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24×12^3 vs 16×8^3 lattices: Controlling finite-size effects in lattice SU(2) including fermions

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We report on the hadron spectrum, chiral-symmetry parameters, and the interquark potential of two-color SU(2) including dynamical Kogut-Susskind fermions. By comparing the results on a 24×12^3 with those on a 16×8^3 lattice, we demonstrate that finite-size effects are small in a range of couplings where approximate scaling is observed.

INTRODUCTION

In this paper we report on an investigation of the low-lying meson masses, the quark condensate in the vacuum, and the potential between heavy quarks in two-color SU(2), including four degenerate Kogut-Susskind fermions. In order to get control over finite-size effects, our calculations are carried out on 16×8^3 and 24×12^3 lattices, thereby extending previous work on the smaller lattice.[1] The fermonic degrees of freedom are incorporated using the pseudofermion technique.[2]

On the 16×8^3 lattice we have taken data at two values of the SU(2) coupling $\beta=4/g^2=1.85$ and 1.95 with quark masses chosen at $ma=0.10$, 0.07, 0.05, and 0.035 (a is the lattice spacing). These results can directly be compared with data at $\beta=1.95$ and masses $ma=0.07$ and 0.05 that we have recently collected on a 24×12^3 lattice with five times the volume. We expect to approach approximate asymptotic scaling in this β range.

Because the details of the theoretical and of the technical setup[1,3] have been described previously, we proceed immediately to the presentation of our results.

MESON MASSES

We discuss the four channels $\pi/0^{+-}$(exotic), ρ/B, ρ'/A_1, and π'/ϵ separately, first on the 16×8^3 lattice. The behavior of the propagators is very similar for both β values as well as for all quark masses so that they can be discussed together. Subsequently, these results will be compared with those on the 24×12^3 lattice. The results are summarized in Table I at the end of this section.

$\pi/0^{+-}$(exotic): No evidence for a 0^{+-} parity partner to the π has been found. All correlation functions rendered a stable π mass. This observation holds true also for the larger lattice; single mass fits for time distances ≥ 4 are excellent, and the values on the larger lattice are very close to those on the smaller lattice.

ρ/B: This channel is dominated by the ρ. Even though an oscillating behavior due to a B contribution is visible in the propagator on 16×8^3, it is so strongly suppressed that for large t values single mass fits work very well. Since fits over the entire range $t>0$ require an excited 1^{--} state, the B mass value must be interpreted rather as an upper limit. The situation improves on the larger lattice. While the previous ρ mass values are reproduced, the B signal can be separated better, resulting in a slightly smaller value than before.

ρ'/A_1: In these correlation functions the oscillating

TABLE I. Comparison of meson masses (in units of the inverse lattice spacing) from two lattices of different size at quark masses $ma=0.07$ and 0.05.

m	0.07		0.05	
	16×8^3	24×12^3	16×8^3	24×12^3
π	0.700(10)	0.705(10)	0.625(13)	0.607(10)
ρ	1.10(4)	1.16(6)	1.00(3)	1.06(4)
A_1	1.54(2)	1.63(16)	1.44(2)	1.63(13)
B	1.71(8)	1.77(21)	1.62(8)	1.55(52)
ϵ	1.02(2)	1.22(2)	0.90(2)	1.15(4)
π'	0.89(6)	0.76(4)	0.90(21)	0.607(7)
ρ'	1.19(6)	1.19(2)	1.17(4)	1.06(10)

TABLE II. Comparison of the quark condensate (in lattice units) for two different lattice sizes.

m	0.05	$\langle \bar{q}q \rangle^{1/3}_{2\text{ flavors}}$ 0.07	0.10
16×8^3	0.503(6)	0.548(2)	0.608(2)
24×12^3	0.517(2)	0.555(2)	

state A_1 has a higher mass but also a slightly bigger amplitude than the ρ'. Thus the ρ' can dominate the correlation functions only at large t values. On the 16×8^3 lattice the resulting ρ' mass is somewhat higher than the ρ mass. (In the continuum limit the ρ' should converge to the ρ mass in order to restore flavor symmetry.) The uncertainty affects also the A_1 value on the 16×8^3 lattice. On 24×12^3, however, a single mass fit for $t \geq 6$ returns a ρ' mass remarkably close to the ρ value, and is therefore in nice agreement with flavor-symmetry restoration. Decreasing the time cut in a series of two-state fits yields a stable A_1 mass value on this lattice.

