Matter-parity constraints on particle spectrum in three-generation Calabi-Yau manifolds

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An analysis of the particle spectrum after intermediate scale breaking in the three-generation Calabi-Yau manifold $\mathrm{CP}^3\times\mathrm{CP}^2/Z_3$ is given. The spectrum is investigated within the framework of intermediate scale breaking which preserves matter parity and breaks the rank-six gauge group $\mathrm{SU(3)}_C\times\mathrm{SU(3)}_L\times\mathrm{SU(3)}_R$ to the standard-model gauge group $\mathrm{SU(3)}_C\times\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$. It is shown that a large number of particles and mirror particles associated with the extra families and mirror families on $\mathrm{CP}^3\times\mathrm{CP}^3/Z_3$ can all be made superheavy through mass generation induced by nonrenormalizable interactions that are present in the superpotential below the compactification scale. The final spectrum of the compactified superstring below the intermediate-mass scale breaking is shown to be the spectrum of the standard $N=1$ supersymmetry theory and a few additional exotic leptons. Some of these exotic leptons are hybrid structures containing equal mixture of leptons and mirror leptons.

I. INTRODUCTION

Since the advent of the $E_8\times E_8'$ heterotic string,[1] its compactification on the manifold $M_4\times K$, where K is the compact six-dimensional Calabi-Yau manifold,[2] has provided an attractive scheme for a unified four-dimensional theory. The Calabi-Yau compactification preserves $N=1$ supersymmetry and simultaneously one of the E_8 groups reduces to E_6 which may be viewed as the grand unification group of particle interactions at the compactification scale. However, the physically relevant manifold with three generations is nonsimply connected: it is the manifold $\mathrm{CP}^3\times\mathrm{CP}^3/Z_3$ (Ref. 3). On this manifold the group E_6 breaks to the group $\mathrm{SU(3)}_C\times\mathrm{SU(3)}_L\times\mathrm{SU(3)}_R$ due to Wilson flux lines.[4,5] The rank-six $[\mathrm{SU(3)}]^3$ symmetry must be broken further to the rank-four standard-model gauge group $\mathrm{SU(3)}_C\times\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$. A standard scenario to achieve this is the so-called intermediate-mass scale breaking at $M_I\approx10^{15}$ GeV. Above this mass scale but below the compactification scale, the massless spectrum of the $\mathrm{CP}^3\times\mathrm{CP}^3/Z_3$ compactification consists of nine families of leptons which belong to the $(1,3,\overline{3})$ representation of $\mathrm{SU(3)}_C\times\mathrm{SU(3)}_L\times\mathrm{SU(3)}_R$, seven families of $(3,\overline{3},1)$ quarks, seven families of $(\overline{3},1,3)$ antiquarks, six families of $(1,\overline{3},3)$ mirror leptons, four families of $(\overline{3},3,1)$ mirror quarks, and four families of $(3,1,\overline{3})$ mirror antiquarks.

The phenomenology on three-generation Calabi-Yau manifolds has been investigated extensively recently[6–10] and a number of difficulties with the model have been pointed out.[7,8] Surprisingly, however, a detailed analysis of the intermediate-mass scale breaking as well as an analysis of the particle spectrum below the intermediate-mass scale has not been carried out. This analysis is rather crucial since the pattern of intermediate scale breaking determines the number of low-lying exotic particles. Too many of these low-lying exotic particles would be phenomenologically disastrous since they could generate Landau singularities as well as produce an unacceptably large value of $\sin^2\theta_W$.

Recently the authors[11] presented an analysis of intermediate scale breaking within the framework of matter-parity invariance[6,12] imposed above the intermediate scale. This is needed to forbid rapid proton decay.[13] The analysis used two different lepton multiplets and their mirror counterparts to generate vacuum expectation values (VEV's) for the SU(5)- and SO(10)-singlet fields ν^c and N. Remarkably the lowest-lying extrema maintain matter parity *after* symmetry breaking and *prevent* $\mathrm{SU(2)}\times\mathrm{U(1)}$ breaking at M_I.

In this paper we discuss the Goldstone and Higgs fields associated with intermediate-mass scale breaking. We discuss the absorption phenomenon which grows masses for the vector bosons in the reduction $\mathrm{SU(3)}_C\times\mathrm{SU(3)}_L\times\mathrm{SU(3)}_R\rightarrow\mathrm{SU(3)}_C\times\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$. It is shown that the absorption involves linear combinations of the lepton multiplets and their mirror counterparts. The unabsorbed components are thus also linear combinations of the lepton multiplets and their mirrors. These hybrid structures are shown to possess electroweak masses and thus may be accessible at accelerator energies.

In addition to the lepton multiplets and their mirrors which enter in intermediate scale breaking and the three standard-model generations of quarks and leptons, there exist extra generations of lepton, quark, and antiquark multiplets and their mirrors which must be removed from the low-energy spectrum to avoid phenomenological disasters such as Landau singularities in coupling constants and an unacceptably large value of $\sin^2\theta_W$. We address this question in this paper and show that it is possible to find models where the unwanted spectrum of particles become superheavy. This is brought about by mass generation from the nonrenormalizable interactions in the superpotential that enter the theory below the compactification scale.

In Sec. II a brief review of symmetry-breaking analysis of Ref. 11 is given. In Sec. III Goldstone and Higgs analysis after intermediate scale symmetry breaking is carried out. Absorption of the lepton multiplets and their mirrors is analyzed. The mass growth of the vector bosons associated with the breaking of $SU(3)_C \times SU(3)_L \times SU(3)_R$ to the standard-model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ is exhibited. The components of the lepton multiplets and their mirrors which are unabsorbed in the symmetry breaking are also discussed. In Sec. IV we show how mass generation from the nonrenormalizable interactions in the superpotential can make the unwanted modes superheavy.

II. SYMMETRY BREAKING ON CALABI-YAU MANIFOLDS AND GENERATION OF N and ν^c VEV's

The three-generation Calabi-Yau manifold is described by $CP^3 \times CP^3/Z_3$, where this manifold is defined through the intersection of the following polynomials involving eight complex coordinates x_0, x_1, x_2, x_3 and y_0, y_1, y_2, y_3:

$$0 = P_1 = \sum x_i^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3 \ , \tag{2.1a}$$

$$0 = P_2 = x_0 y_0 + c_1 x_1 y_1 + c_2 x_2 y_2 + c_3 x_3 y_3 + c_4 x_2 y_3 + c_5 x_3 y_2 \ , \tag{2.1b}$$

$$0 = P_3 = \sum y_i^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3 \ . \tag{2.1c}$$

