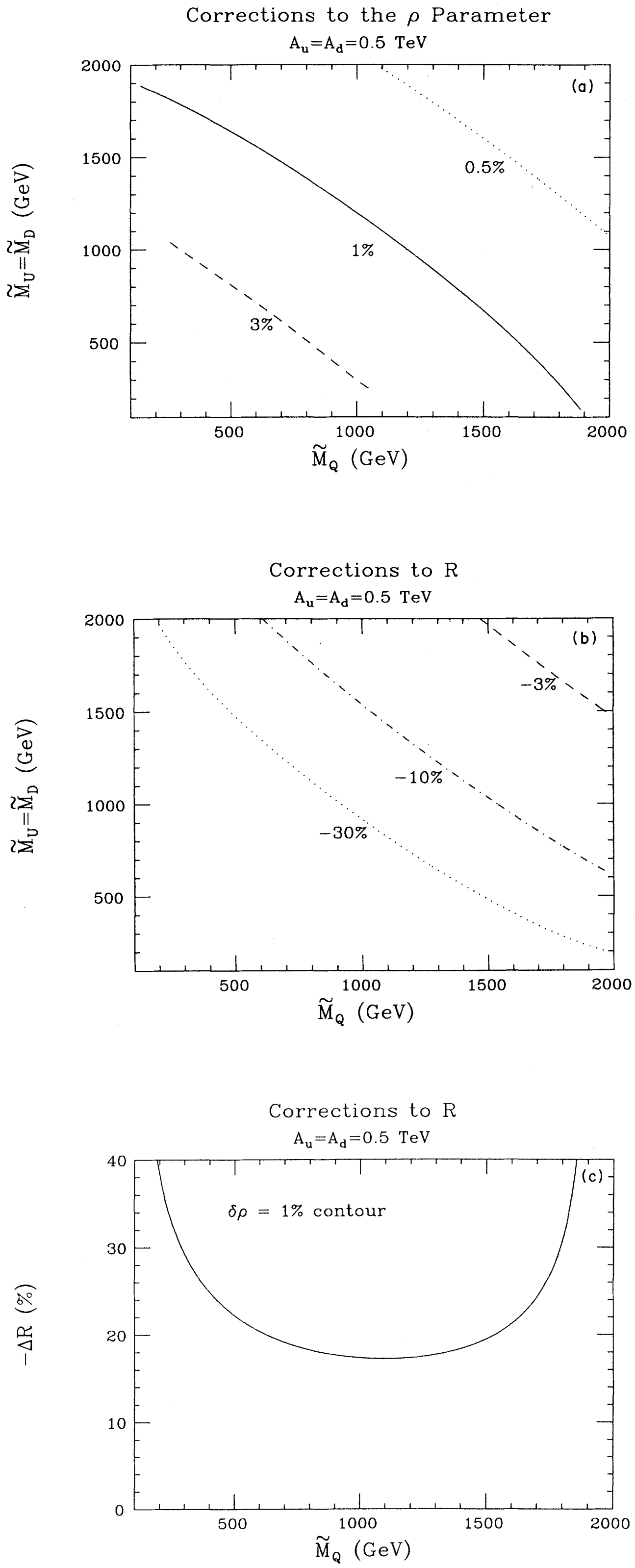

FIG. 4. We consider the parameter configuration of fixed $A_u=A_d=0.5$ TeV and show contours as a function of $\tilde{M}_Q$ and $\tilde{M}_U=\tilde{M}_D$. In (a) we plot contours of fixed $\delta\rho$; in (b) we plot contours of fixed ΔR; and in (c) we plot ΔR as we move along the 1% contour of $\delta\rho$ in the parameter $\tilde{M}_Q$.

