Model-independent extraction of $|V_{ub}/V_{cb}|$ from nonleptonic-B-meson-decay data

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A model-independent extraction of $|V_{ub}/V_{cb}|$ from exclusive nonleptonic-B-meson-decay data is discussed. It is noted that the ratio of the weak decay amplitudes $|A_w(B^0\to K^0D^0)/A_w(B^0\to K^0\bar{D}^0)|$ is given by the ratio $|V_{ub}V_{cs}^*/V_{cb}V_{us}^*|$ without assuming any specific calculation methods of the weak decay amplitudes and it is discussed how to estimate $|A_w(B^0\to K^0D^0)/A_w(B^0\to K^0\bar{D}^0)|$ from the exclusive nonleptonic-B-decay data under the presence of B^0-$\bar{B}^0$ mixing and final-state interaction.

We are eager to know experimentally an exact value of the ratio of the Kobayashi-Maskawa (KM) mixing matrix element V_{ub} to V_{cb}, $|V_{ub}/V_{cb}|$, because the value offers an important clue for studying the magnitude of the CP-nonconservation effect, for checking quark-mass-matrix models, and for searching for new physics beyond the standard model. Recently many experimental studies on $|V_{ub}/V_{cb}|$ have been reported.[1]

However, most of the extraction methods of $|V_{ub}/V_{cb}|$ are more or less model dependent. Especially, the extraction from the exclusive nonleptonic-B-decay data, for example, from $B(B^0\to\pi^+\pi^-)/B(B^0\to\pi^+D^-)$, is highly model dependent, because in the nonleptonic B decays, there are many contributions[2] from different types of quark diagrams (the external W-emission diagram, the internal W-emission diagram, the W-exchange diagram, the W-annihilation diagram, and the penguin diagram), and furthermore, we must take the final-state-interaction effect into consideration. Generally these contributions are very complicated, so that it is not so easy to estimate those exactly.

In this paper we note that the weak decays $B^0\to K^0D^0$ and $B^0\to K^0\bar{D}^0$ are caused by only one same-type diagram[3] (the internal W-emission diagram) as illustrated in Fig. 1, so that both weak decay amplitudes $A_w(B^0\to K^0D^0)$ and $A_w(B^0\to K^0\bar{D}^0)$ are quite identical except for the KM mixing factors $V_{ub}V_{cs}^*$ and $V_{cb}V_{us}^*$:

$$\frac{|A_w(B^0\to K^0D^0)|}{|A_w(B^0\to K^0\bar{D}^0)|}=\left|\frac{V_{ub}}{V_{cb}}\right|\left|\frac{V_{cs}}{V_{us}}\right| , \tag{1}$$

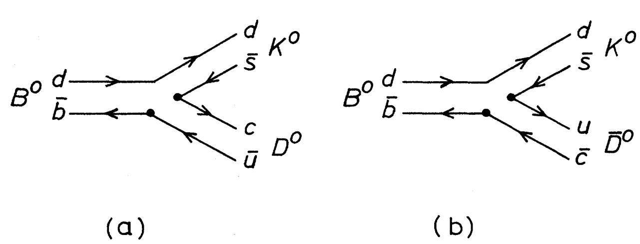

FIG. 1. Quark diagrams for (a) $B^0\to K^0D^0$ and (b) $B^0\to K^0\bar{D}^0$.

where K^0 and/or D^0 ($\bar{D}^0$) can be replaced with K^{*0} and/or D^{*0} ($\bar{D}^{*0}$). Therefore, if we can know the value of $|A_w(B^0\to K^0D^0)/A_w(B^0\to K^0\bar{D}^0)|$ from exclusive nonleptonic-B-decay data and the value of $|V_{cs}/V_{us}|$ from D-decay data, then we can get the value of $|V_{ub}/V_{cb}|$ model independently.

The problem is how to estimate the value of $|A_w(B^0\to K^0D^0)/A_w(B^0\to K^0\bar{D}^0)|$ from exclusive nonleptonic-B-decay data under the presence of B^0-$\bar{B}^0$ mixing and final-state interaction.