The above manifolds depend on nine complex parameters corresponding to the nine deformations on this manifold. This nine-parameter $CP^3 \times CP^3/Z_3$ manifold must be further constrained to obtain a phenomenologically viable manifold. One of the most important phenomenological constraints in superstring model building is the elimination of rapid proton decay.[13] A remarkable invariance which is needed to forbid the existence of dangerously rapid proton decay is that of matter parity. Matter parity is defined by[6,12,14]

$$M_2 = C U_Z \ , \tag{2.2}$$

where C acts on the Calabi-Yau coordinates $(x_0, x_1, x_2, x_3) \otimes (y_0, y_1, y_2, y_3)$,

$$C = (1,1,\sigma) \otimes (1,1,\sigma), \quad \sigma = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} , \tag{2.3}$$

and U_Z is an element of $SU(3)_C \times SU(3)_L \times SU(3)_R$ which is the group symmetry allowed on the manifold $CP^3 \times CP^3/Z_3$:

$$U_Z = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix} . \tag{2.4}$$

The C-invariant Calabi-Yau manifold is still a fairly large manifold depending on five complex parameters: i.e.,

$$0 = P_1 = \sum x_i^3 + a_1 (x_0 x_1 x_2 + x_0 x_1 x_3) \ , \tag{2.5a}$$

$$0 = P_2 = c_0 y_0 + c_1 x_1 y_1 + c_2 (x_2 y_2 + x_3 y_3) + c_3 (x_2 y_3 + x_3 y_2) \ , \tag{2.5b}$$

$$0 = P_3 = \sum y_i^3 + b_1 (y_0 y_1 y_2 + y_0 y_1 y_3) \ . \tag{2.5c}$$

Above the intermediate-mass scale breaking where $[SU(3)]^3$ symmetry is exact, the particle interactions are separately C and M_2 invariant. However, below the intermediate-mass scale C invariance is broken through the VEV of the C-odd but M_2-even SU(5)-singlet field ν^c. Consequently, below the intermediate mass C invariance no longer holds, though, as was seen in Ref. 11, M_2 invariance holds.

Below the compactification scale but above the intermediate scale breaking where the interaction structure of the theory is $[SU(3)]^3$ invariant one may decompose the 27-plet into its $SU(3)_C \times SU(3)_L \times SU(3)_R$ parts:

$$27 = L_r^l(1,3,\bar{3}) \oplus Q_l^a(3,\bar{3},1) \oplus Q_a^{cr}(\bar{3},1,3) \ . \tag{2.6}$$

In Eq. (2.6), L are the leptons, Q the quarks, and Q^c the antiquarks. The indices $a,l,r = 1,2,3$ are the $SU(3)_C$, $SU(3)_L$, and $SU(3)_R$ indices. The $SU(3)_R$ indices may be further decomposed into $SU(2)_L \times SU(2)_R$ indices so that $l = (\lambda, 3)$, $r = (\rho, 3)$ where $\lambda, \rho = 1,2$. The lepton, quark, and antiquark multiplets may now be explicitly written in familiar particle notation:

$$L_\rho^\lambda = (H^\lambda, \epsilon^{\lambda\mu} H'_\mu), \quad L_\rho^3 = (e^c, \nu^c), \quad L_3^\lambda = l^\lambda, \quad L_3^3 = N \ , \tag{2.7a}$$

$$Q_\lambda^a = \epsilon_{\lambda\mu} q^{a\mu}, \quad -Q_3^a = D^a \ , \tag{2.7b}$$

$$Q_a^{c\rho} = (u_a^c, -d_a^c), \quad -Q_a^{c3} = D_a^c \ . \tag{2.7c}$$

In Eq. (2.7) $q^{a\lambda} = (u^{a\lambda}, d^{a\lambda})$ and $l^\lambda = (\nu, l)$ are the quark and lepton $SU(2)_L$ doublets while D^a and D_a^c are the color Higgs triplets where the superscript c stands for the conjugate fields. A similar analysis holds for the $\overline{27}$ decomposition. Under U_Z the components of the 27-plet have even or odd transformations:[6]

$$U_Z \text{ even: } H^\lambda, H'_\lambda, D, D^c, N \ ; \tag{2.8a}$$

$$U_Z \text{ odd: } q^\lambda, l^\lambda, u^c, d^c, l^c, \nu^c \ . \tag{2.8b}$$

The C parity of the different families and mirror families are exhibited in Table I (Ref. 6). Matter parities of the families and mirror families can then be easily obtained using Eqs. (2.8) and Table I. These are listed in Table II. For convenience we introduce the notation

$$(r,s,t, \ldots ; n,m,p, \ldots) = (C \text{ odd}; C \text{ even}) \ . \tag{2.9}$$

It is easily seen from Table I that r,s,t take on values $1-, 3-, 6, 8-$ for leptons, 3,4,5,6 for mirror leptons, $4-, 6-$ for quarks and $1-, 3-$ for mirror quarks, etc. Similarly $m,n,p, \ldots$ take on values $1+, 3+, 5, 7, 8+$ for leptons and 1,2 for antileptons, etc.

TABLE I. List of linear combinations of lepton, quark, and antiquark families and mirror families which are eigenstates of C. We use the notation of Ref. 6 with $L_{1+}=(L_1+L_2)/\sqrt{2}$, etc.

C property	Families	Mirror families
C even	$L_{1+},L_{3+},L_5,L_7,L_{8+}$	$\bar{L}_1,\bar{L}_2$
	$Q_1,Q_2,Q_3,Q_{4+},Q_{6+}$	$\bar{Q}_{1+},\bar{Q}_{3+}$
	$Q_1^c,Q_2^c,Q_3^c,Q_{4+}^c,Q_{6+}^c$	$\bar{Q}_{1+}^c,Q_{3+}^c$
C odd	L_{1-},L_{3-},L_6,L_{8-}	$\bar{L}_3,\bar{L}_4,\bar{L}_5,\bar{L}_6$
	Q_{4-},Q_{6-}	$\bar{Q}_{1-},\bar{Q}_{3-}$
	Q_{4-}^c,Q_{6-}^c	$\bar{Q}_{1-}^c,\bar{Q}_{3-}^c$

As may be seen from Eqs. (2.2) and (2.8), the N and ν^c fields within a given multiplet which is an eigenstate of C have opposite matter parities. Thus one needs two different multiplets, one C even and the other C odd, if only the matter-parity even N and ν^c fields are to grow VEV's so that rapid proton decay be forbidden. In the following we thus utilize two lepton multiplets L_i $(i=1,2)$ and their mirrors $\bar{L}_i$, where $i=1$ is a C-even multiplet and $i=2$ is a C-odd multiplet. If only the N_1 and $\bar{N}_1$, ν_2^c, and $\bar{\nu}_2^c$ fields grow VEV's, matter-parity invariance would be preserved. The effective potential which governs symmetry breaking below the compactification scale has the form

$$V=V_m+V_F+V_D\ , \tag{2.10}$$

where V_m is a mass term arising from $N=1$ supersymmetry breaking, V_F is the F term arising from the superpotential, and V_D is the D term generated by $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R$ gauge transformations. We discuss now each of these terms briefly. It is conventionally assumed that in this class of superstring models supersymmetry is broken so that soft supersymmetry-breaking mass terms are generated in the scalar-boson section. These soft $(\text{mass})^2$ terms can turn negative through renormalization-group (RG) effects which signals spontaneous breaking which generates the intermediate-mass scale. An analysis of the renormalization-group equations below the compactification scale shows[15] that such a phenomenon does indeed occur in simple models, in fact, quite rapidly due to the large number of massless fields that enter below the compactification scale. Further, the evolution of the soft symmetry-breaking $(\text{mass})^2$ need not necessarily be the same for the multiplets and their mirrors. Thus assuming that at a scale not far below the compactification scale, the squared masses of the scalar fields turn negative we may write for V_m the form

$$V_m=-\sum_i m_i^2(x_i+y_i+z_i+w_i)-\sum_i \bar{m}_i^2(\bar{x}_i+\bar{y}_i+\bar{z}_i+\bar{w}_i)\ , \tag{2.11}$$

where

$$x_i=N_iN_i^\dagger,\quad y_i=\nu_i^c\nu_i^{c\dagger},\quad z_i=H_i^2H_i^{2\dagger},\quad w_i=H'_{2i}H'^\dagger_{2i}\ , \tag{2.12}$$

$$\bar{x}_i=\bar{N}_i\bar{N}_i^\dagger,\quad \bar{y}_i=\bar{\nu}_i^c\nu_i^{c\dagger},\quad z_i=\bar{H}_i^2\bar{H}_2^{2\dagger},\quad \bar{w}_i=\bar{H}'_{2i}\bar{H}'^\dagger_{2i}\ .$$

