PHYSICAL REVIEW D VOLUME 40, NUMBER 7 1 OCTOBER 1989

Z-boson bremsstrahlung in heavy-quark decay

V. Barger
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

W.-Y. Keung
Physics Department, University of Illinois at Chicago, Chicago, Illinois 60680

T. G. Rizzo
Physics Department, University of Wisconsin, Madison, Wisconsin 53706
and Ames Laboratory and Physics Department, Iowa State University, Ames, Iowa 50011
(Received 15 May 1989)

We calculate the rate for Z-boson bremsstrahlung in the charged-current decay $Q \to qWZ$ of a heavy fermion Q to a light fermion q. This process may be used to test the WWZ trilinear gauge-boson coupling of the standard model. We evaluate the branching fraction for this decay mode of fourth-generation and E_6 exotic fermions; the branching fraction increases dramatically with increasing m_Q and may reach 1% for TeV fermion masses.

The construction of new multi-TeV hadron colliders, such as the Superconducting Super Collider and the CERN Large Hadron Collider, and e^+e^- colliders in the 1–2-TeV range will allow a search for new quarks and leptons with masses much higher than the potential search limits from existing accelerators. Such heavy fermions (Q), if they exist, will dominantly decay via their charged-current coupling to the W and a lighter fermion q: i.e., $Q \to qW$. As is well known, when $m_Q^2 \gg M_W^2$ the decay rate for $Q \to qW$ is enhanced by a factor of $\sim m_Q^2/M_W^2$ due to the longitudinal W coupling to the $Q\bar{q}$ current. This enhancement is similar to that of the coupling of a Higgs scalar (H) to a heavy quark and it is expected that the decay $Q \to qWH$ with an addition H in the final state may become important at large-m_Q values.[1] Similarly the decay rate for $Q \to qWZ$ will become increasingly significant at high m_Q due to the longitudinal Z coupling as long as the axial-vector coupling to Q is nonzero. The $Q \to qWZ$ decay proceeds through the three diagrams shown in Fig. 1. The presence of the graph involving the WWZ trilinear gauge-boson interaction means that a measurement of $\Gamma(Q \to qWZ)$ could test the standard-model WWZ trilinear gauge coupling.[2]

There are several interesting possibilities for the fermions Q and q. If a fourth family of fermions exists,

$$\begin{pmatrix} a \\ v \end{pmatrix}_L, \quad \begin{pmatrix} \nu_4 \\ L \end{pmatrix}_L, \quad a_R, \quad v_R, \quad L_R \; , \tag{1}$$

the following decay modes could be realized:

$$v \to (t,a)WZ, \quad a \to (v,b)WZ, \quad L \to \nu_4 WZ \; , \tag{2}$$

with the rates dependent on mass heirachies and quark mixing.[3] Since data on the ρ parameter restrict $|m_v - m_a| \lesssim 190$ GeV (Ref. 4) for $m_t > 60$ GeV (Ref. 5) and since $M_W + M_Z \approx 173$ GeV the decays $v \to aWZ$ or $a \to vWZ$ will be highly phase-space suppressed. The ρ-parameter analysis also restricts $|m_L - m_{\nu_4}| \lesssim 330$ GeV so that the phase space in the case of $L \to \nu_4 WZ$ is also limited. The reactions $v \to tWZ$ and $a \to bWZ$ appear more promising since in the limit of small mixing (i.e., a nearly diagonal mixing matrix) the ρ parameter places only very weak constraints on m_v and m_a. However, these fermion masses cannot be arbitrarily large and still satisfy the constraints which come from perturbative unitarity;[6] to this end we assume m_v and $m_a \lesssim 700$ GeV.