Note that even when we can identify one of the two neutral B mesons from $\Upsilon(4S)$ as B^0 ($\bar{B}^0$) by the observation of the exclusive decay mode $B^0\to\pi^+D^-$ ($\bar{B}^0\to\pi^-D^+$), we cannot distinguish whether the observed final state K_SD^0 in the other B decay comes from $B^0\to K^0D^0$ or from $\bar{B}^0\to\bar{K}^0D^0$. In order to know the values of $B(B^0\to K^0D^0)$ and $B(B^0\to K^0\bar{D}^0)$, we must know[4] the value of the B^0-$\bar{B}^0$ mixing parameter r:

$$r=B(B^0\to\bar{B}^0\to\bar{X})/B(B^0\to B^0\to X) . \tag{2}$$

As is well known,[5] in e^+e^- annihilation experiments at $\Upsilon(4S)$, the $B^0\bar{B}^0$ pair is created in a state of odd charge conjugation ($L=1$ orbital angular momentum), so that the ratio of how often one observes B^0B^0 or $\bar{B}^0\bar{B}^0$ in the final state to how often it is $B^0\bar{B}^0$ is given by

$$[N(B^0B^0)+N(\bar{B}^0\bar{B}^0)]/N(B^0\bar{B}^0)=r . \tag{3}$$

The experimental value of r can be obtained from the measurements of the ratio of like-sign to unlike-sign dilepton events and/or the exclusive B decays into π^+D^-/π^-D^+ (π^+D^{*-}/π^-D^{*+}) (Ref. 6).

Then, using this observed value of r, we can get the "true" values of $B(B^0\to K_SD^0)$ and $B(B^0\to K_S\bar{D}^0)$ as follows:

$$B(B^0\to K_SD^0)=\frac{1}{1-r}[B(\text{“}B^0\text{”}\to K_SD^0) -rB(\text{“}B^0\text{”}\to K_S\bar{D}^0)] , \tag{4}$$

$$B(B^0\to K_S\bar{D}^0)=\frac{1}{1-r}[B(\text{“}B^0\text{”}\to K_S\bar{D}^0) -rB(\text{“}B^0\text{”}-K_SD^0)] , \tag{5}$$

where "B^0" means a neutral B meson whose partner in $\Upsilon(4S)$ decay is identified as $\bar{B}^0$ by the exclusive decay modes $\bar{B}^0 \to \pi^- D^+, \pi^- D^{*+}$, and so on.

Another problem is how to estimate the final-state-interaction effect for the weak decay amplitudes $A_w(B^0 \to K^0 D^0)$ and $A_w(B^0 \to K^0 \bar{D}^0)$. The final-state $K^0 \bar{D}^0$ from the weak decay $B^0 \to K^0 \bar{D}^0$ can mix with the final state[7] $K^+ D^-$ from the weak decay $B^0 \to D^- K^+$ by the quark rearrangement $u \leftrightarrow d$. (There is no rescattering channel which is caused by a quark-pair annihilation and creation $q\bar{q} \to q'\bar{q}'$.) We consider that the $K\bar{D}$ rescatting at $E_{\mathrm{c.m.}} = m_B$ has almost elastically taken place. Therefore, the observed decay amplitudes $A(B^0 \to K^0 \bar{D}^0)$ and $A(B^0 \to K^+ D^-)$ are given by

$$A^0_{+-} \equiv A(B^0 \to K^+ D^-) = \tfrac{1}{2}(a_1 e^{i\delta_1} + a_0 e^{i\delta_0}) , \quad (6)$$

$$A^0_{00} \equiv A(B^0 \to K^0 \bar{D}^0) = \tfrac{1}{2}(a_1 e^{i\delta_1} - a_0 e^{i\delta_0}) , \quad (7)$$

where a_n $(n=0,1)$ is the weak decay amplitude of $B \to K\bar{D}$ with isospin component $I=n$, and δ_n is the $K\bar{D}$ phase shift for $I=n$ at $E_{\mathrm{c.m.}} = m_B$. Therefore, when $\delta_1 - \delta_0 \neq 0$, the $K^0\bar{D}^0$-K^+D^- mixing appears. On the other hand, since the final state $K^+\bar{D}^0$ from the weak $B^+ \to K^+ \bar{D}^0$ consists of a single isospin component $I=1$, the observed decay amplitude is expressed as

$$A^+_{+0} \equiv A(B^+ \to K^+ \bar{D}^0) = a_1 e^{i\delta_1} . \quad (8)$$

Therefore, if we measure the magnitudes of these physical decay amplitudes $|A^0_{+-}|$, $|A^0_{00}|$, and $|A^+_{+0}|$, we can get the "pure" weak decay amplitude $|A_w(B^0 \to K^0 \bar{D}^0)|$,

$$\begin{aligned} |A_w(B^0 \to K^0 \bar{D}^0)|^2 &= \tfrac{1}{4}(a_1 - a_0)^2 \\ &= \tfrac{1}{2}(|A^0_{+-}|^2 + |A^0_{00}|^2) \\ &\quad - \tfrac{1}{2}|A^+_{+0}|[2(|A^0_{+-}|^2 + |A^0_{00}|^2) \\ &\quad - |A^+_{+0}|^2]^{1/2} , \end{aligned} \quad (9)$$

