New tripreon models with semisimple metacolor groups

Gongru Lu* and Bing-Lin Young
Ames Laboratory and Department of Physics, Iowa State University, Ames, Iowa 50011

Xinmin Zhang† and Lingde Wan
Department of Physics, Henan Normal University, Xinxiang, Henan, China
(Received 1 March 1988)

A systematic analysis of the *chiral* tripreon models with $SU(N)\times SO(M)$-metacolor symmetry is made. Unique solutions to the 't Hooft anomaly-matching condition with the Bars constraint are obtained for $SU(3)\times SO(5)$ models which also satisfy complementarity. Both the metacolor group and the gauged color-flavor subgroup of the metaflavor group are asymptotically free. A model which has a low-metacolor-energy scale and three generations of composite fermions with the quantum numbers of quarks and leptons of the standard model is obtained.

I. INTRODUCTION

Among the various proposed preon models,[1] the one we call the chiral tripreon model, in which quarks and leptons are composed of three spin-$\frac{1}{2}$ valence preons, are particularly interesting because of the field-theoretical constraints established recently. Tripreon models are characterized by their local and global symmetries and associated energy scales. One symmetry is the metacolor (MC) group G_{MC}, which describes the local gauge interaction among the preons. The other, called the metaflavor (MF) group G_{MF}, is the global symmetry of the preon. Depending on the symmetry-breaking pattern, an invariant subgroup G_{CF} of G_{MF}, called the color-flavor (CF) group, may be needed. In this case, two energy scales are used to describe the breakdown of the global symmetry of the preons and the appearance of local symmetry of the bound states.

The MC interaction is associated with a confinement scale Λ_{MC} above which physics is described by preons and their local gauge interactions and all other interactions are negligible. Below Λ_{MC} preon condensates as well as preon bound states, which are singlets under G_{MC}, are formed to break the G_{MF} into an invariant subgroup G_{CF}. The G_{CF} is subsequently gauged at a lower-energy scale Λ_{CF}. The gauged G_{CF} becomes the local symmetry of the composite fields. Therefore, the standard-model symmetry $SU(3)_C\times SU(2)_L\times U(1)_Y$ is either identified with or contained in G_{CF}.

One can further imagine the existence of a grand unification scale Λ_{GUT}, above which all the interactions are unified. The three energy scales satisfy the hierarchy relationship

$$\Lambda_{CF}<\Lambda_{MC}<\Lambda_{GUT} . \quad (1)$$

In such a scenario, the composite model could be made relevant to physics in the TeV energy region. Despite this very appealing picture, it is difficult to construct preon models which posses a simple dynamics and lead to a realistic description of quarks and leptons.

Important progress in restricting possible theories for tripreon models has been made recently. Weingarten, Nussinov, and Witten[2] (WNW) have shown that in a vectorlike theory containing spin-$\frac{1}{2}$ constituents P, such as the tripreon model, the masses of the low-lying bound states satisfy an inequality of the form

$$M(\bar{P}\gamma_5 P)\le kM(PPP) , \quad (2)$$

where $M(\bar{P}\gamma_5 P)$ is the mass of the lightest, nonsinglet, pseudoscalar bound state, $M(PPP)$ that of the lightest spin-$\frac{1}{2}$ fermionic bound states, and k is a constant of order unity. Since there are no pseudoscalar bosons as light as the electron, or the up and down quarks, the WNW condition eliminates vectorlike MC interactions, invalidating some of the tripreon models proposed early in the literature. For example, all the type-I models of Ref. 3 based on a single irreducible representation of the $SU(N)$ MC group are ruled out.

Progress along the same line was made in the proof by Vafa and Witten[4] (VW) that in a vectorlike gauge theory the associated chiral symmetry is necessarily broken. Then the constituent fermions gain masses of the order of Λ_{MC} and so do all the bound states. This is in contradiction to the fact that quarks and leptons are practically massless in comparison with the composite scale, which is at least of the order of a few TeV. Thus, vectorlike theories are again ruled out as candidates of the MC interactions. This leaves only the possibility of chiral interactions among preons, such as in some of the type-II models of Ref. 3.

In addition to the above rigorous results, two general principles have been used in the construction of preon models. One is the by-now-classical 't Hooft anomaly-matching condition.[5] The MC currents are required to be free of anomalies, while the anomalies of MF currents in the composite sector must be identical to those in the preon sector. With the help of spectator fields, the gauged group G_{CF} is also free of anomalies. In addition to ensuring the renormalizability of the MC and CF interactions, this anomaly matching is a necessary condition in the preservation of chiral symmetry at the composite level to guarantee the existence of massless bound states.

A second principle which has been applied recently is the complementarity principle.[6] This principle, based on results of lattice gauge calculations,[7] asserts that a certain type of gauge theories that undergoes spontaneous symmetry breakdown (the Higgs phase) is equivalent to that of the corresponding confining theories without spontaneous symmetry breakdown (the confining phase). More specifically, this is to assert that a gauge theory which is spontaneously broken by scalar condensates in the fundamental representation of the gauge group can be continued analytically into the confining phase of the theory. The relevance of complementarity to preon models is that it allows an analysis of the symmetry structure of the preon dynamics, which produces bound states and is, therefore, in the confining phase, in terms of the simpler Higgs phase.

In the Higgs-phase analysis,[8] condensates are allowed to form with nontrivial MC quantum numbers and, therefore, break both the MC and MF symmetries. A final global symmetry is obtained, in the way of a tumbling gauge theory,[9] when the MC group cannot be broken down further. The MC-singlet sector of this surviving global group $G_{\rm CF}$ can be used to identify the quantum numbers of the massless composite fermion. A detailed analysis of simple chiral MC groups in terms of complementarity and tumbling can be found in Ref. 8. Recently, Georgi[10] proposed a moose notation, in which the MC group of a given preon is simple, to facilitate the analysis of the Higgs phase. It should be emphasized that the actual physics of preons occurs in the confinement phase. The Higgs phase is a method used to identify the final unbroken global symmetry which will be gauged as the local symmetry of the composite fermions.

