Special points in three-generation moduli space

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We focus on some special points in the moduli space of the three-generation compactification which give rise to a four-dimensional effective Lagrangian at the compactification scale differing markedly from that found in previous studies. In particular, we consider manifolds possessing R symmetries as well as manifolds arising from blowing up an alternative construction of the three-generation class. In addition to the natural matter parities found in previous models, some of these constructions offer the substantial improvements of having fewer or no mirror quarks and exact flat directions (modulo an incomplete understanding of certain singlet modes) leading to three-generation models with SO(10), SU(5), or the standard-model gauge group at the compactification scale.

I. INTRODUCTION

The attractive uniqueness of string theory is swamped by the large vacuum degeneracy; predictability therefore suffers a similar fate. Lacking a dynamical prescription for choosing a vacuum state, it is worthwhile to examine physical restrictions which might help in reducing the set of viable vacuum candidates. A seemingly mild yet surprisingly powerful criterion is to demand that the resulting four-dimensional theory which follows from a particular choice of the vacuum state have three generations of elementary particles. This criterion was applied to (2,2) Calabi-Yau compactifications in Ref. 1 and it was shown that of the *a priori* thousands of topologically distinct complete intersection Calabi-Yau manifolds modded out by free group actions, apparently only one can fit the bill. This manifold is the one initially constructed by Tian and Yau[2,3] and mathematically and phenomenologically analyzed in Ref. 4. (See also Refs. 5–7.) The resulting low-energy phenomenology, as shown in Ref. 4 is surprisingly consistent with observation. Of course, the point of any such phenomenological study is not to build *the* model; our understanding of string theory is, at present, far too limited for such a task. Rather, the philosophy is to push as far as we can within the context of such concrete and well-motivated choices for the vacuum state in order to possibly extract relevant physics as well as to guide further study by exposing both the phenomenological virtues and deficiencies in our present formulation of string theory.

One of the most attractive aspects of such superstring models is that one bypasses many of the *ad hoc* assumptions of conventional model building. After selecting a manifold for compactification, (quasi)topological calculations yield the low-energy particle content, renormalizable Yukawa couplings, discrete symmetries, and even some aspects of gauge symmetry breaking. A number of nonrenormalization theorems ensure that many of these lowest-order computations are not corrected by higher-order effects.[8] Nevertheless, our present level of understanding does require that we make certain arbitrary choices in order to extract low-energy physics. Two prominent choices are the supersymmetry-breaking scenario and the selection of the precise form of the vacuum manifold. In particular, supersymmetry breaking in Ref. 4 was assumed to trigger vacuum expectation values along certain phenomenologically viable directions in field space, and the complex structure of the vacuum manifold was chosen so as to give rise to the maximal discrete symmetry group (for the covering space). An important question to ask is whether by modifying either of these choices we yield phenomenologically viable low-energy models. In this way we can gain an understanding of how special (or generic) acceptable three-generation models are and in the process possibly improve upon existing models. For example, one might hope to reduce the large number of extra generation-antigeneration pairs present at the compactification scale in Ref. 4 which have to be given large mass in order to avoid many phenomenological catastrophes. Although this can be accomplished, these excess states make the model unnecessarily cumbersome and rather delicate. Furthermore, one might hope to go beyond the intricate flat-direction analysis of Ref. 4 by finding exactly flat directions in field space. In fact, one motivation for the present work is to find such exactly flat directions which allow us to deform our theory to a three-generation SO(10) or SU(5) (2,0) model [as opposed to the viable but far less attractive E_6 which arises in (2.2) theories].

The question of modifying the symmetry-breaking pattern was addressed in Ref. 5 where it was shown that a variety of different symmetry-breaking scenarios all give rise to unacceptable physics. It may be possible, of course, that there are some unexplored symmetry-breaking patterns which do give rise to viable phenomenology, but as yet none has been found. We shall not discuss this method of generating new models further. Some work has been done on choosing alternative complex structures.[6] The qualitative features of many such models, though, are readily discerned from the analysis of

Ref. 4. That is, since the covering space in Ref. 4 was chosen to give rise to a maximal set of discrete symmetries D, modifications of the complex structure generically (but not always) give rise to smaller groups which are often contained in D. The prospects for such a modified compactification are then, qualitatively speaking, determined by whether any of the new terms allowed by the reduced symmetry group can mediate phenomenologically disastrous interactions. Although it is important and interesting to study such examples (to shed light, for instance, on how stable the model of Ref. 4 is under small perturbations), in this paper we shall consider points in the moduli space of the three-generation compactification which can give rise to low-energy effective Lagrangians which differ radically from that studied in Ref. 4. We do this in two ways.

