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Renormalization-group-improved unitarity bounds on the Higgs-boson and top-quark masses

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A renormalization-group-improved perturbative unitarity bound for elastic scattering amplitudes is proposed. This prescription leads to upper bounds on the Higgs-boson and top-quark masses as a function of the energy at which perturbative unitarity is violated and new physics enters. Upper bounds on the scale of new physics in models with no Higgs boson are also discussed.

I. INTRODUCTION

Weak interactions are well described by a theory of massive vector bosons: the $W^{\pm}$ and Z. However, it is known that such a theory by itself is nonrenormalizable. In the standard $SU(2)_L \times U(1)$ model,[1] this problem is overcome by appending a scalar particle, dubbed the Higgs boson, which couples to the massive vector bosons in a manner which renders the theory renormalizable. The condition of renormalizability does not determine the Higgs-boson mass, however; it remains a free parameter of the theory.

Similarly, the standard model accommodates but does not predict fermion masses. The top quark is required in the standard model to cancel triangle anomalies, whose presence would otherwise ruin the renormalizability of the theory.[2] This requirement does not determine the top-quark mass, however.

In this paper we present a prescription to determine upper bounds on the Higgs-boson and top-quark masses by combining the ideas of perturbative unitarity and triviality. For clarity we develop and discuss this prescription in the context of the Higgs-boson mass in Sec. II. We then apply the same ideas to the top-quark mass in Sec. III. Section IV summarizes our results.

II. UPPER BOUND ON THE HIGGS-BOSON MASS

There have been various attempts to find an upper bound on the Higgs-boson mass. The simplest approaches in the sense that they require rather minimal assumptions are based on perturbative unitarity and triviality. The perturbative unitarity bound is derived by requiring that the Born amplitude for elastic longitudinal vector-boson scattering not exceed the unitarity limit.[3,4] Although this bound is usually interpreted as the onset of strong coupling, it may also be regarded as a bound on the Higgs-boson mass, as we discuss below. The triviality bound on the Higgs-boson mass is based on the assertion that scalar field theories are trivial, i.e., noninteracting, unless they are regarded as effective, low-energy theories.[5] The triviality bound is then derived by demanding that the Higgs-boson mass not exceed the scale at which the effective-field-theory description breaks down and new physics enters. The triviality bound is generally obtained by using lattice regularization, with the inverse lattice spacing playing the role of the scale of new physics.[6-13]

In this paper we combine the notions of perturbative unitarity and triviality to obtain a bound on the Higgs-boson mass as a function of the scale at which new physics enters. The basic prescription is to apply the condition of perturbative unitarity to longitudinal vector-boson scattering amplitudes in which the coupling is allowed to run with energy. The Goldstone-boson equivalence theorem[4,14-16] tells us that these amplitudes are identical to Goldstone-boson scattering amplitudes. Since the Goldstone bosons are scalars and the coupling of a scalar field theory grows with increasing energy, we will find that for any Higgs-boson mass there is an energy at which perturbative unitarity is violated, and we identify this energy as the scale at which new physics enters. This prescription is linked with the notion of triviality in the sense that the growth of the coupling with increasing energy is related to the triviality of the theory.

The unitarity bound on the Higgs-boson mass has been obtained previously by applying the condition $|a_0| \leq 1$ to the zeroth partial-wave amplitude for longitudinal vector-boson scattering in the limit $s \gg m_H^2$, while ignoring the running of the coupling.[3,4] Since the Born approximation to a_0 is proportional to $G_F m_H^2$, one obtains an upper bound on the Higgs-boson mass, which is $m_H^2 \leq 8\pi\sqrt{2}/3G_F \approx (1\ \text{TeV})^2$.

Recently, Lüscher and Weisz[13] have pointed out that the unitarity bound on the Higgs-boson mass may be improved by applying the stronger condition $|\text{Re}a_0| \leq \frac{1}{2}$. Since a_0 is real in Born approximation, this lowers the unitarity bound on the Higgs-boson mass by a factor of $1/\sqrt{2}$. Since this stronger condition is important for our later discussion, we give a brief outline of its derivation.

