Checking the S matrix in QCD in axial gauges within the generalized Leibbrandt-Mandelstam prescription

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We apply the Leibbrandt-Mandelstam prescription to QCD within a family of gauges comprising the homogeneous axial gauge and the planar gauge. We calculate to one-loop order the UV-divergent parts of the quark self-energy, the quark-quark-gluon vertex, and the four-quark one-particle-irreducible vertex. The results are found to satisfy the Slavnov-Taylor-Lee identity. Summing up the contributions to the S-matrix element for quark-quark scattering, we show that the result is indeed independent of the gauge parameters n_μ and α and of the auxiliary vector n^*_μ. The nonlocal counterterms persisting in Green's functions for $n^2 \neq 0$ are thus compatible with gauge independence of physical quantities. The Lorentz-noninvariant divergent structures (local as well as nonlocal ones) turn out to be proportional to the classical equations of motion.

I. INTRODUCTION

Noncovariant gauge choices containing a gauge-fixing term characterized by a constant four-vector n_μ have been studied and used for more than 15 years. These axial-type gauges[1] have shown their advantages in various fields. Important cases are the light-cone (LC) gauge, $n^2=0$, which has facilitated many considerations in supersymmetric and string theories,[2,3] and the temporal gauge $n_\mu=(1,0,0,0)$ which, for example, has recently been used to investigate QCD at finite temperature.[4] However, already the quantization of Yang-Mills (YM) theories in the homogeneous axial gauge $nA=0$ is a nontrivial task and different techniques have been used for different choices of n_μ. If one agrees to use the translation-invariant gauge field propagator

$$\Delta^{ab}_{\mu\nu}(q)=\frac{-i\delta^{ab}}{q^2+i\epsilon}\left[g_{\mu\nu}-\frac{q_\mu n_\nu+q_\nu n_\mu}{qn}+\frac{n^2 q_\mu q_\nu}{(qn)^2}\right], \tag{1.1}$$

one has to define a prescription to regularize the singularities $(qn)^{-\beta}$, $\beta>0$, in momentum-space Feynman integrals. For a long time, the principal-value (PV) prescription

$$\mathbf{P}\frac{1}{(qn)^\beta}\equiv\lim_{\epsilon\to+0}\frac{1}{2}\left[\frac{1}{(qn+i\epsilon)^\beta}+\frac{1}{(qn-i\epsilon)^\beta}\right] \tag{1.2}$$

was considered the correct technique to handle these poles. It obeys power counting; hence, the usual concepts of renormalization can be applied, allowing us to establish the unitarity and gauge independence of S-matrix elements.[5] Unfortunately, this prescription fails for $n^2=0$ (Refs. 6 and 7). In Wilson loop calculations[8] it turns out to be inconsistent in the temporal gauge too. Until recently[9,10] a consistent treatment of the temporal gauge seemed to require a modification of the propagator (1.1) itself.[11] The PV prescription runs into difficulties above the one-loop level anyway, because $[\mathbf{P}(qn)^{-\beta}]^2$ is not well defined. The consistency of using different vectors n_μ for different loop orders in intermediate stages of the calculation[5] has not been verified as yet.

For the LC gauge two equivalent versions of a prescription have been introduced independently by Leibbrandt[7] and Mandelstam[2] (LM):

$$\left[\frac{1}{(qn)^\beta}\right]_{\mathrm{LM}}=\frac{(qn^*)^\beta}{(qnqn^*+i\epsilon)^\beta}, \quad \epsilon>0, \tag{1.3}$$

where $n_\mu=(n_0,\mathbf{n})$ and $n^*_\mu=(n_0,-\mathbf{n})$ which was shown afterwards to emerge quite naturally from a Hamiltonian quantization of YM theories in this gauge by Bassetto, Dalbosco, Lazzizzera, and Soldati.[12] With the help of a gauge-fixing term containing n^*_μ, this prescription has also been justified by Andrasi and Taylor.[13]

The LM prescription has some advantage over the PV technique, because it regularizes products of poles $(pn)^{-1}(kn)^{-1}$ without any δ functions in the decomposition formula

$$\frac{1}{qn}\frac{1}{pn-qn}=\frac{1}{pn}\left[\frac{1}{qn}+\frac{1}{pn-qn}\right]. \tag{1.4}$$

This very splitting formula, however, together with the presence of a second four-vector n^*_μ now gives rise to *nonlocal* (nonpolynomial) contributions to the divergent parts of one-particle-irreducible (1PI) vertex functions. Thus the renormalization of YM theories in the LC gauge seems to require nonlocal counterterms. Fortunately, it was soon pointed out[14] that S-matrix elements are not affected by them. (Indeed, nonlocal divergences do not show up in the two-component formalism, where only physical degrees of freedom are present.[15]) The structure of these nonlocalities has been investigated by Bassetto, Dalbosco, and Soldati[14] invoking cancellation of Lorentz-noninvariant divergent terms in the S matrix and by Skarke and Gaigg[16] using extended Becchi-Rouet-Stora-Tyutin (BRST) invariance. It was shown that nonlocal divergences are no longer present in Green's functions and that all nonlocal counterterms are generated by expanding $(DFn)(nD)^{-1}(nFn^*)$, where $D_\mu^{ab}=\partial_\mu\delta^{ab}-gf^{abc}A_\mu^c$, in powers of $(n\partial)^{-1}$.