In Eq. (2.11) we have exhibited only the neutral fields which may grow VEV's in the analysis of symmetry breaking at the intermediate scale. The term V_F in Eq. (2.10) which is the F part of the potential arises from a superpotential which in general consists of two parts: the first is a cubic interaction of type $(27)^3+(\overline{27})^3$ while the second is a nonrenormalizable interaction which is an infinite-order expansion in the inverse of the compactification mass. This second part of the interaction is a reflection of the fact that the infinite number of heavy modes that arise in the superstring have been integrated out below the compactification scale. The nonrenormalizable terms in the superpotential are just the remnants of the coupling of these heavy fields to the light sector. Thus we write the superpotential as the sum of a renormalizable and a nonrenormalizable part:

$$W=W_{\mathrm{r}}+W_{\mathrm{nr}}\ , \tag{2.13}$$

where

$$W_{\mathrm{r}}=W_{\mathrm{r}}(L,Q,Q^c)+\overline{W}_{\mathrm{r}}(\bar{L},\bar{Q},\bar{Q}^c)\ . \tag{2.14}$$

We exhibit here explicitly for later use the form of $W_{\mathrm{r}}(L,Q,Q^c)$:

$$\begin{aligned}W_{\mathrm{r}}(L,Q,Q^c)={}&\lambda^1_{ABC}\epsilon_{aa'a''}d^a_A u^{a'}_B D^{a''}_C+\lambda^2_{ABC}\epsilon^{aa'a''}u^c_{aA}d^c_{a'B}D^c_{a''C}+\lambda^3_{ABC}(-H^\lambda_A H'_{\lambda B}N_C-H^\alpha_A\nu^c_B l_{\lambda c}+H'_{\lambda A}l^C_B l^\lambda_C)\\&-\lambda^4_{ABC}(D^a_A N_B D^c_{aC}-D^a_A l^c_B u^c_{aC}+D^a_A\nu^c_B d^c_{aC}+q^{a\lambda}_A l_{\lambda B}D^c_{aC}-q^a_{\lambda A}H^\lambda_B u^c_{aC}-q^{a\lambda}_A H'_{\lambda B}d^c_{aC})\ .\end{aligned} \tag{2.15}$$

The indices A,B,C in Eq. (2.15) run over the various generations. For the Calabi-Yau manifold of Eq. (2.5), many of the couplings in Eq. (2.15) are automatically zero by C invariance. Thus one has[11]

$$\lambda^{1,2,3,4}_{rst}=0=\lambda^{1,2,3,4}_{mnr}\ , \tag{2.16a}$$

$$\lambda^4_{mrn}=0=\lambda^4_{rmn}\ . \tag{2.16b}$$

An expression similar to Eq. (2.15) with restrictions simi-

TABLE II. List of the matter parities of particles and mirror particles in the full set of families and mirror families. r runs over C-odd values while n runs over C-even values. Matter parity of families and mirror families are the same.

Matter parity	Particles
M_2 even	l_r, e_r^c, ν_r^c
	q_r, u_r^c, d_r^c
	D_n, D_n^c, N_n
	H_n, H_n'
M_2 odd	l_n, e_n^c, ν_n^c
	q_n, u_n^c, d_n^c
	D_r, D_r^c, N_r
	H_r, H_r'

lar to Eq. (2.16) hold for the superpotential $\overline{W}_r[\overline{L},\overline{Q},\overline{Q}^c]$. As already stated the nonrenormalizable part of the superpotential is in general an infinite-order expansion in the inverse of the compactification mass M_c. We exhibit here only the part relevant for symmetry breaking. We assume for this purpose a dimension $2n(n\geq 2)$ term in the superpotential of the form

$$W_{\rm nr}=\sum_i \frac{\lambda_i^2}{(M_c)^{2n-3}}(L_{ir}^l \overline{L}_{ir}^l)^n \,. \tag{2.17}$$

In Eq. (2.17) λ_i are the couplings of the heavy fields to the light fields. It is the superpotential of Eq. (2.17) rather than the cubic interactions of W_r which are F flat, that play the dominant role in symmetry breaking at the intermediate scale. The potential V_F arising from Eq. (2.17) is

$$V_F=\frac{n^2}{M_c^{4n-6}}\sum_i \lambda_i^4 (f_i f_i^\dagger)^{n-1} \times(x_i+\overline{x}_i+y_i+\overline{y}_i+z_i+\overline{z}_i+w_i+\overline{w}_i)\,, \tag{2.18a}$$

where

$$f_i=N_i\overline{N}_i+\nu_i^c\overline{\nu}_i^c+H_i^2\overline{H}_i^2+H_{2i}'\overline{H}_{2i}' \,. \tag{2.18b}$$

Finally the D term of the potential in Eq. (2.10) arising from the $SU(3)_L\times SU(3)_R$ gauge transformations is

$$\begin{aligned}V_D=&\frac{g_L^2}{24}\left[\sum_i x_i-\overline{x}_i+y_i-\overline{y}_i-\tfrac{1}{2}(z_i-\overline{z}_i+w_i-\overline{w}_i)\right]^2+\frac{g_L^2}{8}\left[\sum_i(-z_i+\overline{z}_i+w_i-\overline{w}_i)\right]^2\\&+\frac{g_L^2}{2}\left|\sum_i(H_i'^\dagger\nu_i^c-\overline{\nu}_i^{c\dagger}\overline{H}_{2i}')\right|^2+\frac{g_R^2}{6}\left[\sum_i[x_i-\overline{x}_i-\tfrac{1}{2}(y_i-\overline{y}_i+z_i-\overline{z}_i+w_i-\overline{w}_i)]\right]^2\\&+\frac{g_R^2}{8}\left[\sum_i(y_i-\overline{y}_i-z_i+\overline{z}_i+w_i-\overline{w}_i)\right]^2+\frac{g_R^2}{2}\left|\sum_i(N_i^\dagger\nu_i^c-\overline{\nu}_i^{c\dagger}\overline{N}_i)\right|^2 .\end{aligned} \tag{2.19}$$

