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Heavy-Higgs-boson effects in the triplet-Majoron model

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An effective-Lagrangian approach is used to explore the effects of a strongly interacting longitudinal gauge-boson sector within the framework of the triplet-Majoron model of Gelmini and Roncadelli (GR). The leading one-loop effects in such a limit are obtained using a covariant derivative expansion method to secure the manifestly invariant one-loop effective action. Comparison is made with the minimal effects present in the strongly interacting limit of the standard model. Additional effects including enhancements to the production of the charged scalars of the GR model are found to be present. In particular it is found that the model's doubly charged scalar could be searched for at a hadron collider capable of producing center-of-mass energies of order $\sqrt{s}=40$ TeV and an integrated luminosity of order 10^{40} cm^{-2}.

I. INTRODUCTION

The introduction of the minimal Higgs sector and, in particular, the scalar Higgs boson into the standard model provides the mechanism by which that model is made renormalizable.[1] However, the details of this sector remain experimentally unverified. To study its impact on the known lower-energy phenomenology it is possible to use a nonrenormalizable effective model. The radiative corrections to such a model then contain terms which, in general, diverge according to the scale at which the new interactions become important. Thus, these divergences determine the generic manner in which the new physics must enter the theory, whether that new physics be a simple heavy scalar as in the Higgs sector of the standard model or other new matter, and gauge interactions as in technicolor models.

One may then ask what happens if the Higgs sector is altered so that the resulting theory is nonrenormalizable. One method of performing this alteration is to allow the minimal Higgs doublet to become strongly self-interacting. That is, we allow the self-coupling constant of the Higgs doublet to become large. After the usual spontaneous symmetry breaking, this complex doublet corresponds to a triplet of pseudoscalar Goldstone bosons and a massive Higgs scalar. Thus in this limit the Goldstone particles, which, in conjunction with gauged $SU(2)\times U(1)$ interactions, supply the longitudinal components of the gauge bosons, become strongly interacting. Additionally, the mass squared of the Higgs scalar, m_H^2, being linear in the coupling constant, also grows large and ultimately begins to diverge. In this way m_H behaves as a regulator for the theory. If the limit is formally taken to infinity, then the Higgs scalar boson actually disappears from the theory as an independent field, its mass surviving merely as the scale at which new interactions must become important. The theory then loses renormalizability and in fact be represented by a nonlinear σ model.

Spurred by the fact that the Higgs scalar has not yet been found, considerable attention has been given to isolating the most easily visible signatures of a strongly interacting longitudinal gauge-boson system within the framework of the standard model.[2] However, there is no stricture which requires the use of the minimal Higgs sector for such a program. It is possible that more complex Higgs structures could yield more readily observable results in these strongly interacting limits. The model of Gelmini and Roncadelli,[3] constructed independently by Georgi, Glashow, and Nussinov,[4] is an extension of the standard model which has an unusually rich Higgs-boson structure. Here the Higgs sector is enlarged to include a complex scalar triplet as well as the usual complex doublet. The triplet transforms under the adjoint representation of $SU(2)_L$ and thus has invariant couplings to the usual Higgs doublet as well as to the left-handed components of fermions and leptons. We thus have a new source for interactions involving the longitudinal gauge bosons. It is the purpose of this paper to examine the strongly interacting limit in the Gelmini and Roncadelli (GR) model, extracting those effects which depend most strongly on the scale m_H and on those parameters unique to the GR framework.

In Sec. II of this paper we briefly review the effective strongly interacting limit of the standard model and develop general criteria for selecting the low-energy observables most sensitive to the effects. We then sketch the calculation of the leading one-loop corrections to these observables in the effective model using techniques developed by Honerkamp[5] based on the work of Coleman and co-workers.[6] In Sec. III we apply these methods to the GR model comparing the results with those obtained in Sec. II and additionally finding significant enhancement to the production of charged scalar particles. In particular, we find that these charged scalars could be produced in pairs via gauge-boson fusion. Such processes could be looked for at a hadron collider producing center-of-mass energies of order $\sqrt{s}=40$ TeV and an integrated luminosity of order 10^{40} cm^{-2}. The reader already familiar with the formalism of Sec. II may proceed directly to Sec. III without great loss of continuity.

II. HEAVY-HIGGS-BOSON EFFECTS IN THE STANDARD MODEL

The Higgs sector of the standard model may be represented by the Lagrangian

$$\mathcal{L}=D^{\mu}\Phi_D^{\dagger}D_{\mu}\Phi_D-V(\Phi_D)+\mathcal{L}_{\rm Yuk}\ , \tag{2.1}$$

where Φ_D is the usual Higgs doublet (the subscript D is a mnemonic for doublet)

$$\Phi_D=\begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix}\equiv\frac{1}{\sqrt{2}}\begin{bmatrix}\phi_2+i\phi_1\\ \sigma-i\phi_3\end{bmatrix}\ . \tag{2.2}$$

D_μ is the $SU(2)_L\times U(1)_Y$-covariant derivative

$$D_\mu\Phi_D=\left[\partial_\mu-ig\,\mathbf{A}_\mu\cdot\frac{\tau}{2}=-ig'B_\mu\right]\Phi_D \tag{2.3}$$

with τ representing the Pauli matrices. The standard Yukawa coupling to fermions may be given in the convenient form

$$\mathcal{L}_{\rm Yuk}=-\frac{1}{2}\overline{\Psi}_L\left[\sigma+i\frac{\tau}{2}\cdot\boldsymbol{\phi}\right](\Gamma+\Gamma_3\tau_3)\Psi_R+\text{H.c.}\ , \tag{2.4}$$

where, for instance, if Ψ_L is an up-down quark pair then the coupling constants are

$$\Gamma=\Gamma_u+\Gamma_d\ \text{ and }\ \Gamma_3=\Gamma_u-\Gamma_d\ . \tag{2.5}$$

Finally, the potential may be written in the form

$$V(\Phi_D)=\lambda\left[\Phi^{\dagger}\Phi-\tfrac{1}{2}v_D^2\right]^2\ , \tag{2.6}$$

where v_D, analagous to the f_π of pion physics, is the dimensionful decay constant associated with the $\boldsymbol{\phi}$; $v_D\approx 250$ GeV.

The ungauged kinetic terms and the potential are invariant under the $SU(2)_L\times SU(2)$ transformation

$$\sigma+i\frac{\tau}{2}\cdot\boldsymbol{\phi}\rightarrow e^{i\varepsilon_L\cdot\tau/2}\left[\sigma+i\frac{\tau}{2}\cdot\boldsymbol{\phi}\right]e^{-i\varepsilon_R\tau/2}\ , \tag{2.7}$$

where $\tau/2$ are the generators for the two SU(2) groups. The asymmetrical minimum of the potential (2.6) induces the spontaneous breaking of this extra symmetry to the vector SU(2) subgroup, resulting in a Higgs scalar σ with mass squared $m_H^2=2\lambda v_D^2$ and three Nambu-Goldstone fields associated with the three axial, or broken, SU(2) generators. In the presence of the gauge interactions these would-be Goldstone fields

$$w_L^{\pm}=\frac{1}{\sqrt{2}}(\phi_2\pm i\phi_1),\quad z_L=\phi_3 \tag{2.8}$$

become the longitudinal components of the vector gauge bosons $W_\mu^{\pm}$, Z_μ.