First, we seek complex structures on the first realization of the three-generation manifold R (as a codimension-three variety in $CP^3 \times CP^3$) which give rise to discrete R symmetries. (All of the symmetries which arise in Ref. 4 are non-R symmetries.) As discussed in Ref. 9, R symmetries place powerful constraints on renormalizable and nonrenormalizable terms in the effective Lagrangian. Whereas nonzero couplings in the Lagrangian must be invariant under ordinary symmetries, they must transform in a specific nontrivial manner under R symmetries (we will make this precise later). At first sight it seems unlikely that one could find a complex structure on the first realization of the three-generation manifold which respects R symmetries. This is because symmetries on the two CP^3 spaces are tied together by a hyperplane which, as shall be clear in the next section, generically forces them to be non-R symmetries. However, we shall find, surprisingly, that there are *smooth* complex structures giving rise to R symmetries and that, as anticipated, the pattern of allowed couplings is markedly different from that found in Ref. 4. One interesting consequence of these symmetries is the existence of exactly flat directions in the superpotential [up to an incomplete understanding of singlets from $H^1(\text{End}T)$] which allow us to holomorphically deform our initial E_6 (2,2) theory to a three-generation SO(10) or SU(5) (2,0) compactification. With flux breaking, the latter can give rise to the standard-model gauge group at the compactification scale. We will also see that the discrete symmetries of such models can also give rise to phenomenologically essential matter parities.

Second, we shall study string compactification on the second realization of the three-generation manifold K (as a codimension-one variety in $CP^2 \times CP^2$) presented in Ref. 3. (The equivalence of this construction with the first realization was proven in Ref. 10.) Model building on this manifold has been hampered by the increased complexity of the construction which involves resolving quotient singularities and hence requires an understanding of the resulting blow-up modes. We shall see that it is not at all hard to overcome this obstacle. It is not known whether this manifold is continuously connected to R, and in any event, the structure of the effective Lagrangian for the massless modes is very different from that found on R. We shall see that these examples also admit R symmetries and that such symmetries ensure flat directions [again, up to an incomplete understanding of $H^1(\text{End}T)$ modes] leading to SU(5) or SO(10) (2,0) models. Furthermore, we shall also mention some examples with novel Wilson loop symmetry breaking that have the potential to eliminate the phenomenologically troublesome mirror quarks and antiquarks that arise in these models.

In Secs. II and III we shall describe each of these constructions in turn and derive all of the necessary information for detailed model building. In Sec. IV we shall use this information to comment on some of the phenomenological characteristics of these models.

II. THE FIRST REALIZATION

The covering space for the first realization of the three-generation manifold,[3] R_0, is the vanishing locus of a bidegree (3,0), (3,0), and (1,1) homogeneous polynomials in $CP^3 \times CP^3$. Modulo projective general linear transformations in each of the CP^3 factors, R_0 can be written as the intersection of

$$C_1:\ \sum_{i=0}^{3} x_i^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3 + a_3 x_0 x_2 x_3 + a_4 x_1 x_2 x_3 = 0\ , \tag{2.1}$$

$$C_2:\ \sum_{i=0}^{3} y_i^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3 + b_3 y_0 y_2 y_3 + b_4 y_1 y_2 y_3 = 0\ , \tag{2.2}$$

$$H:\ x_0 y_0 + \sum c_{ij} x_i y_j = 0\ . \tag{2.3}$$

For R_0 to be smooth we further require the transversality constraint: $dC_1 \wedge dC_2 \wedge dH$ to be nonvanishing on R_0. This manifold is simply connected and has[4] $h^{2,1}=23$ and $h^{1,1}=14$, where $h^{p,q}=\dim H_{\bar{\partial}}^{p,q}(R_0)$. The number of generations obtained by compactifying on R_0 is thus seen to be 9, and hence we seek a freely acting Z_3 group of holomorphic automorphisms so that $R=R_0/Z_3$ is a smooth three-generation manifold. The Z_3 action G generated by

$$g:(x_0,\ldots,y_3) \to (x_0,\alpha^2 x_1,\alpha x_2,\alpha x_3,y_0,\alpha y_1,\alpha^2 y_2,\alpha^2 y_3)$$

where α is a nontrivial cube root of unity, is a freely acting automorphism so long as $a_3=a_4=b_3=b_4=0$, and the only nonzero c_{ij}'s in addition to c_{00} are c_{11}, c_{22}, c_{33}, c_{23}, and c_{32} (for ease of notation we shall call these $c_0,\ldots,c_5$). Furthermore, these nonzero c's must satisfy

$$c_2 + c_3\delta\delta' + c_4\delta' + c_5\delta \neq 0\ , \tag{2.4}$$

where δ and δ' are cube roots of -1. These conditions ensure that R is a smooth three-generation Calabi-Yau manifold, so long as R_0 is smooth.

In Ref. 4 we made the specific "standard" choice $a_1=a_2=b_1=b_2=0$; $c_0=c_1=1$, $c_2=c_3\equiv c$, and $c_4=c_5=0$. This highly symmetric form of R_0 gives rise to a large group of holomorphic automorphisms. These consist of $(x_i,y_i)\to(x_{p(i)},y_{p(i)})$ where p is an element of the permutation group on four objects: $(x_i,y_i)\to(\alpha^{n_i}x_i,\alpha^{-n_i}y_i)$ and $(x_i,y_i)\to(y_i,x_i)$.