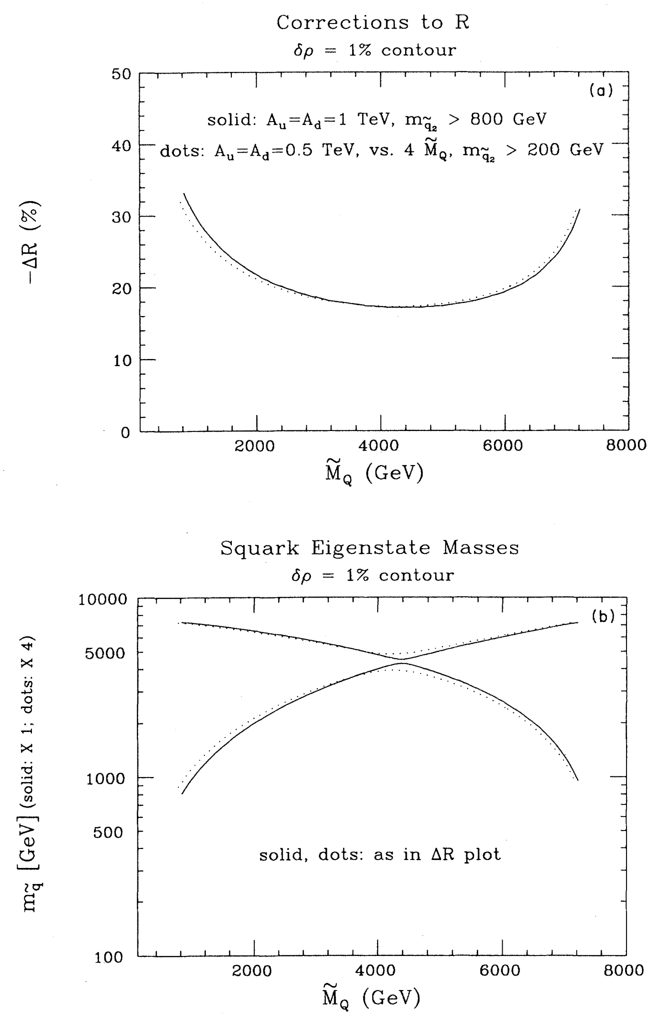

FIG. 5. We compare $\delta\rho=1\%$ results for (i) $A_u=A_d=1$ TeV and (ii) $A_u=A_d=0.5$ TeV. In (a) the solid curve gives ΔR for case (i) as a function of $\tilde{M}_Q$, while the dotted curve gives ΔR for case (ii) as a function of $4\tilde{M}_Q$. In (b) we plot $m_{\tilde{q}_{1,2}}$ values. (1 denotes the heavier eigenstate, and 2 the lighter; in the present case, the u and d eigenstate masses are the same.) The solid curves give $m_{\tilde{q}_{1,2}}$ for case (i) as a function of $\tilde{M}_Q$, while the dotted curves give $4m_{\tilde{q}_{1,2}}$ for case (ii) as a function of $4\tilde{M}_Q$. For these plots we have restricted $m_{\tilde{q}_2}$ to lie above 800 GeV [200 GeV] in case (i) [(ii)], respectively.

tween mass eigenstates of a given I_3. By constraining $\tilde{M}_U=\tilde{M}_D$ we have guaranteed that the up and down squarks have the same mass eigenstates, thereby guaranteeing that $\delta\rho$ will be small. In any case, we see that the squark mass scale need not be numerically small in order to get large ΔR when $\delta\rho \lesssim 1\%$ (Ref. 7).

In summary, we have computed the leading one-loop corrections, arising from squark loops, to the simplest of the Higgs-boson-mass sum rules of the minimal supersymmetric model. The corrections that we compute are leading in the sense that they are the dominant corrections when the A and $\tilde{M}$ parameters of the soft supersymmetry-breaking potential, specifying the size of Higgs-boson–squark–squark trilinear couplings and the size of bilinear soft masses squared are large compared to m_W, quark masses, and the superpotential parameter μ of the MSSM. These leading contributions are finite and well defined. We find that the sum-rule corrections from the squark sector (and, of course, the slepton sector as well) could be quite large, even if we constrain $\delta\rho$ to be less than 1%. Such large corrections do, however, arise only when the Higgs-boson–sfermion–sfermion couplings are allowed to become substantially larger than the m_W mass scale, in agreement with the general screening theorem arguments of Ref. 2. The corrections can become very large, without violating $\delta\rho \lesssim 1\%$, if the up- and down-squark Yukawa couplings are similar in magnitude (so that the up- and down-squark eigenstate masses are approximately the same) but such as to yield a large mass splitting between the heavier (approximately degenerate) up- and down-squark eigenstates and the lighter (approximately degenerate) up- and down-squark eigenstates.