It also turned out[17,18] that the LM prescription yields consistent results off the LC, $n^2\neq 0$, and it was generalized to $n_\mu^*\neq(n_0,-\mathbf{n})$ by Gaigg and Kreuzer[19] and by Leibbrandt.[20] The integrals hitherto obtained are equivalent[21] in these two approaches. This generalization has later been derived in a Hamiltonian quantization procedure by Lazzizzera[9] and in another one where n_μ^* is already included in the Lagrangian by Burnel.[22] The possibly strongest support so far, however, comes from Wilson loop calculations in the temporal gauge, as Hüffel, Landshoff, and Taylor[10] obtained the correct time exponentiation behavior with the propagator (1.1) and prescription (1.3) where $n_\mu=(1,0,0,0)$ and $n_\mu^*=(0,1,0,0)$.

While this prescription seems to allow a uniform treatment of $nA=0$ for all values of n_μ, there are also severe drawbacks, concerning the renormalization procedure. As soon as $n^2=0$ is dropped, the structure of counterterms becomes much more complicated and no stringent ordering principle is seen as yet. Moreover, divergent nonlocalities now persist also in Green's functions.[23] Thus it is of vital importance to check that they do not affect physical quantities, in particular the S matrix. A first check of this fact is the aim of this paper.

The paper is organized as follows. In Sec. II we apply the generalized LM prescription to a one-parameter family of axial gauges comprising the homogeneous axial gauge and the planar gauge. We calculate the UV-divergent parts of the quark self-energy, the quark-quark-gluon vertex, and the four-quark 1PI vertex to one-loop order. The structure of counterterms is more complicated than in the LC gauge, and the results are found to agree with the Slavnov-Taylor-Lee identity. In Sec. III we sum up the UV-divergent contributions to the S-matrix element of quark-quark scattering. All Lorentz-noninvariant structures, in particular the nonlocal terms, vanish, and the UV-divergent part of this matrix element is the same as in covariant gauges. This cancellation is traced back to the fact that Lorentz-noninvariant counterterms are proportional to the classical equations of motion.

II. QCD IN A CLASS OF AXIAL GAUGES

The homogeneous axial gauge $nA=0$ can be generalized to inhomogeneous gauges in various ways.[24] One of these generalizations leads to a simplified gluon propagator[25] if an additional gauge parameter α is chosen appropriately (planar gauge). For the present considerations, however, it is of more interest that this family of gauges yields n_μ-dependent UV divergences already in the PV prescription,[24,25] provided $\alpha\neq 0$. Hence, it is a suitable model to investigate the behavior of such terms in the S matrix. This family of gauges has recently been studied by Kummer,[26] who presented a very concise derivation of the Slavnov-Taylor-Lee identities in a completely ghost-free formulation, using the compact notation of DeWitt.[27] In the first part of this section we shall follow his arguments, allowing for the presence of fermion fields, because in the following we shall need the Lee identity for the fermion-fermion-boson vertex. The total action of QCD in this family of axial gauges is given by

$$S=S_{\text{inv}}+S_{\text{gb}}+j_i\phi_i \ , \tag{2.1}$$

$$S_{\text{gb}}=\frac{1}{2\alpha}N_j^\alpha\phi_i R_{\alpha\beta}N_i^\beta\phi_j \ , \tag{2.2}$$

where $N_i^\alpha=\delta_{a_i}^{a_\alpha}n_{\mu_i}\delta(x_i-x_\alpha)$ in the bosonic sector and zero otherwise, and $R_{\alpha\beta}$ is a symmetric field-independent operator. We have omitted the ghost term as in this class of gauges ghost loops do not contribute to the renormalization of the gauge field in dimensional regularization.[28] The gauge transformation

$$\delta\phi_i=D_i^\alpha(\phi)\delta\omega^\alpha,\quad D_i^\alpha(\phi)=\nabla_i^\alpha+gt_{ij}^\alpha\phi_j \tag{2.3}$$

in the path-integral variables of the generating functional

$$W(j)=\int(d\phi)\exp(iS) \tag{2.4}$$

leads to the Slavnov-Taylor identity

$$\left[\frac{1}{\alpha}N_i^\alpha R_{\alpha\beta}N_j^\beta\frac{\delta}{i\delta j_j}+j_i\right]D_i^\gamma\left[\frac{\delta}{i\delta j}\right]W=0 \ . \tag{2.5}$$