Unitarity of the S matrix implies that the Jth partial-wave amplitude of an elastic scalar scattering process, defined by

$$a_J(s) = \frac{1}{32\pi} \int_{-1}^{1} dz\, P_J(z) T(s,z) \,, \tag{1}$$

where $T(s,z)$ is the amplitude and z is the cosine of the scattering angle, satisfies

$$\text{Im}a_J \geq |a_J|^2 \tag{2}$$

for the scattering of massless particles.[17] Since $\text{Im}a_J$

$\leq |a_J|$ (Schwartz inequality), Eq. (2) implies $|a_J| \leq 1$, which is the usual condition used to obtain the unitarity bound on the Higgs-boson mass. However, if we rewrite Eq. (2) as

$$\mathrm{Im} a_J \geq (\mathrm{Re} a_J)^2 + (\mathrm{Im} a_J)^2 , \tag{3}$$

we obtain

$$(\mathrm{Re} a_J)^2 \leq \mathrm{Im} a_J (1 - \mathrm{Im} a_J) . \tag{4}$$

Since the right-hand side of this equation is bounded by $\frac{1}{4}$, it implies

$$|\mathrm{Re} a_J| \leq \tfrac{1}{2} , \tag{5}$$

as noted in Ref. 13. This inequality is obvious if one envisions the unitarity circle.[18]

The zeroth-partial-wave amplitudes for longitudinal-vector-boson scattering are given in Ref. 4. The unitarity bound on the Higgs-boson mass is obtained by considering the limit $s \gg m_H^2$. In this limit we have[19]

$$a_0(W_L^+ W_L^- \to W_L^+ W_L^-) = -\frac{\lambda}{4\pi} , \tag{6a}$$

$$a_0\left(\frac{1}{\sqrt{2}} Z_L Z_L \to \frac{1}{\sqrt{2}} Z_L Z_L\right) = -\frac{3\lambda}{16\pi} , \tag{6b}$$

$$a_0\left(W_L^+ W_L^- \to \frac{1}{\sqrt{2}} Z_L Z_L\right) = -\frac{\lambda}{8\pi\sqrt{2}} , \tag{6c}$$

where $\lambda = G_F m_H^2/\sqrt{2}$. The combination of $W_L^+ W_L^-$ and $Z_L Z_L$ which has the largest zeroth-partial-wave amplitude and hence yields the tightest bound[20] on the Higgs-boson mass is the (normalized) state $(2W_L^+ W_L^- + Z_L Z_L)\sqrt{6}$, which is the zero-isospin state (Ref. 13). It has a zeroth-partial-wave amplitude of

$$a_0(I=0) = -\frac{5\lambda}{16\pi} . \tag{7}$$

Applying the unitarity condition $|\mathrm{Re} a_0| \leq \frac{1}{2}$ gives

$$m_H^2 \leq \frac{8\pi\sqrt{2}}{5G_F} \approx (780\ \mathrm{GeV})^2 , \tag{8}$$

which is the bound quoted in Ref. 13.

The longitudinal-vector-boson scattering amplitudes may be obtained via the Goldstone-boson equivalence theorem,[4,14-16] which instructs us to replace the external longitudinal vector bosons with the corresponding Goldstone bosons of the R_ξ gauge. The resulting amplitude is equivalent to that of the longitudinal vector bosons up to terms of $O(M_W^2/s)$ which, for our purposes, are negligible. This theorem explains why the amplitudes given in Eqs. (6a)-(6c) are proportional to λ, the scalar field self-coupling in the Higgs sector of the standard model.

The running of the scalar field self-coupling λ is given by the usual renormalization-group equation. At one loop it gives, for $\mu > m_H$,

$$\lambda(\mu) = \frac{\lambda}{1 - (3\lambda/2\pi^2)\ln(\mu/m_H)} , \tag{9}$$

where $\lambda = \lambda(m_H) = G_F m_H^2/\sqrt{2}$. It is well known that $\lambda(\mu)$ becomes infinite at a finite-energy scale $\mu = m_H \exp(2\pi^2/3\lambda)$, the Landau pole. This singularity may be avoided if $\lambda = 0$; i.e., the theory is trivial. It is in this sense that the running of the coupling is related to the triviality of scalar field theories.