The decoupling requirement originally proposed in Ref. 5 is probably too restrictive as a general principle in the construction of preon models.[11] In spite of this, because of the chiral invariance which is maintained in the confinement phase of the MC-singlet sector, the models considered in this paper satisfy the decoupling requirement, as will be discussed later.

In this paper we make a systematic analysis of chiral tripreon models with semisimple MC groups of the form $G_{\rm MC}={\rm SU}(N)\times{\rm SO}(M)$, where all preons are taken to be left handed. The use of semisimple MC groups in the construction of preon models has been considered in Ref. 12 where several interesting models were investigated. An interesting feature of this class of models is that the MC interactions have two mass scales, a higher scale $\Lambda_{\rm MC}^{(H)}$ and a lower scale $\Lambda_{\rm MC}^{(L)}$, for the ${\rm SO}(5)_{\rm MC}$ and ${\rm SU}(3)_{\rm MC}$, respectively. In the intermediate-energy range between $\Lambda_{\rm MC}^{(L)}$ and $\Lambda_{\rm MC}^{(H)}$, the model can behave like a fermion-boson constituent model. This may help simplify the preon dynamics. However, the gap of the two MC scales cannot be too large. Otherwise the gauge interaction associated with $\Lambda_{\rm MC}^{(L)}$ may be very weak at $\Lambda_{\rm MC}^{(H)}$. In such a case the gauge group associated with the lower MC can be treated as global there. Then the model consists practically of a simple MC group.

In Sec. II we make a brief summary of chiral preon models discussed in the recent literature and motivate our model of semisimple groups for the MC symmetry. In Sec. III, we analyze models with $G_{\rm MC}={\rm SU}(N)\times{\rm SO}(M)$ which consist of at least one set of preons transforming nontrivially in both MC groups. These models can potentially produce interesting composite fermion spectra. The complementarity analysis applied to semisimple MC interactions is presented in Sec. IV using model A of Sec. III as an illustration. In Sec. V we make complete analysis of another $G_{\rm MC}={\rm SU}(3)\times{\rm SO}(5)$ model with a MF group which reproduces the quark-lepton spectrum of the standard model without exotic fermions. This model has a low MC energy scale and satisfies the criteria of a realistic preon model. Discussions are made in Sec. VI, where we will briefly discuss, among other things, the two MC scales in terms of their associated running coupling constants. Some technical details needed are given in Appendixes A and B.

II. SUMMARY AND MOTIVATION

Since vectorlike tripreon models are ruled out by the WNW condition,[2] only complex MC-group representations are allowed in theories of the SU(N) type. If we restrict ourselves to simple groups, the possible candidates for $G_{\rm MC}$ are SU(N) for $N\geq 3$, SO($4M+2$) for $M\geq 2$, and E_6. If we further limit ourselves to a single representation of a simple $G_{\rm MC}$ group, only E_6 is allowed. However, if we abandon the limitation which restricts all preons to a single representation, SU(N) groups in which preons occur in at least two complex representations are also viable.

Several chiral tripreon models with the group structure $[E_6]_{\rm MC}\times{\rm SO}(10)_{\rm CF}$ (Refs. 13–16) have been proposed recently, where the CF group SO(10) originates from global MF groups $G_{\rm MF}={\rm SU}(N)$, $N=16$, 18, and 27. However, in the $G_{\rm MF}={\rm SU}(27)$ case, the metacolor sector is not asymptotically free. The $G_{\rm MF}={\rm SU}(16)$ case predicts an excessive number of exotic composite fermions and consequently loss of asymptotic freedom in the CF sector. The $G_{\rm MF}={\rm SU}(18)$ case predicts a MC scale higher than the grand unification scale, violating Eq. (1).

Another class of models[17] with the symmetry groups ${\rm SU}(N)_{\rm MC}\times{\rm SU}(N+K)_{\rm MF}\times U(1)_{\rm MF}$ for $K=4$ and 5 does not admit realistic composite particle spectra. What we mean by a realistic model is that (a) the model is asymptotically free in the MC and CF sectors, (b) it predicts a minimal composite spectrum of three or four quark-lepton families without an excessive number of exotic fermions, and (c) the model possesses a low metacolor scale satisfying Eq. (1). Recently, Geng and Marshak[18] succeeded in constructing such a realistic model. However, some of the composite fermions there are composed of five preons. This will make the preon dynamics very complicated.

The above summary shows that it is difficult to construct chiral preon models with relatively simple MC dynamics which can produce a realistic quark-lepton spectrum when the MC symmetry is restricted to a simple group. A different approach is called for. A possible route is to extend the MC symmetry to semisimple groups. An apparent argument against using semisimple groups for the metacolor is that it contains more than one coupling constant. However, the merit of considering more complicated structures for the MC group has been emphasized by Georgi.[10] Georgi argued that, because of our ignorance of strongly coupled quantum fields, interesting features may be revealed and better understanding gained if we consider the fundamental fermions transforming simultaneously in more than one group.

III. GENERAL ANALYSES OF $SU(3n)\times SO(M)$ METACOLOR GROUPS

Consider the MC symmetry group $G_{\rm MC} = SU(N)_{\rm MC}\times SO(M)_{\rm MC}$, where the preons are taken to be left handed and their $SO(M)$ representations are restricted to be either the vectorial or the minimal spinorial form. The composite fermions are singlet under both $SU(N)_{\rm MC}$ and $SO(M)_{\rm MC}$. The MF groups are formed by the direct product of special unitary groups as determined by the MC anomaly cancellation and by the required low-energy physics.

In general, preons can occur in various representations of $G_{\rm MC}$, denoted by $P_1, P_2, \ldots, P_s$. The case with only one representation of preon $s=1$ is not possible for anomaly-free chiral MC interactions. Models constructed with $s\geq 3$ in general violate Eq. (1). We will concern ourselves in this paper only with the case $s=2$.