Beyond the simple four-generation extension to the standard model (SM), there are other possibilities for $Q \to qWZ$ decays. As an example, we consider the two

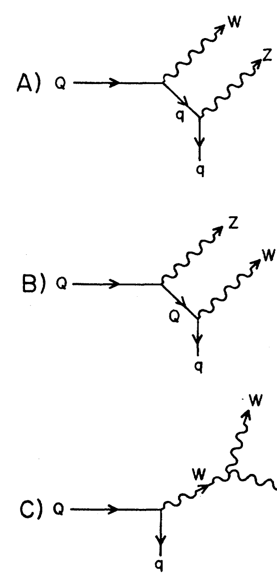

FIG. 1. Feynman diagrams for the heavy-fermion decay $Q \to qWZ$.

 ©1989 The American Physical Society

charged vectorlike fermions (h and E) which occur in E_6 superstring-inspired models. With suitable discrete symmetries applied to the superpotential, h can be a charge $-\frac{1}{3}$ isosinglet quark and E is a charge -1 isodoublet lepton with $T_3=-\frac{1}{2}$. Both can decay by mixing with the ordinary fermions via the modes

$$h \rightarrow (u,c,t)WZ, \quad E \rightarrow (\nu_e,\nu_\mu,\nu_\tau)WZ \; . \tag{3}$$

Since h and E are vectorlike to a good approximation in the mass-eigenstate basis (if ordinary-exotic mixing is small which will assume to be the case[7]) these fermions will not have enhanced couplings to longitudinal Z's.

We consider the general decay $Q \rightarrow qWZ$ whose matrix element is given by

$$\begin{aligned}\mathcal{M}(Q\rightarrow qWZ)=\bar{q}_L &\left[\frac{g_L^q \not{\epsilon}_Z \not{q}^* \not{\epsilon}_W}{q^{*2}-m_q^2} + \frac{g_L^Q \not{\epsilon}_W \not{Q}^* \not{\epsilon}_Z}{Q^{*2}-m_Q^2} \right. \\ &\left. + \frac{g_W \not{P}}{W^{*2}-M_W^2} \right] Q_L \\ &+\bar{q}_R \frac{g_R^q \not{\epsilon}_Z m_q \not{\epsilon}_W}{q^{*2}-m_q^2} Q_L + \bar{q}_L \frac{g_R^Q \not{\epsilon}_W m_Q \not{\epsilon}_Z}{Q^{*2}-m_Q^2} Q_R \; ,\end{aligned} \tag{4}$$

where ϵ_W (ϵ_Z) is the W (Z) polarization vector,

$$Q^*=Q-Z, \quad q^*=q+Z, \quad W^*=Q-q \; , \tag{5}$$

with Q,W,Z,q denoting the corresponding particle four-momenta, and $Q_{L,R}=\frac{1}{2}(1\pm\gamma_5)Q$. In terms of $x_W=\sin^2\theta_W \simeq 0.230$ and $c=g^2[2(1-x_W)]^{-1/2}V_{Qq}$, where V_{Qq} denotes the fermion mixing matrix element, the couplings are

$$\begin{aligned} g_{L,R}^q &= c(T_{3L,R}^q - x_W Q^q) \; , \\ g_{L,R}^Q &= c(T_{3L,R}^Q - x_W Q), \quad g_W = \pm c(1-x_W) \; , \end{aligned} \tag{6}$$

with the plus (minus) sign to be chosen for W^+ (W^-) emission. The vector P in Eq. (4) is defined by the usual standard-model coupling plus an anomalous magnetic moment interaction:[2]

$$\begin{aligned} P^\lambda = \epsilon_W \cdot \epsilon_Z &\left[Z^\lambda \left[2 - \frac{M_Z^2}{M_W^2} \right] - \frac{W^\lambda M_Z^2}{M_W^2} \right] \\ &+ 2(\epsilon_W^\lambda W \cdot \epsilon_Z - \epsilon_Z^\lambda Z \cdot \epsilon_W) \\ &+ (\kappa_Z - 1)[(Z^\lambda + W^\lambda)(\epsilon_Z \cdot W \epsilon_W \cdot Z - \epsilon_W \cdot \epsilon_Z M_Z^2 \\ &\qquad - \epsilon_W \cdot \epsilon_Z W \cdot Z)/M_W^2 \\ &\qquad + Z^\lambda \epsilon_W \cdot \epsilon_Z - \epsilon_Z^\lambda Z \cdot \epsilon_W] \; . \end{aligned} \tag{7}$$

Here κ_Z is the strength of the W^+W^-Z anomalous magnetic moment coupling. It is defined analogously to the $W^+W^-\gamma$ anomalous coupling and has the value of unity in the SM. In our analysis we neglect QCD corrections which are expected to be small for heavy-quark decays.