together with the value of $\delta_I \equiv \delta_1 - \delta_0$,

$$\cos\delta_I = \frac{|A^0_{+-}|^2 - |A^0_{00}|^2}{|A^+_{+0}|[2(|A^0_{+-}|^2 + |A^0_{00}|^2) - |A^+_{+0}|^2]^{1/2}} , \quad (10)$$

where we have used an empirical relation from recent phenomenological studies:[8]

$$|\text{external } W\text{-emission amp.}| > |\text{internal } W\text{-emission amp.}| , \quad (11)$$

i.e., $|A^0_{+-}| > |A^0_{00}|$ (i.e., $a_1 > a_0 > 0$).

Whether or not this treatment is reasonable can be checked by confirming the triangle relation

$$A^0_{+-} + A^0_{00} = A^+_{+0} . \quad (12)$$

In order to estimate the relative ratios $(|A^0_{00}|/|A^0_{+-}|)/|A^+_{+0}|$ from the observed values of $B(B^0 \to K^0 \bar{D}^0)$, $B(B^0 \to K^+ D^-)$, and $B(B^+ \to K^+ \bar{D}^0)$, we must know the value of the lifetime ratio $\tau(B^0)/\tau(B^+)$: for example, the value of $|A^+_{+-}|^2/|A^0_{00}|^2$ is obtained from

$$\frac{|A^+_{+0}|^2}{|A^0_{00}|^2} = \frac{B(B^+ \to K^+ \bar{D}^0)}{B(B^0 \to K^0 \bar{D}^0)} \frac{\tau(B^0)}{\tau(B^+)} . \quad (13)$$

Of course, the ratio may be obtained from direct measurements of the lifetimes $\tau(B^0)$ and $\tau(B^+)$. However, it will be also convenient to use the relations

$$\begin{aligned} \frac{\tau(B^0)}{\tau(B^+)} &= \frac{B(B^0 \to D^- D_s^+)}{B(B^+ \to \bar{D}^0 D_s^+)} \\ &= \frac{B(B^0 \to K^0 \psi)}{B(B^+ \to K^+ \psi)} , \end{aligned} \quad (14)$$

where the relation is also satisfied with the final two meson states with $1^- + 0^-$, $0^- + 1^-$, and $1^- + 1^-$ as well as $0^- + 0^-$. The relation (14) comes from the following reasons: the both weak decays $B^0 \to D^- D_s^+$ and $B^+ \to \bar{D}^0 D_s^+$ ($B^0 \to K^0 \psi$ and $B^+ \to K^+ \psi$) are caused by only one same-type diagram,[9] i.e., the external W-emission diagram (the internal W-emission diagram), and although there are some final-state-interaction effects, the effects on B^0 decay are completely parallel to those on B^+ decay differently from the case of $B \to \pi\bar{D}$ decays.

Next, we discuss the final-state-interaction effect on the $B^0 \to K^0 D^0$ decay. The final state $K^0 D^0$ can mix with a state $\pi^- D_s^+$ through the quark rearrangement $u \leftrightarrow s$. (In this case, too, there is no rescattering channel which is caused by a quark rearrangement $q\bar{q} \leftrightarrow q'\bar{q}'$.)

It is useful to use the language of the old $\mathrm{SU(3)_{flavor}}$. As in the case of the $B \to K\bar{D}$ decay due to $\bar{b} \to \bar{c}u\bar{s}$, where the final states K^+D^- and $K^0\bar{D}^0$ have two isospin components $I=0$ and $I=1$, the final states $\pi^- D_s^+$ and $K^0 D^0$ from the B^0 decays due to $\bar{b} \to \bar{u}c\bar{s}$ have two V-spin components $V=0$ and 1, where V spin is defined as (s,u) is V-spin doublet with $V_3 = (+\frac{1}{2}, -\frac{1}{2})$. However, for the $B^0 \to \pi^- D_s^+ / K^0 D^0$ decays, there is no relation analogous to the relation (12) in the $B \to K\bar{D}$ decays. The weak Hamiltonian for the quark decay $\bar{b} \to \bar{c}u\bar{s}$ has a single isospin component $I = \frac{1}{2}$ [it belongs to **8** of $\mathrm{SU(3)_{flavor}}$], and B^+ and B^0 exist in an isospin doublet, while the weak Hamiltonian for the quark decay $\bar{b} \to \bar{u}c\bar{s}$ has $V=0$ and 1 components [it belongs to **3**+**6*** of $\mathrm{SU(3)_{flavor}}$], and B^0 and B^+ exist in $V=0$ and $\frac{1}{2}$ states, respectively.