The solutions to the extrema equations arising from Eq. (2.10) were analyzed in Ref. 11. It was shown there that the lowest-lying minima corresponded to the ones where matter parity was conserved at the intermediate scale when the $SU(2)_L\times U(1)$ subgroup was left unbroken. To the leading order it was found that

$$\langle N_1\rangle\simeq\langle\overline{N}_1\rangle\simeq\left[\frac{\Sigma_1^2 M_c^{4n-6}}{2n^2(2n-1)\lambda_1^4}\right]^{1(4n-4)}, \tag{2.20a}$$

$$\langle \nu_2^c\rangle\simeq\langle\overline{\nu}_2^c\rangle\simeq\left[\frac{\Sigma_1^2 M_c^{4n-6}}{2n^2(2n-1)\lambda_2^4}\right]^{1/(4n-4)}, \tag{2.20b}$$

$$\langle H_{2i}'\rangle=0=\langle H_i^2\rangle,\quad \langle N_2\rangle=0=\langle \nu_1^c\rangle\,, \tag{2.20c}$$

where

$$\Sigma_i^2\equiv(m_i^2+\overline{m}_i^2)\,. \tag{2.20d}$$

In the nonleading order there arise in Eqs. (2.20a) and (2.20b) terms proportional to Δ_i^2, where

$$\Delta_i^2\equiv(m_i^2-\overline{m}_i^2)\,. \tag{2.21}$$

These terms reflect the deviation of the extrema solution from their D-flat values and are thus dependent on the $SU(3)_L\times SU(3)_R$ gauge coupling constants g_L and g_R. The terms proportional to Δ_i^2 are in general quite important in that they often break the vacuum degeneracy and help locate the lowest minimum. It was also seen that the lowest minimum which automatically preserves matter parity also preserves $SU(2)_L\times U(1)_Y$ invariance.

III. GOLDSTONE AND HIGGS PHENOMENA AND VECTOR-BOSON MASS GROWTH

We consider next the effect of the VEV growth of N and ν^c on the spontaneous breaking of $SU(3)_C\times SU(3)_L\times SU(3)_R$. The N_1 VEV growth yields

$$SU(3)_C\times SU(3)_L\times SU(3)_R \xrightarrow{\langle N_{C\ \rm even}\rangle} SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\,. \tag{3.1}$$

while the ν_2^c VEV growth breaks the group further to the standard-model gauge group: i.e.,

$$SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L} \xrightarrow{\langle \nu^c_{C\ \rm odd}\rangle} SU(3)_C\times SU(2)_L\times U(1)_Y\,. \tag{3.2}$$

We can calculate the vector-boson masses and identify the Goldstone bosons that are absorbed. The vector-boson masses arise from an analysis of the kinetic energy of the lepton multiplet L^l_{ir} and the mirror lepton multiplet $\bar{L}^{\,r}_{il}$ with $i=1,2$ corresponding to C-even and C-odd multiplets. The gauge couplings of these multiplets are

$$L_{L_i}=-\sum_i (D_\mu L^l_{ir})^\dagger (D_\mu L^l_{ir})-\sum_i (D_\mu \bar{L}^{\,r}_{il})^\dagger (D_\mu \bar{L}^{\,r}_{il}) \;, \tag{3.3}$$

where

$$D_\mu L^l_{ir}=\partial_\mu L^l_{ir}-\frac{ig_L}{2}A^\alpha_{\mu L}(\lambda^\alpha_L)^l_{l'}L^{l'}_{ir}+\frac{ig_R}{2}A^\alpha_{\mu R}L^l_{ir'}(\lambda^\alpha_R)^{r'}_r \tag{3.4}$$

and a similar equation holds for the $\bar{L}_i$ multiplet. In Eq. (3.4) $A^\alpha_{\mu L,R}$, $\alpha=1-8$ are the $\mathrm{SU}(3)_{L,R}$ gauge fields. As a consequence of the VEV growth of Eq. (2.20) we find that 12 vector bosons grow masses. These are

$$A^\alpha_{\mu L},\quad A^{1,2}_{\mu R},\quad A^\alpha_{\mu R},\quad A^{\mathrm{I}}_\mu,\quad A^{\mathrm{II}}_\mu,\quad \alpha=4,5,6,7 \;, \tag{3.5}$$

where $A^{\mathrm{I,II}}_\mu$ are two linear combinations of $A^8_{\mu L}, A^8_{\mu R}, A^3_{\mu R}$ orthogonal to the hypercharge boson $[A^Y_\mu=(g_R A^8_{\mu L}+g_L A^8_{\mu R}+\sqrt{3}g_R A^3_{\mu R})/(4g_R^2+g_L^2)^{1/2}]$. The vector-boson masses are

$$M^\alpha_L=\frac{1}{\sqrt{2}}g_L\sqrt{N^2+v^2},\quad M^{1,2}_R=\frac{1}{\sqrt{2}}g_R v \;,$$
$$M^{4,5}_R=\frac{g_R}{\sqrt{2}}N,\quad M^{6,7}_R=\frac{g_R}{\sqrt{2}}\sqrt{N^2+v^2} \;,$$
$$(M^{\mathrm{I,II}})^2=\tfrac{1}{3}(g_L^2+g_R^2)(N^2+v^2) \pm\tfrac{1}{3}[(g_L^2+g_R^2)^2(N^2+v^2)^2 -3N^2v^2(g_R^4+4g_R^2g_L^2)]^{1/2} \;, \tag{3.6}$$

where

$$N^2=N_1N_1^\dagger+\bar{N}_1\bar{N}_1^\dagger,\quad v^2=v_2^c v_2^{c\dagger}+\bar{v}_2^c\bar{v}_2^{c\dagger} \;. \tag{3.7}$$

For the purpose of knowing what the low-energy spectrum of the theory is, it is important to determine what components of the lepton multiplets L_i and the mirror multiplets $\bar{L}_i$ are absorbed by the vector bosons in the process of spontaneous breaking. In general, for an effective potential $V(\boldsymbol{\phi})$, where $\boldsymbol{\phi}=\{\phi_a\}$ is a column symbol of all the scalar fields, the absorbed Goldstone bosons are given by the nonzero vectors $(t^\alpha)_{ab}\langle\phi_b\rangle$, where t^α are the group generators for the representation of $\boldsymbol{\phi}$. For our case $\{\phi_a\}=\{L^l_{ir},L^{l\dagger}_{ir},\bar{L}^r_{il},\bar{L}^{r\dagger}_{il}\}$ and t^α are SU(3) Gell-Mann matrices of $\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R$. The following calculations neglect small corrections of $O(\Delta/M_c;\Sigma/M_c)$. For the bosons absorbed by $A^\alpha_{\mu L}$, $\alpha=4,5,6,7$ we find

$$\cos\theta\frac{1}{\sqrt{2}}(\nu_1+\bar{\nu}_1^\dagger)-\sin\theta\frac{1}{\sqrt{2}}(H_2^{0\prime}+\bar{H}_2^{0\prime\dagger}) \;,$$
$$\cos\theta\frac{1}{\sqrt{2}}(e_1+\bar{e}_1^\dagger)-\sin\theta\frac{1}{\sqrt{2}}(H_2^{-\prime}+\bar{H}_2^{-\prime\dagger}) \;, \tag{3.8a}$$