First we note that we can associate two energy scales $\Lambda_{\rm MC}^{(H)}$ and $\Lambda_{\rm MC}^{(L)}$ with the $G_{\rm MC}$: one with $SO(M)_{\rm MC}$ and the other with $SU(N)_{\rm MC}$. Let us consider the following scenario. At the higher scale $\Lambda_{\rm MC}^{(H)}$ preon-preon bosonic bound states start to form. These bound states are singlets under the group associated with $\Lambda_{\rm MC}^{(H)}$. Then the bosonic bound states combine with a third preon to form the desired composite fermions at the lower scale $\Lambda_{\rm MC}^{(L)}$. This requires that the bosonic bound states transform under the MC group of the lower scale as the conjugate representation of the third preon. In the models considered below, the coupling constant of the $SO(M)_{\rm MC}$ evolves faster than that of the $SU(N)_{\rm MC}$. (See Appendix B for more detail.) We can further imagine that the two coupling constants coincide at a super-high-energy scale, e.g., a grand unification scale $\Lambda_{\rm GUT}$. Then $SO(M)_{\rm MC}$ is associated with $\Lambda_{\rm MC}^{(H)}$ and $SU(N)_{\rm MC}$ with $\Lambda_{\rm MC}^{(L)}$.

We choose the $G_{\rm MC}=SU(N)\times SO(M)$ representations of the two species of preons to be

$$P_1=([\mathbf{n}]_{3n},\mathbf{1}) \text{ and } P_2=([\bar{\mathbf{n}}]_{3n},M) , \tag{3}$$

where $N=3n$, $[\mathbf{n}]_N$ is a representation of $SU(N)$ with n totally antisymmetric indices, and $[\bar{\mathbf{n}}]_N$ is the conjugate of $[\mathbf{n}]_N$. Therefore, unless $n=1$ $(N=3)$, the preon will not be in the fundamental representation or the conjugate of $SU(N)$. Let the multiplicities of P_1 and P_2 be N_1 and N_2, respectively. This corresponds to the global-symmetry group $G_{\rm MF}=SU(N_1)\times SU(N_2)$. The anomaly-free condition in the MC sector requires $N_1=N_2M$, independent of n. Two MC singlets of tripreon states with different fermion-number assignments can be formed: $(P_1P_1P_1)$ and $(P_1\bar{P}_2\bar{P}_2)$ (Ref. 19). We will discuss in the following the simplest cases of $n=1,2$, and $(M,N_1,N_2)=(8,16,2)$ and $(5,15,3)$.

We have already commented in Sec. I that the two MC scales and, hence, their associated interaction strengths cannot be too drastically different. This requirement also makes the model satisfy the decoupling theorem since no mass term can be formed due to invariance under both MC groups. For example, the mass term of the form (P_2P_2) transforms like (□,1) or (⊞,1) which is not invariant under $SU(N)$ and hence is not allowed.

A. $n=1$ and $M=5$

The MC group is $G_{\rm MC}=SU(3)_{\rm MC}\times SO(5)_{\rm MC}$ and the minimal global MF group that produces interesting composite spectrum is $G_{\rm MF}=SU(15)_F\times SU(3)_F$. So the left-handed chiral preons are in the $G_{\rm MC}$ representation $P_1=(3,1)$ and $P_2=(\bar{3},5)$ with the multiplicities $N_1=15$ and $N_2=3$, respectively. The quantum-number assignment of the preons and composite fermions can be found in Table I by setting $M=5$.

The anomaly-matching condition[20] involving three $SU(15)_F$ currents is

$$54l_1^- +189l_1^+ +216l_1^0+3l_2+6l_3=3 \tag{4a}$$

and that of three $SU(3)_F$ currents is

$$15(l_2-7l_3)=15 . \tag{4b}$$

Relevant expressions of anomaly coefficients can be found in Appendix A. With the Bars condition,[12] which assumes minimal values for the 't Hooft indices, i.e., $|l_i|\leq 1$, we obtain a unique solution $l_2=1$ and all other indices are zero. This solution predicts the massless composite fermions to be $(P_1\bar{P}_2\bar{P}_2)$ in the (□,□) representation of the MF group $SU(15)_F\times SU(3)_F$, corresponding to 45 composite fermions.

If we gauge the subgroup $G_{\rm CF}=SU(5)$ of $G_{\rm MF}$ (Ref. 16), then the composite fermions are divided into three families of $\mathbf{5}+\overline{\mathbf{10}}$ representations of the SU(5):

$$\mathbf{15}\to\mathbf{5}+\overline{\mathbf{10}} .$$

On the other hand, if we take $G_{\rm CF}=SU(3)\times SU(2)\times U(1)$ as the gauged low-energy subgroup of the $G_{\rm MF}$, the solution contains the following representations of the $G_{\rm CF}=SU(3)\times SU(2)\times U(1)$:

$$(3,1)(-2)+(1,2)(3)+(1,1)(-6) +(3,1)(4)+(\bar{3},2)(-1) , \tag{5}$$

where numbers in the first set of parentheses of each term denote the SU(3)×SU(2) representation and the number in the second set of parentheses is the U(1) quantum number. These particles can be identified with the three generations of quarks and leptons and the $G_{\rm CF}$ is identified with the standard-model group. There are no exotic composite fermions.

TABLE I. Quantum-number assignment of the preons and composite fermions in model A; $M=5$. (All preons are left handed.)

	$SU(3)_{MC}\times SO(M)_{MC}$		$SU(3M)_F\times SU(3)_F$		't Hooft index
Preon					
P_1	$\Box$	1	$\Box$	1	
P_2	$\bar{\Box}$	M	1	$\Box$	
Composite					
$P_1P_1P_1$	1	1	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}$, $\Box\Box\Box$, $\begin{smallmatrix}\Box\Box\\\Box\end{smallmatrix}$	1	l_1^-,l_1^+,l_1^0
$P_1\bar{P}_2\bar{P}_2$	1	1	$\Box$	$\Box$, $\overline{\Box\Box}$	l_2,l_3

B. $n=1$ and $M=8$

The $G_{MC}=SU(3)_{MC}\times SO(8)_{MC}$ assignments of the two preons are $P_1=(3,1)$ and $P_2=(\bar{3},8)$ with multiplicities 16 and 2, respectively. This leads to the global MF group $G_{MF}=SU(16)_F\times SU(2)_F$. The particle content of the model is given in Table II. Anomalies occur only in $SU(16)_F$. the 't Hooft anomaly-matching equation from three $SU(16)_F$ currents is

$$65l_1^-+209l_1^++247l_1^0+l_2^-+3l_2^+=3\ . \tag{6}$$

Taking into account the Bars condition,[12] we obtain a unique solution $l_2^+=1$, and all the other indices vanish. This solution predicts that the massless composite fermions transform in the ($\Box$,) representation of the $G_{FM}=SU(16)_F\times SU(2)_F$. If we take $G_{CF}=SO(10)$ as the gauged CF subgroup of the G_{FM}, the above solution corresponds to three families of the **16** representation of the $SO(10)_{CF}$.