The Legendre transform from the generating functional of connected Green's functions $Z=-i\ln W$ to the generating functional of 1PI vertices $\Gamma(a)=Z(j)-j_ia_i$ gives the Lee identity

$$D_i^\gamma\Gamma_{,i}=\frac{ig}{\alpha}f_{\gamma\alpha\delta}R_{\alpha\beta}N_j^\beta N_k^\delta X^{jk}(a) \ , \tag{2.6}$$

where an index after a comma indicates functional derivation with respect to a and

$$X^{ij}\equiv\Gamma_{,ij}^{-1} \ . \tag{2.7}$$

A crucial property of this family of axial gauges is that one has

$$N_i^\alpha t_{ij}^\gamma=f_{\alpha\delta\gamma}N_j^\delta \tag{2.8}$$

which is responsible for the comparatively simple result (2.6). Differentiation with respect to a_l and a_m yields

$$\begin{aligned}D_i^\gamma\Gamma_{,ilm}=\frac{ig}{\alpha}&f_{\gamma\alpha\delta}R_{\alpha\beta}N_j^\beta N_k^\delta\\&\times(X^{jp_1}\Gamma_{,p_1lp_2}X^{p_2p_3}\Gamma_{,p_3mp_4}X^{p_4k}\\&\quad+X^{jp_1}\Gamma_{,p_1mp_2}X^{p_2p_3}\Gamma_{,p_3lp_4}X^{p_4k}\\&\quad-X^{jp_1}\Gamma_{,p_1lmp_2}X^{p_2k}) \ .\end{aligned} \tag{2.9}$$

At $a=0$, identifying a_i,a_j as boson and a_l,a_m as fermion lines, the left-hand side (LHS) contains the contributions of the quark-quark-gluon vertex and the quark self-energy. The one-loop contribution to the RHS is diagrammatically represented in Fig. 1.

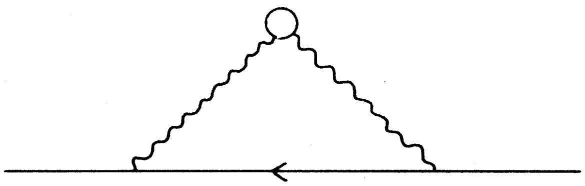

FIG. 1. One-loop contribution (2.25) of the Lee identity (2.9).

For the following actual computations (2.9) will be written explicitly in terms of the explicit forms of $S_{\rm inv}$ and $S_{\rm gb}$:

$$S_{\rm inv}=\int dx\left[-\tfrac{1}{4}(F^a_{\mu\nu})^2+\sum_j \bar\psi^j(i\gamma^\mu\mathcal{D}_\mu-m_j)\psi^j\right], \quad (2.10)$$

$$S_{\rm gb}=\frac{1}{2\alpha}\int dx(n^\mu A^a_\mu)\frac{\partial^2}{n^2}(n^\mu A^a_\mu)\ . \quad (2.11)$$

The quark flavors ψ^j have masses m_j, the field strength and the covariant derivative are defined by

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu A^c_\nu\ , \quad (2.12)$$

$$\mathcal{D}_\mu=\partial_\mu-ig\tau^a A^a_\mu\ , \quad (2.13)$$

and the group generators are represented by Hermitian matrices:

$$[\tau^a,\tau^b]=if^{abc}\tau^c\ . \quad (2.14)$$

For the gluon propagator we obtain

$$\Delta^{ab}_{\mu\nu}(q)=\frac{-i\delta^{ab}}{q^2+i\epsilon}\left[g_{\mu\nu}-\frac{q_\mu n_\nu+q_\nu n_\mu}{qn}+\frac{(1+\alpha)n^2q_\mu q_\nu}{(qn)^2}\right], \quad (2.15)$$

while the fermion propagator and the vertices are the same as in covariant gauges. In (2.15), the planar gauge is given by $\alpha=-1$ while for $\alpha=0$, we recover the propagator (1.1) of the homogeneous axial gauge.[24] We employ dimensional regularization in a space of 2ω dimensions to calculate the UV-divergent parts of the fermionic vertices contributing to the S-matrix element of quark-quark scattering in the generalized LM prescription. The respective integrals are listed in the Appendix where we also comment on the allowed range of values for n_μ and n^*_μ. We shall use the expressions "Abelian" and "non-Abelian" for the graphs which appear in QED and the additional ones of QCD, respectively. Group-theoretic factors C_F and C_A are defined as

$$C_F\delta_{ij}=(\tau^a\tau^a)_{ij},\quad C_A\delta^{ad}=f^{abc}f^{bcd}\ . \quad (2.16)$$

With the further abbreviations

$$\alpha_s=\frac{g^2}{4\pi},\quad \epsilon=4-2\omega\ , \quad (2.17)$$

$$R=\sqrt{(nn^*)^2-n^2(n^*)^2}\ , \quad (2.18a)$$

$$\hat n_\mu=n_\mu-\frac{kn}{k^2}k_\mu\ , \quad (2.18b)$$

$$\hat n^*_\mu=n^*_\mu-\frac{kn^*}{k^2}k_\mu\ , \quad (2.18c)$$

$$\breve n^*_\mu=n^*_\mu-\frac{kn^*}{kn}n_\mu=\hat n^*_\mu-\frac{kn^*}{kn}\hat n_\mu\ , \quad (2.18d)$$

$$a_1=\left[1-\frac{nn^*}{R}\right], \quad (2.18e)$$

$$a_2=\frac{n^2}{R}\frac{kn^*}{kn}\ , \quad (2.18f)$$

we find, for the UV-divergent part of the quark self-energy,

$$\Sigma(p)=C_F\frac{\alpha_s}{2\pi\epsilon}\left[\not p+2m+\frac{2}{R}(\not n pn^*-\not n^* pn)+2a_1(1+\alpha)(\not p-m)\right]. \quad (2.19)$$