It is the custodial vector symmetry mentioned above that ensures the proper value for the mass relation between the gauge bosons

$$\rho\equiv\frac{m_W^2}{m_Z^2\cos^2\theta_W}=1\ , \tag{2.9}$$

where $\theta_W=\arctan(g'/g)$ is the Weinberg angle. We will denote such relations as ρ, which are determined at the tree level solely by the group structure of the theory, by the term natural relations. To probe the theory's structure further in terms of natural relations it is necessary to look at radiative corrections. These corrections must come from terms which do not respect the custodial symmetry responsible for the tree value. Hence, to continue our example, we could look for corrections to ρ from the gauge and Yukawa interactions, neither of which share the left-right symmetry of the rest of the Higgs sector.[7-9]

We therefore have two main sources for observable signatures in the large-λ limit. The first comes from corrections to natural relations which grow with large λ; as above the sources for these effects are those terms, principally the gauge and Yukawa interactions, which do not share the larger symmetries of the rest of the Lagrangian. The other principal source, rather obviously, comes from enhancements to the scattering of longitudinal gauge bosons, due to the fact that the self-coupling of those components is becoming large.

To study these effects it is expedient to take the formal limit $\lambda\rightarrow\infty$ at the outset. This constrains the fields to lie identically at the minimum of the potential (2.6), and if the limit is taken in such a way as to preserve the symmetries of the Lagrangian, we must have

$$\sigma=(v_D^2-\boldsymbol{\phi}^2)^{1/2}\ . \tag{2.10}$$

We thus obtain a gauged nonlinear σ model, and the theory is determined entirely by the Goldstone fields associated with the broken, or coset, generators of the custodial symmetry. This effective model may then be used to calculate the leading one-loop corrections in either model for the strongly interacting (heavy-Higgs-boson) limit.[10]

The identification (2.10) forces the Goldstone fields to transform nonlinearly. Using the transformation law (2.7) above we see that the variations of the fields may be written in terms of Killing vectors $A_\alpha^i(\phi)$ as

$$\delta\phi^i=\varepsilon_V^a A_a^i(\phi)+\varepsilon_A^k A_k^i(\phi)\equiv\varepsilon^\alpha A_\alpha^i(\phi)\ , \tag{2.11}$$

where

$$A_a^i(\phi)=\epsilon_{aij}\phi^j,\quad A_j^i(\phi)=\delta^{ij}(v_D^2-\boldsymbol{\phi}^2)^{1/2}\ . \tag{2.12}$$

Here $\varepsilon_V=\varepsilon_R+\varepsilon_L$ and $\varepsilon_A=\varepsilon_R-\varepsilon_L$. Latin subscripts near the beginning of the alphabet are used to denote quantities pertaining to the vector subgroup to which the theory is broken and letters near the middle of the alphabet for those which pertain to the axial coset space under which the whole theory is not invariant. Greek letters are used for indices which run over all group values.

These nonlinear transformations are compensated for in the kinetic terms of the Lagrangian by the presence of a function

$$g_{ij}(\phi)=\delta_{ij}+\frac{\phi^i\phi^j}{v_D^2-\boldsymbol{\phi}^2} \tag{2.13}$$

which operates as a covariant metric defining a curved-space manifold in terms of the fields ϕ. This function may be given in terms of the Killing vectors as the inverse of the contravariant metric

$$g^{ij}(\phi)\equiv\frac{1}{v_D^2}A^i_{\alpha}(\phi)A^j_{\alpha}(\phi) \tag{2.14}$$

which satisfies the defining equation

$$g^{ij}{}_{,k}A^k_{\alpha}-g^{ik}A^j_{\alpha,k}-g^{jk}A^i_{\alpha,k}=0\ . \tag{2.15}$$

A comma is used to denote differentiation by the fields $g_{,k}=\partial g/\partial\phi^k$. The fact that the Goldstone fields may be taken as the coordinates for the coset manifold is due to Coleman, Wess, and Zumino.[6] Essentially this allows the group transformations (2.11) to be represented as general coordinate transformations $\phi^i\rightarrow\phi'^i(\phi)$. Under these reparametrizations the variations of the coordinates must transform as contravariant vectors. In particular

$$\partial_\mu\phi^i\rightarrow\partial_\mu\phi^{i'}=\frac{\partial\phi^{i'}}{\partial\phi^k}\partial_\mu\phi^k \tag{2.16}$$

and similarly for the covariant metric

$$g_{ij}\rightarrow g'_{ij}=\frac{\partial\phi^l}{\partial\phi^{i'}}\frac{\partial\phi^k}{\partial\phi^{j'}}g_{lk}\ . \tag{2.17}$$

The most general Lagrangian invariant under these transformations with at most two space-time derivatives may therefore be written in the form

$$\mathcal{L}=\tfrac{1}{2}\partial^\mu\phi^i g_{ij}\partial_\mu\phi^j-v_j^\mu(x,\phi)\partial_\mu\phi^j+u(x,\phi)\ , \tag{2.18}$$

where we have explicitly allowed for external vector and scalar sources, $v_j^\mu(x,\phi)$ and $u(x,\phi)$, respectively. By comparison with (2.1) we see that these sources are

$$\begin{aligned} v_k^\mu(x;\phi)&=g_{kj}A^j_{\alpha}V^\mu_{\alpha}\ ,\\ u(x;\phi)&=\tfrac{1}{2}V^\mu_{\alpha}m_{\alpha\beta}V_{\beta\mu}+\mathcal{L}_{\rm Yuk}\ , \end{aligned} \tag{2.19}$$

where

$$V_{b\mu}=\begin{cases}\frac{g}{2}A_{1\mu}\equiv\frac{e}{2\sqrt{2}\sin\theta_W}(W^+_\mu+W^-_\mu),\\ \frac{g}{2}A_{2\mu}\equiv\frac{ie}{2\sqrt{2}\sin\theta_W}(W^+_\mu-W^-_\mu)\ ,\\ \frac{g}{2}A_{3\mu}+\frac{g'}{2}B_\mu\equiv-e(A_\mu+\cot2\theta_W Z^0_\mu)\ ,\end{cases}$$
$$V_{k\mu}=\begin{cases}\frac{g}{2}A_{1\mu}\equiv\frac{e}{2\sqrt{2}\sin\theta_W}(W^+_\mu+W^-_\mu),\\ \frac{g}{2}A_{2\mu}\equiv\frac{ie}{2\sqrt{2}\sin\theta_W}(W^+_\mu-W^-_\mu),\\ \frac{g}{2}A_{3\mu}-\frac{g'}{2}B_\mu\equiv\frac{e}{\sin2\theta_W}Z^0_\mu\ .\end{cases} \tag{2.20}$$

The function

$$m_{\alpha\beta}=\frac{1}{v_D^2}A^i_{\alpha}g_{ij}A^j_{\beta} \tag{2.21}$$

plays the geometrical role of a projector onto the coset space

$$m_{\alpha\beta}A^j_{\beta}=A^j_{\alpha},\ \text{etc.}\ , \tag{2.22}$$

which invariantly couples the Goldstone-boson currents to the vector-boson fields on the coset manifold. The formalism used above is reviewed in Ref. 11, and the interested reader is directed there for further details.