As shown in Ref. 4, though, none of these symmetries are R symmetries. A simple way to see this is to recall, as pointed out in Ref. 11, that an R symmetry necessarily acts nontrivially on the holomorphic (3,0) form ω. Following Candelas[12] we can write ω as

$$\omega=\oint\oint\oint \frac{\epsilon_{ijkl}x_i dx_j \wedge dk_k \wedge dx_l \epsilon_{nmpq} y_n dy_m \wedge dy_p \wedge dy_q}{C_1 C_2 H}, \tag{2.5}$$

where the contour integrals are about C_1, C_2, and H. Now, it is easy to verify that all of the symmetries above leave ω invariant. The reason for this is clear: the hyperplane H is such that any group action leaving it invariant necessarily also leaves ω invariant.

The question then arises: are there other choices for H which give rise to *smooth* manifolds respecting discrete R symmetries? At first sight it might appear that the answer to this question is no as smoothness often requires all coordinates to appear in the defining equations; this in turn implies that symmetries of R_0 (which, in particular, preserve H) will always pair off symmetries in the x coordinates with their inverse action on the y coordinates—thus preserving ω. As we now see, though, for the case at hand, we can choose H so that R_0 (and R) respects R symmetries.

To this end, we first note the fact that if in addition to c_0 and c_1 at least one of the c's in (2.4) is nonzero, say exactly one, then R_0 is nonsingular almost everywhere in this one-dimensional subspace of the moduli space. This follows directly from an examination of the transversality condition. Namely, for $dC_1 \wedge dC_2 \wedge dH$ to vanish at some point, either $dH=0$ at some point (dC_1 and dC_2 never vanish) or $i_x(dH)$ is proportional to dC_1 and $i_y(dH)$ is proportional to dC_2 where $i_{x,y}$ are the evaluation maps on $\sum_i (\partial/\partial x_i)\otimes dx_i$ and $\sum_i (\partial/\partial y_i)\otimes dy_i$, respectively. Now, without loss of generality, take c_0, c_1, c_2 to be the only nonzero coefficients. This implies that $dH=0$ only if x_3 and y_3 are the only nonzero coordinates. Such points do not lie on C_1 and C_2. Furthermore, $i_x(dH)\propto dC_1$ and $i_y(dH)\propto dC_2$ imply that $y_i=\kappa x_i^2$, $x_i=\kappa' y_i^2$ for $i=0,1,2$ and $x_3=y_3=0$, for some constants κ and κ'. However, none of these points lie on C_1 and C_2, for almost all values of the coefficient c_2, and hence the manifold is nonsingular. We also point out that we need at least one nonzero c_i in addition to c_0 and c_1 for if not, it is straightforward to see that dH vanishes on R_0.

With this understanding of the transversality constraint, we now see that the smooth manifold with H defined by nonzero c_0, c_1, and c_2 (with all other c's zero) allows us to *independently* scale r_x:$x_3 \to \alpha x_3$ and r_y:$y_3 \to \alpha y_3$ by cube roots of unity, thus giving us an R symmetry. (Although we have not done a complete study, many choices of H giving rise to permutation R symmetries are not smooth.)

From our introductory remarks, we anticipate that such a choice for the manifold R will give rise to couplings which are quite different from the usual x-y-symmetric choice of H.[13] In order to extract the effective Lagrangian (both renormalizable and nonrenormalizable terms) we need the transformation properties of the lepton and quark fields. Recall from Ref. 4 that after flux breaking via

$$g \to \text{diag}(1,1,1)\text{diag}(\alpha,\alpha,\alpha)\text{diag}(\alpha,\alpha,\alpha)$$

[where the latter is the subgroup $H=\text{SU}(3)_{\text{color}}\times\text{SU}(3)_L \times\text{SU}(3)_R$ of E_6] the generations consist of nine leptons, seven quarks, and seven antiquark multiplets [where these names refer to the $(1,3,\bar{3})$, $(3,1,\bar{3})$, and $(\bar{3},3,1)$ representations of H] and the antigenerations consist of six leptons, four quarks, and four antiquarks. The polynomial representatives used in Ref. 4 for the generations are still valid in this context and hence can be used to directly determine the transformation properties of these fields. The only new symmetries which arise on the quotient space from the nonstandard choice of H made here are r_x and r_y. We list their action on the generations in Table I. One important point to bear in mind is that the monomial representatives are more precisely described as parametrizing $H^1(T)$, and correspond to the transformations of the scalar component of the **10** of SO(10) contained in the **27** of E_6.