The UV-divergent part of the quark-quark-gluon vertex contains Abelian and non-Abelian parts:

$$\Lambda_\mu(p',p)=\Lambda_\mu(p'-p)\equiv\Lambda_\mu(k)=\Lambda^{(1)}_\mu(k)+\Lambda^{(2)}_\mu(k)\ . \quad (2.20)$$

The result for the Abelian graph is found to be

$$\Lambda^{(1)}_\mu(k)=\left[\frac{C_A}{2}-C_F\right]\frac{\alpha_s}{2\pi\epsilon}A_\mu\ , \quad (2.21)$$

while the non-Abelian graph contributes

$$\Lambda^{(2)}_\mu(k)=-\frac{C_A}{2}\frac{\alpha_s}{2\pi\epsilon}(A_\mu+2B_\mu)\ . \quad (2.22)$$

The expressions A_μ and B_μ evaluate to

$$A_\mu=\left[\gamma_\mu+\frac{2}{R}(\not n n^*_\mu-\not n^* n_\mu)+2a_1(1+\alpha)\gamma_\mu\right] \quad (2.23)$$

and

$$\begin{aligned}B_\mu=\Bigg\{&\left[-\not n\breve n^*_\mu\left[\frac{2}{R}+\frac{n^2}{R^3}(n^*\breve n^*)\right]+\not n^*\breve n^*_\mu\frac{n^2}{R^3}(n\breve n^*)-a_2\left[\gamma_\mu-\frac{\not k n_\mu}{kn}\right]\right]\\&+\alpha\left[a_1\gamma_\mu+a_2\frac{\not k n_\mu}{kn}+\not n^* n_\mu\frac{n^2}{R^3}(\breve n^*)^2+\not n n_\mu\left[-a_1\frac{2}{n^2}-\frac{(n^*)^2}{R^3}(n\breve n^*)+\frac{n^2}{R^3}\frac{kn^*}{kn}(n^*\breve n^*)\right]\right]\\&+(1+\alpha)^2\frac{k^2n^2}{(kn)^2}a_2\left[\left[\gamma_\mu-\frac{\not k k_\mu}{k^2}\right]+\frac{1}{R^2}[\not n^*\hat n^*_\mu n^2-(\not n^*\hat n_\mu+\not n\hat n^*_\mu)nn^*+\not n\hat n_\mu(n^*)^2]\right]\Bigg\}\ . \end{aligned} \quad (2.24)$$

The non-Abelian part of the quark-quark-gluon vertex[29] is the final result of a long calculation which would have nearly been intractable without the help of the algebraic program package REDUCE.

Translating (2.9) back into the noncompact notation, a Fourier transform yields the following Lee identity in momentum space:

$$(p'_\mu - p_\mu)[-ig\tau^a\Lambda_\mu(p'-p)] - ig\tau^a\Sigma(p') + ig\tau^a\Sigma(p)$$
$$= \frac{g}{\alpha} f^{abc} \frac{n_\mu n_\nu}{n^2} \int \frac{d^{2\omega}q}{(2\pi)^4} \Delta^{bd}_{\mu\rho}(q)(-ig\gamma_\rho\tau^d) \frac{i}{\not{q}+\not{p}-m+i\epsilon}(-ig\gamma_\sigma\tau^e)\Delta^{ec}_{\sigma\nu}(q+p-p')[q^2-(q+p-p')^2] , \tag{2.25}$$

where the form of the last term has been generated by the antisymmetric structure constant f^{abc}. As $n^\mu \Delta^{ab}_{\mu\nu}$ is proportional to α, the integral on the RHS of (2.25) is found to be of order α^2. Hence, the validity of (2.25) for $\alpha=0$ is obvious from (2.18b)–(2.18d), and evaluating the integral shows that (2.25) holds for $\alpha\neq 0$ as well.

The four-fermion 1PI vertex consists of the "box graph" and the crossed graph. Because of the noncommuting color matrices, the UV-divergent parts of the two graphs do not cancel each other as in QED (Ref. 18), but give

$$\Gamma^{(4)}_{\mu\nu}(p_1,p_2,p_3,p_4) = \bar{u}(p_3)\tau_a\tau_b\gamma^\mu u(p_1)\bar{u}(p_4)[\tau_a,\tau_b]\gamma^\nu u(p_2)B_{\mu\nu} = -\frac{C_A}{2}\bar{u}(p_3)\tau_c\gamma^\mu u(p_1)\bar{u}(p_4)\tau_c\gamma^\nu u(p_2)B_{\mu\nu} . \tag{2.26}$$

Thus the UV-divergent contributions from the gluon self-energy, the non-Abelian vertex, and the box graph all are proportional to C_A. For $B_{\mu\nu}$ we obtain, as in the Abelian case,[18]

$$B_{\mu\nu} = -i\frac{4\alpha_s^2}{\epsilon}\frac{(n^2)^2 kn^*}{R^3(kn)^3}(1+\alpha)^2[R^2 g_{\mu\nu} - nn^*(n_\mu n^*_\nu + n_\nu n^*_\mu) + n^2 n^*_\mu n^*_\nu + (n^*)^2 n_\mu n_\nu] . \tag{2.27}$$

If such a term would not cancel in a physical observable quantity, it would require a nonlocal counterterm with four fermion fields in the Lagrangian. Since nonlocal divergences are not restricted by $n^2=0$, similar "counterterms" can be expected for 1PI vertex functions involving more than four external fermion lines.