where

$$\cos\theta\equiv g_L\langle N_1\rangle(g_L^2\langle N_1\rangle^2+g_R^2\langle\nu_2^c\rangle^2)^{1/2} \;,$$
$$\sin\theta\equiv g_R\langle\nu_2^c\rangle/(g_L^2\langle N_1\rangle+g_R^2\langle\nu_2^c\rangle^2)^{1/2} \tag{3.8b}$$

and $(H_2^{0\prime},H_2^{-\prime})$ is the Higgs $\mathrm{SU}(2)_L$ doublet. Note that the fields of Eq. (3.8a) form a $\mathrm{SU}(2)_L$ doublet. The components absorbed by $A^\alpha_{\mu R}$, $\alpha=1,2$ are

$$\frac{1}{\sqrt{2}}(e_2^c+\bar{e}_2^{c\dagger}) \tag{3.8c}$$

and those absorbed by $A^\alpha_{\mu R}$, $\alpha=4,5,6,7$ are

$$\frac{1}{\sqrt{2}}(e_1^c+\bar{e}_1^{c\dagger}),\quad \frac{1}{\sqrt{2}}(\nu_1^c+\bar{\nu}_1^{c\dagger}) \;. \tag{3.8d}$$

Finally, the components absorbed in making A^{I}_μ and A^{II}_μ heavy are

$$\frac{1}{\sqrt{2}}(\mathrm{Im}N_1-\mathrm{Im}\bar{N}_1),\quad \frac{1}{\sqrt{2}}(\mathrm{Im}\nu_2^c-\mathrm{Im}\bar{\nu}_2^c) \;. \tag{3.8e}$$

These are all $\mathrm{SU}(2)_L$ singlets [though Eq. (3.8d) is an $\mathrm{SU}(2)_R$ doublet].

There are 72 Hermitian components of the lepton multiplets $L_i,\bar{L}_i$ $(i=1,2)$ of which the 12 of Eqs. (3.8) are the massless absorbed Goldstone bosons. As seen from Eqs. (3.8), it is linear combinations of C-even and C-odd leptons and their mirrors that get absorbed. Associated with these are 12 superheavy bosons of mass $O(M_I)$. The general mass matrix is $M_{ab}=\langle\partial^2V/\partial\phi_a\partial\phi_b\rangle$, where V is defined in Eq. (2.10) with V_m and V_F given in Eqs. (2.11) and (2.18). The full D term is

$$V_D=\tfrac{1}{2}\sum_\infty(D^\alpha_L D^{\alpha\dagger}_L+D^\alpha_R D^{\alpha\dagger}_R) \;, \tag{3.9}$$

where

$$D^\alpha_L=\tfrac{1}{2}g_L\sum(L^{\lambda\dagger}_{i\rho}L^{\lambda'}_{i\rho}-\bar{L}^{\,\lambda}_{i\rho}\bar{L}^{\,\lambda'\dagger}_{i\rho})(t^\alpha)^\lambda_{\lambda'} \;,$$
$$D^\alpha_R=-\tfrac{1}{2}g_R\sum(L^{\lambda\dagger}_{i\rho}L^\lambda_{i\rho'}-\bar{L}^{\,\lambda}_{i\rho}\bar{L}^{\,\lambda\dagger}_{i\rho'})(t^\alpha)^{\rho'}_\rho \;. \tag{3.10}$$

The leading piece of the two nonvanishing VEV's is given in Eqs. (2.20a) and (2.20b). This implies that the contribution of V_m and V_F to M_{ab} is of electroweak size, i.e., $O(\Sigma^2;\Delta^2)$. Only V_D can produce large mass contributions and only for those fields having $\langle N_1\rangle^2$ or $\langle\nu_2^c\rangle^2$ as coefficients in M_{ab}. It is straightforward to pick these heavy fields out. The pieces in Eq. (3.9) containing D^α_L, $\alpha=4,5,6,7$ yield

$$\phi_{L(45)}=\cos\theta\frac{1}{\sqrt{2}}(\nu_1-\bar{\nu}_1^\dagger)-\sin\theta\frac{1}{\sqrt{2}}(H_2^{0\prime}-\bar{H}_2^{0\prime\dagger}) \;,$$
$$\phi_{L(67)}=\cos\theta\frac{1}{\sqrt{2}}(e_1-\bar{e}_1^\dagger)-\sin\theta\frac{1}{\sqrt{2}}(H_2^{-\prime}-\bar{H}_2^{-\prime\dagger}) \;, \tag{3.11a}$$

where

$$M_{L(45)}=M_{L(67)}=g_L(\langle N_1\rangle^2+\langle\nu_2^c\rangle^2)^{1/2} \;. \tag{3.11b}$$

$\phi_{L(45)}$ and $\phi_{L(67)}$ form an $\mathrm{SU}(2)_L$ doublet. Terms arising from D^α_R, $\alpha=1.2$ give

$$\phi_{R(12)}=\frac{1}{\sqrt{2}}(e_2^c-\bar{e}_2^{c\dagger}),\quad M_{R(12)}=g_R\langle\nu_2^c\rangle \tag{3.12}$$

while the heavy fields from D^α_R, $\alpha=4,5,6,7$ are

$$\phi_{R(45)}=\frac{1}{\sqrt{2}}(e_1^c-\bar{e}_1^{c\dagger}),\quad \phi_{R(67)}=\frac{1}{\sqrt{2}}(\nu_1^c-\bar{\nu}_1^{c\dagger}) \;, \tag{3.13a}$$

where

$$M_{R(45)}=g_R\langle N_1\rangle\ . \tag{3.13b}$$

$$M_{R(67)}=g_R\langle [N_1^2+(\nu_2^c)^2]^{1/2}\rangle\ . \tag{3.13c}$$

The heavy fields arising from the $D^8_{R,L}$ parts of V_D are more complicated to treat as the mass matrix links four fields: $\mathrm{Re}N_1,\mathrm{Re}\bar{N}_1,\mathrm{Re}\nu_2^c,\mathrm{Re}\bar{\nu}_2^c$. A detailed discussion of this sector is given in the Appendix. There it is shown that the heavy fields are

$$\phi_\pm=N_\pm(\mathrm{Re}N_1-\mathrm{Re}\bar{N}_1)\pm N_\mp(\mathrm{Re}\nu_2^c-\mathrm{Re}\bar{\nu}_2^c) \tag{3.14a}$$

with masses

$$M_\pm=\{a+b\pm[(a-b)^2+4c^2]^{1/2}\}^{1/2}\ , \tag{3.14b}$$

where

$$N_\pm=\frac{1}{2}\left[1\mp\frac{b-a}{\sqrt{(b-a)^2+4c^2}}\right]^{1/2},$$

$$a\equiv\tfrac{1}{3}(g_L^2+g_R^2)\langle N_1\rangle^2,\quad b\equiv\tfrac{1}{3}(g_L^2+g_R^2)\langle\nu_2^c\rangle^2\ , \tag{3.14c}$$

$$c\equiv\tfrac{1}{6}(2g_L^2-g_R^2)\langle N_1\nu_2^c\rangle\ .$$

Note that $M_\pm^2>0$.

