Strange matter at finite temperature

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We investigate strange matter at temperatures between 0 and 30 MeV. Increasing temperature destabilizes systems stable at $T=0$. The electric charge carried by quarks and the Coulomb barrier change significantly. The pressure dependence of the system is also presented.

I. INTRODUCTION

Strange matter is quark matter with strangeness per baryon close to unity. It is possible that large lumps of strange matter are absolutely stable.[1,2] However, it requires a very-high-order weak transition to produce it from ordinary nuclei under normal conditions. On the other hand, we believe that under high enough pressure nuclei will convert to quark matter, and consequently strange matter will be formed. Once it is produced it will remain stable even at much lower pressure. Hence, natural places to find strange matter are dense objects produced in final stages of stellar evolution. In particular, it is possible that what we call neutron stars are in fact strange stars.[3,4] They emerge as remnants of a supernova explosion, a process in which pressure is of the order of 100 MeV/fm^3 and temperature rises up to tens of MeV. It is therefore interesting to examine the influence of temperature on the properties of strange matter and on the transition between normal and strange matter.

In this paper we investigate the properties of "hot" strange matter. We consider temperatures up to 30 MeV and pressures up to 200 MeV/fm^3. Cold strange matter under zero pressure has been studied by Farhi and Jaffe.[2]

The quark matter is taken as a Fermi gas of up, down, and strange quarks contained in the MIT bag. There are also some electrons in the system to neutralize the electric charge. Flavor equilibrium is maintained by weak interactions in which neutrinos are produced. Even if they are bound to the strange matter they contribute very little to the energy and pressure of the system. Thus we neglect them entirely. Electrons are also rather scarce but, as they do not feel the bag surface, they generate a Coulomb barrier in finite systems, e.g., quark stars.

We present the basic formulas and discuss the equilibrium conditions in Sec. II. In Sec. III a method of approximate solution of the equation of state for low temperatures is given. The numerical results are presented in Sec. IV. We compare there the exact and approximate solutions and give a simple parametrization of the temperature and pressure dependence. In the last section we summarize the results.

II. EQUATION OF STATE

In this section we calculate the basic thermodynamic properties of the bulk strange matter for temperatures between 0 and 30 MeV. We consider a Fermi gas of u,d,s quarks and electrons. For finite temperature we also have antiparticles in the system. However, the chemical potential of antiparticles is equal to $-\mu$, and we obtain the following bounds for the density n_i of massless antifermions:

$$1-\frac{e^{-\mu_i/T}}{8}<\frac{n_i}{(g_i/\pi^2)T^3e^{-\mu_i/T}}<1\ , \tag{1}$$

where g_i is the number of spin-color degrees of freedom. As the chemical potentials of electrons and quarks in the strange matter are of order 10 and 250 MeV, respectively, the densities of antiquarks are negligibly small in comparison with the densities of quarks, electrons, and positrons. Hence positrons are the only antiparticles we will consider.

Long-range strong interactions are approximated by the bag pressure B, whereas α_s corrections are neglected. Flavor equilibrium is maintained via the weak reactions

$$\begin{aligned}
&d\to u+e^-+\bar\nu,\quad s\to u+e^-+\bar\nu\ ,\\
&u\to d+e^++\nu,\quad u\to s+e^++\nu\ ,\\
&u+e^-\leftrightarrow d+\nu,\quad u+e^-\leftrightarrow s+\nu\ ,\\
&s+e^+\leftrightarrow u+\bar\nu,\quad d+e^+\leftrightarrow u+\bar\nu\ ,\\
&s+u\leftrightarrow d+u\ .
\end{aligned}\tag{2}$$