The UV-divergent part of the gluon self-energy for a pure YM theory has been calculated by Gaigg, Kreuzer, and the present author in Ref. 23 and reads

$$\Pi_{\mu\nu}(k) = iC_A\frac{\alpha_s}{2\pi\epsilon}k^2 C_{\mu\nu} , \tag{2.28}$$

where $C_{\mu\nu}$ is given by

$$C_{\mu\nu} = \Bigg\{ \Bigg[\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right](\tfrac{11}{3} - 2a_2) - (\hat{n}_\mu \breve{n}^*_\nu + \hat{n}_\nu \breve{n}^*_\mu)\left[\frac{2}{R} + \frac{n^2}{R^3}(n^*\breve{n}^*) \right] + (\hat{n}^*_\mu \breve{n}^*_\nu + \hat{n}^*_\nu \breve{n}^*_\mu)\frac{n^2}{R^3}(n\breve{n}^*) \Bigg]$$
$$+ \alpha\Bigg[2a_1\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] + (n_\mu \hat{n}^*_\nu + n_\nu \hat{n}^*_\mu)\frac{n^2}{R^3}(\breve{n}^*)^2$$
$$+ (n_\mu \hat{n}_\nu + n_\nu \hat{n}_\mu)\left[-a_1\frac{2}{n^2} - \frac{(n^*)^2}{R^3}(n\breve{n}^*) + \frac{n^2}{R^3}\frac{kn^*}{kn}(n^*\breve{n}^*) \right] \Bigg]$$
$$+ (1+\alpha)^2\frac{k^2 n^2}{(kn)^2}a_2\left[\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] + \frac{1}{R^2}[\hat{n}^*_\mu \hat{n}^*_\nu n^2 - (\hat{n}_\mu \hat{n}^*_\nu + \hat{n}_\nu \hat{n}^*_\mu) nn^* + \hat{n}_\mu \hat{n}_\nu (n^*)^2] \right] \Bigg\} . \tag{2.29}$$

Special cases of this formula have been obtained by various authors.[20,30] The abbreviations (2.18b)–(2.18f) allowed us to write Eqs. (2.24) and (2.29) in a compact form exhibiting structures transverse to k_μ, but they might be unsuitable to determine the structure of the bosonic "counterterms." Obviously, this structure is much more complicated than in the LC gauge. An ansatz obeying the requirements of extended BRST invariance has been given in Ref. 23. Unfortunately, it is not likely to be complete. Off the LC, n-point 1PI functions $n \geq 3$ might require new structures not contributing to the two-point function. As in the LC gauge, a deeper understanding of the bosonic "counterterms" probably requires additional information[16] about the specific form of the LM prescription. The new nonlocal fermionic "counterterms," on the other hand, are related one to one to the respective bosonic ones, as we shall see in the next section.

III. CANCELLATION OF GAUGE-DEPENDENT TERMS

In spite of the complex structure of the 1PI vertices given above, we want to demonstrate that no gauge-dependent UV divergences are present in the S-matrix element of quark-quark scattering. To keep the number of graphs down, we consider two quarks of different flavor. As the external fermions are taken on shell, we have

$$k^\mu \bar{u}(p_3)\gamma_\mu u(p_1)=k^\mu \bar{u}(p_4)\gamma_\mu u(p_2)=0\ , \tag{3.1}$$

where $k=p_3-p_1=p_2-p_4$ is the momentum transfer. This current conservation serves to annihilate gauge-dependent parts of the gluon propagator (2.15). For example, it is obvious that the fermion loop of the gluon self-energy depicted in Fig. 2(l) cannot give a gauge-dependent contribution, being transverse to the transferred momentum $k_\mu \Pi_f^{\mu\nu}=0$ and being gauge independent itself. Thus we only have to sum up the contributions from the graphs depicted in Figs. 2(a)–2(k).

First we treat the parts proportional to C_F. The self-energy and the contribution of the Abelian vertex can be written in the form

$$\Sigma(p)=\delta m+\bar{\mathcal{M}}(\not p-m)+(\not p-m)\mathcal{M}\ , \tag{3.2}$$

$$\Lambda_\mu^F(k)=-\bar{\mathcal{M}}\gamma_\mu-\gamma_\mu\mathcal{M}\ , \tag{3.3}$$

where

$$\delta m=mC_F\frac{3\alpha_s}{2\pi\epsilon}\ , \tag{3.4}$$

$$\mathcal{M}=C_F\frac{\alpha_s}{2\pi\epsilon}\left[(\tfrac{3}{2}+\alpha)+\frac{1}{R}[\not n^*\not n-(2+\alpha)nn^*]\right]\ , \tag{3.5}$$