It is a simple matter to use our previous results in Ref. 4 to work out the analogous transformation laws for the antigenerations. By the Lefshetz hyperplane theorem, seven of the 14 (1,1) forms are contributed by C_1 and the other seven by C_2. Since in this sense they are insensitive to the hyperplane, we can take over the (simultaneous x-y) scaling results of Ref. 4 making the replacement that the x-space forms are invariant under r_y and vice versa. This gives Table II. We note that the simplest way to derive these results from scratch (or for some other symmetry) is to make use of the *explicit* coordinate based representatives for the (1,1) forms (more precisely, their homological duals) found in Ref. 14. Alternatively, one can also make use of the Lefshetz fixed-point theorems as in Ref. 4.

We should emphasize at this point that the three-point antigeneration couplings are topological (up to nonperturbative sigma model effects) and hence are independent of the choice of H. Of course, many of the nonrenormalizable couplings (which we shall discuss) do depend on H and the transformation properties above are a powerful

TABLE I. Transformation properties of $H^1(R,T)$ fields under discrete R symmetries.

27 field	r_x	r_y	**27** field	r_x	r_y	**27** field	r_x	r_y
λ_1	1	1	q_1	α	1	Q_1	α	1
λ_2	α	1	q_2	1	α	Q_2	1	α
λ_3	1	1	q_3	1	1	Q_3	1	1
λ_4	1	α	q_4	1	1	Q_4	1	1
λ_5	1	1	q_5	1	α	Q_5	1	α
λ_6	1	1	q_6	1	1	Q_6	1	1
λ_7	α	α	q_7	α	1	Q_7	α	1
λ_8	1	α						
λ_9	α	1						

TABLE II. Transformation properties of $H^1(R,T^*)$ fields under discrete R symmetries.

$\overline{27}$ field	r_x	r_y	$\overline{27}$ field	r_x	r_y	$\overline{27}$ field	r_x	r_y
$\bar{\lambda}_1$	1	1	$\bar{q}_1$	α^2	1	$\bar{Q}_1$	α^2	1
$\bar{\lambda}_2$	1	1	$\bar{q}_2$	α	1	$\bar{Q}_2$	α	1
$\bar{\lambda}_3$	α	1	$\bar{q}_3$	1	α	$\bar{Q}_3$	1	α
$\bar{\lambda}_4$	1	α^2	$\bar{q}_4$	1	α^2	$\bar{Q}_4$	1	α^2
$\bar{\lambda}_5$	α^2	1						
$\bar{\lambda}_6$	1	α						

analytic tool for dealing with them. We will use this information in the last section to extract some phenomenological properties of this compactification. For now we consider the second construction of the three-generation manifold.

III. THE SECOND REALIZATION

The second realization of the Tian-Yau manifold, K, is presented as the second example in the Appendix of Ref. 3. This construction of the three-generation manifold has received little attention in the string literature for two reasons. First, although it was initially thought[3] that this manifold was a distinct from the first three-generation example in Ref. 3, it was later shown that the two examples are actually diffeomorphic.[10] Second, the construction is technically more difficult than the first (involving passing through a singular manifold which one can then resolve) apparently making model building a more formidable task. The sets of models, though, realizable on each manifold are almost certainly not identical as, for example, it may not even be possible to smoothly deform the complex structure of R to K. Furthermore, we shall see below that the technical obstacles are readily overcome.

The construction of K (Ref. 3) begins with the simply connected $h^{2,1}=83$, $h^{1,1}=2$ codimension one variety, K_0, in $CP^2\times CP^2$ defined by a bidegree (3,3) homogeneous equation, say, $\sum x_i^3y_i^3+\delta(\sum(x_i^3y_{i+1}^3+x_i^3y_{i-1}^3))=0$ for almost any choice of complex parameter δ. The x's and y's here are homogeneous $CP^2\times CP^2$ coordinates. We then quotient this manifold by the *nonfreely* acting group of order 27 generated by

$$\sigma_1:(x_1,x_2,x_3,y_1,y_2,y_3)\to(x_2,x_3,x_1,y_2,y_3,y_1)\ ,$$

$$\sigma_2:(x_1,x_2,x_3,y_1,y_2,y_3)\to(x_1,\alpha x_2,\alpha^2x_3,y_1,\alpha y_2,\alpha^2y_3)\ ,$$

$$\sigma_3:(x_1,x_2,x_3,y_1,y_2,y_3)\to(x_1,x_2,x_3,y_1,\alpha y_2,\alpha^2y_3)$$

to arrive at a three-generation manifold, K. The latter manifold has[10] fundamental group Z_3 and $h^{1,1}=6$ and $h^{2,1}=9$. In this construction it is important to bear in mind that the fixed-point sets are the six tori, $[x_1^3+\delta(x_2^3+x_3^3)=0]\times[1,0,0]$ and $[1,0,0]\times[y_1^3+\delta(y_2^3+y_3^3)=0]$ (along with the other four obtained by cyclic permutation on the coordinates) and hence their resolution does not change the Euler characteristic of the quotient space (but does change the individual Hodge numbers). For more details see Ref. 10.

