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Electromagnetic properties of neutrinos in a medium

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We show that, contrary to the situation in the vacuum, a Majorana neutrino can have electric and magnetic dipole moments in a medium. This is because of new contributions, equal for a particle and its antiparticle, that can arise only in a material background. For Dirac neutrinos, these contributions make the magnitudes of the dipole moments of the particle and the antiparticle unequal. We discuss the conditions which give rise to such effects, with particular attention to the role played by the discrete symmetries C, P, and T.

The peculiarities of the behavior of neutrinos within a material medium has been a subject of great recent interest. It has been shown that, owing to the interactions with the particles in the background, the effective mass of a neutrino in a medium could be very different from its mass in the vacuum.[1] More recently, it has been pointed out that this fact might solve the solar-neutrino puzzle.[2]

In this paper we address another interesting aspect of neutrinos in a medium: viz., their electromagnetic properties. In particular, contrary to the situation in the vacuum, a Majorana neutrino can have electric- and magnetic-dipole-moment interactions in a material background. The reason for this is that there are contributions to the dipole moments that can arise only in material background which are the same for the particle and the antiparticle and therefore can be nonzero even for a Majorana neutrino. Even if the neutrino is a Dirac particle, these contributions imply that the dipole moments can be very different from the corresponding values in the vacuum. In what follows, we spell out the exact conditions which give rise to such effects and then discuss the possible physical ramifications of the effects.

We begin by introducing the notation for the electromagnetic vertex of a neutrino:

$$\langle \nu(k')|j_\alpha^{\rm EM}|\nu(k)\rangle \equiv \bar{u}(k')\Gamma_\alpha(k,k',v)u(k) \; . \tag{1}$$

Notice that, in general, Γ_α depends not only on the momenta of the incoming and outgoing neutrinos but also on the parameters quantifying the background. In the simplest case of an isotropic background, we have only one new four-vector in the problem, viz., the velocity of the center of mass of the medium v^μ. This is indicated in Eq. (1). In the vacuum, of course, the dependence on v^μ disappears.

Conservation of electromagnetic current implies

$$q^\alpha \Gamma_\alpha(k,k',v)=0 \; , \tag{2}$$

where

$$q \equiv k-k' \tag{3}$$

is the photon momentum. In the vacuum, the matrix element of Eq. (1) must be proportional to the electric charge of the fermion in the limit $k \to k'$, so that, in our case,

$$\Gamma_\alpha(k,k,v=0)=0 \; . \tag{4}$$

In the vacuum, the most general structure of Γ_α consistent with (3) and (4) is[3]

$$\Gamma_\alpha=(q^2\gamma_\alpha - q_\alpha \not{q})(R+r\gamma_5)+i\sigma_{\alpha\beta}q^\beta(D_M+D_E\gamma_5) \; , \tag{5}$$

which defines the various electromagnetic form factors of a neutrino. Several restrictions on the form factors follow[3,4] from the requirement that the current $j_\alpha^{\rm EM}$ is a Hermitian operator. CP invariance, if it is applicable, puts more constraints. In addition, if the neutrino is a Majorana particle, there arise further constraints. For example, since the operators $\bar{\nu}\gamma_\alpha\nu$, $\bar{\nu}\sigma_{\alpha\beta}\nu$, and $\bar{\nu}\sigma_{\alpha\beta}\gamma_5\nu$ vanish identically by the Majorana condition, we have[3,4]

$$R=D_M=D_E=0 \tag{6}$$

so that only r can be nonzero for a Majorana neutrino. This implies that in the vacuum, a Majorana neutrino can have an axial charge radius but can have neither the charge radius, nor the electric or magnetic dipole moments.