and $\bar{\mathcal{M}}=\gamma_0\mathcal{M}^\dagger\gamma_0$. This entails that the mass and the field renormalization are given by $m_0=m-\delta m$ and $\psi^{(0)}=(1-\mathcal{M})\psi$. A gauge-dependent matrix $(1-\mathcal{M})$ replaces the familiar renormalization constant $Z_2^{-1/2}$. Hence, we have to reexamine the Lehmann-Symanzik-Zimmermann (LSZ) reduction technique.[31] We find that we must put $\bar{u}(p)(1-\mathcal{M})$ and $(1-\bar{\mathcal{M}})u(p)$ at the external lines. Summing up all contributions to order g^4 including the effects of mass and wave-function renormalization, we obtain

$$\bar{u}(p_3)\left\{[\delta m+\bar{\mathcal{M}}(\not p_3-m)+(\not p_3-m)\mathcal{M}]\frac{1}{\not p_3-m}\gamma_\mu+\gamma_\mu\frac{1}{\not p_1-m}[\delta m+\bar{\mathcal{M}}(\not p_1-m)+(\not p_1-m)\mathcal{M}]\right.$$
$$\left.-\bar{\mathcal{M}}\gamma_\mu-\gamma_\mu\mathcal{M}-\delta m\frac{1}{\not p_3-m}\gamma_\mu-\gamma_\mu\frac{1}{\not p_1-m}\delta m-\mathcal{M}\gamma_\mu-\gamma_\mu\bar{\mathcal{M}}\right\}u(p_1)\Delta_{\mu\nu}\bar{u}(p_4)\gamma^\nu u(p_2) \tag{3.6}$$

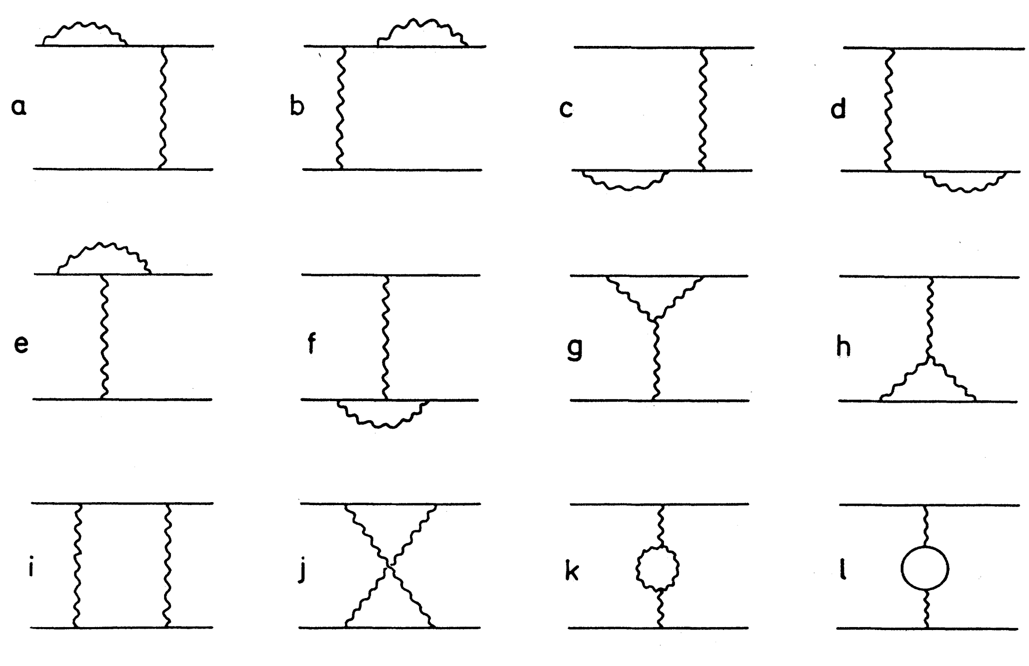

FIG. 2. One-loop Feynman diagrams contributing to the S-matrix element of quark-quark scattering.

and corresponding expressions from the graphs turned upside down. With the help of

$$\frac{1}{\not p-m}\overline{\mathcal{M}}(\not p-m)u(p)$$
$$=\left[\frac{1}{\not p+m}\overline{\mathcal{M}}(\not p-m)+\overline{\mathcal{M}}\frac{1}{1+\frac{\not p-m}{2m}}\right]u(p)$$
$$=\overline{\mathcal{M}}u(p) \tag{3.7}$$

the contributions from the upper (lower) fermion line are seen to vanish identically, just as in QED in covariant gauges. Hence, the form of the gluon propagator is irrelevant in this case and we conclude that the UV-divergent part of the S-matrix element contains no gauge-dependent terms proportional to C_F.