We now determine the transformation properties of all of the modes in this compactification scheme under the (covering space) symmetry group D which is generated by $(x_i,y_i)\to(x_{p(i)},y_{p(i)})$ where p is a permutation symmetry, $S_i^x:x_i\to\alpha x_i$, $S_i^y:y_i\to\alpha y_i$, $J:(x_i,y_i)\to(y_i,x_i)$.

On K_0, there are *a priori* 100 monomial deformations of the complex structure which arise as bidegree (3,3) monomials in x and y coordinates (the ten cubic x-space monomials times the ten in the y coordinates). Using the equivalence[12] $q\sim q+C^A\partial_AP+\delta P$ where C^A is a linear function, we can eliminate 17 of the 100 combinations leaving us with the desired 83 representatives of $H^{2,1}$. The two (1,1) forms are the pullbacks of each of the Kahler forms on the ambient CP^2 spaces. The latter are invariant under all symmetries (except the Z_2 symmetry J under which they form a regular representation), while the transformations of the (2,1) forms are directly determined from this explicit coordinate representation. For any choice of flux breaking it is standard to determine which modes descend to K.

We now turn to the blow-up modes. Recall from Ref. 10 that we resolve the singular tori by extracting $T^2\times B_4$ (where B_4 is the four ball) and gluing back $T^2\times Q^4$ where Q^4 is the plum product of the two disc bundles each over S^2 with first Chern number -2. Since Q^4 deformation retracts to $S^2\wedge S^2$ (two spheres touching at a point) it is straightforward to determine that we get additional cohomology arising from $H^{1,1}(T^2\times(S^2\wedge S^2))$ and $H^{2,1}(T^2\times(S^2\wedge S^2))$. Each of these contributes two elements;[10] these arise from $H^{0,0}(T^2)\times H^{1,1}(S^2\wedge S^2)$ and $H^{1,0}(T^2)\times H^{1,1}(S^2\wedge S^2)$. With this explicit identification of the origin of the blow-up modes, we can determine how these fields transform under the action of the discrete symmetry group. We first consider the (1,1) forms. The scaling symmetries do not mix the tori, and act trivially on $H^{0,0}(T^2)$. The two (1,1) forms arising from $S^2\wedge S^2$ (which is simply two spheres touching at a point) may be identified with the Kahler forms on these spaces, which again are invariant under the scalings. The permutation symmetries are a little more difficult to deal with as they interchange the singular tori. The most straightforward way to deal with these symmetries is to use the cyclic Z_3 symmetry to label the forms in the following way. Let the ith x-space torus be the one with $y_i=1$, and let the two (1,1) forms associated with this torus be denoted $a_1^{(i)}$ and $a_2^{(i)}$. We choose the assignment of the subscripts so that the cyclic Z_3 action induces the transformation $a_1^{(i)}\to a_1^{(i+1)}$ and $a_2^{(i)}\to a_2^{(i+1)}$. The remaining generator of the S_3 symmetry is a Z_2 coordinate swap, say

$$P_{12}:(x_1,x_2,x_3,y_1,y_2,y_3)\to(x_2,x_1,x_3,y_2,y_1,y_3)\ .$$

This map interchanges the first and second tori (in each CP^2) and maps the third onto itself. The local normal bundle coordinates of the third tori are interchanged by this action thus implying that the plumbed S^2's are interchanged as well. The first two tori are in a similar situation except that in addition to the S^2's interchanging, the tori do so as well. With this analysis, the transformation properties of the (1,1) forms are readily determined (and shall be given momentarily).

A similar analysis applies to the (2,1) forms except that

TABLE III. Transformation properties of (2,1) blow-up modes on K. Blow-up (1,1) modes transform identically except for being invariant under the scaling symmetries.

27 field	σ_1	P_{12}	S_i^x	S_i^y	J
L_1	L_3	L_4	α	1	L_7
L_2	L_4	L_3	α	1	L_8
L_3	L_5	L_2	α	1	L_9
L_4	L_6	L_1	α	1	L_{10}
L_5	L_1	L_6	α	1	L_{11}
L_6	L_2	L_5	α	1	L_{12}
L_7	L_9	L_{10}	1	α	L_1
L_8	L_{10}	L_9	1	α	L_2
L_9	L_{11}	L_8	1	α	L_3
L_{10}	L_{12}	L_7	1	α	L_4
L_{11}	L_7	L_{12}	1	α	L_5
L_{12}	L_8	L_{11}	1	α	L_6

we must also take into account the nontrivial transformation properties of the one-form on the torus. Including this effect gives the list of transformation properties in Table III. In this table, for ease of presentation we use the following notation: the two (2,1) forms on the ith x-space torus are labeled L_{2i-1} and L_{2i}; similarly for the y-space forms with the indices increased by 6.

The transformation properties of the 12 blow-up (1,1) modes are identical except for the fact derived above that they are all invariant under the scaling symmetries $S_i^{x,y}$. We thus see that much of the information required for model building in this example can be extracted without much effort.[15] Of course, we would ideally determine the values of the renormalizable Yukawa couplings. For the (1,1) forms these (tree-level) couplings are topological and hence the results of Ref. 14 apply here (after a suitable change of basis). The (2,1) couplings offer a more difficult obstacle as we cannot immediately apply the monomial manipulation method of Ref. 12 to the blow-up modes. Nevertheless, much can be said about the structure of low-energy phenomenology without the precise knowledge of the values of all of the renormalizable couplings.