π'/ϵ: The results for this channel must be interpreted with some caution. (i) We have investigated only the connected part of the singlet propagator although the disconnected part contributes in the continuum limit. (ii) Admixtures of the 0^{++} glueball must be expected. The mixing should be enhanced in an unquenched simulation which has built in the possibility of virtual fermion loops. Such a glueball analysis, however, would require high-statistics and/or link averaging techniques which are either unavailable at the moment or are not applicable to a theory with the highly nonlocal fermionic part included in the action. (For an attempt in this direction, yet based on a large quark mass value 0.10, see Ref. 4.) (iii) At finite lattice spacing a local (lattice) non-Goldstone pion is coupled to the singlet state, a consequence of the Kogut-Susskind formulation of the fermions. This π' is

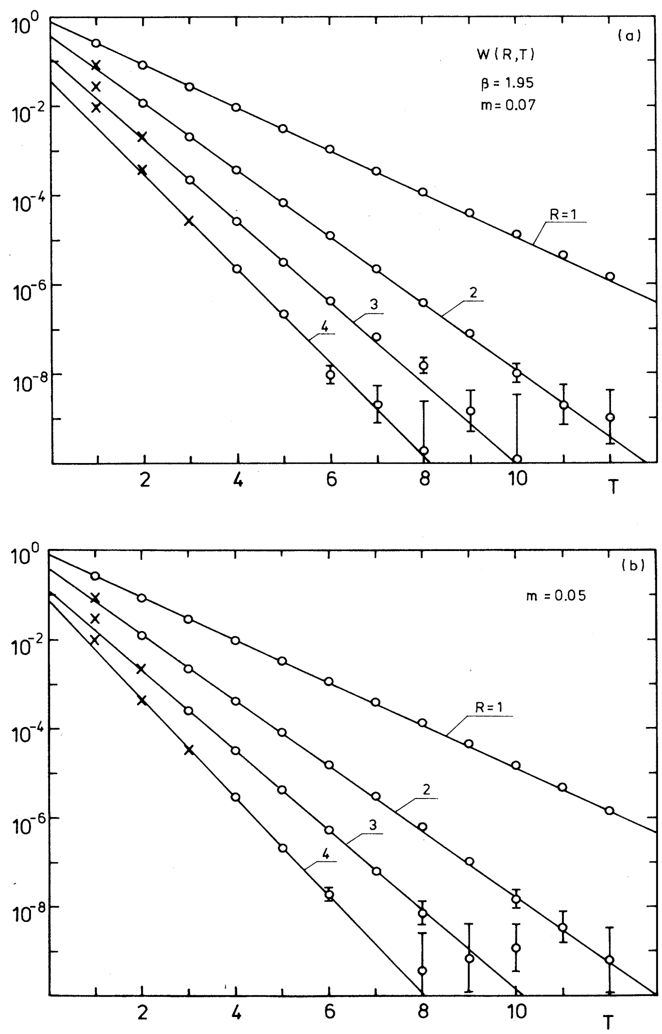

FIG. 1. The logarithms of Wilson loops, $\ln W(R,T)$, as linear functions of T on the 24×12^3 lattice; quark masses being $ma = 0.07$ in (a) and 0.05 in (b).