Since our group invariance has now taken the form of a generalized coordinate reparametrization invariance, the one-loop radiative corrections to the Lagrangian above may be compactly calculated in manifestly invariant form simply by applying normal coordinate methods to the perturbative expansion and evaluation of the effective action $\Gamma[\varphi]$. We obtain this from the background-field (φ^i) generating functional

$$Z[\varphi]\equiv e^{i\Gamma[\varphi]}=\int[d\phi]_{\rm inv}\exp\left[i\int d^4x\,\mathcal{L}(\varphi+\pi)\right]\ . \tag{2.23}$$

The integration above is performed over the quantum fluctuations π^i, $\phi^i=\varphi^i+\pi^i$, of the Goldstone fields in a manifestly group-invariant manner, and $[d\phi]_{\rm inv}$ is an appropriately chosen invariant volume element for the coset manifold.[12]

We develop the perturbative expansion using the methods of Honerkamp,[5] as related by Boulware and Brown.[11] Here the action is expanded about its stationary point, defined in terms of the classical background field φ^i, the solution to the Euler-Lagrange equations applied to the Lagrangian (2.18). The quantum fluctuations from the classical field values are parametrized by an affine parameter λ, defining a path on the coset manifold such that $\phi^i(x;\lambda)|_{\lambda=0}=\varphi^i(x)$ and $\phi^i(x;\lambda)|_{\lambda=1}=\phi^i(x)$. The expansion to one loop may then be accomplished by expanding the effective Lagrangian in (2.23) along the fluctuation curves, keeping the first nonzero correction to the classical value: i.e.,

$$\begin{aligned}\mathcal{L}(\phi)&=\mathcal{L}(\varphi)+\sum_{n=1}^{\infty}\frac{1}{n!}\frac{d^n}{d\lambda^n}\mathcal{L}(\phi(x;\lambda))\Big|_{\lambda=0}\\ &\approx\mathcal{L}(\varphi)+\frac{1}{2}\frac{d^2}{d\lambda^2}\mathcal{L}(\phi(x;\lambda))\Big|_{\lambda=0}\ .\end{aligned} \tag{2.24}$$

To simplify the form of the expansion and to present it in manifestly coordinate invariant form we may choose the fluctuation curves $\phi^i(x;\lambda)$ to be the geodesics of the manifolds given by

$$\frac{d^2\phi^i}{d\lambda^2}+\Gamma^i_{kl}\frac{d\phi^k}{d\lambda}\frac{d\phi^l}{d\lambda}=0\ , \tag{2.25}$$

where Γ^i_{kl} is the affine connection

$$\Gamma^i_{kl}=\tfrac{1}{2}g^{ij}(g_{lj,k}+g_{jk,l}-g_{kl,j})\ . \tag{2.26}$$

The derivatives $d/d\lambda$ in the expansion (2.24) may then, since the Lagrangian is a scalar, be replaced with covariant derivatives $\mathcal{D}_\lambda$ along the geodesic

$$\mathcal{D}_\lambda v^j = \frac{dv^j}{d\lambda} + \Gamma^j_{kl}\sigma^k v^l \tag{2.27}$$

for v^j a contravariant vector, and so forth. We have defined $\sigma^k \equiv d\phi^k/d\lambda|_{\lambda=0}$. These covariant derivatives may, in turn, be replaced by covariant space-time derivatives D_μ since

$$D_\lambda \partial_\mu \phi^k = \partial_\mu \sigma^k + \Gamma^k_{jl}\sigma^l \partial_\mu \phi^j = D_\mu \sigma^k . \tag{2.28}$$

We note that σ transforms contravariantly under the field redefinitions (2.11). The result of the expansion above may then, after a little algebra, be written in the manifestly invariant form

$$\mathcal{L}(\phi) = \mathcal{L}(\varphi) + \tfrac{1}{2}\nabla^\mu \sigma^k g_{kl}(\varphi)\nabla_\mu \sigma^l + \tfrac{1}{2}\sigma^k U_{kl}(\varphi)\sigma^l + \text{higher-order terms} . \tag{2.29}$$

We have defined

$$\begin{aligned} \nabla_\mu \sigma^k &= D_\mu \sigma^k - v^k_{\mu;l}\sigma^l , \\ U_{kl} &= u_{k;l} - v^\mu_{m;k} v^m_{\mu;l} - R_{klmn}\partial_\mu \varphi^m \partial^\mu \varphi^n \\ &\quad - \tfrac{1}{2}\partial_\mu \varphi^m (v^\mu_{m;l;k} + v^\mu_{m;k;l}) \\ &\quad - \tfrac{1}{2} R_{klmn}(\partial_\mu \varphi^m v^{\mu n} + \partial_\mu \varphi^n v^{\mu m}) . \end{aligned} \tag{2.30}$$

A semicolon is used to represent covariant differentiation by the fields:

$$u_{;k} = u_{,k} \quad \text{for } u \text{ a scalar function of } \varphi , \tag{2.31}$$

$$v_{j;k} = v_{j,k} - \Gamma^m_{jk} v_m \quad \text{for } v \text{ a covariant function of } \varphi ,$$

and so on, and we have defined the curvature tensor $R_{plmn} = g_{pk} R^k_{lmn}$ by

$$R^k_{lmn} \equiv \Gamma^k_{ln,m} - \Gamma^k_{lm,n} + \Gamma^k_{jm}\Gamma^j_{ln} - \Gamma^k_{jn}\Gamma^j_{lm} . \tag{2.32}$$

Also, since, together with the classical field φ^i, the initial tangent σ^i uniquely determines the curve on which the quantum fluctuations are parametrized, we may choose this tangent field as the variable of integration in the generating functional. Thus, to one loop, we may write

$$\begin{aligned} Z[\varphi] &= \exp\left[i \int d^4x\, \mathcal{L}(\varphi) \right] \int [d\sigma]_{\text{inv}} \exp\left[i \int d^4x \left[\tfrac{1}{2}\nabla^\mu \sigma^k g_{kl} \nabla_\mu \sigma^l + \tfrac{1}{2}\sigma^k U_{kl}(\varphi)\sigma^l \right] \right] \\ &= e^{i\Gamma^{(0)}(\varphi)} \text{Det}^{-1/2}[(\nabla^2 - U)^l_k] , \end{aligned} \tag{2.33}$$

where the subscripts and superscripts are included in the last line to indicate that the determinant is of the mixed tensor covariant in the first index and contravariant in the second, and $\Gamma^{(0)}(\varphi) = \int d^4x\, \mathcal{L}(\varphi)$ is the tree-level effective action. The one-loop effective action $\Gamma^{(1)}(\varphi)$ is accordingly given by

$$\begin{aligned} i\Gamma^{(1)}(\varphi) &= \ln \text{Det}^{-1/2}[(\nabla^2 - U)^l_k] \\ &\equiv i \int d^4x\, \mathcal{L}^{(1)}(\varphi) . \end{aligned} \tag{2.34}$$

The determinant in this expression is, of course, infinite and must be regulated using, as per the Introduction, m_H as the regulator. The leading relevant effects, i.e., those which diverge with large m_H, must then be extracted and evaluated. All this may be done using Schwinger's proper-time techniques.[13] However, the details of the calculation are not material to the construction of the theory here. Therefore, we merely present the results of the calculation, referring the interested reader to the literature.[11,14]

Hence, we identify the one-loop effective Lagrangian in the action (2.34) as

$$\begin{aligned} \mathcal{L}^{(1)}(\varphi) &= \frac{m_H^4}{64\pi^2}\delta^k_k + \frac{m_H^2}{32\pi^2} U^k_k \\ &\quad + \frac{1}{32\pi^2}\ln\frac{m_H^2}{\mu^2}\left(-\tfrac{1}{12}F^{\mu\nu k}{}_l F_{\mu\nu}{}^l{}_k + \tfrac{1}{2}U^k_l U^l_k\right) \\ &\quad + \text{finite terms} . \end{aligned} \tag{2.35}$$