At 30 MeV temperature and density above 100 MeV/fm^3 the mean free path of neutrinos is much below the radius of a typical quark star,[3] so we should consider neutrinos to be bound to the system. Nevertheless, they contribute very little to the energy density and pressure and can be neglected in the study of the equilibrium properties of the strange matter. Hence, the chemical potentials are related as follows:

$$\mu_e\equiv\mu_{e^-}=-\mu_{e^+},\quad \mu_s=\mu_d=\mu_u+\mu_e\ . \tag{3}$$

We parametrize them by means of μ and x:

$$\mu_s=\mu_d=\mu,\quad \mu_e=x\mu,\quad \mu_u=(1-x)\mu\ . \tag{4}$$

There are two conserved quantities in the system: the baryon number A and electric charge. Their chemical potentials are equal to μ_A and $-\mu_e$, where

$$\mu_A=(3-x)\mu\ . \tag{5}$$

As the system must be locally neutral, we have

$$2n_u - n_d - n_s - 3n_e = 0 \ , \tag{6}$$

where $n_e = n_{e^-} - n_{e^+}$. This leaves one independent chemical potential, say μ.

The equilibrium condition under external pressure p is

$$p_u + p_d + p_s + p_{e^+} + p_{e^-} = p + B \ , \tag{7}$$

where B is the bag constant.

The partial pressures of each species of particles are given by their thermodynamic potentials ω_i calculated per unit volume:

$$p_i = -\omega_i = \frac{g_i T}{2\pi^2} \int_{m_i}^{\infty} d\epsilon\, \epsilon(\epsilon^2 - m_i^2)^{1/2} \ln(1 + e^{-(\epsilon-\mu_i)/T}) \ , \tag{8}$$

where $g_i = 2$ for electrons and 6 for quarks, T is the temperature of the system, and m_i is the mass of the ith species. We take electrons and u and d quarks to be massless and in the following denote the s-quark mass by m.

Number densities n_i are given by

$$n_i = -\frac{\partial}{\partial \mu_i} \omega_i \bigg|_T \ . \tag{9}$$

Now, inserting Eqs. (8) and (9) into (7) and (6) we can find μ and x, and thus find all thermodynamic parameters of the system.

The baryon density n_A is simply calculated as

$$n_A = \tfrac{1}{3}(n_u + n_d + n_s) \tag{10}$$

and the free energy per baryon reads

$$f_A = \mu_A - \frac{p}{n_A} \tag{11}$$

with μ_A given by Eq. (5). The entropy density and entropy per baryon can be derived as

$$s = -\frac{\partial \omega}{\partial T} \bigg|_{\mu_A} , \quad s_A = \frac{s}{n_A} \ , \tag{12}$$

where $\omega = \sum_i \omega_i$. Using (11) and (12) we obtain energy per baryon E_A and energy density ρ

$$E_A = f_A + T s_A, \quad \rho = E_A n_A \tag{13}$$

or explicitly

$$\rho = 4B + 3p + \frac{3m^2}{\pi^2} \int_m^{\infty} d\epsilon \frac{\sqrt{\epsilon^2 - m^2}}{1 + e^{(\epsilon - \mu_s)/T}} \ . \tag{14}$$

At $T = 0$, the energy per baryon determines whether the system is stable against decay into neutrons or nuclei. For E_A lower than 930 MeV the strange matter cannot decay even into ^{56}Fe nuclei. When T is greater than 0 a finite-size strange-matter lump starts evaporating. As long as E_A is below 930 MeV, emission of, e.g., an α particle goes via energy fluctuations. For higher E_A these fluctuations are not needed and the process goes much faster. In the following we will call strange matter stable if its E_A is below 930 MeV.

In general, as T is much smaller than the quark chemical potentials (about 300 MeV), the quark content of the strange matter does not change much with T. The electrons, however, are quite sensitive to T because their chemical potential is of order of 10 MeV. To see the effect we look at two quantities depending on the electron content of the strange matter: the quark electric charge and the Coulomb barrier.