Let us now investigate the contributions proportional to C_A. If we consider for the moment the special case[24] of the homogeneous axial gauge $\alpha=0$, we can use that according to the Ward identities the C_A-dependent part of $\Lambda^1_\mu(k)+\Lambda^2_\mu(k)$ is transverse to k^μ and that the gluon self-energy even satisfies $k^\mu\Pi_{\mu\nu}=k^\nu\Pi_{\mu\nu}=0$. Therefore, the gauge-dependent parts of the gluon propagator (2.15) vanish either at the 1PI vertices or by current conservation (3.1), leaving only the "Feynman part" $-i\delta^{ab}g_{\mu\nu}/k^2$. We can thus compare directly Eqs. (2.24) and (2.29), and set to zero all terms k^μ, k^ν, and $\not k$. In this way we observe that the gauge-dependent part from the gluon self-energy cancels with corresponding expressions of the quark-quark-gluon vertices, up to a part proportional to $(kn)^{-3}$ which exactly matches the contribution of the four-quark 1PI vertex (2.27). Hence, for $\alpha=0$, the S-matrix element to one-loop order contains no gauge-dependent UV divergences. If we not turn to the case $\alpha\neq 0$, we need only consider the two blocks in Eqs. (2.24) and (2.29) having α in front, because these are the only nontransverse parts where the above argument is insufficient. These pieces have to be contracted with $g_{\mu\nu}-k_\mu n_\nu(kn)^{-1}$ instead of a mere $g_{\mu\nu}$. If this is done, their contributions again cancel exactly, thus gauge independence holds also for $\alpha\neq 0$.

Examining once again the C_A-dependent divergences which we have just observed to vanish in the S matrix, we find a one-to-one correspondence of terms $\not n,\not n^*$ to terms $\hat{n}_\mu,\hat{n}^*_\mu$ which holds in an analogous way for $\gamma_\mu\leftrightarrow g_{\mu\nu}-k_\mu k_\nu/k^2$ and $\not k\leftrightarrow 0$. In other words, the sum of the C_A-dependent counterterms can be written as

$$(D^{ab}_\mu F^b_{\mu\nu}-g\bar\psi\tau^a\gamma^\nu\psi)\Omega^a_\nu\ . \tag{3.8}$$

Ω^a_μ itself contains terms $\bar\psi\tau^a\gamma^\rho\psi$ and the structure of nonlocalities is much richer, but aside from that, things are quite analogous to the LC gauge.[14] All gauge-dependent terms turn out to be proportional to the classical equations of motion, the C_F-dependent ones to

$$\bar\psi(i\not{\mathcal{D}}-m)=0,\quad (i\not{\mathcal{D}}-m)\psi=0\ , \tag{3.9}$$

as can be seen from (3.2) and (3.3), and the C_A-dependent ones as in (3.8). This implies that they can be absorbed in a redefinition of the fields A_μ, ψ, and $\bar\psi$. The usual proofs that the S matrix is not affected by local field redefinitions[32] are not immediately applicable, since Ω_ν in (3.8) is nonlocal. However, our calculation has revealed that the gauge independence of the S matrix is due to the fact that all nonlocal gauge-dependent terms combine into (3.8).

IV. CONCLUSIONS

Investigating the consistency of the generalized LM prescription for values of n_μ and n^*_μ as specified in the Appendix, we have considered QCD in a family of gauges comprising the homogeneous axial gauge and the planar gauge. The calculation of all 1PI vertices contributing to gauge-dependent UV divergences of the S-matrix element for quark-quark scattering has been performed to one-loop order. The result of this calculation which made extensive use of the program package REDUCE is that all terms dependent on the gauge parameters n_μ and α or on the auxiliary vector of the LM prescription n^*_μ cancel in this matrix element. Moreover, our results for the quark-quark-gluon vertex are found to agree with the Slavnov-Taylor-Lee identity for arbitrary values of α. Thus the generalized LM prescription, in spite of the vast number of nonlocal divergent structures in 1PI vertex functions and even in Green's functions, appears to be a consistent technique to handle the unphysical poles $(qn)^{-\beta}$ for a large family of axial gauges, including the LC gauge and the temporal gauge. The nonlocal "counterterms" generated by this prescription seem to cause no harm for the physical (observable) quantities, as they satisfy the "minimum requirement" of proportionality to the equations of motion. A formal proof of this behavior, however, is considerably more difficult than in the special case $n^2=0$, since the structure of bosonic "counterterms" is not fully understood as yet. Another interesting issue would be to check whether this cancellation of gauge-dependent terms also works for infrared divergences as has been demonstrated for the homogeneous axial gauge in the PV prescription by Konetschny.[33]

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APPENDIX

Here we list the UV-divergent terms of the integrals that have been used to derive Eqs. (2.19)–(2.27). The real Lorentz vectors n_μ and n^*_μ can be chosen independently, as long as

$$R^2=(nn^*)^2-n^2(n^*)^2>0 \tag{A1}$$

and additionally on the LC

$$n^2=0\Longrightarrow nn^*>0\ . \tag{A2}$$

Therefore, differentiation with respect to n_μ where n^*_μ is kept fixed yields consistent results.[19] The expressions obtained by the procedure outlined in Ref. 23 read

$$\int d^{2\omega}q \frac{1}{[(q-p)^2-m^2](qn)^2}\Bigg|_{\text{div}} = \frac{2}{R}\left[\frac{nn^*-R}{n^2}\right]\bar{I} , \tag{A3}$$