The renormalization-group parameter μ is chosen to be the mass m_W of the vector bosons, and $F^{\mu\nu k}{}_l$ is a non-Abelian field-strength tensor defined as

$$\begin{aligned} F^{\mu\nu k}{}_l &= -i[\nabla^\mu, \nabla^\nu]^k_l \\ &= -iR^k{}_{lmn}\partial^\mu \varphi^m \partial^\nu \varphi^n + i(\nabla^\mu v^{\nu k}{}_{;l} - \nabla^\nu v^{\mu l}{}_{;k}) . \end{aligned} \tag{2.36}$$

The quartically divergent term merely counts the coset fields. The quadratically divergent term, using the definition (2.30), can be seen to be

$$\frac{m_H^2}{32\pi^2 v_D^2}[4\mathcal{L}(\varphi) - \mathcal{L}_{\text{Yuk}}(\varphi)] \tag{2.37}$$

and thus amounts to a rescaling of v_D and the Yukawa couplings. Under suitable normalization conditions these terms may therefore be taken to vanish. The physically observable effects, in accordance with Veltman's screening theorem,[15] are to be found in the remaining logarithmically divergent term. Although the full form of that term is rather cumbersome, it is a straightforward matter to isolate the relevant terms. To extract the mass terms, we may set the Goldstone fields to zero. Thus we note that in this model there is no contribution to the relation $\rho = 1$ arising from the quantum fluctuations of the Goldstone fields. For the contributions to interactions involving longitudinal gauge bosons, the calculations can be simplified by use of the equivalence theorem of Ref. 16, which states that, letting $\sqrt{s}$ be the center-of-mass energy for a given process involving longitudinal gauge bosons, the S-matrix element for that process is equivalent to leading order in m_W^2/s to the same process with the longitudinal gauge boson replaced by the corresponding Goldstone boson; $w_L^\pm$ or z_L. Thus, in the limit $\sqrt{s} \approx m_H \gg m_W$, the error involved in performing calculations using the Goldstone fields as interpolating fields for the full longitudinal components becomes negligible. The leading terms at these energies are those enhanced by the extra powers of momenta associated with the Goldstone fields in our expression (2.35). Isolating these we obtain

$$\mathcal{L}^{(1)}=\frac{1}{96\pi^2 v_D^4}\ln\frac{m_W^2}{m_H^2}\{-[2(\partial^\mu\varphi^i g_{ij}\partial_\mu\varphi^j)^2+(\partial^\mu\varphi^i g_{ij}\partial_\nu\varphi^j)^2] +6(\partial^\mu\varphi^i g_{ij}\partial_\mu\varphi^j)\mathcal{L}_{\rm Yuk} -3\mathcal{L}_{\rm Yuk}^2\}+\cdots \; . \tag{2.38}$$

The Yukawa terms are included primarily to dismiss them. The smallness of the traditional Yukawa coupling as well as the additional powers of the decay constant in the denominators of each term effectively suppress any potentially visible effects, thus yielding no effectual constraint on either m_H or the Yukawa couplings. The other terms contribute solely to the scattering of longitudinal gauge bosons—a negligible process in the standard model with a light-Higss-boson mass. Here, however, they can have significant effects. These terms have been evaluated by Gaillard and by Cheyette,[17] who found the invariant amplitudes (tree plus one loop) to be

$$\begin{aligned}\mathcal{A}(w_L^+w_L^-\to w_L^+w_L^-)&=-\frac{1}{v_D^2}u+\frac{1}{24\pi^2 v_D^4}\ln\frac{m_H^2}{m_W^2}(s^2+t^2+u^2)\;,\\ \mathcal{A}(w_L^+w_L^-\to z_L z_L)&=\frac{1}{v_D^2}s+\frac{1}{48\pi^2 v_D^4}\ln\frac{m_H^2}{m_W^2}(s^2+t^2+u^2)\;,\\ \mathcal{A}(z_L z_L\to z_L z_L)&=0+\frac{1}{16\pi^2 v_D^4}\ln\frac{m_H^2}{m_W^2}(s^2+t^2+u^2)\;.\end{aligned} \tag{2.39}$$

In the first amplitude we have taken u to be the square of the difference between incoming w_L^+ and outgoing w_L^- and t to be the square of the difference between incoming and outgoing w_L^+. For m_H and $\sqrt{s}$ in the TeV realm, the corrections become comparable in size with the tree values. Of special note is the amplitude for longitudinal Z^0 scattering which is negligible at tree values, but of the same size as the other channels at one loop.

III. HEAVY-HIGGS-BOSON EFFECTS IN THE GR MODEL

The reactions described in the previous section provide unambiguous signals for a strongly interacting $w_L^\pm, z_L$ sector, if fairly limited in variety. It is possible that other models of electroweak interactions offer a wider diversity of possible signatures. Further, if the longitudinal gauge bosons do prove to be strongly interacting, these signatures could provide valuable constraints on the alternate models.

Thus we consider the model of Gelmini and Roncadelli,[3,4] which provides an unusual richness of new interactions involving the longitudinal bosons considered above. Here the standard model is extended solely in the Higgs sector by the inclusion of a complex, weak hypercharge-1 triplet $\mathbf{\Phi}_T$ transforming under the adjoint representation of $SU(2)_L$. This transformation may be put in a convenient form by defining the matrix

$$\mathcal{M}=\tau\cdot\mathbf{\Phi}_T,\quad {\rm Tr}\mathcal{M}^\dagger\mathcal{M}=2|\mathbf{\Phi}_T|^2 \tag{3.1}$$

so that under an $SU(2)_L$ transformation

$$\mathcal{M}\to\mathcal{M}'=e^{i\varepsilon_L\cdot\tau/2}\mathcal{M}e^{-i\varepsilon_L\cdot\tau/2}\;. \tag{3.2}$$

The triplet may therefore have an invariant Yukawa coupling of the form

$$\mathcal{L}_{\Phi{\rm Yuk}}=-\tfrac{1}{2}\Gamma_{LL}(\overline{\Psi})_L^c\mathcal{M}\Psi_L+{\rm H.c.}\;, \tag{3.3}$$

where, for example, $\Psi_L^T=(\nu_L,e_L)$. The assignation of lepton number -2 to the triplet allows small Majorana masses to be formed for the neutrinos through the spontaneous breakdown of the lepton-number symmetry $U(1)_{\rm lept}$. Under this process the triplet develops a nonzero vacuum expectation value

$$\langle 0|{\rm Tr}\mathcal{M}^\dagger\mathcal{M}|0\rangle=v_T^2\;. \tag{3.4}$$

The decay constant v_T, by the evolution of stellar objects, is constrained to be not more than approximately 10 keV (Refs. 18 and 19). The ratio $v=v_T/v_D\lesssim 10^{-7}$ therefore becomes a measure of smallness in the theory.

Since the triplet obeys the usual Gell-Mann–Nishijima formula we may write $\mathcal{M}$ in terms of its charge eigenstates as

$$\mathcal{M}=\sqrt{2}\begin{bmatrix}\frac{1}{\sqrt{2}}\Phi^+ & \Phi^{++}\\ \Phi^0 & -\frac{1}{\sqrt{2}}\Phi^+\end{bmatrix}\;. \tag{3.5}$$

The Gelmini and Roncadelli (GR) model therefore corresponds to the addition of new charged scalar fields into the theory. Under the symmetry breaking above

$$\mathcal{M}\to\begin{bmatrix}\Phi^+ & \sqrt{2}\Phi^{++}\\ v_T+\sigma_T+i\Phi_3 & -\Phi^+\end{bmatrix}\;. \tag{3.6}$$

The same groups are gauged in the kinetic terms as in the standard model. Hence the additional breaking of the $U(1)_{\rm lept}$ symmetry gives rise to a true Goldstone boson, the Majoron χ (essentially Φ_3).