The density of electric charge carried by quarks is, by Eq. (6), equal to the electron density n_e. When calculated per baryon it is given by $Q_A \equiv n_e / n_A$, and measures the number of electrons minus positrons. As we will see in Sec. IV it grows fast with temperature.

In finite systems the quarks are confined inside the bag. Electrons, however, are bound to the system only by the Coulomb force, so they spread outside the bag. This results in a Coulomb barrier V_C for positively charged particles.[4] Any particle with charge Q (measured in proton charge units) must have kinetic energy higher than QV_C to get inside the bag. In order to calculate V_C we have to find the Coulomb potential at the bag surface. Let r measure the distance from the bag center outwards and R be the bag radius. Then $V_C = eV(R)$. Quark charge density ρ_q is assumed uniform inside the bag, whereas electron-positron charge density ρ_e is fixed by thermodynamic balance. Introducing a local electron chemical potential $\mu_e(r)$, we have

$$\mu_e(r) - eV(r) = \text{const} = 0 \ . \tag{15}$$

The last equality follows from the fact that at $r = \infty$ there are no electrons nor positrons and $V(\infty) = 0$. We take R big enough to assume that the total charge density vanishes for $r \ll R$ and the bag surface is flat. Then $eV(0) = \mu_e(0) \equiv \mu_e^0$, where μ_e^0 is the electron chemical potential in the bulk limit. In this approximation the Poisson equation reads

$$-\frac{1}{4\pi} \frac{d^2 V}{dr^2} = \rho(r) = e n_q \Theta(R - r) - e n_e(r) \ . \tag{16}$$

As $V(r)$ is a monotonic function of r and $V'(r=0) = 0$, we get

$$e \int_0^{V(0)} n_e dV = e \int_{V(0)}^{V(R)} n_q dV = n_q (V_C - \mu_e^0) \ . \tag{17}$$

Using $n = dp/d\mu$, Eq. (15), and omitting index 0 at μ_e^0 we get

$$V_C = \mu_e - \frac{p_e - p_e(\mu_e = 0)}{n_e} = \mu_e \frac{3 + 2\pi^2 T^2/\mu_e^2}{4 + 4\pi^2 T^2/\mu_e^2} \ , \tag{18}$$

where $p_e = p_{e+} + p_{e-}$.

III. APPROXIMATE SOLUTION

The temperatures discussed in this paper are much smaller than the quark chemical potentials. In such a case the effects of temperature can be expressed as simple $O(T^2/\mu^2)$ corrections to the thermodynamic functions.

First, we have to calculate the T dependence of the chemical potentials. To this end we parametrize the possible changes of μ and x with temperature as

$$\mu=\mu_0-\Delta\mu, \quad x=x_0+\Delta x \ , \tag{19}$$

where μ_0 and x_0 denote values of μ and x at $T=0$.

In order to estimate the T dependence of $\Delta\mu$ and Δx we use (6) and (7) with densities and pressure expanded to the lowest order in $\Delta\mu/\mu_0$, Δx, and T^2/μ_0^2. We get

$$\Delta\mu=a\pi^2\frac{T^2}{\mu_0^2}\mu_0, \quad \Delta x=b\pi^2\frac{T^2}{\mu_0^2} \ , \tag{20}$$

where a and b depend on μ_0, x_0, and m and are solutions of the following system of linear equations:

$$Aa+Bb=Q, \quad Ca+Db=P \ . \tag{21}$$

The coefficients A,B,C,D and right-hand sides Q,P are given by the formulas

$$\begin{aligned} A&=1-\sqrt{1-z^2}-3x_0(2-2x_0+x_0^2) \ , \\ B&=2-x_0(4-3x_0) \ , \\ C&=1+(1-x_0)^4+(1-z^2)^{3/2}+\tfrac{1}{3}x_0^4 \ , \\ D&=(1-x_0)^3-\tfrac{1}{3}x_0^3 \ , \\ Q&=\frac{1}{3}\left[1-\frac{1-\frac{1}{2}z^2}{\sqrt{1-z^2}}-3x_0^2\right] \ , \\ P&=\tfrac{1}{2}[1+(1-x_0)^2+\sqrt{1-z^2}+\tfrac{1}{3}x_0^2] \ , \end{aligned} \tag{22}$$

where $z=m/\mu_0$. The above formulas are valid for $\mu_0-m \gg T$.