The MC symmetry contains another CF group $G_{CF}=SU(4)_C\times SU(2)_L\times SU(2)_R$ as the low-energy gauged subgroup of the G_{MF}. Then the solution gives the following representation of $G_{CF}=SU(4)_C\times SU(2)_L\times SU(2)_R$:

$$3[(\bar{4},1,2)+(4,2,1)]\ , \tag{7}$$

which corresponds to three generations of quarks and leptons of the Pati-Salam[21] type with massive neutrinos. There are no exotic fermions.

C. $n=2$ and $M=8$

The MC and MF symmetries are

$$G_{MC}\times G_{MF}=SU(6)_{MC}\times SO(8)_{MC}\times SU(16)_F\times SU(2)_F\ .$$

The preon and composite fermion representation contents are similar to those given in Table II with the ($\Box$) and ($\bar{\Box}$) representations of the $SU(3)_{MC}$ being replaced by the **15** ($\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$) and $\overline{\mathbf{15}}$ ($\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$) representations of the $SU(6)_{MC}$, respectively. The 't Hooft anomaly-matching equation is

$$65l_1^-+209l_1^++247l_1^0+3l_2^++l_2^-=15\ . \tag{8}$$

We found that l_1^-, l_1^+, and l_1^0 are zero and the pair, l_2^+ and l_2^-, can have the values

$$(l_2^+,l_2^-)=(5,0),(4,3),(3,6),(2,9),(1,12),(0,15)\ . \tag{9}$$

All sets of solutions in Eq. (9) can potentially give rise to fifteen generations of composite fermions which are included in fifteen **16** representations of the $SU(16)_F$. If we impose the Bars condition,[12] there are no solutions to Eq. (8).

The case $n=2$ and $M=5$ will not be discussed here. We just point out that the solution of the 't Hooft anomaly-matching condition consists of fifteen **15** repre-

TABLE II. Quantum-number assignment of the preons and composite fermions in model B. (All preons are left handed.)

	$SU(3)_{MC}\times SO(8)_{MC}$		$SU(16)_F\times SU(2)_F$		't Hooft index
Preon					
P_1	$\Box$	1	$\Box$	1	
P_2	$\bar{\Box}$	8	1	$\Box$	
Composite					
$P_1P_1P_1$	1	1	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}$, $\Box\Box\Box$, $\begin{smallmatrix}\Box\Box\\\Box\end{smallmatrix}$	1	l_1^-,l_1^+,l_1^0
$P_1\bar{P}_2\bar{P}_2$	1	1	$\Box$	1, $\Box\Box$	l_2^-,l_2^+

sentations of the SU(5). However, there are no solutions when the Bars condition is imposed.

To summarize, we note that models A and B with $n=1$ have nontrivial solutions under the Bars condition[12] and can give rise to three generations of quark and leptons. Furthermore, they have all the required properties of a realistic preon model: (1) The MC dynamics of the preons is chiral and free of MC anomalies; (2) the $n=1$ models can produce reasonable composite fermion spectra; (3) as discussed in Appendix B, both the MC and CF sectors are asymptotically free. This is contrast with some of the $[E_6]_{MC}\times[SO(10)]_{CF}$ models[13–15] which are not asymptotically free.

IV. COMPLEMENTARITY ANALYSIS

The complementarity analysis of the $[E_6]_{MC}$ and $SU(N)_{MC}$ models has been done by several authors.[8,17,18,22] The symmetry structures of the preons in our model is more complicated than that considered in Ref. 10 but is not more complicated in performing the complementarity analysis. In the following we will consider model A of the preceding section as an illustration.

We will first consider the $SO(M)_{MC}$ with a general M and then set $M=5$. The symmetry of the preons is given by the MC and MF groups

$$G_T=G_{MC}\times G_{MF}$$
$$=[SU(3)\times SO(M)]_{MC}\times[SU(3M)\times SU(3)]_{MF}\,,$$

where representations of the two species of preons are

$$P_1=(\mathbf{3},\mathbf{1};\mathbf{3M},\mathbf{1})\ \text{ and }\ P_2=(\bar{\mathbf{3}},\mathbf{M};\mathbf{1},\mathbf{3})\,. \tag{10}$$

We will show that the Higgs and confining phases lead to the same composite particle spectrum and therefore this model satisfies complementarity.

1. Higgs phase

As already noted in Sec. I, the Higgs phase does not correspond to a physical situation. The Higgs-phase analysis is a method used to identify the final unbroken global symmetry.

The appearance of condensates will generally break some of the preon symmetries. We will look for the condensate channel which preserves maximal allowed symmetries. There are two ways to form two-preon condensates: $\langle p_1p_2\rangle$ and $\langle p_2p_2\rangle$. The $\langle p_1p_2\rangle$ condensate is obtained via the $SU(3)_{MC}$ interaction and is an SU(3) singlet. However, this condensate breaks all the symmetry except the $SU(3)_{MC}$. Only the $\langle p_2p_2\rangle$ condensate can produce interesting results. This situation is consistent with the following dynamical consideration. We imagine that the $SU(3)_{MC}$ and the $SO(5)_{MC}$ coupling strengths are comparable at some superhigh energy. As energy decreases to $\Lambda_{MC}^{(H)}$ the $SO(M)$ interaction becomes strong and reaches its critical value. On the other hand, the coupling of the SU(3) which evolves more slowly is still below its critical value at $\Lambda_{MC}^{(H)}$. Hence, condensates will first appear in the p_2p_2 channel.