There are 72 real components in the multiplets of $L_i,\bar{L}_i$ $(i=1,2)$ of which we have seen above 12 real components are the absorbed Goldstone modes and 12 components become superheavy. These 24 components are combinations of C-even and C-odd lepton states and their mirrors. Thus the remaining 48 components must also be linear combinations of leptons and their mirrors. If these fields have low mass, one might be able to sample *remnants of the mirror world* in accelerator experiments. These exotic leptons would thus carry the imprint of the Calabi-Yau manifold on which the superstring has compactified.

Of the 48 remaining components, two combinations are shown in the Appendix to be massless at M_I:

$$\frac{1}{\sqrt{2}}(\mathrm{Im}N_1+\mathrm{Im}\bar{N}_1),\quad \frac{1}{\sqrt{2}}(\mathrm{Im}\nu_2^c+\mathrm{Im}\nu_2^c)\ . \tag{3.15}$$

They are expected to grow electroweak size masses at the M_W scale from RG corrections and $\mathrm{SU(2)}\times\mathrm{U(1)}$ breaking. The above discussion has been model independent. To proceed further one must make additional detailed assumptions concerning the structure of the Calabi-Yau manifold. Thus N_1 and ν_2^c couple to the lepton multiplets in the renormalizable cubic interactions producing a mass term of the type

$$-\lambda^3_{nm1}H_n^\lambda H'_{\lambda m}\langle N_1\rangle-\lambda^3_{rs1}H_r^\lambda H'_{\lambda s}\langle N_1\rangle\ . \tag{3.16}$$

Thus any Higgs doublet for which λ^3_{nm1} or λ^3_{rs1} are nonzero will become superheavy. Phenomenologically, one requires at least *one* pair of Higgs doublets *not* to couple to N_1 and hence remain massless. This would be the doublet pair that at the M_W scale accomplishes the $\mathrm{SU(2)}\times\mathrm{U(1)}$ breaking. Presumably this pair lies in the C-even sector so that matter parity be maintained. To have a Higgs doublet *not* coupling to N_1 requires a zero in the coupling constant matrix λ^3_{mn1}. One may check that such an "accident" does indeed occur for the symmetric Calabi-Yau manifold where the cubic Yukawa couplings are known.[16,9] The simplest assumption is that there is only a single zero in λ^3_{mn1} (and no zero in λ^3_{rs1}). There are 32 real components associated with $H_i^\lambda,H'_{\lambda i},\bar{H}_i^\lambda,\bar{H}'_{\lambda i}$, $i=1,2$. Thus the above assumption implies that (aside from the usual fields associated with the three generations of the standard model) the fields in $L_i,\bar{L}_i$, $i=1,2$ that will remain light are $72-(12+12+32)=16$ in number.

Equation (2.15) also shows renormalizable coupings of ν_2^c to $H^\lambda l_\lambda$ and the matter-parity constraint (2.16) limits mass growth from these couplings to

$$-\lambda^3_{r2m}(H_r^\lambda l_{\lambda m}\langle\nu_2^c\rangle+H_m^\lambda l_{\lambda r}\langle\nu_2^c\rangle)\ . \tag{3.17}$$

Thus if the coupling constant λ^3_{221} is nonzero, both $l_{\alpha 1},l_{\alpha 2}$ will become superheavy, while if λ^3_{221} vanishes, they will remain light. Again which of these two possibilities occur depends on the details of the Calabi-Yau manifold. We list in Table III the fields that remain light in $L_i,\bar{L}_i$, $i=1,2$ (over and above the three generations of the standard model) for the two possibilities.

IV. SPECTRUM OF EXTRA GENERATIONS

As discussed in the Introduction, the massless spectrum of the theory at the intermediate scale is rather large. It consists of nine families of $(1,3,\bar{3})$ leptons and six families of mirror $(1,\bar{3},3)$ leptons; seven families of $(3,\bar{3},1)$ quarks and four families of $(\bar{3},3,1)$ mirror quarks; and seven families of $(\bar{3},1,3)$ antiquarks and four families of $(3,1,\bar{3})$ mirror antiquarks. Below intermediate-mass scale breaking, one would like to have as few of the extra generations or mirror generations as possible. This is because in $N=1$ supersymmetric theory there is little room beyond three generations if one wants to achieve consistency with the current experimental limit on $\sin^2\theta_W$.

In Sec. III we have seen that the breaking of $\mathrm{SU(3)}_C\times\mathrm{SU(3)}_L\times\mathrm{SU(3)}_R$ symmetry at the intermediate scale requires at least two lepton multiplets L_i $(i=1,2)$, one C even and the other C odd, and their mirrors $\bar{L}_i$. Certain linear combinations of these are absorbed while of the remaining, many become superheavy leaving only 8 real fields with electroweak masses in the low-energy

TABLE III. Additional (exotic) light particles of electroweak mass when only a single zero remains in λ^2_{mn1}. The two columns refer to possibilities discussed in text. l_2^α and $\bar{l}_2^\alpha$ are $\mathrm{SU(2)}_L$ doublets. All other particles are $\mathrm{SU(2)}_L\times\mathrm{U(1)}_Y$ invariant.

$\lambda^3_{221},\bar{\lambda}^3_{221}=0$	$\lambda^3_{221}\neq 0$
$\frac{1}{\sqrt{2}}(\mathrm{Im}\nu_2^c+\mathrm{Im}\bar{\nu}_2^c)$	$\frac{1}{\sqrt{2}}(\mathrm{Im}\nu_2^c+\mathrm{Im}\bar{\nu}_2^c)$
$\frac{1}{\sqrt{2}}(\mathrm{Im}N_1+\mathrm{Im}\bar{N}_1)$	$\frac{1}{\sqrt{2}}(\mathrm{Im}N_1+\mathrm{Im}\bar{N}_1)$
$\mathrm{Re}N_2,\mathrm{Im}N_2$	$\mathrm{Re}N_2,\mathrm{Im}N_2$
$\mathrm{Re}\bar{N}_2,\mathrm{Im}\bar{N}_2$	$\mathrm{Re}\bar{N}_2,\mathrm{Im}\bar{N}_2$
$\frac{1}{\sqrt{2}}(\mathrm{Re}N_1+\mathrm{Re}\bar{N}_1)$	$\frac{1}{\sqrt{2}}(\mathrm{Re}N_1+\mathrm{Re}\bar{N}_1)$
$\frac{1}{\sqrt{2}}(\mathrm{Re}\nu_2^c+\mathrm{Re}\bar{\nu}_2^c)$	$\frac{1}{\sqrt{2}}(\mathrm{Re}\nu_2^c+\mathrm{Re}\bar{\nu}_2^c)$
$l_2^\lambda,\bar{l}_2^{\lambda\dagger}$	

domain. As discussed in Ref. 11, one also has that below the intermediate-mass scale breaking the mass matrix has the general form

$$W_{\text{mass}}=(M_{rs}^{(l)}l_r\bar{l}_s+M_{mn}^{(l)}l_m\bar{l}_n)+(M_{rs}q_r\bar{q}_s+M_{mn}^{(q)}q_m\bar{q}_n)\ . \tag{4.1}$$

An examination of the C-odd and C-even lepton and quark states of Table I shows that three generations of leptons and quarks remain massless below the intermediate-mass scale. Also barring accidental zeros in the mass matrix, the remaining families and mirror families would become massive. However, unless these extra families and mirror families have superheavy masses, the theory is not phenomenologically viable since it would lead to Landau singularity in the color coupling constant below the intermediate-mass scale and yield unacceptably large values of $\sin^2\theta_W$. This is precisely the situation that occurs if one straightforwardly extends the superpotential of Eq. (2.17) to run over all the families and mirror families. Such an extension yields electroweak masses for all the extra families and mirror families which would lead to the phenomenological problem discussed above.