The calculation now proceeds in a straightforward manner and makes use of standard helicity techniques;[8] the squaring of the matrix element and the phase-space integrations are both performed numerically with the results independently checked by calculations using the algebraic-manipulation program REDUCE. The helicity amplitude calculation is easily performed in the rest frame of the initial heavy fermion Q, with the z axis chosen along the momentum of q. We use the convention

$$u_L = \begin{bmatrix} \chi_L \\ 0 \end{bmatrix}, \quad u_R = \begin{bmatrix} 0 \\ \chi_R \end{bmatrix} . \tag{8}$$

The two-component spinor χ of helicity λ for Q is

$$\chi_L^i(Q,\lambda) = \chi_R^i(Q,\lambda) = m_Q \delta_{i\lambda} \quad (i,\lambda = \pm 1) \; . \tag{9}$$

Here the left (L) and right (R) decompositions are equal because the spinor is evaluated in the Q rest frame. The q spinors are given by

$$\chi_{L,R}^i(q,\lambda) = \sqrt{q_0 \mp \lambda |\mathbf{q}|}\, \delta_{i\lambda} \; . \tag{10}$$

The decay width is calculated from the sum of squares of the helicity amplitudes. In presenting the results we will normalize the decay width for $Q \rightarrow qWZ$ to that for $Q \rightarrow qW$ to remove overall mixing-angle and coupling-constant factors. This ratio is essentially the $Q \rightarrow qWZ$ branching fraction.

Let us first consider the decay $L \rightarrow \nu_4 WZ$. Figure 2 shows $\Gamma(L\rightarrow\nu_4 WZ)/\Gamma(L\rightarrow\nu_4 W)$ for $m_{\nu_4}=0$. Data on the ρ parameter restrict the mass of L to be $\lesssim 330$ GeV for $m_{\nu_4}=0$. For this limited L mass range the ratio of decay rates is never much larger than 10^{-3} and would be virtually impossible to observe. The separate contributions from the trilinear WWZ vertex and from initial-state Z bremsstrahlung to the ratio of decay rates are also shown in this figure. One sees that Z bremsstrahlung from the heavy initial fermion and Z production from the WWZ vertex yield comparable contributions to the overall rate.

Figures 3 and 4 show the corresponding ratio of widths for v- and a-quark decays, respectively; here the ratios

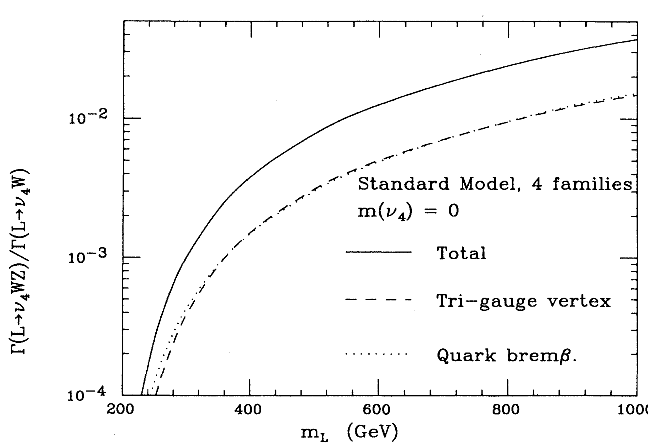

FIG. 2. The ratio of decay rates $\Gamma(L\rightarrow\nu_4 WZ)/\Gamma(L\rightarrow\nu_4 W)$ vs m_L assuming ν_4 to be massless. The individual contributions from the WWZ vertex and from emission from the heavy charged lepton are shown separately.