In the presence of a background, one can have more terms in Γ_α than the ones present in Eq. (5). These new terms (denoted by a prime) must have some explicit dependence on v^μ in their tensor structure. We want to consider two such terms here: viz.,

$$\Gamma'_\alpha(k,k',v)=iD'_E(k,k',v)(\gamma_\alpha v_\beta-\gamma_\beta v_\alpha)q^\beta\gamma_5 + iD'_M(k,k',v)\epsilon_{\alpha\beta\lambda\rho}\gamma^\beta\gamma_5 q^\lambda v^\rho \; , \tag{7}$$

where D'_E and D'_M are new form factors. Just as the effect of the last two terms of (5) can be described by the operators

$$O_E = D_E \bar{\nu}\sigma_{\alpha\beta}\nu\tilde{F}^{\alpha\beta}, \quad O_M = D_E \bar{\nu}\sigma_{\alpha\beta}\nu F^{\alpha\beta} \tag{8}$$

in momentum space, the terms in (7) can be described by

$$O'_E = D'_E \bar{\nu}\gamma_\alpha\gamma_5\nu v_\beta F^{\alpha\beta} ,$$
$$O'_M = D'_M \bar{\nu}\gamma_\alpha\gamma_5\nu v_\beta \tilde{F}^{\alpha\beta} \tag{9}$$

which might arise because of interactions with the particles constituting the background. If we go to the rest frame of the medium so that $v_\beta = (1,\mathbf{0})$, using $\tilde{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\lambda\rho}F_{\lambda\rho}$ and the representation of the Dirac matrices,[5] we find the following nonrelativistic limit of the operators in (9):

$$O'_E \to d'_E \phi^\dagger \sigma\phi\cdot\mathbf{E}, \quad O'_M \to d'_M \phi^\dagger \sigma\phi\cdot\mathbf{B} , \tag{10}$$

where ϕ is the so-called *large component* of the neutrino spinor, and $d'_{E,M}$ denotes $D'_{E,M}$ evaluated when $q=0$. Therefore, we have the possibility of extra contributions to the electric and magnetic dipole moments through D'_E and D'_M, respectively. In addition, notice that in (9), the fermion bilinears are of the form $\bar{\nu}\gamma_\alpha\gamma_5\nu$ which is nonvanishing for a Majorana neutrino, so that D'_E and D'_M can be nonzero even for a Majorana neutrino.

We now discuss the precise conditions that have to be satisfied in order that operators such as O'_E and O'_M appear in the effective interaction Lagrangian of the neutrinos. This requires a discussion of the effects of the discrete operations P, T, and C on O'_E and O'_M. The analysis becomes particularly simple and intuitive if, for the moment, we neglect any momentum dependence of the form factors involved. Using the usual transformation rules for the fermion and the electromagnetic fields,[5] we then obtain the results summarized in Table I.

Looking at the table, we notice that although under P and T, O'_E and O'_M transform the same way as O_E and O_M, respectively, their C transform is different. This has far-reaching consequences. For a collection of nonrelativistic particles and antiparticles, $O_E + O_M$ reduce to

$$d_E(\mathbf{s}-\bar{\mathbf{s}})\cdot\mathbf{E} + d_M(\mathbf{s}-\bar{\mathbf{s}})\cdot\mathbf{B} ,$$

where $\mathbf{s}$ and $\bar{\mathbf{s}}$ are the spin expectation values for the particles and antiparticles. Thus, the contributions from O_E and O_M to the dipole moments of the particle and the antiparticle are opposite. Since a Majorana neutrino is its own antiparticle, it thus follows that $d_E = d_M = 0$ for them.[3,4] On the other hand, the reduction of $O'_E + O'_M$ for nonrelativistic particles and antiparticles is

$$d'_E(\mathbf{s}+\bar{\mathbf{s}})\cdot\mathbf{E} + d'_M(\mathbf{s}+\bar{\mathbf{s}})\cdot\mathbf{B}$$

which is odd under C and also under CPT. But for the same reason, these contributions are equal for the particle and the antiparticle—which also explains why these contributions can be nonzero even for a Majorana neutrino. For Dirac neutrinos, even $d_{E,M}$ can be nonzero, so that the dipole moment of the neutrino will be $d + d'$, whereas that of the antineutrino will be $-d + d'$, so that they are not exactly opposite of each other in a medium.