The condensate

$$\Phi=\langle P_2P_2\rangle\neq 0 \tag{11}$$

which is produced via the $SO(m)$ interaction is in the representation $(\mathbf{n}_1,\mathbf{1};\mathbf{1},\mathbf{n}_2)$ of the G_T, where $\mathbf{n}_1=\mathbf{3}$ or $\bar{\mathbf{6}}$ and $\mathbf{n}_2=\bar{\mathbf{3}}$ or $\mathbf{6}$. The $(\mathbf{3},\mathbf{1};\mathbf{1},\bar{\mathbf{3}})$ representation preserves the maximal allowed symmetries[23] which agrees with the requirement of complementarity.[9] This $(\mathbf{3},\mathbf{1};\mathbf{1},\bar{\mathbf{3}})$ representation breaks the $SU(3)_{MC}$ and $SU(3)_F$ into a diagonal $\widetilde{SU}(3)_F$. Thus, the group G_T breaks down to $G_I=SO(M)_{MC}\times SU(3M)_F\times\widetilde{SU}(3)_F$. The preons P_1 and P_2 are then decomposed according to the representations of the G_I:

$$P_1\to P_{11}(\mathbf{1};\mathbf{3M},\mathbf{3})\,, \tag{12a}$$

$$P_2\to P_{21}(\mathbf{M};\mathbf{1},\mathbf{1})\ \text{ and }\ P_{22}(\mathbf{M};\mathbf{1},\mathbf{8})\,. \tag{12b}$$

Note that the original P_2 branches into P_{21} and P_{22} according to the $\mathbf{1}$ and $\mathbf{8}$ representations of $\widetilde{SU}(3)_F$ obtained from the $\overline{\square}\times\square$ of the $SU(3)_{MC}\times SU(3)_F$. According to Georgi's survival principle,[24] the P_{21} and P_{22}, which belong to real representations of G_I, become massive, leaving $P_{11}=(\mathbf{1};\mathbf{3M},\mathbf{3})$ as the only surviving massless fermion. The tumbling is ended here.

2. Confining phase

The group representation content of the preons and the composite fermions are similar to that of model A of Sec. III and can be found in Table I.. The 't Hooft anomaly-matching conditions are

$$3=l_1^+A(\square\square\square)_{3M}+l_1^-A\left(\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}\right)_{3M}+l_1^0A\left(\begin{smallmatrix}\square\square\\\square\end{smallmatrix}\right)_{3M}$$
$$+l_2D(\square)_3+l_3D(\overline{\square\square})_3 \tag{13a}$$

for three $SU(3M)_F$ currents and

$$3M=3M[l_2A(\square)_2+l_3A(\overline{\square\square})_3] \tag{13b}$$

for three $SU(3)_F$ currents. In the above expressions $A(\ )_N$ and $D(\ )_N$ denote the anomaly coefficient and the dimension of the indicated representations of $SU(N)$. Their values can be found in Appendix A. The solution is $l_2=1$ and all other indices are zero. Then the massless fermions belong to the $(\mathbf{3M},\mathbf{3})$ representation of the $G_{MF}=SU(3M)_F\times SU(3)_F$, identical to the massless particle spectrum in the Higgs phase, i.e., Eq. (12a). Thus, this model satisfies complementarity. In the case $M=5$, the model can give rise to three generations of quarks and leptons under gauged color flavor $G_{CF}=SU(5)$ or $SU(3)\times SU(2)\times U(1)$ (Ref. 16). This second possibility leads directly to the fermion spectra of the standard model and can potentially have a low-energy scale satisfying Eq. (1). However, the first possibility which leads to SU(5) as the color-flavor group violates Eq. (1).

V. A REALISTIC LOW-ENERGY-SCALE TRIPREON MODEL WITH COMPLEMENTARITY

In this section we make a complete analysis, including the 't Hooft anomaly matching and complementarity, of a model similar to model A of Sec. III but with a different MF group. We will show that this model has a low-energy scale and satisfies all the criteria of a realistic preon model stated in the Introduction.

Consider a chiral model with $G_{\rm MC}={\rm SU}(3)_{\rm MC}\times{\rm SO}(5)_{\rm MC}$ and $G_{\rm MF}=G_{321F}\times{\rm SU}(3)'_F$, where $G_{321F}={\rm SU}(3)_F\times{\rm SU}(2)_F\times{\rm U}(1)_F$. The preons are denoted by P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, and P_2. Their representation content are given in Table III.

1. Higgs phase

Following an analysis similar to that in the preceding section we argue that condensates appear in the P_2P_2 channel

$$\Phi'=\langle P_2P_2\rangle\neq0 \qquad (14)$$

which has the representation $(3,1;1,1,0;\bar{3})$ under the preon symmetry group

$$[{\rm SU}(3)\times{\rm SO}(5)]_{\rm MC}\times[G_{321F}\times{\rm SU}(3)'_F]_{\rm MF}\ .$$

The ${\rm SU}(3)_{\rm MC}\times{\rm SU}(3)'_F$ breaks down to the diagonal $\widetilde{\rm SU}(3)$ due to the condensate Eq. (14). The original preons branch into seven states, p_{1j}, $j=1,\ldots,5$, p_{21}, and p_{22}, according to the remaining invariant group

$${\rm SO}(5)_{\rm MC}\times{\rm SU}(3)_F\times{\rm SU}(2)_F\times{\rm U}(1)_F\times\widetilde{\rm SU}(3)_F\ .$$

The representations of the seven preon states are shown in Table IV. The preons P_{21} and P_{22} become massive and all other preons remain massless. Since all the massless fermions are ${\rm SO}(5)_{\rm MC}$ singlets, the tumbling is completed.