$$\begin{aligned}\int d^{2\omega}q \frac{q_\mu}{[(q-p)^2-m^2]qn}\Bigg|_{\text{div}} = \frac{1}{R}\Bigg[& m^2\left[n^*_\mu - \frac{nn^*-R}{n^2}n_\mu\right] + 2\left[pn^* - pn\frac{nn^*-R}{n^2}\right]p_\mu \\ & + \frac{(pn^*)^2n^2-2(pn)(pn^*)(nn^*)+(pn)^2(n^*)^2}{R^2}n^*_\mu \\ & + \Bigg[\frac{-(pn^*)^2(nn^*)n^2+2(pn)(pn^*)n^2(n^*)^2-(pn)^2(nn^*)(n^*)^2}{n^2R^2} \\ & + 2(pn)^2\frac{nn^*-R}{(n^2)^2}\Bigg]n_\mu\Bigg]\bar{I} ,\end{aligned} \tag{A4}$$

$$\int d^{2\omega}q \frac{q_\mu}{(q-p_1)^2[(q-p_2)^2-m^2](qn)}\Bigg|_{\text{div}} = \frac{1}{R}\left[n^*_\mu - \frac{nn^*-R}{n^2}n_\mu\right]\bar{I} , \tag{A5}$$

$$\begin{aligned}&\int d^{2\omega}q \frac{q_\mu q_\nu}{(q-p_1)^2[(q-p_2)^2-m^2](qn)^2}\Bigg|_{\text{div}} \\ &\quad = \frac{1}{R}\left[\frac{1}{R^2}\left[nn^*n^*_\mu n^*_\nu - (n^*)^2(n_\mu n^*_\nu + n_\nu n^*_\mu) + \frac{nn^*(n^*)^2}{n^2}n_\mu n_\nu\right] + \frac{nn^*-R}{n^2}\left[g_{\mu\nu} - \frac{2}{n^2}n_\mu n_\nu\right]\right]\bar{I} ,\end{aligned} \tag{A6}$$

$$\begin{aligned}&\int d^{2\omega}q \frac{q_\mu q_\nu}{(q-p_1)^2[(q-p_2)^2-m^2](qn)}\Bigg|_{\text{div}} \\ &\quad = \frac{1}{2R}\Bigg\{\frac{1}{R^2}\Bigg[(pn^*n^2-pn\,nn^*)n^*_\mu n^*_\nu - [pn^*nn^* - pn(n^*)^2](n_\mu n^*_\nu + n_\nu n^*_\mu) + \left[pn^*(n^*)^2 - \frac{pn\,nn^*(n^*)^2}{n^2}\right]n_\mu n_\nu\Bigg] \\ &\qquad + (g_{\mu\nu}pn^* + p_\mu n^*_\nu + p_\nu n^*_\mu) - \frac{nn^*-R}{n^2}\left[p_\mu n_\nu + p_\nu n_\mu + pn\left[g_{\mu\nu} - \frac{2}{n^2}n_\mu n_\nu\right]\right]\Bigg\}\bar{I} ,\end{aligned} \tag{A7}$$

where

$$\bar{I} = \frac{i\pi^2}{2-\omega}, \quad p = p_1 + p_2 , \tag{A8}$$

and agree completely with Leibbrandt's formulas,[29] provided those are expressed in terms of n_μ and n^*_μ (Ref. 21). The range of values for n_μ and n^*_μ as specified by (A1) and (A2) is the one given in Ref. 19 where the LM prescription was generalized by analytic continuation to all values of n_μ and n^*_μ allowing Wick rotation. For the actual computation it is convenient to restrict oneself further by

$$n^2(n^*)^2 > 0 \text{ or } n^2 = (n^*)^2 = 0 , \tag{A9}$$

$$nn^* > 0 . \tag{A10}$$

It is then easy to show that there exists a Lorentz frame where n_μ and n^*_μ take the form

$$\begin{aligned}& n_\mu = (n_0, 0, 0, n_3), \quad n^*_\mu = \lambda(n_0, 0, 0, -n_3) , \\ & \lambda > 0, \quad n_0 \neq 0, \quad n_3 \neq 0 .\end{aligned} \tag{A11}$$

We calculate

$$I(\alpha,\beta) = \int d^{2\omega}q \frac{1}{(q^2+2pq-L)^\alpha}\left[\frac{qn^*}{qn^*qn+i\epsilon}\right]^\beta \tag{A12}$$

(which contains only Lorentz-invariant quantities) in the Lorentz frame (A11) absorbing the positive scaling factor λ in the $i\epsilon$ term. The result can be written in terms of L, p^2, pn, pn^*, n^2, $(n^*)^2$, and nn^* which in every Lorentz frame take the same values as in the particular frame (A11). Thus, we are able to calculate $I(\alpha,\beta)$ for all choices of n_μ and n^*_μ obeying (A1), (A9), and (A10) with the methods developed in Ref. 17. Using Feynman parameters and differentiation with respect to p_μ, as outlined in Refs. 18 and 23, we obtain (A3)–(A8) from (A12).

However, (A3)–(A8) are still valid if (A9) and (A10) are dropped. Equation (A1) alone is sufficient for the existence of a Lorentz frame where

$$n_\mu = (n_0, 0, 0, n_3), \quad n^*_\mu = (n^*_0, 0, 0, n^*_3) . \tag{A13}$$

This fact can be used to compute (A12) directly, performing a change of variables $dq_0 dq_3 \to d(qn)d(qn^*)$ where R appears as the corresponding Jacobian. In this way, we obtain expressions for (A12) whose divergent parts again lead to (A3)–(A8).

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