IV. PHENOMENOLOGY

We now briefly discuss some of the new phenomenological characteristics of the three-generation manifolds we have been discussing.

We first review the discussion of Ref. 9 regarding the strong restrictions which R symmetries place on couplings in the effective Lagrangian. Let r be a holomorphic automorphism of a chosen Calabi-Yau compactification which maps $\omega \rightarrow \gamma^{-1}\omega$ where γ is some complex phase. By a redefining the lift of the symmetry r to the vacuum gauge bundle, the symmetry r can be chosen to act on all components of 27's and $\overline{27}$'s of E_6 according to $27 \rightarrow \gamma^{-1/3}\beta 27$, $\overline{27} \rightarrow \gamma^{1/3}\beta'\overline{27}$ where β (β') are the transformation matrices for the corresponding Calabi-Yau moduli. With these redefinitions, any superpotential term must transform by a factor of γ. Terms of the form $\overline{27}^3$, for example, can certainly appear as long as the wedge product of the corresponding (1,1) forms is invariant under the action of the R symmetry. This is precisely as expected, since the topological formula[16] for the $\overline{27}$'s is simply the integral of the wedge product of the three corresponding (1,1) forms over the manifold of compactification. Similarly, 27^3 couplings can occur if the product of the corresponding monomial representatives transforms by a factor of γ^2, which is the familiar condition originally shown in Ref. 11. More generally, terms of the form $27_1 \cdots 27_m \overline{27}_1 \cdots \overline{27}_{m+3k}$ can occur so long as $\beta_1 \cdots \beta_m \beta'_1 \cdots \beta'_{m+3k}$ equals γ^{1-k}. An R symmetry can, for example, leave many fields invariant. We see, therefore, that couplings between such fields with $k=0$ (in contrast with the situation for non-R symmetries) are then forbidden by the symmetry.

For the manifold R, we were able to choose the complex structure so as to respect the two R symmetries r_x, r_y. It is straightforward to determine which three-point couplings amongst the generations are allowed by these R symmetries. The product of the monomial representatives for such a nonzero coupling must be invariant under all of the symmetries except r_x and r_y under which it must transform by a factor of α. As compared with the usual choices of the complex structure[4] which have no R symmetries, the pattern of allowed couplings is substantially altered. For example, only about half as many three-point couplings are nonzero as compared with the standard choice (more precisely, there are 42 nonzero three-point generation couplings as compared with 85 nonzero couplings for the standard choice[17,18]). Furthermore, it is a simple matter to determine the values of these couplings using the method of Ref. 12. For the case at hand this amounts to a straightforward generalization of the results of Refs. 17 and 18.

Only a detailed study of the low-energy model emerging from this compactification, which we shall present elsewhere, will allow us to pass conclusive judgment on the prospects for acceptable phenomenology. However, we would like to emphasize one particular phenomenological application of these R symmetries.

We have found that modifying the choice of H from that made in Ref. 4 to that described in Sec. II has a pronounced effect on the three-point couplings; the potential restrictiveness of R symmetries, however, is even more apparent when we consider nonrenormalizable superpotential terms.[9] In particular, consider the nonrenormalizable superpotential terms which can lift an F-flat direction in field space. These are couplings of the form $A^p(27\overline{27})^n$ where A is a generic E_6-singlet field and $p=0$ or 1. Now, if one can find, say, a 27 and a $\overline{27}$ such that their product is invariant under an R symmetry, then all $(27\overline{27})^n$ terms are forbidden and hence the only way a flatness spoiling superpotential term can arise is if the A field transforms under this R symmetry by a factor of γ. If there are no such A fields we can deform the theory along this flat direction to a (2,0) model. In Ref. 9 this method was used to deform the standard (2,2) theory on the quintic hypersurface in CP^4, $Y_{4,5}$, to a (2,0) theory.

It was argued some time ago[8] that (2,0) theories offer a generically more promising starting point for realistic low-energy phenomenology. Although a number of such

(2,0) theories have recently been constructed[19] [which are not deformations of (2,2) theories], none have a realistic number of generations. The satisfying aspect of (2,0) theories which are deformations of (2,2) theories (on complete intersection Calabi-Yau spaces) is that, since the number of generations is preserved by the deformation, the (class of) manifolds we have been discussing are the unique (2,2) starting point for deforming to a three-generation (2,0) theory.