There are also allowed couplings involving only the two Higgs multiplets. The most general potential invariant under $SU(2)_L\times U(1)_Y\times U(1)_{\rm lept}$ is of the form

$$V(\Phi_D,\Phi_T)=\lambda_0(\tfrac{1}{2}\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M})+\lambda_1(\Phi_D^\dagger\Phi_D-\tfrac{1}{2}v_D^2)^2+\lambda_2(\tfrac{1}{2}\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M})^2+\lambda_3(|\Phi_D|^2(\tfrac{1}{2}\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M}-\tfrac{1}{2}v_T^2))$$
$$+\lambda_4(|\Phi_D|^2\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M}-\Phi_D^\dagger\mathcal{M}\mathcal{M}^\dagger\Phi_D)+\lambda_5((\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M})^2-\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M}\mathcal{M}^\dagger\mathcal{M})\ . \quad (3.7)$$

There is a minimization condition relating three of the new parameters

$$\lambda_0+\lambda_2 v_T^2+\tfrac{1}{2}\lambda_3 v_D^2=0\ . \quad (3.8)$$

The Lagrangian for this Higgs sector may then be written as

$$\mathcal{L}=D^\mu\Phi_T^\dagger D_\mu\Phi_T+D^\mu\Phi_D^\dagger D_\mu\Phi_D+\mathcal{L}_{\mathrm{Yuk}}+\mathcal{L}_{\Phi\mathrm{Yuk}}$$
$$-V(\Phi_D,\Phi_T)\ . \quad (3.9)$$

A traditional one-loop renormalization-group analysis of this sector[20] shows that the GR model may be "trivial" in the same sense that $\lambda\phi^4$ theory is trivial;[21] that is, the only values of the potential's coupling constants for which the theory is consistent to all energies are the trivial ones, $\lambda_i=0$. In practice, this is evidenced by the fact that the ratio of the quartic coupling constants to the U(1) gauge coupling does not tend to a finite value in the ultraviolet limit or, equivalently, to a fixed point of the renormalization-group equations.[22]

Hence, an interacting GR model would tend to encounter a number of renormalization problems, such as trapping, in which, as one of the coupling constants λ_i begins to grow large at some scale, feedback in the renormalization-group equations causes all the other coupling constants to grow large at the same scale. That is, as one term of the Lagrangian becomes strongly interacting at some scale, all the other terms would also become strongly interacting at the same scale. These problems may be simply circumvented by viewing the theory as an effective low-energy theory, valid only up to some cutoff scale where new physics enters to modify the theory. The problems of triviality may then be avoided provided the ratio λ_i/g' for $i=1,2,\ldots,5$ is well behaved over the region of the theory's applicability.[23] Such a renormalization procedure has been applied to the GR model, allowing a maximum value of 6π for any quartic coupling constant in the region of the model's applicability, and resulting in large regions of allowed coupling constants.[24]

We are thus led to an effective Lagrangian viewpoint. One may also ask what has become of the $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ symmetry which protected the natural relation $\rho=1$ in the standard model. It is violated by both the doublet and triplet gauge terms and by the Yukawa interactions, but the scale to which this violation is important is set either by v_T or by the Yukawa coupling. For example, the violation is already evident at the tree level, for the masses of the gauge bosons are given by

$$m_W^2=\tfrac{1}{4}g^2(v_D^2+2v_T^2)\ \text{and}\ m_Z^2=\tfrac{1}{4}(g^2+g'^2)(v_d^2+4v_T^2)\ , \quad (3.10)$$

and thus

$$\rho\approx 1-2v^2\neq 1 \quad (3.11)$$

even at the tree level. But, as noted, because of the stringent astrophysical constraints on v_T, there is no contradiction with the experimental value. More significantly, however, the λ_4 term in the potential directly violated this symmetry. In the strongly interacting limit we may therefore look for corrections both to natural relations and to the various interactions of the longitudinal gauge bosons arising from the term

$$V_{\lambda_4}\equiv\lambda_4(|\Phi_D|^2\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M}-\Phi_D^\dagger\mathcal{M}\mathcal{M}^\dagger\Phi_D)\ . \quad (3.12)$$

As in Sec. II we study the strongly interacting $w_L^\pm$ and z_L limit by taking the formal limit $\lambda_1\to\infty$ at the outset. This forces the nonlinear constraint

$$\Phi_D=\begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix}\equiv\frac{1}{\sqrt{2}}\begin{bmatrix}\phi_2+i\phi_1\\ (v_D^2-\phi)^{1/2}-i\phi_3\end{bmatrix} \quad (3.13)$$

and the potential becomes

$$V(\Phi_D,\Phi_T)=(\lambda_0+\tfrac{1}{2}\lambda_3 v_D^2)(\tfrac{1}{2}\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M})+\lambda_2(\tfrac{1}{2}\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M})^2$$
$$+\lambda_4\left[\frac{v_D^2}{2}\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M}-\Phi_D^\dagger\mathcal{M}\mathcal{M}^\dagger\Phi_D\right]$$
$$+\lambda_5((\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M})^2-\mathrm{Tr}\mathcal{M}^\dagger\mathcal{M}\mathcal{M}^\dagger\mathcal{M})\ . \quad (3.14)$$

The triplet breaking proceeds in the normal manner.

At the tree level the mass eigenstates and masses in this limit are given as follows: For the new scalars,

$$\chi^{++}=\Phi^{++},\quad m_{\chi^{++}}^2=\lambda_4 v_D^2+\lambda_5 v_T^2\ ,$$
$$\chi^+=\frac{1}{\sqrt{1+2v^2}}(\Phi^+-\sqrt{2}v\phi^+)=\Phi^++O(v)\ , \quad (3.15)$$
$$m_{\chi^+}^2=\tfrac{1}{2}\lambda_4(v_D^2+2v_T^2)\ .$$

The fact that χ^+ has not been observed at DESY PETRA implies $m_{\chi^+}>21$ GeV, while, for new physics entering the theory at the TeV level, the cutoff-dependent renormalization procedure discussed above leads to the constraint $m_{\chi^+}<730$ GeV (Ref. 24). More specific upper limits are in general dependent on the exact manner in which the new physics enters the theory. The bounds for $m_{\chi^{++}}$ follow from the relation $m_{\chi^{++}}\approx\sqrt{2}m_{\chi^+}$. The Higgs scalar associated with the triplet in this limit is σ_T with mass squared $m_{\sigma_T}^2=2\lambda_2 v_T^2$. The minimization criteria (3.8) has been used throughout. The massless Goldstone boson associated with the spontaneous breakdown of the $\mathrm{U}(1)_{\mathrm{lept}}$ symmetry is the Majoron

$$\chi=\frac{1}{\sqrt{1+4v^2}}(\Phi_3+2v\phi_3)=\Phi_3+O(v) \quad (3.16)$$

and the fields associated with the longitudinal components of the gauge bosons are

$$\begin{aligned} w_L^+ &= \frac{1}{\sqrt{1+2v^2}}(\phi^+ + \sqrt{2}v\Phi^+) \\ &= \phi^+ + O(v) = \frac{1}{\sqrt{2}}(\phi_2 + i\phi_1) + O(v)\ , \\ z_L &= \frac{1}{\sqrt{1+4v^2}}(\phi_3 - 2v\Phi_3) = \phi_3 + O(v)\ . \end{aligned} \tag{3.17}$$

Thus, to order v, we may disregard the mixings between the multiplets in our calculations.