In order to get approximate formulas for thermodynamic quantities we first expand Eq. (8) for ω_i to the lowest order in T/μ and then we use the formulas of Sec. II with μ and x given by (19). For example, we get

$$\mu_A(T)=\mu_A(0)-[(3-x_0)a+b]\frac{\pi^2T^2}{\mu_0} \ , \tag{23}$$

$$n_A(T)=n_A(0)-[3(1-x_0)^3a+3(1-x_0)^2b + \tfrac{1}{3}x_0+x_0^2(b-ax_0)-1]T^2\mu_0 \ . \tag{24}$$

As we have already observed in Sec. II the charge density of quarks (equivalent to the electron density) is quite sensitive to temperature. Here the zero-temperature term is of order of $(x_0\mu_0)^3$ and the first correction is of order of $x_0T^2\mu_0$. Because the latter term is in fact the leading one we keep also the T^4 correction in the expression for n_e:

$$n_e(T)=n_e(0)+[\tfrac{1}{3}x_0+x_0^2(b-ax_0)]T^2\mu_0 + \frac{\pi^2}{3}(b-ax_0)\frac{T^4}{\mu_0} \ . \tag{25}$$

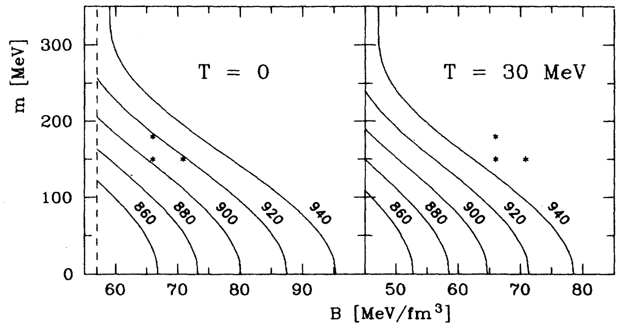

FIG. 1. E_A contours for $E_A=860$, 880, 900, 920, 940 MeV, $T=0$, and 30 MeV. The asterisks mark the (A), (B), and (C) sets of B and m (see text).

IV. NUMERICAL RESULTS

Here we present the numerical results for the quantities discussed in previous sections. All the results were obtained by solving exactly the equations of Sec. II. We have compared these results with those obtained by means of approximate formulas of Sec. III and have found that they agree with accuracy better than 1%.

In order to make an explicit calculation we have to choose the parameters B and m. To this end, following Ref. 2, we look at the stability[5] regions in the B-m plane. Figure 1 shows the lines of constant energy per baryon E_A under zero external pressure ($p=0$) for $T=0$ and 30 MeV.

The region to the left of the vertical lines is excluded by experiment as for such a low B the nonstrange quark matter would be stable against decay into nuclei. These lines correspond to $E_A=930$ MeV and $m\rightarrow\infty$. We observe that with growing temperature T the stability region moves to the lower values of B and m.

Let us present now the temperature dependence of thermodynamic quantities for the following sets of B and m:

$$\begin{aligned} &(A) \quad B=66.0 \text{ MeV/fm}^3, \quad m=150 \text{ MeV} \ , \\ &(B) \quad B=66.0 \text{ MeV/fm}^3, \quad m=180 \text{ MeV} \ , \\ &(C) \quad B=70.8 \text{ MeV/fm}^3, \quad m=150 \text{ MeV} \ . \end{aligned} \tag{26}$$