Triviality may be avoided if we regard the running coupling $\lambda(\mu)$ as describing an effective theory at energies below the scale at which it is subsumed by some deeper theory. We propose to identify this scale of new physics as the energy at which perturbative unitarity is violated. If we call this scale Λ, we then find from Eqs. (5) and (7) that

$$\frac{5\lambda(\Lambda)}{16\pi} \leq \frac{1}{2} , \tag{10}$$

for $\Lambda \gg m_H$. Using Eq. (9), this yields

$$m_H^2 \leq \frac{2\pi^2\sqrt{2}}{3G_F}\left(\ln\frac{\Lambda}{m_H} + \frac{5\pi}{12}\right)^{-1} . \tag{11}$$

This equation gives the renormalization-group-improved unitarity bound on the Higgs-boson mass for a given value of Λ/m_H. Although Eq. (11) is strictly valid only for $\Lambda \gg m_H$, the high-energy approximation to the amplitudes [(6a)-(6c)] is rather good even for energies just above the Higgs-boson mass, so we may apply Eq. (11) for modest values of Λ/m_H.[21]

The solid curve in Fig. 1 shows the bound on the Higgs-boson mass as a function of the ratio of the scale of new physics, Λ, to the Higgs-boson mass. For comparison, the dashed curve shows the results of a nonperturbative study by Lüscher and Weisz[13] using lattice regularization. In their calculation the scale Λ is the inverse of the lattice spacing, which provides an effective cutoff. Unfortunately, it is difficult to determine the quantitative relationship between our definition of Λ and theirs. Thus the remarkable agreement between their results and ours should not be taken too seriously. We can only say that there is qualitative agreement.

It has been suggested that one may estimate the scale of

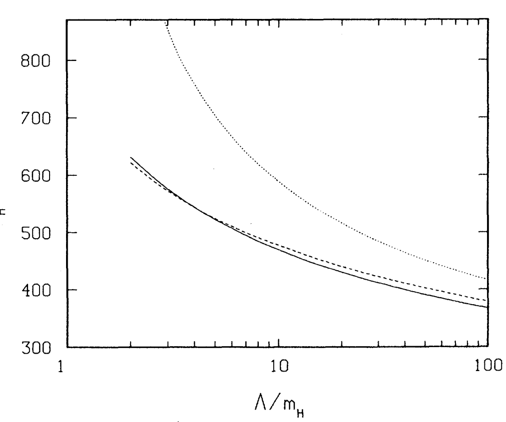

FIG. 1. Upper bound on the Higgs-boson mass as a function of Λ/m_H, where Λ is the scale of new physics. The solid curve is the renormalization-group-improved unitarity bound, given by Eq. (11). The dashed curve is the result of a lattice calculation of Ref. 13. The dotted curve is obtained by identifying Λ with the Landau pole, and is given by Eq. (12).

new physics Λ as the energy at which the running coupling $\lambda(\mu)$ becomes infinite.[6,9] Using Eq. (9), we find

$$m_H^2 \leq \frac{2\pi^2\sqrt{2}}{3G_F \ln(\Lambda/m_H)} . \tag{12}$$

This inequality yields the dotted curve in Fig. 1. For $\Lambda \gg m_H$, e.g., $\Lambda = M_{\rm Pl}$ or $M_{\rm GUT}$ (Refs. 22 and 23) the result of this prescription is indistinguishable from ours, while for small values of Λ/m_H, the bound on m_H is weaker than ours. However, this approach may be criticized on the grounds that it entails evolving the running coupling to infinity, thus entering the strong-coupling regime where the use of the one-loop approximation to the running coupling, Eq. (9), is unjustified. Our approach has the advantage that the running coupling is evolved to only a finite value, $\lambda(\mu)/\pi^2 = 8/5\pi$. Perturbative unitarity is violated for this value of $\lambda(\mu)$, so it presumably corresponds to strong coupling. However, throughout most of the range of integration of the renormalization-group equation, $\lambda(\mu)$ is much smaller than this value, and the one-loop approximation is valid.