As we now see, the R symmetries do protect such flat directions for the manifold R, up to our incomplete understanding of all of the modes arising from $H^1(\mathrm{End}T)$. A brief glance at Tables I and II shows that any product of $\lambda_1,\lambda_3,\lambda_5,\lambda_6$ with $\overline{\lambda_1},\overline{\lambda_2}$ is invariant under r_x and r_y. Furthermore, $\lambda_2\overline{\lambda_5},\lambda_4\overline{\lambda_6},\lambda_8\overline{\lambda_6},\lambda_9\overline{\lambda_5}$ are also invariant. Are there any gauge singlets which can lift these flat directions? Amongst the complex structure and Kahler moduli, the answer is no, as can again be seen using Tables I and II. This leaves the singlets coming from $H^1(\mathrm{End}T)$. Unfortunately, although this cohomology group is well understood and calculable for some Calabi-Yau manifolds,[20] no one has succeeded in computing it for the three-generation manifold. Using deformation theory[21,20] we can construct 17 representatives but there is no reason to believe that this exhausts $H^1(\mathrm{End}T)$. Any such $H^1(\mathrm{End}T)$ singlet accessible by deformation theory transforms such as a bidegree (3,0), (0,3), or (1,1) monomial and hence cannot lift our flat directions. Thus, up to our ignorance of the full structure of $H^1(\mathrm{End}T)$ we have exact flat directions. If we deform along one we get an SO(10) model; if we deform along two independent ones (which are mutually flat) we get a three-generation (2,0) model with an SU(5) gauge group [SU(3) $\times$SU(2)$\times$U(1) with the flux breaking described earlier] at the compactification scale. Furthermore, it is easily seen that deforming along the flat directions above gives mass to a number of the excess generation-antigeneration modes while necessarily leaving light leptons which can act as Higgs doublets. This compactification scale theory is thus a more promising starting point than that of Ref. 4. A detailed study is required in order to judge the low-energy prospects for this theory. [For example, can we give mass to all of the excess modes through further vacuum expectation values (VEV's) while keeping the Higgs leptons light?]

Even if future work shows that the End$T$ singlets do lift these flat directions, these compactifications have the potential to lead to viable low-energy models. [We note that the model in Ref. 4 has also been constructed subject to this incomplete understanding of $H^1(\mathrm{End}T)$.] In particular we note that, as discussed in Refs. 4 and 22, an essential ingredient in obtaining viable low-energy models is the presence of a matter parity amongst the discrete and/or gauge symmetries which prevents catastrophic dimension-four baryon- and lepton-number-violating processes. In Ref. 4 a Z_2 matter parity was found. Our choice of H does not respect this Z_2 map so we must look for such a matter parity amongst our other symmetries. Happily, we can directly take over the work of Ref. 22 in which it was noted that the Z_3 symmetry B mapping $x_2\to\alpha x_2$ and $y_2\to\alpha^2 y_2$ (which exists for both the standard choice of H and the one studied here) also provides a matter parity and hence R meets the first important test for yielding a viable model.

The manifold K also gives rise to some interesting phenomenological features. R symmetries, for example, abound in this realization. Writing the holomorphic three-form as

$$\omega=\oint\frac{\epsilon_{ijk}x_i dx_j\wedge dx_k\epsilon_{pqr}y_p dy_q\wedge dy_r}{K_0} \tag{4.1}$$