Of course, if we assume $\mathrm{SU(4)_{flavor}}$ symmetry, considering that $D^0 D_s^+$ state from B_c^+ decay exists in a single V-spin state $V=1$ (because $V=0$ state is forbidden due to Bose statistics), we can get similar relations to Eqs. (6)–(8) and (12):

$$A(B^0 \to \pi^- D_s^+) = \tfrac{1}{2}(a_{V=1} e^{i\delta_{V=1}} + a_{V=0} e^{i\delta_{V=0}}) , \quad (15)$$

$$A(B^0 \to K^0 D^0) = \tfrac{1}{2}(a_{V=1} e^{i\delta_{V=1}} - a_{V=0} e^{i\delta_{V=0}}) , \quad (16)$$

$$\sqrt{2} A(B_c^+ \to D^0 D_s^+) = a_{V=1} e^{i\delta_{V=1}} , \quad (17)$$

$$A(B^0 \to \pi^- D_s^+) + A(B^0 \to K^0 D^0) = \sqrt{2} A(B_c^+ \to D^0 D_s^+) . \quad (18)$$

If we get the values of $|A(B^0\to\pi^- D_s^+)|$, $|A(B^0\to K^0 D^0)|$, and $|A(B_c^+\to D^0 D_s^+)|$ from the observed values of the corresponding B decay branching ratios and $\tau(B^0)/\tau(B_c^+)$, we can estimate the "pure" weak decay amplitude $|A_w(B^0\to K^0 D^0)|$. However, SU(4)$_{\rm flavor}$ is not such a good symmetry yet at $E_{\rm c.m.}\sim m_B$ in contrast with SU(3)$_{\rm flavor}$, and, in addition, it seems to be hard to measure $\tau(B_c^+)$ and $B(B_c^+\to D^0 D_s^+)$ precisely even in the near future.

When we have no experimental value of $\delta_V\equiv\delta_{V=1}-\delta_{V=0}$, we can only get the relation

$$|A(B^0\to K^0 D^0)|>|A_w(B^0\to K^0 D^0)| \tag{19}$$

from Eq. (16) and the empirical relation (11), so that we can only get the upper limit of $|V_{ub}/V_{cb}|$ from

$$\frac{|A(B^0\to K^0 D^0)|}{|A_w(B^0\to K^0\bar{D}^0)|}>\frac{|A_w(B^0\to K^0 D^0)|}{|A_w(B^0\to K^0\bar{D}^0)|}=\left|\frac{V_{ub}}{V_{cb}}\right|\left|\frac{V_{cs}}{V_{us}}\right| . \tag{20}$$

In order to get a sizable value of $|V_{ub}/V_{cb}|$, we must know the value of δ_V.

There is another idea for estimating the value of δ_V: we notice that SU(3)$_{\rm flavor}$ is a good symmetry for strong interactions (not for weak interactions) at $E_{\rm c.m.}\simeq m_B$. Then, we can take the value of δ_V as $\delta_V\simeq\delta_I$ approximately (of course, we cannot regard $a_{V=1}\simeq a_{I=1}$ and $a_{V=0}\simeq a_{I=0}$). Therefore, we can get the value of $|A_w(B^0\to K^0 D^0)|$ from the observed values of $|A(B^0\to\pi^- D_s^+)|$, $|A(B^0\to K^0 D^0)|$ and $\cos\delta_I$ [given by Eq. (10)], as follows:

$$\begin{aligned}|A_w(B^0\to K^0 D^0)|^2 &=\tfrac{1}{4}(a_{V=1}-a_{V=0})^2\\ &=\tfrac{1}{2}|A(B^0\to K^0 D^0)|^2\left[1+\frac{1}{\cos\delta_I}\right]\\ &\quad+\tfrac{1}{2}|A(B^0\to\pi^- D_s^+)|^2\left[1-\frac{1}{\cos\delta_I}\right] .\end{aligned} \tag{21}$$