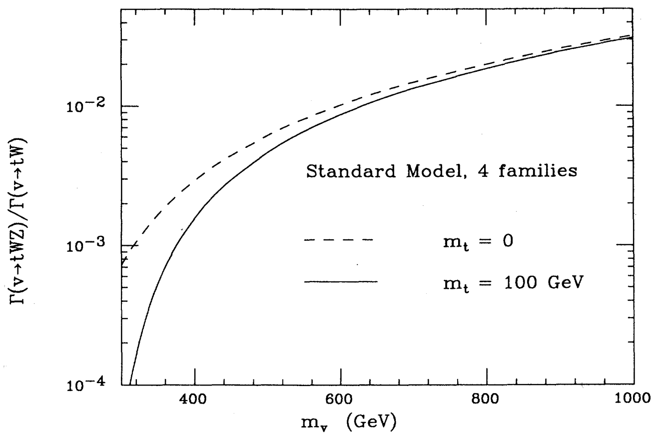

FIG. 3. The ratio of decay rates $\Gamma(v\to tWZ)/\Gamma(v-tW)$ vs m_v.

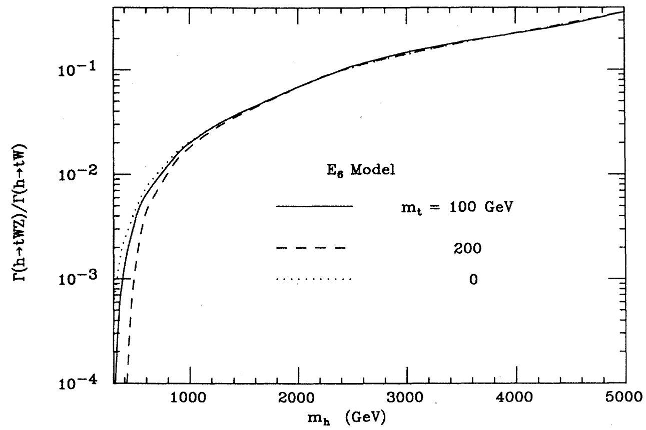

FIG. 5. The ratio of decay rates $\Gamma(h\to tWZ)/\Gamma(h\to tW)$ vs m_h.

can be as large as $\simeq 10^{-2}$ for the mass range allowed by perturbative unitarity. Branching fractions this large for the three-body final state may be observable as will be discussed below.

For the case of the E_6 exotic fermions h and E, which decay via mixing with the ordinary fermions, there is no upper bound on their masses in the limit of small mixing (subject however to possible cosmological and astrophysical constraints). The decay matrix elements for the three-body decays $h\to tWZ$ and $E\to\nu WZ$ are given by Eq. (4) with $T^h_{3L}=T^h_{3R}=0$ and $T^E_{3L}=T^E_{3R}=-\frac{1}{2}$. Although the couplings of the exotic fermions to Z are vectorial, the WWZ trilinear gauge-boson couplings remain and give rising branching fractions as the masses of E and h increase. The width ratios for these decays are shown in Figs. 5 and 6. For the h or E mass in TeV range the ratio of branching fractions exceeds 10^{-2} and is larger than 10^{-1} for masses above $\simeq 3$ TeV.

The additional interesting decay channels $E\to lZZ$ and $h\to bZZ$ involve double neutral-current interactions. Their matrix elements are given by Eq. (4) with $g_W=0$ (since there is no $3Z$ vertex) with an overall change in normalization given by $g/\sqrt{2}\to g/2c_W$. A further factor of $\frac{1}{2}$ must be included in the decay rate to account for the two identical Z's in the final state. The longitudinal enchancements found in previous processes are not present in these channels.

Figure 7 shows the influence of deviations in the value of κ_Z away from unity on the relative size of the branching fraction for the decay $v\to tWZ$. Note that the relative size of the branching function increases substantially for $\kappa_Z\neq 1$ with a factor $\simeq 2.3$ increase for $|\Delta\kappa_Z|=1$ over the $\kappa_Z=1$ prediction. This result is a general feature of all the decays we have examined. Any increase in event rate above the standard-model prediction must be statistically significant and reasonably high statistics would be necessary to probe $|\Delta\kappa_Z|\simeq 0.2-0.3$. To this end we make some estimates for the number of events with an additional Z that one might expect.