At present, theoretical evaluations from first principles of the nonrenormalizable interactions of the superpotential on Calabi-Yau manifolds are not available. Here we proceed phenomenologically to determine the desired constraints on the nonrenormalizable part of the superpotential which can avoid the problem discussed above. These represent the constraints that the Calabi-Yau manifold of Eq. (2.5) must satisfy if it is to be phenomenologically viable. The desired form for the leading part of the nonrenormalizable interactions of the superpotential for extra generations $A\neq i$ is

$$\Delta W_{\text{nr}}=\sum_{A\neq i}\frac{\lambda_{AB}^2}{(M_c)^{2n_{AB}-3}}(L_i\bar{L}_i)^{n_{AB}-1}(L_A\bar{L}_B)\ . \tag{4.2}$$

The condition needed to avoid the phenomenological difficulties is

$$n_{AB}<n_i,\quad A,B\neq i\ , \tag{4.3}$$

where $i=1,2$ refers to the C-even and C-odd multiplets that enter in the intermediate scale breaking of Eq. (2.17) and $A\neq i$ run over all other lepton and quark generations. Since the VEV's of L_A and $\bar{L}_A$ are zero, Eq. (4.2) does not contribute to the extrema equations. Thus results of Eq. (2.20) hold with Eq. (4.2) included. The growth of N_1 and ν_2^c VEV's then generate mass terms in Eq. (4.2). For simplicity, we will assume from now on that Eq. (4.2) is diagonal in A and B and write $\lambda_{AB}=\lambda_A\delta_{AB}$, $n_A=n_{AA}$. One has then

$$W_{\text{nr}}^{\text{mass}}=\sum_{A\neq i}m_A(L_A\bar{L}_A)\ , \tag{4.4}$$

where

$$m_A=\frac{\lambda_A^2}{M_c^{2n_A-3}}\left[\left[\frac{\Sigma_1^2}{\lambda_1^4}+\frac{\Sigma_2^2}{\lambda_2^4}\right]\times\frac{M_c^{4n_i-6}}{2n_i^2(2n_i-1)}\right]^{(n_A-1)/(2n_i-2)}\ . \tag{4.5}$$

In Table IV we present an estimate of m_A setting M_c equal to the Planck mass $M_{\text{Pl}}=2.4\times10^{18}$ GeV. One sees that values of $n_i\geq3$ are phenomenologically acceptable. Thus with the superpotential of Eqs. (2.17) and (4.2) we can break the $[\text{SU(3)}]^3$ symmetry spontaneously at the intermediate scale and make all the extra families and mirror families superheavy with masses $\gtrsim10^{14}$ GeV with $n_i\geq3$.

The three massless generations that arise from the diagonalization of the mass matrix of Eq. (4.1) contain extra particles not found in the standard model. First they contain six Higgs doublets which are probably too many to give a satisfactory value for $\sin^2\theta_W$. However, as discussed in Sec. III, the cubic interactions in Eq. (2.15) of type $HH'N$ would be expected to give superheavy masses to the extra Higgs doublets. In addition there are color triplets D^a, D_a^c which can mediate proton decay through dimension five operators. From Eq. (2.15), however, one finds that the VEV growth for the N_1 field generates a mass term for D^a, D_a^c of the form

$$-\lambda_{A1B}^4D_A^aD_{aB}^c\langle N_1\rangle \tag{4.6}$$

and thus gives superheavy masses to all the D^a,D_a^c quarks. A recent analysis[17] shows that the proton-decay mediated by the D-quark exchange can be made consistent with the existing proton lifetime lower limits from Irvine-Michigan-Brookhaven (IMB) and KAMIOKANDE[18] experiments with a D-quark mass as low as 10^{13}–10^{14} GeV. This is the mass range for the D quarks expected from the mass growth of Eq. (4.6).

V. CONCLUSION

In this paper we have investigated the particle spectrum of the three-generation Calabi-Yau compactification of the $E_8\times E_8'$ superstring below the intermediate-mass scale. The analysis was done under the assumption that the symmetry breaking at the intermediate-mass scale preserves matter parity which is needed to forbid the dangerous proton-decay arising from dimension-four operators. An analysis of the Goldstone and Higgs fields

TABLE IV. The mass m_A of unwanted generations as a function of n_i with $n_A=2$, $\lambda_A=10^2$, $\Sigma_1=\Sigma_2=10^3$ GeV for two different values of $\lambda_1=\lambda_2=\lambda$.

	$\lambda=1$		$\lambda=10^{-2}$	
n_i	$\langle N_1\rangle$ (GeV)	m_A (GeV)	$\langle N_1\rangle$ (GeV)	m_A (GeV)
2	2.2×10^{10}	2.9×10^{6}	2.2×10^{12}	2.9×10^{10}
3	2.0×10^{14}	1.9×10^{14}	2.0×10^{15}	1.9×10^{16}
4	4.2×10^{15}	8.2×10^{16}	1.9×10^{16}	1.8×10^{18}
5	2.0×10^{16}	1.7×10^{18}	6.2×10^{16}	1.7×10^{19}

and the mass growth of vector boson associated with the breakdown of the $[SU(3)]^3$ symmetry to the standard-model symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ was carried out. The unabsorbed components of two lepton families and two mirror lepton families which provide the Higgs fields in intermediate scale breaking were classified. It was shown that a number of unabsorbed components become superheavy through the cubic superpotential after the N and ν^c VEV growth leaving a few fields with electroweak size masses. Further, it is was shown that all the remaining unwanted families and mirror families of quarks and leptons can be removed from the low-energy spectrum by generating superheavy masses for these particles through nonrenormalizable interactions that appear in the superpotential. The final low-energy spectrum of the theory was shown to be just the spectrum of the minimal $N=1$ supersymmetry theory except for the few exotic leptons listed in Table III. The phenomenology expected from these extra particles will be discussed elsewhere.