2. Confining phase

There are six constraint equations on the anomaly indices obtained from triangle graphs which involve flavor currents $[{\rm SU}(3)_F]^3$, $[{\rm SU}(3)_{\rm CF}]^3$, $[{\rm SU}(3)_F]^2{\rm U}(1)$, $[{\rm SU}(3)_{\rm CF}]^2{\rm U}(1)$, $[{\rm SU}(2)_{LF}]^2{\rm U}(1)$, and $[{\rm U}(1)_Y]^3$:

$$3l_1+2l_2+3l_3+6l_4+l_5-7(3l'_1+2l'_2+3l'_3+6l'_4+l'_5)=15\ , \qquad (15a)$$

$$6l'_1+3l_1+(6l'_3+3l_3)-2(6l'_4+3l_4)+\cdots=0\ , \qquad (15b)$$

$$5l'_1+l_1-(5l'_2+l_2)-2(5l'_3+l_3)+(5l'_4+l_4)+(5l'_5+l_5)=0\ , \qquad (15c)$$

$$2l'_1+l_1-2(2l'_3+l_3)+(2l'_4+l_4)+\cdots=0\ , \qquad (15d)$$

$$6l'_2+3l_2-(6l'_4+3l_4)+\cdots=0\ , \qquad (15e)$$

$$6(6l'_1+3l_1)+9(6l'_2+3l_2)-32(6l'_3+3l_3)+(6l'_4+3l_4)+36(6l'_5+l_5)+\cdots=0\ . \qquad (15f)$$

The solutions are

$$l_1=l_2=l_3=l_4=l_5=1 \qquad (16)$$

and zero for all other indices. There are five massless composite fermions: $(P_{1j}\bar{P}_2\bar{P}_2)$, $j=1,\ldots,5$. Their representation contents under ${\rm SU}(3)_F\times{\rm SU}(2)_F\times{\rm U}(1)_F\times{\rm SU}(3)'_F$ can be found in Table III, where they are all in the fundamental representation (□) of the ${\rm SU}(3)'_F$. Since these composite fermions are identical to those in the Higgs phase, complementarity holds.

The standard-model identification of the composite fermions are given in Table III, where u_f, d_f, ν_f, and l_f, $f=1,2,3$, denote the three generations of quarks and leptons. The superscript c means charge conjugation; hence, u^c_{fL}, etc., $f=1,2,3$, correspond to the three right-handed up-type quarks, etc. In this identification, the global groups ${\rm SU}(3)_F\times{\rm SU}(2)_F\times{\rm U}(1)_F$ are gauged at the scale $\Lambda_{\rm CF}$ to become the standard-model group ${\rm SU}(3)_C\times{\rm SU}(2)_L\times{\rm U}(1)_Y$.

We summarize some of the important features of this

TABLE III. Quantum-number assignment of the preons and composite fermions for the ${\rm SU}(3)_{\rm MC}\times{\rm SO}(5)_C\times G_{321F}\times{\rm SU}(3)'_F$ model. (All preons are left handed.)

	${\rm SU}(3)_{\rm MC}$	$\times{\rm SO}(5)_{\rm MC}$	$\times{\rm SU}(3)_{\rm CF}$	$\times{\rm SU}(2)_{LF}$	$\times{\rm U}(1)_{YF}$	$\times{\rm SU}(3)_F$	't Hooft index	SM particle
Preon								
P_{11}	□	1	$\bar{\square}$	1	$\frac{2}{3}$	1		
P_{12}	□	1	1	□	-1	1		
P_{13}	□	1	$\bar{\square}$	1	$-\frac{4}{3}$	1		
P_{14}	□	1	□	□	$\frac{1}{3}$	1		
P_{15}	□	1	1	1	2	1		
P_2	$\bar{\square}$	5	1	1	0	□		
Composite								
$P_{11}\bar{P}_2\bar{P}_2$	1	1	$\bar{\square}$	1	$\frac{2}{3}$	$\bar{\square}\bar{\square}$,□	l'_1,l_1	d^c_{fL}
$P_{12}\bar{P}_2\bar{P}_2$	1	1	1	□	-1	$\bar{\square}\bar{\square}$,□	l'_2,l_2	ν_{fL},l_{fL}
$P_{13}\bar{P}_2\bar{P}_2$	1	1	$\bar{\square}$	1	$-\frac{4}{3}$	$\bar{\square}\bar{\square}$,□	l'_3,l_3	u^c_{fL}
$P_{14}\bar{P}_2\bar{P}_2$	1	1	□	□	$\frac{1}{3}$	$\bar{\square}\bar{\square}$,□	l'_4,l_4	u_{fL},d_{fL}
$P_{15}\bar{P}_2\bar{P}_2$	1	1	1	1	2	$\bar{\square}\bar{\square}$,□	l'_5,l_5	l^c_{fL}
$P_{1i}\bar{P}_{1j}\bar{P}_{1k}$	1	1				1	$l,\ldots$	

model. (1) The MC dynamics of the preon is chiral and anomaly-free. (2) Using the treatment of Sec. III, we can show that the gauged CF group is asymptotically free in both the preon and the composite sectors. (3) Since the gauged color-flavor subgroup is asymptotically free and is weak at the MC scale, this model is a low-scale model satisfying Eq. (1). (4) The model satisfies complementarity. Therefore, this model is a possible candidate of a realistic preon model.

VI. DISCUSSIONS

We have discussed in detail preon models with the semisimple MC group $SU(3)\times SO(5)$. The models satisfy complementarity and possess unique solutions to the 't Hooft anomaly-matching condition subject to the Bars constraint. Their composite fermion spectra consist of three families and each family contains 15 composite fermions which have the standard-model quantum numbers of quarks and leptons. There are no exotic composite fermions. In addition, these models have the following interesting features. (a) There are two MC energy scales: $\Lambda_{MC}^{(H)}$ and $\Lambda_{MC}^{(L)}$. The models can behave like a fermion-boson model in the energy region between the two MC scales. Hence the dominant confining forces are two-body forces. A detailed analysis of two-scale models will be presented in a separate publication. (b) The models are minimal in the sense that they contain a relatively small number of preon degrees of freedom.

