

Evidence for higher twist from $R = \sigma_L / \sigma_T$ data in deep-inelastic electron scattering

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(Received 28 February 1989)

We compare recent precision results and corrected previous data for $R = \sigma_L / \sigma_T$ with the predictions of QCD. We find evidence for a higher-twist contribution of the expected size, according to the inclusive theory.

In the recent precision data of $R = \sigma_L / \sigma_T$, the longitudinal and transverse photon (γ^*) absorption cross-section ratio in electron scattering on deuterium, there is room for high-twist effects when compared with the correct QCD predictions including target-mass contributions.¹ In fact we show that the combined SLAC experiments² favor a dynamical twist-4 contribution of the expected magnitude, as given by the nucleon size $(0.2 \text{ GeV})^{-1}$. That is to say that the data cannot be reproduced with the logarithmic dependence on the invariant momentum transfer, $\ln Q^2$, of the leading-twist QCD (ordinary parton model) and the Q^{-2} powers from the target-mass corrections in a rigorous perturbative analysis. This discrepancy can be explained with the theory of inclusive power corrections in terms of the only relevant twist-4 contribution, with a strength (assuming a well-understood x dependence) corresponding to a parton correlation around 1 fm. This kind of signal is not so compelling as in other predictions of perturbative QCD as befits the stronger nonperturbative content of higher twist.³ But it is the best one can expect from the general theory of inclusive scattering,^{4,5} since in contrast with other structure functions, R is especially sensitive to these contributions, and taking also into account that no new data are expected in the near future, apart from the few points at large x of the E140 Collaboration which are still being analyzed.⁶ Moreover because of the fortuitous simplicity of twist 4 in the longitudinal structure function $F_L = (1 + 4x^2 M^2 / Q^2) F_2 - 2xF_1$, which can be expressed in terms of only one unknown matrix element, the observable $R = F_L / 2xF_1$ (where $x = Q^2 / 2p \cdot q$ and $p^2 = M^2$ is the target mass) is the clearest example of twist analysis in structure functions. The twist-4 contribution to the standard transverse and parity-violating structure functions has several unknown quark-quark correlations in addition to the previous quark-gluon matrix elements, which prohibits an analogous phenomenological analysis and obscures the interpretation of the effective parametrizations used in the ordinary fits.

R vanishes for free spin- $\frac{1}{2}$ quarks in the massless case (scaling limit)⁷ and its value, which is a correction to the Callan-Gross relation⁸ (helicity conservation for photon couplings of massless quarks), has to be small and decreasing as $1/\ln Q^2$ for ordinary gluon radiation, and with Q^{-2} for parton correlations (referred heuristically as transverse momentum) corresponding to higher-twist and mass corrections, which are the only three contributions

in QCD. Their dependence on Q^2 is organized by different expansions in the running strong coupling constant $\alpha(Q^2)$, M^2/Q^2 , and α_{eff}/Q^2 , respectively. The first two are well established in principle, but due to the delicate consistency of the approximations one uses and the conflicting prescriptions, it is important to specify clearly their expressions which, after comparison with data, shall decide on the hypothetical higher-twist contribution governed by an unknown effective coupling constant α_{eff} we include just to stress its dynamical character. In any case the expressions define precisely what we call twist 4 in this paper and allow the comparison with other criteria or prescriptions.

We denote by $f_i^{\text{QCD}}(x, Q^2)$ the ordinary leading-twist structure functions which evolve in Q^2 according to the Altarelli-Parisi (AP) equations, with kernels which are series in $\alpha(Q^2)$ and control the perturbative consistency of the approach. The longitudinal structure function $f_L^{\text{QCD}} = f_2^{\text{QCD}} - 2xf_1^{\text{QCD}}$ has always an additional factor $\alpha(Q^2)$ (because of the Callan-Gross relation) and it can be expressed in terms of f_2^{QCD} and the gluon distribution function $G(x, Q^2)$ (Ref. 9). So f_L^{QCD} is a pure QCD second-order effects once f_2^{QCD} and G are fixed through a fit of the AP equations to the data. At the leading order (LO), which is most likely not corrected by more than 10% by the controversial next-to-LO (NLO) results,¹⁰ one has

$$f_L^{\text{LO}}(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} \frac{8}{3} f_2^{\text{LO}}(y, Q^2) + 4 \left[\sum e_i^2 \right] \frac{\alpha(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} (1-x/y)y G^{\text{LO}}(y, Q^2), \tag{1}$$

$$f_2^{\text{LO}}(x, Q^2) = 2xf_1^{\text{LO}}(x, Q^2) = \sum_i e_i^2 x(q_i + \bar{q}_i), \tag{2}$$