APPENDIX

We discuss here in detail the mass spectrum of the C-even field N_1 and the C-odd field ν_2^c which grow VEV's at the intermediate scale M_I to break $[SU(3)]^3$ to the standard model. The effective potential for these fields are given by Eqs. (2.10)–(2.12), (2.18), (2.20d), and (2.21) with V_D given by

$$V_D=\tfrac{1}{6}(g_L^2+g_R^2)(x_1-\bar{x}_1+y_2-\bar{y}_2)^2 -\tfrac{1}{2}g_R^2(x_1-\bar{x}_1)(y_2-\bar{y}_2)\ . \tag{A1}$$

We see that V_D produces coupling between the N_1 and ν_2^c fields. It is convenient to decompose $N_1,\bar{N}_1,\nu_2^c,\bar{\nu}_2^c$ into real and imaginary parts:

$$\begin{aligned} N_1&=n_1+in_1',\quad \bar{N}_1=\bar{n}_1+i\bar{n}_1'\ ,\\ \nu_2^c&=\nu_2+i\nu_2',\quad \bar{\nu}_2^c=\bar{\nu}_2+i\bar{\nu}_2'\ , \end{aligned} \tag{A2}$$

where the VEV's of $n_1',\bar{n}_1',\nu_2',\bar{\nu}_2'$ all vanish. One may easily verify that the mass matrix elements of the imaginary parts all vanish, e.g.,

$$\left\langle \frac{\partial^2 V}{\partial n_1'^2}\right\rangle = 4\langle n_1'\rangle^2\left\langle\frac{\partial^2 V}{\partial x_1^2}\right\rangle+2\left\langle\frac{\partial V}{\partial x_1}\right\rangle=0 \tag{A3}$$

since $\langle \partial V/\partial x_1\rangle$ vanishes at the extrema. Thus in this approximation all four imaginary parts are massless. Two combinations are the Goldstone bosons of Eq. (3.8e), and the two remaining combinations are entries in Table III. The four real parts, couple in the mass matrix M^2. Labeling rows and columns by $n_1,\bar{n}_1,\nu_2,\bar{\nu}_2$ we find

$$M^2=\begin{bmatrix} A+a & B-(a\bar{a})^{1/2} & c & -d\\ B-(a\bar{a})^{1/2} & \bar{A}+\bar{a} & -\bar{d} & \bar{c}\\ c & -d & C+b & D-(b\bar{b})^{1/2}\\ -d & \bar{c} & D-(b\bar{b})^{1/2} & \bar{D}+\bar{d} \end{bmatrix}, \tag{A4}$$

where

$$\begin{aligned} a&=\tfrac{1}{3}(g_L^2+g_R^2)x_1,\quad b=\tfrac{1}{3}(g_L^2+g_R^2)y_2\ ,\\ c&=\tfrac{1}{6}(2g_L^2-g_R^2)\sqrt{x_1y_2},\quad d=\tfrac{1}{6}(2g_L^2-g_R^2)(x_1\bar{y}_2)^{1/2} \end{aligned} \tag{A5}$$

and

$$\begin{aligned} A&=\frac{4n^2(n-1)}{M_c^{4n-6}}\lambda_1^2(x_1\bar{x}_1)^{n-2}[nx_1\bar{x}_1+(n-2)\bar{x}_1^2]\ ,\\ B&=\frac{4n^3(n-1)}{M_c^{4n-6}}\lambda_1^2(x_1\bar{x}_1)^{n-2}\sqrt{x_1N}\,(x_1+\bar{x}_1)\ . \end{aligned} \tag{A6}$$

In Eq. (A4), the overbar notation means replace x_1,y_2 by $\bar{x}_1,\bar{y}_2$ [e.g., $\bar{a}=(1/3q)(g_L^2+g_R^2)\bar{x}_1$, etc.] and C,D can be obtained from A,B by the replacement $x_1,\bar{x}_1,\lambda_1$ by $y_2,\bar{y}_2,\lambda_2$. [x_1,y_2, etc., are defined in Eq. (2.12).] It is understood in the mass matrix that all fields are VEV's. The leading terms for these VEV's are[11]

$$(x_1)^{2n-2}\simeq(\bar{x}_1)^{2n-2}\simeq\frac{(\Sigma_1)^2M_c^{4n-6}}{2n^2(2n-1)\lambda_1^4}\ , \tag{A7a}$$

$$(y_2)^{2n-2}\simeq(\bar{y}_2)^{2n-2}\simeq\frac{(\Sigma_2)^2M_c^{4n-6}}{2n^2(2n-1)\lambda_2^4} \tag{A7b}$$

while $\bar{x}_1-x_1=O(\Delta_i^2)$, $\bar{y}_1-y_1=O(\Delta_i^2)$ are electroweak in size.[11]

It is convenient to introduce the combinations

$$\tilde{x}_1=\tfrac{1}{2}(x_1+\bar{x}_1),\quad \tilde{y}_1=\tfrac{1}{2}(y_1+\bar{y}_1) \tag{A8}$$

and rewrite (A4) as

$$M^2=M_0^2+M_1^2\ , \tag{A9}$$

where M_0^2 is the matrix M^2 with A,B,C,D set to zero and a and $\bar{a}$ set to $\tilde{a}$, b and $\bar{b}$ set to $\tilde{b}$, and c and d set to $\tilde{c}$, where the tilde quantities are the original ones with x_1,y_2, etc., replaced by $\tilde{x}_1,\tilde{y}_2$ [e.g., $\tilde{a}=\tfrac{1}{3}(g_L^2+g_R^2)\tilde{x}_1$, etc.]. M_1^2 is then the remainder, i.e., $M_1^2\equiv M^2-M_0^2$. From Eqs. (A7) and (A8) it is clear that M_0^2 is the leading part of M^2 for large M_c, while M_1^2 is of (small) electroweak size. We may thus diagonalize M^2 perturbatively, by first solving for the eigenvalues and eigenvectors of M_0^2. One finds easily two superheavy eigenvalues and two massless values:

$$\lambda_\pm^{(0)}=\tilde{a}+\tilde{b}\pm[(\tilde{a}-\tilde{b})^2+4\tilde{c}^{\,2}]^{1/2}\ ,$$
$$\lambda_{3,4}^{(0)}=0 \tag{A10}$$

with corresponding eigenfunctions given in Eq. (3.14a) and

$$\phi_3=\frac{1}{\sqrt{2}}(n_1+\bar{n}_1),\quad \phi_4=\frac{1}{\sqrt{2}}(\nu_2+\bar{\nu}_2)\ . \tag{A11}$$

One may find the corrections to $\lambda_\pm^{(0)}$ by perturbation theory where M_1^2 is the perturbation. One easily sees that

$$\Delta\lambda_\pm=\lambda_\pm-\lambda_\pm^{(0)}\simeq-\frac{8(n-1)}{2n-1}(N_\pm\Sigma_1^2+N_\mp\Sigma_2^2)\ , \tag{A12}$$

where $N_\pm$ is given in Eq. (3.14c). A convenient procedure to calculate the nonzero correction to λ_3 and λ_4 is to take the determinant and trace of Eq. (A4). To leading order we have

$$\det M=\lambda_3\lambda_4\lambda_+\lambda_-\simeq 16(n-1)\Sigma_1^2\Sigma_2^2\lambda_+^{(0)}\lambda_-^{(0)} \tag{A13}$$

and

$$\mathrm{tr}M=\lambda_3+\lambda_4+\lambda_++\lambda_-$$
$$\simeq\lambda_+^{(0)}+\lambda_-^{(0)}+\frac{8(n-1)^2}{2n-1}(\Sigma_1^2+\Sigma_2^2) \tag{A14}$$

showing that $\lambda_{3,4}>0$ and hence nontachyonic.

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