The number of events at $\sqrt{s}=1$ or 2 TeV e^+e^- colliders of the kind that interest us is unfortunately not large. From $Q\bar{Q}$ production the decays $Q\to qW$ and $\bar{Q}\to\bar{q}WZ$ give a final state consisting of two jets, $2W$'s, plus a Z.

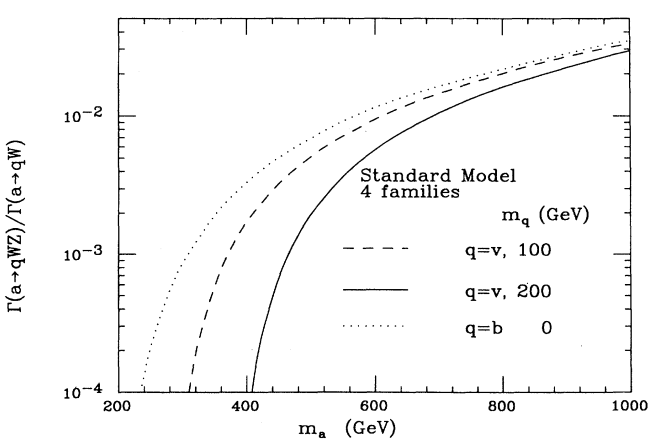

FIG. 4. The ratio of decay rates $\Gamma(a\to qWZ)/\Gamma(a\to qW)$ vs m_a. The case $q=b$ assumes that $a\to vW$ is kinematically forbidden.

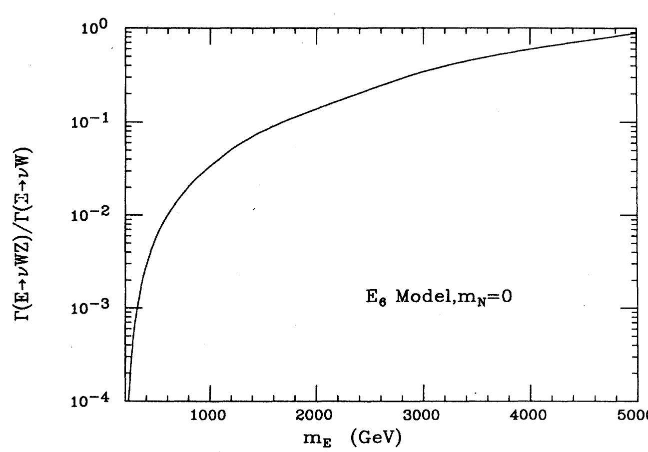

FIG. 6. The ratio of decay rates $\Gamma(E\to\nu WZ)/\Gamma(E\to\nu W)$ vs m_E.

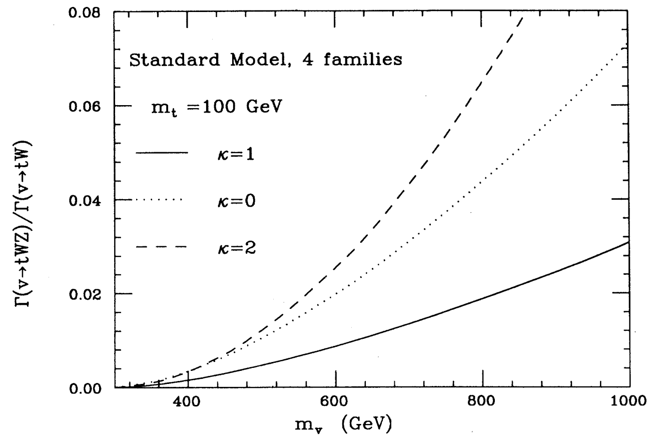

FIG. 7. Anomalous-magnetic-moment κ_z dependence of the ratio of decay rates $\Gamma(v \to tWZ)/\Gamma(v - tW)$ as a function of m_v, with $m_t = 100$ GeV.