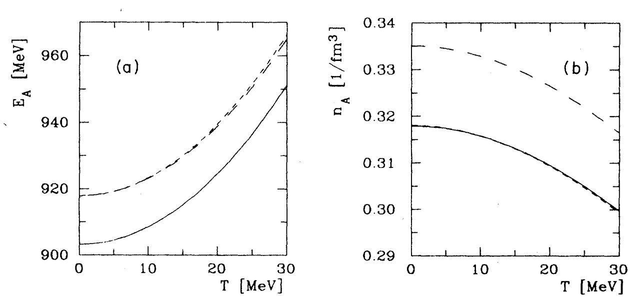

FIG. 2. Temperature dependence of (a) energy per baryon, (b) baryon density for $p=0$ and three sets of B and m: (A) (solid line), (B) (short dashed line) and (C) (long dashed line).

TABLE I. Coefficients of temperature dependence of the energy per baryon E_A, baryon density n_A, and entropy per baryon s_A; used in Eqs. (27) and (28).

m (MeV)	B (MeV/fm^3)	$E_A^{(0)}$ (MeV)	$E_A^{(1)}$ (MeV^{-1})	$n_A^{(0)}$ (fm^{-3})	$10^5 n_A^{(1)}$ (fm^{-3} MeV^{-2})	$s_A^{(1)}$ (MeV^{-1})
150	66.0	903	0.053	0.318	−2.04	0.107
180	66.0	918	0.054	0.318	−2.10	0.108
150	70.8	918	0.052	0.335	−2.08	0.105

Set (A) is the one used by Farhi and Jaffe.[2] Sets (B) and (C) are chosen in such a way that they both give $E_A = 918$ MeV, which still lies within the stability region for $T=0$. As can be seen from Fig. 1 in all three cases the strange matter ceases to be stable at $T=30$ MeV and $p=0$.

Figure 2 shows the temperature dependence of the energy per baryon E_A and baryon density n_A for $p=0$. We observe, e.g., for set (A), that up to $T=22.5$ MeV the energy per baryon is below 930 MeV, i.e., the strange matter is stable. At the same time, the baryon density decreases with T, so that the energy density $\rho = E_A n_A$ remains practically constant for T changing from 0 to 30 MeV (see Table I).

Both E_A and n_A are quadratic in T with accuracy much better than 0.1%, i.e.,

$$E_A = E_A^{(0)} + E_A^{(1)} T^2, \quad n_A = n_A^{(0)} + n_A^{(1)} T^2 , \tag{27}$$

whereas the entropy per baryon, s_A, is linear in T,

$$s_A = s_A^{(1)} T . \tag{28}$$

The coefficients used in Eqs. (27) and (28) are given in Table I.

The temperature effects for the quantities discussed so far are rather small. Much stronger effects are seen for the quantities depending on the electron content of the strange matter, like the quark electric charge per baryon, Q_A, and the Coulomb barrier, V_C. They are shown in Fig. 3. Q_A grows by a factor of order 20, whereas V_C goes down by about 30%.

The increase in the number of electrons per baryon implies higher neutrino emissivity from the strange matter.[6] It also improves the heat transport across the bag surface in a system such as a neutron star with the strange-quark-matter core.

A smaller Coulomb barrier results in a higher probability for a positively charged particle to get inside a finite-size strange-matter bag.

We have also investigated the effects of external pressure on the strange matter. In Fig. 4 we show E_A and n_A versus pressure for $m=150$ MeV and $B=66$ MeV/fm^3. We compare there our results with the Bethe-Johnson (model I) equation of state for neutron matter at $T=0$ (Ref. 7). For low p the strange-quark matter is stiffer than the neutron one and for high p it is softer. The reason for such behavior of strange matter is that at low p the bag pressure B dominates, and as p grows we come closer to a free relativistic gas limit [see Eqs. (29) below].