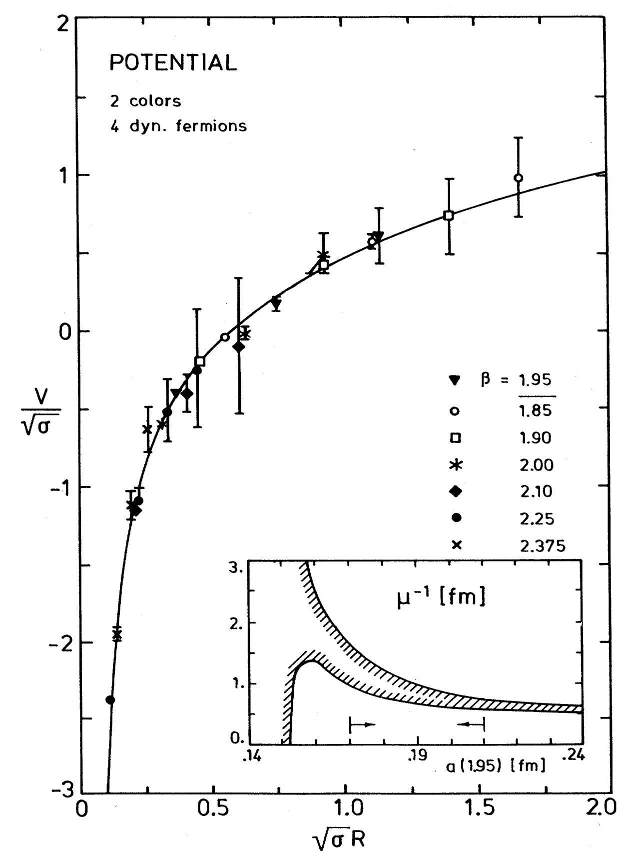

FIG. 2. The potential in physical units ($\sigma^{-1/2} \approx 0.5$ fm) after extrapolating the quark mass to $m = 0$. The physical scale has been fixed by the ρ mass. Assuming approximate asymptotic scaling we compare the data at $\beta = 1.95$ on 24×12^3 with the results from the 16×8^3 lattice, that were obtained at a variety of β values. The line corresponds to a fit based on a screened form of the potential, $V(R) = (-\alpha/R + \sigma R)[1-\exp(-\mu R)]/\mu R$ (see Ref. 1); the inset shows the change of the screening length when the lattice spacing is varied around the value extracted from the ρ mass.

not connected to any continuous global chiral symmetry on a lattice with finite spacing. Its mass should converge to the value of the standard π in the limit of flavor-symmetry restoration for small quark masses and $a \rightarrow 0$. At $[\beta, m]=[1.95, 0.07]$ we obtained an ϵ mass of 1.02(3) and a π' mass of 0.90(15) on the smaller lattice. The ϵ mass did change significantly to a value of 1.22(2) on the 24×12^3 lattice.

We collect all our results on the 24×12^3 lattice for both quark masses in Table I and compare them with the values obtained on the 16×8^3 lattice. The good agreement between the numbers from the two lattice sizes (with the exception of ϵ as mentioned above) corroborates similar observations in SU(3) (Ref. 3 and 5).

We extrapolated the masses linearly to zero quark mass since the linear extrapolation worked well for ρ on the small 16×8^3 lattice. Correcting the fitted masses m_ρ for lattice artifacts through $M_\rho/2=\sinh m_\rho/2$ (masses in lattice units) and taking the ρ as the reference scale, we can extract the lattice spacing in physical units, $a[1.95]=0.20$ fm. Thus the entire spatial lattice volume is roughly $(2.4\ \mathrm{fm})^3$, providing a reasonable number of computational points in the spatial box. The ratio of the lattice spacings is compatible with approximate asymptotic scaling as outlined in detail in Ref. 1.

CHIRAL QUARK CONDENSATE

As a by-product of the conjugate-gradient inversion of the Dirac matrix M, we obtain an estimate of the quark condensate $\langle \bar{q}q \rangle \propto \langle \mathrm{tr}(D+m)^{-1} \rangle$. These values agree with the pseudofermionic computations of the same observable within the error bars. Relating the 24×12^3 data with the results from the smaller lattice, we observe a slight increase of the chiral condensate due to the fivefold increase of the lattice size, Table II.

INTERQUARK POTENTIAL

In addition to the meson masses and the chiral condensate, we have measured the heavy-quark potential on the 24×12^3 lattice. Our motivation was first to check our previous results[1] for finite-size effects and second to explore the interquark forces for separations beyond spatial and time distances $R,T=4,8$ up to $R,T=6,12$.

As shown in Fig. 1, the logarithms of the Wilson loops follow nicely a linear behavior in T up to the largest T values for $R=1$ and 2. For $R=3$ and 4 we can go up to $T=10$ and 8, respectively; for $R=5$ or 6 the signal, except for small T, is swamped unfortunately by noise despite the large statistics of about 400 measurements. With the condition $T\geq R$ we fit a linear T dependence to the logarithm of the Wilson loops $W(R,T)$, the slope of which is to be identified with the potential. The values for the potential $V(R)$ agree with the previous 16×8^3 results. Guided by earlier experience we extrapolate the data linearly to zero quark mass. Applying the same procedure as in Ref. 1, we adopt the ρ mass as our reference scale to convert the potential from lattice to physical units. Assuming *approximate* asymptotic scaling we compare the 24×12^3 data to the previously reported 16×8^3 data in Fig. 2. The results are nicely consistent.

SUMMARY

We conclude that for properly chosen couplings not only approximate scaling can be observed[1] but also that finite-size effects are under control on a lattice of size 24×12^3. This supports and corroborates similar analysis in SU(3) (Refs. 3 and 5). Even though some problems (such as satisfactory flavor-symmetry restoration) require smaller quark masses and smaller lattice spacings (larger lattices correspondingly), we obviously find quite reasonable results for some of the most fundamental properties of hadron physics emerging from analyses of the presently available lattice capacities.

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