From Table I as well as the nonrelativistic reduction discussed above, it is however apparent that both O'_E and O'_M violate CPT. The meaning of this and other statements regarding the violation of any combination of C, P, and T must be clearly understood.[6] In a medium, the Green's functions are defined not as vacuum expectation values of products of field operators but as ensemble averages. This averaging procedure can introduce asymmetries with respect to certain operation even if the Lagrangian were symmetric under the operation. For any operation other than CPT, the violation can thus come either from the Lagrangian or from the background, or maybe both. But since there are strong theoretical reasons to believe that CPT is conserved by the Lagrangian, any breaking of CPT must come from the background. Thus, the particles constituting the medium must have some chemical potentials associated with them; otherwise the background is CPT symmetric and hence D'_E and D'_M vanish.

TABLE I. The transformation properties of different operators under the discrete symmetries P, T, and C, assuming that the form factors are invariant.

	O_E	O_M	O'_E	O'_M
P	$-$	$+$	$-$	$+$
T	$-$	$+$	$-$	$+$
C	$+$	$+$	$-$	$-$

All these statements are true, subject to the condition that the form factors are independent of v^μ. To obtain the more general conditions satisfied by the form factors, we go back to the momentum-space language of Eqs. (5) and (7). Consider parity, for example. Its effect can be summarized into the statement

$$\Gamma_\alpha(k,k',v) \xrightarrow{P} \Gamma^P_\alpha(k,k',v) , \tag{11a}$$

where Γ^P_α is obtained from Γ_α by multiplying every quantity that appears in Γ_α by its parity phase η_P given in Table II. Using similar notation, we obtain

$$\Gamma_\alpha(k,k',v) \xrightarrow{T} -\Gamma^T_\alpha(-k,-k',-v) , \tag{11b}$$

$$\Gamma_\alpha(k,k',v) \xrightarrow{C} -\Gamma^C_\alpha(-k',-k,v) . \tag{11c}$$

From these, one can also deduce the effects of the combined operations, e.g.,

$$\Gamma_\alpha(k,k',v) \xrightarrow{CP} -\Gamma^{CP}_\alpha(-k',-k,v) , \tag{11d}$$

$$\Gamma_\alpha(k,k',v) \xrightarrow{CPT} \Gamma^{CPT}_\alpha(k',k,-v) . \tag{11e}$$

If the Lagrangian and the background are both symmetric under any of the above discrete symmetries, the arrow in the corresponding relation in (11) should be replaced by an equality sign. Such equations then produce constraints on the form factors. In addition, if the neutrino in question is a Majorana particle, we obtain the constraint

$$\Gamma_\alpha(k,k',v) = \Gamma^C_\alpha(-k',-k,v) . \tag{12}$$

On top of all these conditions which may or may not be

TABLE II. The transformation properties of different quantities appearing in Γ_α under various discrete symmetries.

	η_P	η_T	η_C	η_{CP}	η_{CPT}
1	+	+	+	+	+
i	+	−	+	+	−
γ_5	−	+	+	−	−
γ_α	+	−	−	−	+
$\gamma_\alpha\gamma_5$	−	−	+	−	+
$\sigma_{\alpha\beta}$	+	−	−	−	+
$\sigma_{\alpha\beta}\gamma_5$	−	−	−	+	−
$\epsilon_{\alpha\beta\lambda\rho}$	−	−	+	−	+

satisfied in a particular case, we have the Hermiticity condition

$$\Gamma_\alpha(k,k',v)=\gamma_0\Gamma_\alpha^\dagger(k',k,v)\gamma_0 , \tag{13}$$

which is true irrespective of the nature of the neutrino or the status of the discrete symmetries.