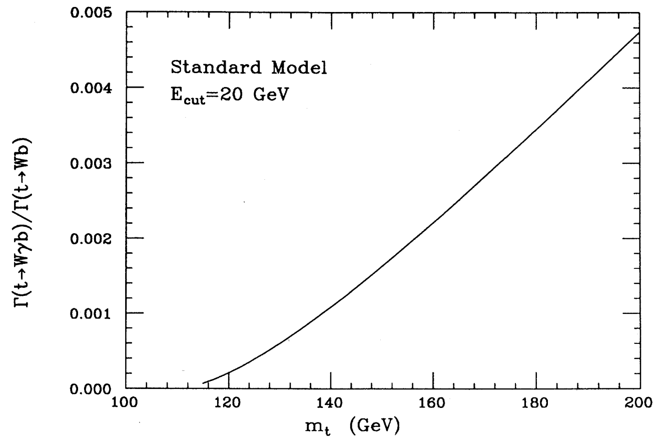

FIG. 8. The ratio $\Gamma(t \to bW\gamma)/\Gamma(t \to bW)$ of decay rates vs m_t assuming $E_\gamma > 20$ GeV.

The number N of such events for a given integrated luminosity L is $N \simeq 2B\sigma L$ where B is the $Q \to qWZ$ branching fraction from Figs. 3 and 4 and σ is given, for example, in Ref. 9. For m_v (m_a) $=600$ GeV we expect $N=30$ (50) events at a $\sqrt{s}=2$ TeV e^+e^- collider with $L=100$ fb^{-1}. However reconstruction efficiencies may only be $\simeq 10\%$ (Ref. 10) for the WWZ final state. For $m_L = 300$ GeV a similar calculation for a $\sqrt{s}=1$ TeV collider yields $N=27$ events. For exotics, which can decay by either W or Z emission, B is given by $\Gamma(Q \to qWZ)/[\Gamma(Q \to qW) + \Gamma(Q \to /q'Z)]$ with $\Gamma(Q - q'Z) = \frac{1}{2}\Gamma(Q - qW)$ as $m_Q \to \infty$. This amounts to an additional suppression factor of $\frac{2}{3}$ in these cases, e.g., for $m_E = 800$ GeV at $\sqrt{s}=2$ TeV collider[9] one finds $N=40$ events which is comparable to the numbers above for v and a. Somewhat smaller N values are obtained in the case of h production and subsequent decay.

An analysis of backgrounds to the $Q \to qWZ$ signals would depend on the Q and q masses, which would be determined from studies of the dominant $Q \to qW$ signal.

It is also of interest to consider photon bremsstrahlung in heavy-quark decay, such as $t \to bW\gamma$. Since $\Gamma(t \to bW\gamma)$ is infrared divergent, the result will depend on the choice of cutoff in the photon energy, $E_{\rm cut}$. At TeV linear colliders, a reasonable choice may be $E_{\rm cut} = 20$ GeV which we will assume in our discussion. The matrix element is given by Eqs. (4)–(6) with the replacements $Z \to A$, $g^q_{L,R} \to cQ^q$, $g^Q_{L,R} \to cQ$, $g_W = \pm c$ where now $c = g^2\sqrt{x_W}/2V_{Qq}$. The corresponding expression for P^λ is given by Eq. (7) with $Z \to A$, M_Z^2 set to zero, and κ_Z set to unity. Figure 8 shows the predicted result. Although the $t \to W\gamma b$ branching fraction grows with m_t it is never larger than $\simeq 4.7 \times 10^{-3}$ for $m_t \lesssim 200$ GeV. Hence the $t \to bW\gamma$ decay mode will not be easily observed.