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APPENDIX: EXPRESSIONS FOR LEADING CORRECTIONS TO MASS SHIFTS

In this appendix, we give expressions for $\Delta m^2_{H^+}-\Delta m^2_{A^0}$ and for Δm^2_W deriving from a single (up-down) family of squark loops, that are valid in the limit where the squark masses are much larger than m_W and the Higgs-boson masses themselves. The contributing one-loop diagrams are of two types: (a) those containing two three-point vertices (Higgs-boson–$\tilde{q}$–$\tilde{q}$ or W-$\tilde{q}$-$\tilde{q}$), and (b) those containing one four-point vertex (Higgs-boson–Higgs-boson–$\tilde{q}$–$\tilde{q}$ or W-W-$\tilde{q}$-$\tilde{q}$). In the notation of Ref. 1 the former diagrams are termed $b1$ diagrams while the latter are called $b2$ diagrams. As mentioned in the text, the correction Δm^2_W is finite on its own, whereas the $1/\epsilon$ corrections to $\Delta m^2_{H^+}$ and $\Delta m^2_{A^0}$ cancel in the difference $\Delta(m^2_{H^+}-m^2_{A^0})$ that enters into the sum rule of Eq. (3). As a result, we are guaranteed that all dependence on the renormalization scale μ_0^2 also cancels. The most convenient form of the final expressions appears below. It will be helpful to define the shorthand notation

$$c_{u,d}\equiv\cos\theta_{u,d}\quad s_{u,d}\equiv\sin\theta_{u,d}\ , \tag{A1}$$

where $\theta_{u,d}$ are the angles that arise in diagonalizing the u- and d-squark mass matrices of Eq. (6). At the same time we obtain the mass eigenstates $\tilde{u}_{1,2}$ and $\tilde{d}_{1,2}$, where

$$\tilde{u}_1=c_u\tilde{u}_L+s_u\tilde{u}_R,\quad \tilde{u}_2=-s_u\tilde{u}_L+c_u\tilde{u}_R\ , \tag{A2}$$

with a similar expression for the $\tilde{d}$'s. The masses of these states are denoted by m's with an appropriate subscript. In terms of these quantities we have

$$\begin{aligned}\frac{1}{m_W^2}\Delta(m^2_{H^+}-m^2_{A^0})|_{b1}&=\frac{3}{m_W^2}\left[\sum_{ij}\Sigma^{b1}_{H^+}(m_{\tilde{u}_i},m_{\tilde{d}_j},m^2_{H^+})-2\Sigma^{b1}_{A^0}(m_{\tilde{u}_1},m_{\tilde{u}_2},m^2_{A^0})-2\Sigma^{b1}_{A^0}(m_{\tilde{d}_1},m_{\tilde{d}_2},m^2_{A^0})\right]\\&=-\frac{\eta}{m_W^4}[(c_us_d\tan\beta A_d^2+s_uc_d\cot\beta A_u^2)^2F(m_{\tilde{u}_1},m_{\tilde{d}_1})+(c_uc_d\tan\beta A_d^2-s_us_d\cot\beta A_u^2)^2F(m_{\tilde{u}_1},m_{\tilde{d}_2})\\&\qquad+(-s_us_d\tan\beta A_d^2+c_uc_d\cot\beta A_u^2)^2F(m_{\tilde{u}_2},m_{\tilde{d}_1})\\&\qquad+(-s_uc_d\tan\beta A_d^2-c_us_d\cot\beta A_u^2)^2F(m_{\tilde{u}_2},m_{\tilde{d}_2})\\&\qquad-\cot^2\beta A_u^4F(m_{\tilde{u}_1},m_{\tilde{u}_2})-\tan^2\beta A_d^4F(m_{\tilde{d}_1},m_{\tilde{d}_2})]\ ,\end{aligned} \tag{A3}$$

where

$$F(m_1,m_2)\equiv\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\frac{m_2^2}{m_1^2}-\ln(m_1^2m_2^2)\ . \tag{A4}$$