We now apply Eqs. (11)–(13) on Eq. (7). In particular, we consider the static limit $q\to 0$ in which case the form factors depend only on $\omega=k\cdot v$ and $\kappa=\sqrt{\omega^2-k^2}$. Writing the static limit of D_E' and D_M' by the lower-case letters as before, we obtain the relations

$$\text{Hermiticity}\Longrightarrow d_{E,M}' \text{ real (Ref. 7)} , \tag{14}$$

$$\text{Majorana}\Longrightarrow d_{E,M}'(-\omega,\kappa)=d_{E,M}'(\omega,\kappa) , \tag{15}$$

whereas the symmetries under discrete operations give

$$P:\ d_E'=0 , \tag{16a}$$

$$T:\ d_E' \text{ imaginary},\ d_M' \text{ real} , \tag{16b}$$

$$C:\ d_{E,M}'(-\omega,\kappa)=-d_{E,M}'(\omega,\kappa) , \tag{16c}$$

$$CP:\ d_E'(-\omega,\kappa)=d_E'(\omega,\kappa) ,$$
$$d_M'(-\omega,\kappa)=-d_M'(\omega,\kappa) , \tag{16d}$$

$$CPT:\ d_{E,M}'^{*}(-\omega,\kappa)=-d_{E,M}'(\omega,\kappa) . \tag{16e}$$

Thus, if the background is *CPT* symmetric, we obtain, using (14),

$$d_{E,M}'(-\omega,\kappa)=-d_{E,M}'(\omega,\kappa) . \tag{17}$$

If we assume now, as we did before, that the form factors are independent of v^μ and hence of ω, then (17) implies that $d_{E,M}'$ vanish in the *CPT*-symmetric case, in accordance with what we obtained before. But in general, it is possible to obtain nonzero contributions to $d_{E,M}'$ even in the *CPT*-symmetric case. Such contributions are odd in the variable ω.

However, if the neutrino is a Majorana particle, then (15) applies and this is in direct contradiction with (17). Thus, for Majorana neutrinos to have magnetic or electric dipole moments, it is necessary that the background is *CPT* asymmetric, so that (17) does not apply and we obtain a contribution obeying (15). This is consistent with the results obtained by previous authors that the dipole moments of Majorana neutrinos are zero when there is no *CPT*-odd effects.

Although we have been talking about neutrinos, it is more than obvious that the entire discussion is equally appropriate for any other neutral fermion, e.g., photinos or gravitinos,[8] if they exist. Our choice of neutrinos as the paradigm example of neutral fermions is dictated solely by the feeling that the possible physical consequences of the magnetic moment of a neutrino are more dramatic than that of other hypothetical particles.

It has been suggested[9] that a large magnetic moment of the electron neutrino might solve the solar-neutrino problem. In the present context, it should be emphasized that even if d_M' is large for a neutrino, it has no relevance for this solution. This is because the main ingredient of the solution is the helicity flip in a dipole moment interaction. However, the effective dipole moment interactions such as in (9) do not change the helicity of the neutrino. Nevertheless, it is possible that, owing to large chemical potentials inside the Sun, the effective magnetic moment d_M' is so large that the path of the neutrinos are bent preferentially by the magnetic field in and around the Sun. In such a case, it is possible that the flux of neutrinos on Earth is depleted.

Large values of $d_{E,M}'$ could also affect the dynamics of supernovae in important ways. However, a quantitative study of these effects and their consequences depends on particular models of particle physics and stellar astrophysics. These model-dependent estimates are subjects of further study.

To summarize, then, we have shown that in a medium, neutrinos can have new, extra contributions to their magnetic and electric dipole moments. These contributions can be nonzero even for Majorana neutrinos and can arise if the particles constituting the background have chemical potentials associated with them.

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[7]To be more precise, $d'^{*}_{E,M}(\omega - i0,\kappa) = d'_{E,M}(\omega + i0,\kappa)$.

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