A separate measure of the validity of the one-loop approximation to the running coupling, Eq. (9), for a given value of $\lambda(\mu)$ is the relative size of the one- and two-loop terms in a perturbative expansion of the β function,[24] defined by

$$\frac{d}{d\ln\mu}\lambda(\mu) = \beta(\lambda(\mu)) . \tag{13}$$

The two-loop β function is given by[25]

$$\beta(\lambda) = \frac{3\lambda^2}{2\pi^2}\left[1 - \frac{13\lambda}{16\pi^2}\right] , \tag{14}$$

so the ratio of the magnitude of the two-loop term to the one-loop term is $13\lambda(\mu)/16\pi^2$. For $\lambda(\mu)/\pi^2 = 8/5\pi$, the value at which perturbative unitarity is violated, this ratio is $13/10\pi \approx 0.4$ which, although non-negligible, suggests that the one-loop approximation is not unreasonable even for this value of $\lambda(\mu)$, which is the maximum value considered. The improved unitarity condition, $|\mathrm{Re}\, a_0| \leq \frac{1}{2}$, halves the maximum value of $\lambda(\mu)$ compared with the usual condition, $|a_0| \leq 1$. This improved condition thus helps justify our use of the one-loop β function.

We conclude that if the standard Higgs model is correct, the Higgs-boson mass must be less than the unitarity bound, $m_H \lesssim 780$ GeV. Furthermore, there is necessarily another threshold in the weak interaction, the scale of new physics, Λ. For a given Higgs-boson mass, the maximum value of this threshold may be deduced from the solid curve in Fig. 1. If we consider the W-boson and Higgs-boson masses as the first and second thresholds of the weak interaction, this represents a third threshold.[26]

It is also possible that the standard Higgs model is not correct. Without the Higgs boson, the amplitudes for longitudinal-vector-boson scattering are proportional to $G_F s$. We may use unitarity to estimate the scale Λ at which this description of the weak interaction breaks down. The largest zeroth-partial-wave amplitude is again the zero-isospin combination of $W_L^+ W_L^-$ and $Z_L Z_L$ (Ref. 15). We find

$$a_0(I=0) = \frac{G_F s}{8\pi\sqrt{2}} . \tag{15}$$

Using Eq. (5), we obtain

$$\Lambda^2 \leq \frac{4\pi\sqrt{2}}{G_F} \approx (1.2\ \mathrm{TeV})^2 . \tag{16}$$

Thus, in the absence of any specific model of the symmetry-breaking sector of the electroweak interaction, new physics should enter before about 1.2 TeV. A specific model may require new physics at a lower scale, such as the standard Higgs model, which requires $m_H \lesssim 780$ GeV.

III. UPPER BOUND ON THE TOP-QUARK MASS

We may also apply our prescription to the top-quark mass. At high energies, the zeroth-partial-wave amplitude for elastic $t\bar{t}$ scattering[27] is proportional to $G_F m_t^2$, i.e., the square of the Yukawa coupling of the top quark.[28] For large m_t, the renormalization-group equation tells us that the top-quark Yukawa coupling becomes infinite at a finite energy, which leads us to speculate that new physics must enter prior to this energy scale. Nonperturbative studies of the Higgs-boson–top-quark system support this conjecture.[29,30]

The zeroth-partial-wave amplitude for color-singlet, elastic, same-helicity $t\bar{t}$ scattering in the limit $s \gg m_t^2, m_H^2$ is[28]

$$a_0(t\bar{t} \to t\bar{t}) = -\frac{3\kappa}{4\pi} , \tag{17}$$

where $\kappa = G_F m_t^2/\sqrt{2}$. Applying the unitarity condition $|\mathrm{Re}\, a_0| \leq \frac{1}{2}$ yields the bound

$$m_t^2 \leq \frac{2\pi\sqrt{2}}{3G_F} \approx (500\,\mathrm{GeV})^2 , \tag{18}$$

which improves the bound on m_t of Ref. 28 by $1/\sqrt{2}$.

The one-loop renormalization-group equation for $\kappa(\mu)$ is[22]

$$\frac{d}{d\ln\mu}\kappa(\mu) = \frac{9}{4\pi^2}\kappa^2(\mu) - \frac{4}{\pi}\alpha_s(\mu)\kappa(\mu) . \tag{19}$$

Note that we have included one-loop QCD effects, which are qualitatively important. If $\kappa(\mu) \gtrsim 16\pi/9\alpha_s(\mu)$, the right-hand side of Eq. (19) is positive, and the Yukawa coupling grows with increasing energy. Ignoring the running of α_s, we find that $\kappa(\mu)$ grows without bound unless

$$m_t^2 \leq \frac{16\pi\sqrt{2}}{9G_F}\alpha_s \approx (270\,\mathrm{GeV})^2 , \tag{20}$$

where we have used $\alpha_s \approx 0.1$.