Let us present now a simple parametrization of baryon density n_A, energy density ρ, entropy density s, and electron density n_e. In general n_A, $\rho - B$, s, and n_e depend on p and B only via $P \equiv p + B$. Thus only m remains a free parameter. For $m=150$ MeV we fit following formulas which work with accuracy better than 1% for P in the range 50–350 MeV/fm^3:

$$\begin{aligned} n_A &= 0.014\,11 P^{0.7435} - 7.731\times10^{-6} T^2 P^{0.2306} , \\ \rho &= B + 3P + 1.557 P^{0.6455} - 0.001\,583 T^2 P^{0.09619} , \\ n_e &= 4.464\times10^{-4} P^{-0.6726} + 1.944\times10^{-6} T^2 P^{-0.2172} , \\ s &= 0.003\,851 T P^{0.5052} , \end{aligned} \tag{29}$$

where $P = p + B$.

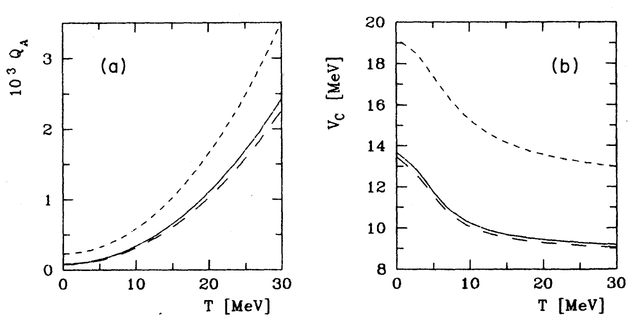

FIG. 3. Temperature dependence of (a) quark electric charge per baryon and (b) Coulomb barrier for $p=0$. B and m are as in Fig. 2.

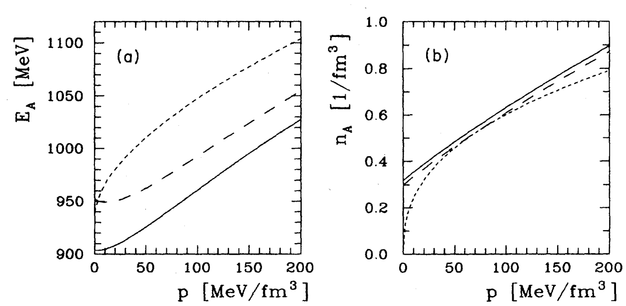

FIG. 4. Pressure dependence of (a) energy per baryon, (b) baryon density for $m=150$ MeV, and $B=66$ MeV/fm^3, $T=0$ (solid line) and $T=30$ MeV (long dashed line). Short-dashed line is the Bethe-Johnson neutron matter.

V. SUMMARY

We have investigated the properties of strange matter at temperatures between 0 and 30 MeV and external pressures up to 200 MeV/fm^3. We have found that the temperature does not affect much the characteristics connected with quarks. On the other hand, the quantities which depend on electron abundance (hadronic charge per baryon, Coulomb barrier) change significantly. In both cases the T dependence can be expressed in terms of T^2/μ^2 corrections.

It also turned out that for practical purposes, the pressure dependence of basic thermodynamical functions is well described by a simple power-law fit.

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[1]E. Witten, Phys. Rev. D **30**, 272 (1984).
[2]E. Farhi and R. L. Jaffe, Phys. Rev. D **30**, 2379 (1984).
[3]P. Haensel, J. L. Zdunik, and R. Schaeffer, Astron. Astrophys. **160**, 121 (1986).
[4]C. Alcock, E. Farhi, and A. V. Olinto, Astrophys. J. **310**, 261 (1986).
[5]The term "stability" is used in the sense defined in Sec. II.
[6]N. Iwamoto, Phys. Rev. Lett. **44**, 1637 (1980); A. Burrows, Phys. Rev. D **20**, 1816 (1979); Phys. Rev. Lett. **44**, 1640 (1980).
[7]H. A. Bethe and M. B. Johnson, Nucl. Phys. **A230**, 1 (1974).