We have argued in the preceding sections that the gap between the two MC energy scales should be reasonably close so that the coupling strength of the gauge interaction associated with the lower-energy scale is not negligible at the higher-energy scale. It is straightforward to demonstrate that our models possess this property. Consider the case $G_{MC}=SU(3)\times SO(5)$ with their fine-structure constant denoted by α_3 and α_5. Take the super energy scale Λ_{GUT}, where the two simple MC groups are "unified," i.e., $\alpha_3(\Lambda_{GUT})=\alpha_5(\Lambda_{GUT})=0.12$. At their respective MC energy scales $\Lambda_{MC}^{(H)}$ and $\Lambda_{MC}^{(L)}$ the two couplings reach their critical values $\alpha^c=\pi/[3C_2(R)]$: $\alpha_5^c=\pi/6$ and $\alpha_3^c=\pi/4$. Then we have $\Lambda_{GUT}/\Lambda_{MC}^{(H)}=3\times10^3$, $\Lambda_{MC}^{(H)}/\Lambda_{MC}^{(L)}=6\times10^2$, and $\alpha_3[(\Lambda_{MC}^{(H)})^2]=0.28\alpha_5^c=0.15$. Taking, e.g., $\Lambda_{MC}^{(L)}=10$ TeV, we have $\Lambda_{MC}^{(H)}=6\times10^3$ TeV and $\Lambda_{GUT}=2\times10^7$ TeV. Hence, we have reasonable gaps among Λ_{GUT}, $\Lambda_{MC}^{(H)}$, and $\Lambda_{MC}^{(L)}$.

TABLE IV. Preon decomposition in the Higgs phase for the $SU(3)_{MC}\times SO(5)_{MC}\times G_{321F}\times SU(3)'_F$ model. (All preons are left handed.)

Preon	$SO(5)_{MC}$	$\times SU(3)_F$	$\times SU(2)_F$	$\times U(1)_F$	$\times \widetilde{SU}(3)'_F$
P_{11}	1	$\overline{\square}$	1	$\frac{2}{3}$	$\square$
P_{12}	1	1	$\square$	-1	$\square$
P_{13}	1	$\overline{\square}$	1	$-\frac{4}{3}$	$\square$
P_{14}	1	$\square$	$\square$	$\frac{1}{3}$	$\square$
P_{15}	1	1	1	2	$\square$
P_{21}	5	1	1	0	1
P_{22}	5	1	1	0	$\begin{array}{ll}\square&\square\\ \square&\end{array}$

Furthermore, the SU(3) MC interaction is not negligible in comparison with that of the SO(5).

Counting particles and antiparticles, helicities, metacolor, and metaflavor, the two $SU(3)_{MC}\times SO(5)_{MC}$ models considered in this paper have 180 degrees of freedom each in the preon sector. In contrast, in a minimal model proposed, for instance, in the first paper of Ref. 22, there are 240 degrees of freedom. The standard model of quarks and leptons has 90 degrees of freedom for three generations and massless neutrinos. This feature of a large number of degrees of freedom is common to all recently known preon models which can produce a reasonable composite fermion spectrum. It has been argued from cosmological considerations[25] that the preon degrees of freedom D_P is greater than that of the quark leptons D_{QL} (including both fermions and gauge bosons) if the transition from the preon phase to the quark-lepton phase is of first order. Hence in the context of the cosmological argument, $D_P > D_{QL}$ is not unreasonable.

There exists a class of models[14–16] which requires an intermediate invariant global-symmetry group $H_{MF}\subset G_{MF}$ and an additional energy scale Λ_{ch}. In these models the anomaly-matching conditions are not satisfied at the level of the MF group. The preon condensates is assumed to take place at $\Lambda_{ch}<\Lambda_{MC}$ to break the global symmetry of the preon. At Λ_{ch} the MF group G_{MF} is broken into an invariant subgroup H_{MF} in which the anomaly matching is enforced. In our model, as in some other models, the anomalies are matched at the level of G_{MF}, the group H_{MF} and the scale Λ_{ch} are not needed, and the energy scales of preon condensates and preon confinement are expected to be the same order of magnitude, which is a property shared by QCD.

APPENDIX A

The dimension and the anomaly coefficient formulas for several representations of the SU(N) group used in our calculation are given by

$$D(\square)=N,\quad D(\square\square)=\frac{N(N+1)}{2}\ .$$

The anomaly coefficients are normalized to $A(\square)=1$, then

$$A(\square\square\cdots\square)=\frac{(N+2m)(N+m)!}{(N+2)!(m-1)!}\ ,$$

$$A(\overline{\square\square\cdots\square})=-A(\square\square\cdots\square)\ ,$$

$$A\left(\begin{array}{c}\square\\ \square\\ \vdots\\ \square\end{array}\right)=\frac{(N-2m)(N-3)!}{(N-m-1)!(m-1)!}\ ,$$

where the above representations are made of m fundamental representations ($\square$), and

$$A\left(\begin{array}{ll}\square&\square\\ \square&\end{array}\right)=D(\square)A\left(\begin{array}{c}\square\\ \square\end{array}\right)+D\left(\begin{array}{c}\square\\ \square\end{array}\right)A(\square)-A\left(\begin{array}{c}\square\\ \square\\ \square\end{array}\right)-A\left(\begin{array}{c}\square\\ \square\\ \square\end{array}\right)=N^2-9\ .$$

APPENDIX B

To lowest order, the β function is given by the well-known formula

$$\beta = -\left[11C_2(G) - 2\sum T(R)\right] \Big/ 48\pi^2 .$$

For SU(N), $C_2(G)=N, T(R)=\frac{1}{2}$ for the fundamental representation, and $T(R)=(N-1)/2$ for the antisymmetric rank-2 tensor representation. For SO(M), $C_2(G)=M-2$, and $T(R)=1$ for the **M** representation. The sum in $T(R)$ is over all the "flavors" and all participating representations R of the fermions involved.

In the model being considered, each species of preons or composite fermions occur only in one representation (including the complex-conjugate representation) of a given group. The condition of asymptotic freedom (ASF) is, therefore, determined by the multiplicity, i.e., the number of flavors, of a given representation. We enumerate the maximum multiplicities N_j of fermions of each relevant representation which ensures ASF: $N_2=21$ for fundamental representation of SU(2); $N_3=32$ for the fundamental representation of SU(3); $N_5=16$ for SO(5) in the **5** representation; $N_6=16$ for the **15** representation of SU(6); and $N_8=32$ for SO(8) in the **8** representation.