where q^{LO} and G^{LO} are the quark and gluon densities which are the solution of the AP equations at the LO (Ref. 11). Notice that one is implicitly neglecting corrections of order $\alpha(Q^2)$ in Eq. (2) and $\alpha^2(Q^2)$ in Eq. (1) (Ref. 12). One has to include the charm-quark c effects which modify both the AP equations and Eq. (1) due to the process $\gamma^* + \text{gluon} \rightarrow c\bar{c}$. The modification in Eq. (1) amounts to replacing $(\sum e_i^2)$ by $\frac{2}{3}$ and to add the contribution¹³

$$\begin{aligned} & \frac{\alpha}{2\pi} \theta(1-ax) \int_{ax}^1 \frac{dy}{y} 4\left(\frac{a}{y}\right) f_c(x/y, Q^2) y G^{\text{LO}}(y, Q^2), \\ f_c(z, Q^2) &= z^2(1-z)v \frac{2m_c^2}{Q^2} z^3 \ln \frac{1+v}{1-v}, \\ a &= 1 + \frac{4m_c^2}{Q^2}, \quad v^2 = 1 - \frac{4m_c^2}{Q^2} \frac{z}{1-z}. \end{aligned} \quad (3)$$

These effects, which have been included with $m_c = 1.5$ GeV are very small.

To account for the target-mass corrections one has to

$$\int_0^1 dx x^{n-2} F_2(x, Q^2) = \sum_{j=0}^{\infty} (M^2/Q^2)^j \frac{(n+j)!}{j!(n-2)!} \frac{1}{(n+2j)(n+2j-1)} \sum_{i,\tau} (1/Q^2)^{(\tau-2)/2} C_{n+2j,\tau}^i(Q^2) A_{n+2j,\tau}^i.$$

From this expression one can either project out the contribution of a given spin to obtain the Nachtmann moments^{14,16} in the variable $\xi = 2x / (1 + \sqrt{1 + 4x^2 M^2 / Q^2})$ of ordinary structure functions, or combine summations in j with moment inversion techniques to obtain ξ -dependent expressions for the structure functions.¹⁷ This procedure, as discussed extensively in many places,¹⁸ implies a mismatch between the physical range of x ($0 \leq x \leq 1$) and the final expressions for the structure functions,^{19,20} which is ignored in practice in many analyses. A natural and convenient solution for these ξ -scaling well-known pathologies,¹⁹ especially in a higher-twist analysis,¹⁸ is not to perform the summation of the series in M^2/Q^2 but retain terms up to the relevant order in $1/Q^2$ (given by the maximum twist one includes in the analysis). This procedure has been considered in covariant models,²¹ but it has been seldom used in practice.²² The expressions containing the LO leading-twist (LT) contributions and the $O(M^2/Q^2)$ target-mass corrections we compare with the data read

$$\begin{aligned} F_2^{\text{LT}}(x, Q^2) &= f_2^{\text{LO}}(x, Q^2) \\ &+ \frac{x^2 M^2}{Q^2} \left[6x \int_x^1 \frac{dy}{y^2} f_2^{\text{LO}}(y) - 4f_2^{\text{LO}}(x) \right. \\ &\quad \left. - x \frac{d}{dx} f_2^{\text{LO}}(x) \right], \\ 2xF_1^{\text{LT}}(x, Q^2) &= f_2^{\text{LO}}(x, Q^2) + \frac{x^3 M^2}{Q^2} \left[2 \int_x^1 \frac{dy}{y^2} f_2^{\text{LO}}(y) \right. \\ &\quad \left. - \frac{d}{dx} f_2^{\text{LO}}(x) \right], \\ F_L^{\text{LT}}(x, Q^2) &= f_L^{\text{LO}}(x, Q^2) + \frac{x^3 M^2}{Q^2} \left[4 \int_x^1 \frac{dy}{y^2} f_2^{\text{LO}}(y) \right. \\ &\quad \left. - \frac{d}{dx} f_L^{\text{LO}}(x) \right]. \end{aligned} \quad (4)$$

start from the operator-product expansion (OPE) of the two currents which gives the inelastic cross sections in terms of c -number coefficients C (related to partonic hard processes) and the corresponding local operator matrix elements A whose traces contain explicitly the target mass. Handling the tensor algebra and dispersion relations¹⁴ the moments of the structure functions are obtained as power series in M^2/Q^2 , whose terms are themselves expansions in powers of $1/Q^2$ given by the twist τ (canonical dimension minus spin of the operators¹⁵), with additional logarithmical dependence on Q^2 . In the case of, e.g., F_2 they read

We use the next-order contributions $O(M^4/Q^4)$ to control the uncertainty of the approximation, considering just the data in a kinematical range where these corrections are less than a certain value (we choose 20%) in order for the expansion to make sense.²³ We are neglecting terms of the order $\alpha(Q^2)$, $\alpha(Q^2)M^2/Q^2$, and m^4/Q^4 in F_2^{LT} and $2xF_1^{\text{LT}}$, and $\alpha^2(Q^2)$, $\alpha(Q^2)M^2/Q^2$, and M^4/Q^4 in F_L^{LT} . We treat independently the expansions in $\alpha(Q^2)$ and M^2/Q^2 , since their relative value depends very much on Q^2 . Notice that even if f_L vanishes there is a nonvanishing contribution to R , but smaller than the naive $4x^2 M^2/Q^2$.