The one-loop corrections to our effective model may be obtained from Eq. (2.35) by fitting the relevant terms in our Lagrangian to the general model (2.18):

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu \phi^i g_{ij} \partial_\mu \phi^j - v_j^\mu(x,\phi)\partial_\mu \phi^j + u(x,\phi)\ . \tag{2.18}$$

The only new term which contributes to the loop in the longitudinal Goldstone fields is the term mentioned above: V_{λ_4}. Therefore, to follow the procedures of the previous section, our identification for $u(x;\phi)$ must be changed to

$$u(x;\phi) = \tfrac{1}{2}V_\alpha^\mu m_{\alpha\beta} V_{\beta\mu} + \mathcal{L}_{\text{Yuk}} - V_{\lambda_4}\ . \tag{3.18}$$

The expressions for $v_j^\mu(x,\phi)$, g_{ij}, and the other quantities in Sec. II remain unchanged. Using the fact that

$$\mathcal{M}_{,i;j} = -g_{ij}\mathcal{M}\ , \tag{3.19}$$

it is straightforward to show that the quadratic and logarithmic divergences in the one-loop effective Lagrangian are

$$\begin{aligned} \mathcal{L}^{(1)}(\varphi) = &-\frac{m_H^2}{32\pi^2 v_D^2}[4\mathcal{L}_{\Phi_{D,\text{kinetic}}^{\text{gauge}}} + 3\mathcal{L}_{\text{Yuk}} \\ &\qquad - 8V_{\lambda_4} + 4\lambda_4(\tfrac{1}{2}\text{Tr}\mathcal{M}^\dagger\mathcal{M})] \\ &- \frac{1}{32\pi^2}\ln\frac{\mu^2}{m_H^2}(-\tfrac{1}{12}F^{\mu\nu k}{}_l F_{\mu\nu}{}^l{}_k + \tfrac{1}{2}U_l^k U_k^l) \\ &+ \text{finite terms}\ . \end{aligned} \tag{3.20}$$

Again the quadratic term is merely a rescaling of the parameters in the low-energy effective model. In particular the decay constant v_D, the Yukawa couplings Γ and Γ_3, and the two parameters λ_4 and λ_3 each undergo renormalizations of the form $\xi_{\text{ren}} = Z_\xi \xi_{\text{bare}}$, where

$$\begin{aligned} &Z_{v_D} = 1 - \frac{m_H^2}{16\pi^2 v_D^2},\quad Z_\Gamma = Z_{\Gamma_3} = 1 - \frac{m_H^2}{32\pi^2 v_D^2}\ , \\ &Z_{\lambda_4} = 1 - \frac{m_H^2}{8\pi^2 v_D^2},\quad Z_{\lambda_3} = 1 + \frac{\lambda_4 m_H^2}{4\lambda_3 \pi^2 v_D^2} + \frac{m_H^2}{8\pi^2 v_D^2}\ . \end{aligned} \tag{3.21}$$

The logarithmic terms, as before, contain the new physics. Also, as before, it is a straightfoward matter to extract the terms which give the leading corrections. Isolating the coefficients of $W_\mu^+ W^{-\mu}$ and $Z^\mu Z_\mu$ we see

$$\Delta m_W^2 = \frac{1}{64\pi^2}\ln\frac{m_W^2}{m_H^2} g^2 \lambda_4 v_T^2$$

and (3.22)

$$\Delta m_Z^2 = \frac{1}{32\pi^2}\ln\frac{m_W^2}{m_H^2}(g^2 + g'^2)\lambda_4 v_T^2$$

and thus

$$\rho \approx 1 - 2v^2 - \frac{\lambda_4 v^2}{16\pi^2}\ln\frac{m_W^2}{m_H^2}\ . \tag{3.23}$$

Therefore V_{λ_4} does lead to a correction to the relation $\rho = 1$. Because of the smallness of the ratio $v = v_T/v_D$, this does not lead to further constraint on the parameter λ_4 (and thus on the masses of the new charged scalars). Of course, the terms finite in the large-m_H limit also give corrections to ρ. These terms have been calculated in the weakly interacting, light-Higgs-boson model[25] with the leading result

$$\rho \approx 1 + \frac{g^2}{8\pi^2}\left[\frac{m_{\chi^+}}{m_W}\right]^2 (1 - \ln 2) \tag{3.24}$$

which leads to the limit $m_{\chi^+} < 400$ GeV (Ref. 25). However, the finite terms resulting from our effective theory are not generic in nature, depending in general on the exact manner in which the new physics enters the theory.[26] For our purposes then, the limit $m_{\chi^+} < 730$ GeV will be more applicable.

The logarithmically divergent terms (2.38) which led to the amplitudes (2.39) are present in this model, also, and lead to the same enhancements. (The model in Sec. II is, after all, a minimal model for the strongly interacting longitudinal gauge-boson sector.)

Lastly, we may isolate the leading contributions in the limit $\sqrt{s} \approx m_H \gg m_W$ to interactions involving the longitudinal gauge bosons and the new charged scalars. Logically, these should be those terms in (3.20) which arise from the inclusion of V_{λ_4} in $u(x;\phi)$ and which are also enhanced by the presence of extra powers of momenta. Presented in terms of the charge eigenstates these terms can be seen to be

$$\begin{aligned} \mathcal{L}^{(1)} = -\frac{\lambda_4}{32\pi^2 v_D^2}\ln\frac{m_H^2}{m_W^2}\Bigg\{ &4(\partial^\mu w_L^+ \partial_\mu w_L^- + \partial^\mu z_L \partial_\mu z_L)\Phi^{++\dagger}\Phi^{++} - 2(\partial^\mu w_L^+ \partial_\mu w_L^- + \partial^\mu z_L \partial_\mu z_L)[\sigma_T^2 + (\Phi_3)^2] \\ &- \left[2i\partial^\mu w_L^+ \partial_\mu z_L \left[\Phi^{+\dagger}\frac{\sigma_T + i\Phi_3}{\sqrt{2}} - \Phi^{++\dagger}\Phi^+\right] + \text{H.c.}\right]\Bigg\} + \cdots \end{aligned} \tag{3.25}$$

and correspond to the production of charged scalars by the longitudinal gauge bosons. The relevant tree-level terms are

$$\mathcal{L}^{(0)}=-\lambda_4\Bigg[-2w_L^+w_L^-\Phi^{++\dagger}\Phi^{++}+w_L^+w_L^-[\sigma_T^2+(\Phi_3)^2] - \left[iw_L^+z_L\left[\Phi^{+\dagger}\frac{\sigma_T+i\Phi_3}{\sqrt{2}}-\Phi^{++\dagger}\Phi^+\right] +\text{H.c.}\right]\Bigg]+\cdots . \quad (3.26)$$