In conclusion, we have proposed a means of extracting the value of $|V_{ub}/V_{cb}|$ from exclusive nonleptonic-B-decay data without assuming any specific calculation methods of the weak decay amplitudes: the value of $|V_{ub}/V_{cb}||V_{cs}/V_{us}|$ is given by the ratio of the weak decay amplitudes $|A_w(B^0\to K^0 D^0)|/|A_w(B^0\to K^0\bar{D}^0)|$, as shown in Eq. (1); the value of $|A_w(B^0\to K^0\bar{D}^0)|$ can be obtained from the measurements of $B(B^0\to K^0\bar{D}^0)$, $B(B^0\to K^+D^-)$, $B(B^+\to K^+\bar{D}^0)$, and $\tau(B^0)/\tau(B^+)$, as shown in Eq. (9); the value of $|A_w(B^0\to K^0 D^0)|$ can be estimated from the data of $B\to K\bar{D}$ decays and $B^0\to K^0D^0/\pi^-D_s^+$, as shown in Eq. (21). Here, the final states $K\bar{D}$ and $K^0D^0/\pi^-D_s^+$ mean[7] not only 0^-+0^- meson states, but also 0^-+1^- and 1^-+1^-. The observations of 1^-+0^- states can give $B(B^0\to K^{*0}D^0)$ and $B(B^0\to K^{*0}\bar{D}^0)$ without any information on the B^0-$\bar{B}^0$ mixing. However, as already stated (see Ref. 7), the study of the decay modes into 1^-+0^- also requires that of those into 0^-+1^- in order to estimate the final-state-interaction effect. Since the observation of the final states which contain D^{*0} is not so easy, the extraction of $|V_{ub}/V_{cb}|$ from the $B\to 0^-+1^-$ decay data is not practical.

There can be an objection to this proposal: the decay modes $B\to K\bar{D}$ are Cabibbo-suppressed modes, so that we cannot, for a time, except sufficient decay event numbers to determine the values of the branching ratios precisely; from the experimental point of view, for example, a precise measurement of $B(B^0\to\pi^+\pi^-)/B(B^0\to\pi^+D^-)$ will be available in a short time. However, even if we have the precise value of $B(B^0\to\pi^+\pi^-)/B(B^0\to\pi^+D^-)$, we cannot extract the value of $|V_{ub}/V_{cb}|$ from the experimental value without assuming a specific calculation method of the weak decay amplitudes and using theoretical values for some unknown parameters [for example, QCD enhancement and suppression factors, decay constants, weak decay form factors $f_\pm(q^2)$, and so on]. Thus, we sooner or later need a model-independent measurement of the value of $|V_{ub}/V_{cb}|$. In the future, the precise data of $B^0\to K^0D^0$ and $B^0\to K^0\bar{D}^0$ will become more available, so that the measurement of $|V_{ub}/V_{cb}|$ based on Eq. (1) will become practical.

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[2]For a detailed and systematical study of the two-body heavy meson decays, for example, see M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).

[3]Exactly speaking, another diagram (the external W-emission diagram) can contribute to these decays because of the QCD renormalization effects. The sum of the contributions from two diagrams can be expressed by a factor $C_2+\xi C_1[C_1=(C_+ +C_-)/2,\ C_2=(C_+ -C_-)/2]$ in the ter-

minology of Ref. 2. However, for convenience, hereafter we call the contribution $C_2+\xi C_1$ as the contribution from the internal W-emission diagram, and $C_1+\xi C_2$ as that from the external W-emission diagram.

[4]However, of course, when we observe $B^0 \to K^{*0}D^0$ and $B^0 \to K^{*0}\bar{D}^0$ instead of $B^0 \to K^0D^0$ and $B^0 \to K^0\bar{D}^0$, we can distinguish between $K^{*0}D^0$ from B^0 and $\bar{K}^{*0}D^0$ from $\bar{B}^0$ by the decay mode $K^{*0} \to \pi^- K^+$, so that we need not know the value of the parameter r.

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[7]When in the description of the final state $K\bar{D}$ we distinguish the final state with 1^-+0^- from that with 0^-+1^-, the decay mode which is corresponding to the weak decay $B^0 \to K^0\bar{D}^0$ is not $B^0 \to K^+D^-$ but $B^0 \to D^-K^+$ as seen from the quark diagrams, while in the study of the final-state-interaction effect, i.e., in the analysis of the isospin states, the state which is corresponding to $K^0\bar{D}^0$ is not D^-K^+ but K^+D^-. Therefore, in the study of the final-state-interaction effect, we hereafter understand, for example, "the final state K^+D^- with 0^-+1^-" as "$K^+D^{*-}+K^{*+}D^-$."

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[9]Exactly speaking, the penguin diagram with $b \to s+g$ can contribute to the $B \to DD_s^+$ decays, but the contribution can be neglected compared to that from the dominant decay diagram, i.e., the external W-emission diagram with KM factor $V_{cb}V_{cs}^*$.