Equations (1)–(4) show clearly that F_L^{LT} is fixed by the theory once F_2^{LO} and G^{LO} are known. To account for the theoretical uncertainty on these structure functions we have used different parametrizations of the parton distributions including the one used in Ref. 1, but we only present the results obtained with Ref. 24 (DO1 and DO2, with $\Lambda_{\text{QCD}} = 200$ and 400 MeV, respectively), which produce the largest band in which the results from the other parametrizations are contained. This uncertainty band is signaled by the shadowed area in Figs. 1 and 2 (bottom band) meant to visualize the results. In fact the size of this band is larger than the uncertainty coming from the not-considered NLO effects ($\sim 10\%$) (Ref. 10), and of the same order as the uncertainty from the $O(M^4/Q^4)$ effects ($< 20\%$ in our case). So we take it as an estimate of the total theoretical uncertainty of leading-twist and target-mass contributions at this level. Some experimental data from other higher-energy experiments²⁵ are included for comparison in these figures, which show the main part of the SLAC experimental data for the kinematical range where the $O(M^4/Q^4)$ correction is less than 20% (34 for DO1 and 36 for DO2). Moreover the figures give the predictions of both ordinary QCD with target mass and our twist-4 fits (see below) for the kinematical range where the new E140 data will be soon available.⁶

One sees that the QCD predictions for R at leading twist with target-mass corrections we have discussed above are not in agreement with the data (we shall give a

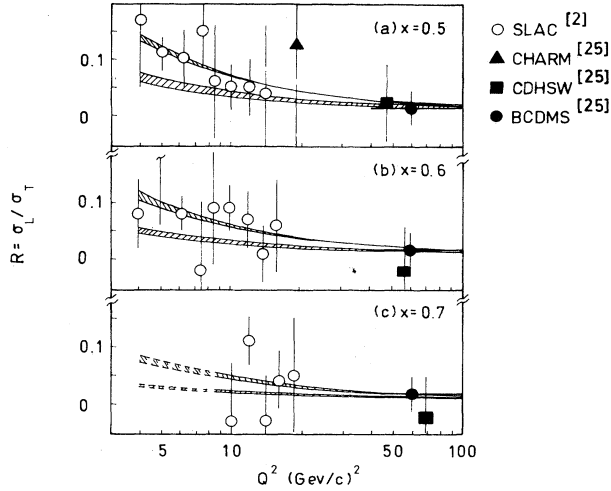


FIG. 1. The values of R at (a) $x=0.5$, (b) $x=0.6$, and (c) $x=0.7$ are shown as a function of Q^2 . The bottom band corresponds to the prediction of perturbative QCD with target-mass effects using different standard parametrizations for the parton distribution functions (Ref. 24). The top band includes the twist-4 contribution we have fitted. Dot-shaded bands are in the kinematical range where the $O(M^4/Q^4)$ corrections are larger than 20%, and hence have not been considered in the fits.

probabilistic meaning to this statement below) and one has then to consider the next higher-twist term with $\tau=4$ (Ref. 26), which has been worked out long since.⁵ It is only an (extremely) technical complication of the $\tau=2$ ordinary analysis, which implies that there are too many independent matrix elements (or parton correlation functions) to carry out a similar analysis as the standard leading-twist one (AP evolution). Fortunately enough, the result for the twist-4 tree-level contribution to F_L is simple:^{5,27}

$$F_L^{\tau 4}(x, Q^2) = \frac{8\kappa^2}{Q^2} T_-(x), \quad (5)$$

where κ^2 is the unknown twist scale and $\kappa^2 T_-(x)$ can be interpreted as the correlation function of a quark and a