In summary, we have calculated the rate for the Z-boson bremsstrahlung process $Q \to qWZ$ in the charged-current decay of a heavy fermion Q for several possible choices of Q and q. When Q is a member of a left-handed weak isodoublet the ratio $\Gamma(Q \to qWZ)/\Gamma(Q \to qW)$ is quantitatively similar for $Q = L, v, a$ and can reach a value of $\simeq 10^{-2}$ for $m_Q \simeq 600$ GeV. The rate is enhanced by the axial-vector coupling of the Q to the longitudinal Z boson. When Q is vectorlike (e.g., the h and E exotic fermions), this axial-vector coupling is absent and the ratio is smaller than that for ordinary fermions. However, whereas the mass range of ordinary fermions are bounded by perturbative unitarity, the masses of the vectorlike fermions are not so constrained. For exotic fermion masses in the TeV range, branching fractions for $Q \to qWZ$ of order 1% can be realized. At TeV e^+e^- colliders, decays of this kind are relatively background-free but do not occur with very large rates even for integrated luminosities in the 100-fb^{-1} range. We have also calculated the ratio $\Gamma(t - bW\gamma)/\Gamma(t - bW)$ for the photon bremsstrahlung in top-quark decay, subject to a photon energy cut of 20 GeV.

Two of the authors (W.Y.K. and T.G.R.) would like to thank the Phenomenology Institute at the University of Wisconsin for its hospitality. This work was supported in part by the U.S. Department of Energy under Contracts Nos. DE-AC02-76ER00881, DE-FG02-84ER40173, and W-7405-ENG82 as well as the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

[1]T. G. Rizzo, Phys. Rev. D **35**, 1067 (1987); V. Barger and W.-Y. Keung, Phys. Lett. B **202**, 393 (1988).
[2]For a general discussion of the trilinear gauge-boson vertex, see K. Hagiwara, R. Peccei, D. Zeppenfeld, and K. Hikasa, Nucl. Phys. **B282**, 253 (1987).
[3]X.-G. He and S. Pakvasa, Phys. Lett. **156B**, 236 (1985); Nucl. Phys. **B278**, 905 (1986); U. Türke *et al.*, *ibid.* **B258**, 313 (1985); W. S. Hou, A. Soni, and H. Steger, Phys. Lett. B **192**,

441 (1987); J. L. Hewett, *ibid.* **193**, 327 (1987); J. L. Hewett and T. G. Rizzo, Mod. Phys. Lett. A **3**, 975 (1988); E. A. Paschos and U. Türke, University of Dortmund Report No. 89-0163 (unpublished); S. Pakvasa, H. Sugawara, and S. F. Tuan, Z. Phys. C **4**, 53 (1980).

[4]U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987); G. Costa *et al.*, Nucl. Phys. **B297**, 244 (1988); J. Ellis and G. Folgi, Phys. Lett. B **213**, 526 (1988); V. Barger, H. Baer, K. Hagiwara, and R. J. N. Phillips, Phys. Rev. D **30**, 947 (1984). For an analysis where intergenerational mixing is included, see V. Barger, J. L. Hewett, and T. G. Rizzo, University of Wisconsin Report No. MAD/PH/481, 1989 (unpublished).

[5]See talks given by the CDF Collaboration, G. P. Yeh, UA2 Collaboration, A. Parker, and UA1 Collaboration, D. Cline, at the Symposium on the Fourth Family of Quarks and Leptons, Santa Monica, California, 1989 (unpublished).

[6]M. Chanowitz, M. A. Furman, and I. Hinchliffe, Phys. Lett. **78B**, 285 (1978); Nucl. Phys. **B153**, 402 (1979).

[7]J. L. Hewett and T. G. Rizzo, University of Wisconsin Report No. MAD/PH/446, 1988 (unpublished).

[8]K. Hagiwara and D. Zeppenfeld, Nucl. Phys. **B274**, 1 (1986); K. Hagiwara *et al.*, *ibid.* **B282**, 253 (1987).

[9]T. G. Rizzo, Phys. Rev. D **40**, 1424 (1989).

[10]V. Barger and T. Han, Phys. Lett. B **212**, 117 (1988); V. Barger, T. Han, and R. J. N. Phillips, Phys. Rev. D **39**, 146 (1989).