To properly treat the running of κ, we must also include the running of α_s. For six flavors, the one-loop renormalization-group equation for α_s is

$$\frac{d}{d\ln\mu}\alpha_s(\mu) = -\frac{7}{2\pi}\alpha_s^2(\mu) . \tag{21}$$

The solution to the coupled equations (19) and (21) is[31]

$$\kappa(\mu)=\frac{2\pi\alpha_s^{8/7}(\mu)}{9\alpha_s^{1/7}(\mu)+C}, \tag{22}$$

where C is a constant of integration, and

$$\alpha_s(\mu)=\frac{\alpha_s(\mu_0)}{1+(7/2\pi)\alpha_s(\mu_0)\ln(\mu/\mu_0)}. \tag{23}$$

The constant C is fixed by the initial condition $\kappa(m_t)=G_F m_t^2/\sqrt{2}$. For the initial condition on α_s we use $\alpha_s(\mu_0=M_W)=0.11$.

Since α_s is asymptotically free, Eq. (22) tells us that $\kappa(\mu)$ is asymptotically free if $C\geq 0$. For $C<0$, however, $\kappa(\mu)$ becomes infinite at the scale μ given by $9\alpha_s^{1/7}(\mu)=|C|$. Thus the requirement that $\kappa(\mu)$ vanish asymptotically implies[31]

$$m_t^2\leq\frac{2\pi\sqrt{2}}{9G_F}\alpha_s(m_t)\approx(95\text{ GeV})^2. \tag{24}$$

This is significantly smaller than the value obtained in Eq. (20) where we ignored the running of α_s.

As in Sec. II, we propose that the scale of new physics, Λ, be identified with the energy at which unitarity is violated. Using the running Yukawa coupling, Eq. (22), in the zeroth-partial-wave amplitude for elastic $t\bar{t}$ scattering, Eq. (17), and imposing $|\text{Re}a_0|\leq\frac{1}{2}$, we generate a bound on the top-quark mass as a function of Λ/m_t. This bound is given by the solid line in Fig. 2. For $m_t\leq 95$ GeV, the Yukawa coupling is asymptotically free, and it is not necessary to postulate new physics. For 95 GeV $\leq m_t\leq 170$ GeV, the Yukawa coupling runs very slowly, and although it eventually becomes large enough that perturbative unitarity is violated, this violation occurs at an energy in excess of the Planck scale.

The preceding analysis assumes the standard Higgs model is responsible for the generation of the top-quark mass. In the absence of the Higgs boson, the amplitude for same-helicity $t\bar{t}$ annihilation into longitudinal vector bosons is proportional to $G_F m_t\sqrt{s}$, and will therefore violate unitarity at sufficiently high energy.[32] The zeroth-partial-wave amplitudes are[28,32]

$$a_0(t_\pm\bar{t}_\pm\to W_L^+W_L^-)=a_0(t_\pm\bar{t}_\pm\to Z_LZ_L)=\pm\frac{G_F m_t\sqrt{s}}{8\pi\sqrt{2}}, \tag{25}$$

where the subscripts on t, $\bar{t}$ indicate the helicity. The combination of states which yields the largest zeroth-partial-wave amplitude is the color-singlet, spin-zero combination of $t\bar{t}$ and the zero-isospin combination of $W_L^+W_L^-$ and Z_LZ_L, for which

$$a_0(I=0)=\frac{3G_F m_t\sqrt{s}}{8\pi\sqrt{2}}. \tag{26}$$

It is this amplitude to which we will apply the unitarity condition.

Since the amplitude under consideration, Eq. (26), is inelastic, we must generalize our previous derivation of the unitarity condition, which was for elastic amplitudes only. Unitarity of the S matrix implies

$$\text{Im}a_J\geq|a_J|^2+|a_J^{\text{in}}|^2, \tag{27}$$

where a_J is the Jth partial-wave amplitude of an elastic scattering process, and a_J^{in} is the Jth partial-wave amplitude for an inelastic process where the initial state is the same as that of a_J, and the final state is any two-body state different from that of a_J. Following the same steps as before, we arrive at

$$|a_J^{\text{in}}|\leq\tfrac{1}{2}, \tag{28}$$

which implies $|\text{Re}a_J^{\text{in}}|\leq\frac{1}{2}$. Thus the conditions on the real part of the elastic and inelastic amplitudes are identical.