The appropriate amplitudes, tree plus one loop, are therefore

$$\mathcal{A}(z_Lz_L\to\chi^{--}\chi^{++})=0+\frac{\lambda_4}{8\pi^2}\frac{s}{v_D^2}\ln\frac{m_H^2}{m_W^2} ,$$

$$\mathcal{A}(w_L^+w_L^-\to\chi^{--}\chi^{++})=2\lambda_4+\frac{\lambda_4}{16\pi^2}\frac{s}{v_D^2}\ln\frac{m_H^2}{m_W^2} ,$$

$$\mathcal{A}(w_L^+w_L^-\to\sigma_T\sigma_T)=-\lambda_4-\frac{\lambda_4}{16\pi^2}\frac{s}{v_D^2}\ln\frac{m_H^2}{m_W^2} =\mathcal{A}(w_L^+w_L^-\to\chi\chi) , \quad (3.27)$$

$$\mathcal{A}(z_Lz_L\to\sigma_T\sigma_T)=0-\frac{\lambda_4}{8\pi^2}\frac{s}{v_D^2}\ln\frac{m_H^2}{m_W^2} =\mathcal{A}(z_Lz_L\to\chi\chi) ,$$

$$\mathcal{A}(w_L^+z_L\to\chi^{++}\chi^-)=-2i\lambda_4+\frac{\lambda_4 i}{16\pi^2}\frac{s}{v_D^2}\ln\frac{m_H^2}{m_W^2} =-\mathcal{A}(w_L^+z_L\to\chi^{++}\chi^-) =-\sqrt{2}\mathcal{A}(w_L^+z_L\to\sigma_T\chi^+) =-i\sqrt{2}\mathcal{A}(w_L^+z_L\to\chi\chi^+) ,$$

and so on. Here, as before, $\sqrt{s}$ represents the center-of-mass energy for the process, and, in reminder, we neglect all corrections to the tree terms of order less than logarithmic. Had the entire derivative expansion been summed, we would have found[27] that the normalization parameter in the logarithms is more appropriately given according to the replacement

$$\ln m_W^2\to\ln\partial^2\approx\ln(-s)=i\pi+\ln s . \quad (3.28)$$

The imaginary portion determines the absorptive part of the amplitude for any of the processes considered above, and, in general, will add to the cross section for a given process at a rate approximately on the order of the other one-loop contributions. We will generally disregard this factor in obtaining numerical estimates, and so our estimates may be regarded as conservative in this regard.

At momenta such that $\sqrt{s}\approx m_H \gg m_W$ the reactions in Eq. (3.27) could provide clear signals for the production of the charged scalars in the Gelmini and Roncadelli (GR) model, and therefore, constraints on the masses of χ^{++} and χ^+. At these energy levels the $U(1)_{\text{lept}}$ symmetry would be restored. Those processes involving χ or σ_T would therefore be replaced by processes involving the neutral scalar Φ_T^0, introducing a factor of $\sqrt{2}$ into the appropriate amplitude for each exchange. Of special note are the amplitudes for the reactions $z_Lz_L\to\chi^-\chi^+$ and $w_L^+w_L^-\to\chi^-\chi^+$ which, in this limit, are of order zero both at tree and one-loop levels. By way of comparison we note, for example, that at the tree level in the weakly interacting GR model with a Higgs-doublet mass $m_H \ll 1$ TeV, the amplitudes in the large center-of-mass energy limit arising from the λ_1 and λ_4 terms in the potential are

$$\begin{aligned}\mathcal{A}(z_Lz_L\to\chi^-\chi^+)&=-\tfrac{1}{2}\lambda_4 ,\\ \mathcal{A}(w_L^+w_L^-\to\chi^-\chi^+)&=-\lambda_4 ,\\ \mathcal{A}(z_Lz_L\to\chi^{--}\chi^{++})&=-\lambda_4 ,\\ \mathcal{A}(w_L^+w_L^-\to\chi^{--}\chi^{++})&=0 .\end{aligned} \quad (3.29)$$

and so forth. Hence, the strongly interacting limit corresponds to a relative enhancement of a source of the doubly charged scalars over that of the singly charged scalars.

One of the most promising means for realizing these reactions is boson-boson fusion occurring via double bremsstrahlung in pp collisions. For example, the amplitude for the process $w_L^+w_L^-\to\chi^{--}\chi^{++}$ [Eq. (3.27)] may be used to approximate the effective cross section as

$$\sigma_{\text{eff}}(w_L^+w_L^-\to\chi^{--}\chi^{++}) \approx\lambda_4^2\frac{|\mathbf{p}'|}{|\mathbf{p}|}\left[\frac{\hat{s}}{2^{12}\pi^5v_d^4}\ln^2\frac{m_H^2}{\hat{s}}+\frac{1}{64\pi^3v_D^2}\ln\frac{m_H^2}{\hat{s}} +\frac{1}{4\pi\hat{s}}\right]\tau\frac{dL}{d\tau}\bigg|_{pp/w^+w^-} , \quad (3.30)$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the center-of-mass three-momenta of the initial and final particles, respectively, and $(\hat{s})^{1/2}$ is the center-of-mass energy of the diboson system. The factor $\tau\, dL/d\tau$ represents the effective luminosity of the gauge-boson pair in the hadron system. Here τ is the ratio of the square of the diboson mass, $m_{VV}\approx(\hat{s})^{1/2}$, to the pp center-of-mass energy squared.

For numerical estimates, we will assume $\sqrt{s_{pp}}=40$ TeV and $m_H=3$ TeV/c. We also take $m_{VV}=2$ TeV/c and $m_{VV}=1$ TeV/c as representative values for the diboson mass. The momentum ratio is approximated by unity. The effective luminosity of the gauge-boson pair has been calculated for $m_{VV}=2$ TeV/c by Rosenfeld and Rosner[28] with the result

$$\tau\frac{dL}{d\tau}\bigg|_{pp/w^+w^-}=\begin{cases}3.3\times10^{-7} & \text{for } \sqrt{s_{pp}}=17 \text{ TeV} ,\\ 5.2\times10^{-6} & \text{for } \sqrt{s_{pp}}=40 \text{ TeV} ,\end{cases} \quad (3.31)$$

which will justify the requirement $\sqrt{s_{pp}}=40$ TeV. A similar calculation for a diboson mass of 1 TeV/c has also been presented[14] with the result

$$\tau\frac{dL}{d\tau}\bigg|_{pp/w^+w^-}=4.5\times10^{-5} . \quad (3.32)$$

TABLE I. Cross sections and estimated yields to reconstructible leptonic final states for the reaction $w_L^+ w_L^- \to \chi^{--}\chi^{++}$ at various values of $m_{\chi^{++}}$ and $(\hat{s})^{1/2}$. Estimates assume a pp collider with $(\hat{s}_{pp})^{1/2}=40$ TeV and an integrated luminosity of 10^{40} cm^{-2}.

$m_{\chi^{++}}$ (GeV/c)	$(\hat{s})^{1/2}$ (TeV)	σ_{eff} (10^{-40} cm^2)	Decay products	Detectable yields $l=e,\mu/l=\mu$
1000	2	150	$l^\pm l^\pm+X$	18/2.1
			$l^+l^+l^-l^-+X$	0.6
500	2	9.4	$l^\pm l^\pm+X$	1.1/0.1
	1	300	$l^\pm l^\pm+X$	36/4.1
			$l^+l^+l^-l^-+X$	1.2
250	2	0.59	$l^\pm l^\pm+X$	
	1	18	$l^\pm l^\pm+X$	2.2/0.26

The results for the cross section are summarized in the first three columns of Table I. For $m_{\chi^{++}}=1.0$ TeV/c, corresponding to the maximum mass allowed by the renormalization-group limit, we find an approximate cross section of 1.5×10^{-38} cm^2. This would correspond to a yield of about 150 events at a hadron collider of integrated luminosity of order 10^{40} cm^{-2}, such as could be achieved by a collider of luminosity 10^{33} cm^{-2}s^{-1} operating for one year. For $m_{\chi^{++}}=500$ GeV/c, a value near the limit allowed by the finite one-loop correction to ρ, we find a total cross section of 9.4×10^{-40} cm^2 for a 2-TeV/c diboson mass and 3.0×10^{-38} cm^2 for a diboson mass of 1 TeV/c. Even at masses as low as 250 GeV/c the cross section can obtain comparable values. Similar results hold for the processes yielding pairs consisting of one doubly and one singly charged scalar.