The above condition on the multiplicities of the preons and composite fermions can be compared with the actual multiplicities of models A and B in Sec. III: $N_3=30$ and $N_5 \to 9$ in model A; $N_3=32$ and $N_6=6$ in model B. The ASF of the CF groups of model A is obvious since the composite fermion spectra are similar to that of the quarks and leptons of the standard model. For model B, the CF group has $N_4=12$ and $N_2=12$ for both the left- and right-handed SU(2). Hence both the CM and CF groups are asymptotically free.

We also note that in model A of Sec. III and the model of Sec. V, $\beta=-1/16\pi^2$ and $-5/16\pi^2$ for the $SU(3)_{MC}$ and $SO(5)_{MC}$, respectively. For model B of Sec. III, $\beta=-1/48\pi^2$ and $-11/48\pi^2$. Hence, the SO(M), $M=5$ and 8 will evolve at a faster rate than SU(3) as energy changes.

ACKNOWLEDGMENTS

This work was supported by U.S. Department of Energy Contract No. W-7405-Eng-82, Office of Energy Research (KA-01-01), Division of High Energy and Nuclear Physics, and by the National Natural Science Foundation, China.

---

*On sabbatical leave from the Department of Physics, Henan Normal University, Xinxiang, Henan, China.

†Present address: DESY, D-2000, Hamburg, Federal Republic of Germany.

[1]For a review of early literature, see L. Lyons, Prog. Part. Nucl. Phys. **10**, 227 (1982). For a more recent development, see W. Buchmüller, Acta Phys. Austriaca Suppl. **XXVII**, 517 (1985); R. D. Peccei, in *Selected Topics in Electroweak Interactions,* proceedings of the Second Winter Institute on New Frontiers in Particle Physics, Lake Louise, Canada, 1987, edited by J. M. Cameron *et al.* (World Scientific, Singapore, 1987); H. Harari, in *Proceedings of the XXIIIrd International Conference on High Energy Physics,* Berkeley, California, 1986, edited by S. C. Loken (World Scientific, Singapore, 1987).

[2]D. Weingarten, Phys. Rev. Lett. **51**, 1830 (1983); S. Nussinov, *ibid.*, **51**, 2081 (1983); E. Witten, *ibid.*, **51**, 2351 (1983).

[3]C. H. Albright, Phys. Rev. D **24**, 1969 (981).

[4]C. Vafa and E. Witten, Nucl. Phys. **B234**, 173 (1984).

[5]G. 't Hooft, in *Recent Developments in Gauge Theories,* edited by G. 't Hooft *et al.* (Plenum, New York, 1980).

[6]S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. **B173**, 208 (1980).

[7]K. Osterwalder and E. Seiler, Ann. Phys. (N.Y.) **110**, 440 (1978); E. Fradkin and S. H. Shenkar, Phys. Rev. D **19**, 3682 (1979); T. Banks and E. Rabinovici, Nucl. Phys. **B160**, 349 (1979).

[8]J. L. Goity, R. D. Peccei, and D. Zeppenfeld, Nucl. Phys. **B262**, 95 (1985).

[9]S. Raby, S. Dimopoulos, and L. Susskind, Nucl. Phys. **B169**, 373 (1980).

[10]H. Georgi, Nucl. Phys. **B266**, 274 (1986).

[11]J. Preskill and S. Weinberg, Phys. Rev. D **24**, 1059 (1981).

[12]I. Bars, Nucl. Phys. **B208**, 77 (1982).

[13]J. M. Gipson, Y. Tosa, and R. E. Marshak, Phys. Rev. D **32**, 284 (1985).

[14]V. Silverira and A. Zee, Phys. Lett. **157B**, 191 (1985).

[15]Y. Okamoto and R. E. Marshak, Phys. Lett. **162B**, 333 (1985).

[16]Xiaoyuan Li and R. E. Marshak, Nucl. Phys. **B268**, 383 (1986).

[17]C. Q. Geng and R. E. Marshak, Phys. Rev. D **35**, 2278 (1987).

[18]C. Q. Geng and R. E. Marshak, Z. Phys. C **35**, 513 (1987).

[19]Singlet representations are contained in the products

$$\mathbf{3}\times\mathbf{3}\times\mathbf{3}=\mathbf{1}+2(\mathbf{8})+\mathbf{10} \quad \text{for SU(3)} ,$$

$$\mathbf{15}\times\mathbf{15}\times\mathbf{15}=\mathbf{1}+2(\mathbf{35})+\mathbf{175}+3(\mathbf{189})+\mathbf{280} + \overline{\mathbf{490}}+\mathbf{896} \quad \text{for SU(6)} ,$$

$$\mathbf{8}\times\mathbf{8}=\mathbf{1}+\mathbf{28}+\mathbf{35} \quad \text{for SO(8)} ,$$

and

$$\mathbf{5}\times\mathbf{5}=\mathbf{1}+\mathbf{10}+\mathbf{14} \quad \text{for SO(5)} .$$

[20]Equations (4a) and (4b) are special cases of Eqs. (13a) and (13b).

[21]J. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).

[22]T. Kobayashi, Phys. Lett. B **180**, 107 (1986); H. P. Hill and S. Raby, Nucl. Phys. **B189**, 93 (1981); J.-M. Gerard, Y. Akamoto, and R. E. Marshak, Phys. Lett. **169B**, 386 (1986).

[23]The representations $(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{6})$ and $(\overline{\mathbf{6}},\mathbf{1};\mathbf{1},\overline{\mathbf{3}})$ will break all the symmetries except the $SO(m)_{MC}\times SU(3M)_{MF}$. The representation $(\overline{\mathbf{6}},\mathbf{1};\mathbf{1},\mathbf{6})$ which has the symmetry $SO(M)_{MC}\times SU(3M)_{MF}\times SU(3)$ similar to the (3,1;1,3) case would leave too many Goldstone bosons not absorbed by gauge bosons.

[24]H. Georgi, Nucl. Phys. **B156**, 126 (1979).

[25]H. Nishimura and Y. Hayashi, Phys. Rev. D **35**, 3151 (1987).