we see therefore that the scaling symmetries $\prod(S_i^x)^{n_i}(S_j^y)^{m_j}$ are R symmetries unless $\sum(n_i+m_i)=0(\mathrm{mod}3)$. It is again a straightforward exercise to work out the nonzero Yukawa couplings; these will be used elsewhere in a detailed study of this model. Here, as in the previous example, we focus on potential exact flat directions in the superpotential. To do so we focus on a particular $27,\overline{27}$ pair; for the latter we choose the restriction to K_0 of one of the CP^2 Kahler forms, which we shall denote by $\Omega_{\overline{27}}$ and for the former we take the 27 whose polynomial representative is $(x_1^3+\alpha x_2^3+\alpha^2 x_3^3)(y_1^3+\alpha^2 y_2^3+\alpha y_3^3)$, which we shall denote by Φ_{27}. Under the scaling R symmetries, it is easy to see that $\Omega_{27}\Omega_{\overline{27}}\to\Phi_{27}\Omega_{\overline{27}}$. We are thus in the situation described in the previous section: no $(\Phi_{27}\Omega_{\overline{27}})^n$ terms can appear in the superpotential. We now need to see if there is a gauge singlet field A which can spoil this would-be flat direction. The two moduli associated with harmonic (1,1) forms certainly do not transform by a factor of γ under the R symmetry. This leaves the complex structure moduli and the fields from $H^1(\mathrm{End}T)$. To deal with the former, we note that they transform like bidegree (3,3) polynomials and hence cannot spoil the flat direction for which they would have to transform under scalings such as $(x_1x_2x_3y_1y_2y_3)^2$. Finally, to deal with the End$T$ fields, we recall that in Ref. 20 it was shown that $h^1(\mathrm{End}T)=176$. Of these,[20] 160 can be reached by the deformation algorithm of Ref. 21, that is, by tensors of the form p_{abcdef} which are symmetric on the last three indices as well as on the second and third indices, but which vanish if symmetrized on the first three. Such tensors transform under scalings such as the monomial $x_a x_b x_c y_d y_e y_f$, and hence none of these 160 fields transforms in the appropriate way so as to spoil the flat direction. The remaining 16 elements of $H^1(\mathrm{End}T)$ come from $H^1(\mathrm{CP}^2\times\mathrm{CP}^2,\mathrm{End}T_{\mathrm{CP}^2\times\mathrm{CP}^2})$. These in turn come from the eight elements in $H^1(\mathrm{CP}^2,T^*_{\mathrm{CP}^2})\times H^0(\mathrm{CP}^2,T_{\mathrm{CP}^2})$ and the eight elements in $H^0(\mathrm{CP}^2,T_{\mathrm{CP}^2})\times H^1(\mathrm{CP}^2,T^*_{\mathrm{CP}^2})$. It is straightforward to see that none of these 16 fields transforms in a manner which permits a dangerous coupling. Having exhausted the possibilities we see that we can deform the (2,2) E_6 compactification on K_0 into an SO(10) (2,0) compactification. Our real interest, of course, lies not in constructing a (2,0) model on K_0 but rather on K. The fields $\Omega_{\overline{27}}$ and Φ_{27} were chosen so as to survive the passage from K_0 to K, but we must ensure that the new fields associated with blowing up the singularities do not spoil the flat direction. To accomplish this, we make use of the R symmetries on K. In passing from K_0 to K, the subgroup of the full discrete symmetry group D of K_0 which lies in the normalizer of G survives as the discrete symmetry of K (Ref. 11). It is easy to see that the scaling symmetries satisfy this criterion. (Of course, they are no longer all independent on K.) It is now easy to see, using Table III, that none of the new moduli associated with blowing up the singularities in this construction spoil the flat direction we have discussed above. [In checking this, it is important to bear in mind that the complex structure moduli transform like $\overline{\omega}$ times the corresponding (2,1) forms.] We are not quite done since one expects new singlet mode contributions to $H^1(\mathrm{End}T)$ from blowing up the tori as well. Determining the number of such modes and their transformation properties seems to be a difficult computation[23] so we are unable to check if they can lift the flat directions. Thus, modulo this uncertainty in the blow-up End$T$ modes, we can deform this example to a three-generation SO(10) (2,0) model. In fact, we can go further. There is another $\overline{\mathbf{27}}$ field on K_0 which transforms in precisely the same way as $\Omega_{\overline{27}}$, namely, the restriction to K_0 of the other CP^2 Kahler form. There is also another **27** field with essentially the same properties as Φ_{27}; the **27** whose polynomial representative is $(x_1^3+\alpha^2 x_2^3+\alpha x_3^3)(y_1^3+\alpha y_2^3+\alpha^2 y_3^3)$. Following the same line of reasoning, it is straightforward to see that these two fields provide another exact flat direction. Deforming along this direction as well gives us a three-generation SU(5) model.

Finally, we would like to briefly mention some interesting possibilities for introducing Wilson lines into compactification on K. The fundamental group of K, after resolving the toroidal singularities, is Z_3 (Ref. 10) associated with freely acting symmetry σ_1. We can use σ_1 to flux break E_6 to $SU(3)^3$ in precisely the same way we used g to flux break in the case of R. Using the polynomial representatives and the blow-up mode transformation properties, one sees that the resulting spectrum consists of precisely that found on R: three generations of leptons, quarks, and antiquarks plus additional generation and/or mirror generation pairs totaling six for the leptons and four for the quarks and antiquarks. In constructing K, though, we made use of three Z_3's; we can equally well modify these symmetries to act nontrivially on the gauge degrees of freedom. (As always we must ensure modular invariance via level matching—this is simple to check for the cases we consider.[24]) Although certainly consistent as conformal theories, the spectrum for these Calabi-Yau orbifolds with flux breaking is somewhat delicate to extract. If we try do so geometrically by resolution, we not only have to blow up the manifold singularities but also deal with a bundle (more precisely, a sheaf) whose nontrivial holonomy is concentrated at these singularities. Under the assumption that the nontriviality of this bundle after resolution does not contribute any additional cohomology, we find some models with phenomenologically attractive spectra. For an example, if we embed σ_2 as we did earlier for σ_1 we find that no mirror quarks survive, in contrast with previous models. Potentially, this is a substantial improvement over previous models which are burdened by an excess of particular–mirror-particle pairs that must gain large masses to avoid spoiling the low-energy phenomenology. We emphasize that we have no rigorous justification for our simplifying assumption about the cohomology of the resolved bundle (nor any justification that such a resolution exists) and hence our conclusions regarding the resulting spectra are tentative. The most appropriate forum for extracting the spectrum from these geometrically subtle compactifications is in the exactly soluable minimal model counterpart[25] which exists for a close relative[26] of the manifold considered here.[27,28] It would be interesting to resolve this question and work out the phenomenology of these special points in three-generation moduli space.

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