Detection of these events would proceed via their principal decay products, which may be leptonic

$$\chi^{++}\to l^+l^+,\quad \chi^+\to l^+\bar{\nu} \tag{3.33}$$

or weak

$$\chi^{++}\to\chi^+W^+,\quad \chi^+\to\Phi^0W^+\ . \tag{3.34}$$

These decays have been discussed for various values of the scalar masses in Refs. 4 and 29. The partial χ^{++} decay widths in the large-$m_{\chi^{++}}$ limit are

$$\frac{1}{64\pi}|\Gamma_{ll}|^2 m_{\chi^{++}} \tag{3.35}$$

for leptonic decay (3.33) and, to leading order in the χ^{++} mass, approximately

$$\left[\frac{g^2}{4\pi}\right]^2 m_{\chi^{++}} \tag{3.36}$$

for the weak decay (3.34). A generically allowed value of the Yukawa coupling Γ_{ll} may be given by[29]

$$|\Gamma_{ll}|^2\approx2.7\times10^{-4}\ . \tag{3.37}$$

Thus we see that at large values of the scalar mass $m_{\chi^{++}}$, the decays would be principally via the weak current. We would therefore expect the decay products from the pair production process considered above to be of the form $W^+W^+W^-W^-\Phi^0\bar{\Phi}^0$. These events would be fairly spherical and involve end products of four quark or lepton pairs. The two light Φ^0 would be essentially unobservable, and thus the events would have well over 20% missing energy.

The cleanest signals would involve primary decays of the $W^\pm$ into leptons, which we will assume could be reconstructed with an efficiency of approximately one. In particular we focus on the detection of four leptons and of same-charge lepton pairs, events which have no quark-antiquark background. We consider only direct decays of the $W^\pm$ into leptons. With these assumptions, an observability factor may be computed as the product of the relevant branching functions for the $W^\pm$ decay. For example, the observability factor for a four-lepton event would simply be $[B(W\to e\bar{\nu}_e,\mu\bar{\nu}_\mu)]^4$. A more exact factor would include the effects of cuts in the rapidity. However, because of the structure of the events, we would not expect appropriate rapidity cuts to significantly alter these factors. Further, the contribution of the absorptive terms serves to offset any reductions in expected yields. Additionally, since detection of electron charge would require special effort[30] while that of muon charge would probably not, we have separately included estimations for decays to same-charge dimuon events ($W^\pm W^\pm\to\mu^\pm\nu\mu^\pm\nu;W^\mp W^\mp\to$anything).

The yields of reconstructable events for the reaction $w_L^+w_L^-\to\chi^{++}\chi^{--}$ may then be estimated using these observability factors. The results, assuming an integrated luminosity of 10^{40} cm^{-2}, are presented in the final two columns of Table I. We see that, for both $m_{\chi^{++}}=1.0$ TeV/c and $m_{\chi^{++}}=50$ TeV/c, even the more spectacular four-lepton events can begin to become visible at integrated luminosities on the order of a few times 10^{40} cm^{-2}. In addition, those processes producing one doubly and one singly charged scalar contribute further to the expected yields of dimuon events. We would also expect such reactions to produce detectable yields of trimuon events at rates of the same order as those predicted in Table I for the other leptonic processes.

Detection of hadronic decays of the $W^\pm$ is a more nebulous affair. However, even here a certain number of events might be reconstructed. For jet-jet decays of the vector bosons where at least one of the jets comes from a heavy quark (c,b,t), the strong production background is of the same magnitude as that produced via the weak processes.[30] Appropriate jets are distinguished by the presence of a muon in the jet. Further, an ISAJET study[31] has shown that hadronic decays can be distinguished from the strong background at a signal/noise ratio of 1:1 with efficiency 0.2. This, coupled with the unusual structure of the events, could allow detection of a percentage of the hadronic decays also.

We therefore conclude that, in a strongly interacting longitudinal gauge system, the doubly charged boson could begin to be looked for at the next generation of colliders. If the charged scalars were relatively light, then they could hopefully be produced via the dominant mechanisms in the weakly interacting system, primarily e^+e^- annihilation and gauge-boson decay.[29] However, the absence of trimuon events at CERN argues against scalar masses that are appreciably lighter than the gauge-boson masses.[32]

Additional reactions characteristic of a strongly interacting sector, such as the multiple productions of pairs of longitudinal $W^{\pm}$ and Z in each of the processes described above are also possible at the energies we have been considering.[33] However, these signatures are considerably weaker than those of the other events described above, being damped by additional powers of v_D. The presence of the second scale v_T additionally serves to damp the observable effects that the presence of the Higgs triplet may generate, both at tree and loop levels. This was reflected not only in our expression for ρ, but also in the fact that we were able to neglect the mixings between the multiplets, allowing a model with strong self-interactions only at the higher scale to be represented by a nonlinear σ model essentially as in the case of a single Higgs doublet.

One could also ask what happens if the triplet becomes strongly interacting at the second scale v_T, which would correspond to the limit $\lambda_2 \to \infty$ in the potential (3.7). As in the doublet case, this forces the nonlinear constraint

$$\sigma_T = (v_T^2 - \mathbf{\Phi}^2)^{1/2} , \quad (3.38)$$

where $\mathbf{\Phi}$ represents the five remaining real component fields. A simple scaling by the factor $v = v_T / v_D$ shows that this would provide an effective theory valid only up to an energy of approximately 130–260 MeV, represented by the mass m_{σ_T}. At this scale, the effective model would then predict enhancements to the scattering of the triplet scalars, including the Majoron. However, production and detection of such interactions at these levels would seem improbable at best. Interestingly, though, the model does predict a correction to ρ which is considerably larger. We find

$$\Delta\rho = -\frac{7\lambda_4}{16\pi^2} \ln \frac{\mu_T^2}{m_{\sigma_T}^2} . \quad (3.39)$$

We choose the normalization parameter μ_T to be the mass of the singly charged scalar, m_{χ^+}. Requiring that the value of ρ agrees to within one standard deviation of the experimental value gives

$$m_{\chi^+} < 51 \text{ GeV}, \quad m_{\chi^{++}} < 72 \text{ GeV} \quad (3.40)$$

which, together with the lower bounds discussed earlier, would allow a fairly narrow mass range for the scalars.

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[7]The corrections from these terms do indeed contribute to the value of ρ. From the quantum fluctuations of the U(1) gauge field one obtains the well-known one-loop correction (Ref. 8)

$$\Delta\rho = \frac{3g^2}{64\pi^2} \tan^2\theta_W \ln \frac{m_H^2}{m_W^2}$$

while from one loop in the fermions of the Yukawa sector we have

$$\Delta\rho = \frac{3e^2}{64 m_W^2 \pi^2} \left[\frac{2 m_u^2 m_d^2}{m_u^2 - m_d^2} \ln \frac{m_u^2}{m_d^2} + m_u^2 + m_d^2 \right]$$

which may be used to put limits on the masses and mass splittings of the fermions (Ref. 9). Although with the minimal Higgs sector there is no contribution to ρ from the quantum fluctuations of the Goldstone fields at one loop, this is not necessarily true of extensions of the standard model.

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$$[d\phi]_{\text{inv}} = [d\phi] \det{}^{1/2}[g_{ij}(\phi)]$$
$$= [d\phi] \exp\left[\delta^4(0) \int d^4x \ln\det g_{ij}(\phi)\right] .$$

The inclusion of this exponential does not affect the expansion at one loop and so may be disregarded. Alternatively, dimensional regularization may be used so that $\delta^4(0)=0$ identically.

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