Applying the unitarity condition, Eq. (28), to the amplitude in Eq. (26) and denoting the energy at which unitarity is violated by Λ, we arrive at

$$m_t\leq\frac{4\pi\sqrt{2}}{3G_F\Lambda}. \tag{29}$$

As before, we identify Λ as the scale at which new physics enters. The dotted line in Fig. 2 gives the bound on m_t as a function of Λ/m_t. This bound is stronger than the corresponding bound derived from the standard model with a Higgs boson.

The bounds we have obtained on the top-quark mass, given in Fig. 2, are weaker than bounds which have been obtained via other considerations. Data from neutral-current phenomenology and vector-boson masses imply $m_t\leq 200$ GeV (for $m_H\leq 1$ TeV), from top-quark loop effects.[33] Stability of the vacuum state in the standard model yields $m_t\leq 95\text{ GeV}+0.6m_H$ since heavy fermions make a negative contribution to the effective potential.[34] However, both of these bounds may be circumvented in extensions of the standard model.

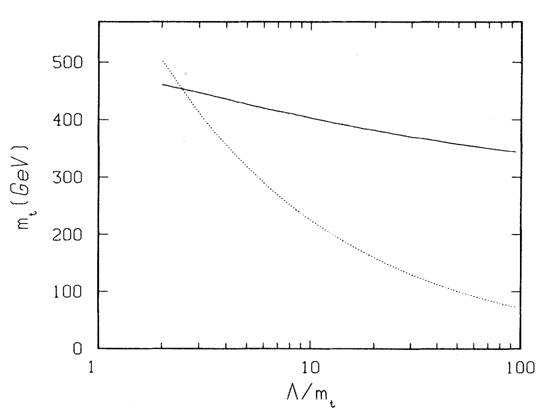

FIG. 2. Upper bound on the top-quark mass as a function of Λ/m_H, where Λ is the scale of new physics. The solid curve is the renormalization-group-improved unitarity bound. The dotted curve is the bound in the absence of the Higgs boson, and is given by Eq. (29).

IV. CONCLUSION

In this paper we used renormalization-group-improved perturbative unitarity to obtain upper bounds on the

Higgs-boson and top-quark masses. The bound on m_H is given as a function of the scale Λ at which perturbative unitarity is violated and new physics enters. If we require $\Lambda \gtrsim m_H$, this leads to the bound $m_H \lesssim 780$ GeV. For larger values of Λ/m_H, the bound on m_H is tightened. In the absence of the Higgs boson, unitarity yields a bound on the scale at which new physics enters of $\Lambda \lesssim 1.2$ TeV.

Similarly, there is a bound on the top mass as a function of the scale of new physics. If we require $\Lambda \gtrsim m_t$, we find $m_t \lesssim 500$ GeV. For $m_t \lesssim 170$ GeV, the scale of new physics exceeds the Planck mass. If $m_t \lesssim 95$ GeV, the Yukawa coupling is asymptotically free, and no new physics is required. In the absence of the Higgs boson, we obtain a bound on the scale of new physics as a function of m_t. For $m_t \approx 100$ GeV, this bound is $\Lambda \lesssim 5$ TeV.

Since the bounds quoted above are based on the assumption that the violation of tree-level unitarity signals the onset of new physics, they cannot be considered rigorous. Nevertheless, they strengthen our conviction that both the Higgs boson and the top quark are within the reach of the proposed Superconducting Super Collider.

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[18]See, e.g., G. Barton, *Introduction to Dispersion Techniques in Field Theory* (Benjamin, New York, 1965).

[19]A normalization factor $1/\sqrt{2}$ is associated with each $Z_L Z_L$ state because the unitarity condition $\mathrm{Im} a_J \gtrsim |a_J|^2$ applies only to normalized states.

[20]In Ref. 4 the state *HH* is included as well, which leads to a slightly tighter bound on the Higgs-boson mass. We do not include this state for reasons we will mention later.

[21]For $\Lambda/m_H = 1$, Eq. (11) corresponds to the bound in Eq. (8). The bound in Eq. (8) therefore corresponds to $s \approx m_H^2$, in which case the state *HH* is not kinematically accessible. This is why we do not include this state in our analysis (see Ref. 20).

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