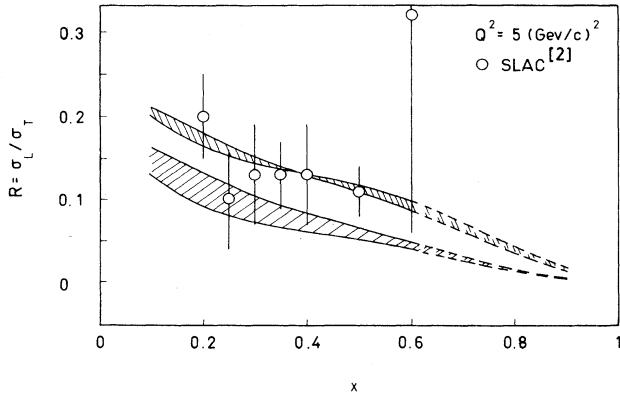


FIG. 2. The values of R at $Q^2=5 \text{ GeV}^2$ are shown as a function of x . The meaning of the shaded bands is the same as in Fig. 1.

gluon with opposite helicity signs and total hadron momentum fraction x . It is known to satisfy a bound in terms of the target mass which ensures $F_L \geq F_L^{\text{LT}}$ (Ref. 28). One can now assume that $T_-(x)$, the probability for a quark and a gluon to carry a given momentum fraction, is given in the large- x limit (relevant for higher twist) essentially by the quark distribution.²⁹ This corresponds to the approximate equality of the $\tau=4$ quark-gluon operator matrix elements in the OPE (Ref. 30) and it leaves only one unknown parameter: the twist-4 quark-gluon correlation scale κ^2 , given here by the proportionality factor between the matrix elements of $\tau=4$ and $\tau=2$ operators. Then the expression of the twist-4 contribution to F_L reads

$$F_L^{\tau 4}(x, Q^2) = \frac{8\kappa^2}{Q^2} f_2^{\text{LO}}(x) \quad (6)$$

which can be fitted to the discrepancies between the data and F_L^{LT} . Strictly, this procedure gives a bound for the additional contributions including higher twists, but with the previous assumption, and the arguments for the consistency of the LO and the presumably small NLO contribution, it can serve as a good estimation of the twist-4 contribution.

The results of a standard minimum- χ^2 fit to the SLAC data^{1,2} with $Q^2 \geq 4 \text{ GeV}^2$ gives for the scale $\kappa^2 = 0.05 \pm 0.01 \text{ GeV}^2$ ($\chi^2 = 22/33$) for DO1, and $0.03 \pm 0.01 \text{ GeV}^2$ ($\chi^2 = 23/35$) for DO2, which are shown in the figures (top band). The higher-energy data included in these figures are clearly compatible with the result, which in practice means that they are not sensitive to twist-4 contributions of this size.

Once the optimal value of κ^2 is obtained, we analyze the degree of confidence of the evidence of higher twist it implies and the accuracy of its order of magnitude in detail. We do it using standard decision theory and interval estimation based on the Kolmogoroff test applied to the deviations of experimental and theoretical values of R through their standardized variable, taking the errors as Gaussian. At a given confidence level, the test is passed according to the maximum distance between the Gaussian probability distribution and a rising, staircaselike function constructed from the data.³¹

This way one can ensure with more than 99% confidence that F_L^{LT} does not explain the data. Taking into account that we have considered the theoretical uncertainty on F_L^{LT} as well, this result is clear evidence that additional contributions are required by the data, and hence of higher twist.

Concerning the magnitude of the contribution, we have calculated confidence intervals for κ^2 assuming these extra contributions are given by Eq. (6). For a probability content of 99% the results are $\kappa^2 = 0.05^{+0.02}_{-0.03}$ for DO1, and $\kappa^2 = 0.03^{+0.04}_{-0.02}$ for DO2, showing clearly that $\kappa^2 \neq 0$.

Therefore we conclude that the new precision data¹ and the revised previous results² for R , which is the structure function where the largest higher-twist signal is expected, favor in fact a twist-4 dynamical component, which is partially understood theoretically and with a strength $\kappa^2 \simeq 0.05 \text{ GeV}^2$, within a reasonable parametriza-

tion. This quark-gluon correlation scale corresponds to the expected sizes of a p_T of 200–300 MeV in the naive parton picture, and it is not in conflict with other deep-inelastic data analysis. In fact, the well understood sign of this gluon twist-4 contribution relative to the mass correction is different for F_2 and F_3 which could explain the apparent contradiction between neutrino and muon analysis.²⁸ But more important in principle, one has to consider also the additional twist-4 quark-quark contributions, making a complete analysis, which is in progress, very difficult. A tentative μ -data fit³² gives the same mag-

nitude for the gluon contribution, but it is not yet conclusive for the quarks. In any case, since they can be related in part by the equations of motion, their order of magnitude cannot be very different.

We thank A. Bodek, S. Dasu, and S. Rock for useful details on the E140 experiment, R. G. Roberts for comments, and G. Parente for carefully checking the computations. This work was partially supported by Comisión Interministerial de Ciencia y Tecnología (CICYT), Spain.

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