Note that the squared masses appearing in the last logarithm of Eq. (A4) are in general each divided by μ_0^2. However, as mentioned earlier, the dependence on μ_0^2 cancels in the full expression, and we have chosen to drop this scale (or equivalently, choose it to be 1 in appropriate units). For the $b2$ diagrams we obtain

$$\begin{aligned}\frac{1}{m_W^2}\Delta(m^2_{H^+}-m^2_{A^0})|_{b2}&=\frac{3}{m_W^3}\left[\sum_{q_i=u_i,d_i,i=1,2}[\Sigma^{b2}_{H^+}(m_{\tilde{q}_1})-\Sigma^{b2}_{A^0}(m_{\tilde{q}_i})]\right]\\&=\frac{2\eta\cos2\beta}{m_W^4}(c_u^2m^2_{\tilde{u}_1}\ln m^2_{\tilde{u}_1}+s_u^2m^2_{\tilde{u}_2}\ln m^2_{\tilde{u}_2}-c_d^2m^2_{\tilde{d}_1}\ln m^2_{\tilde{d}_1}-s_d^2m^2_{\tilde{d}_2}\ln m^2_{\tilde{d}_2})\ .\end{aligned} \tag{A5}$$

Finally, for Δm_W^2 we find (see Ref. 1 for diagram notation)

$$\delta\rho=\frac{\Delta m_W^2}{m_W^2}=\frac{3}{m_W^2}\left[\sum_{ij}\Pi^{b1}(m_{\tilde{u}_i},m_{\tilde{d}_j},m_W^2)+\sum_{q_i=u_i,d_i,i=1,2}\Pi^{b2}(m_{\tilde{q}_i})\right]$$

$$=\frac{\eta}{m_W^2}\left\{c_u^2m_{\tilde{u}_1}^2+s_u^2m_{\tilde{u}_2}^2+c_d^2m_{\tilde{d}_1}^2+s_d^2m_{\tilde{d}_2}^2-c_u^2c_d^2\frac{2m_{\tilde{u}_1}^2m_{\tilde{d}_1}^2}{m_{\tilde{u}_1}^2-m_{\tilde{d}_1}^2}\ln\frac{m_{\tilde{u}_1}^2}{m_{\tilde{d}_1}^2}-c_u^2s_d^2\frac{2m_{\tilde{u}_1}^2m_{\tilde{d}_2}^2}{m_{\tilde{u}_1}^2-m_{\tilde{d}_2}^2}\ln\frac{m_{\tilde{u}_1}^2}{m_{\tilde{d}_2}^2}\right.$$

$$\left.-s_u^2c_d^2\frac{2m_{\tilde{u}_2}^2m_{\tilde{d}_1}^2}{m_{\tilde{u}_2}^2-m_{\tilde{d}_1}^2}\ln\frac{m_{\tilde{u}_2}^2}{m_{\tilde{d}_1}^2}-s_u^2s_d^2\frac{2m_{\tilde{u}_2}^2m_{\tilde{d}_2}^2}{m_{\tilde{u}_2}^2-m_{\tilde{d}_2}^2}\ln\frac{m_{\tilde{u}_2}^2}{m_{\tilde{d}_2}^2}\right\}. \quad \text{(A6)}$$

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[7] Of course, it is possible to choose parameters so that one of the physical squark mass eigenstates has extremely small mass. In the present case, this is achieved by taking $\widetilde{M}_Q$ to be *very* small or *very* large. In this limit we can still keep $\delta\rho=1\%$ [$\delta\rho$ is nonsingular in such a limit, see Fig. 4(a)], but ΔR will blow up logarithmically. This is illustrated, also, by the symmetric limit Eqs. (12) and (13).