Electromagnetic polarizabilities of pseudoscalar Goldstone bosons

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We calculate the electromagnetic polarizabilities of the charged and neutral pions and kaons within the framework of a generalized SU(3) Nambu–Jona-Lasinio model. We obtain strong constraints on the results from chiral symmetry. The possible effects of nonlinearities in the strange-current-quark mass are studied. The values obtained for the charged-pion electromagnetic polarizabilities are within the error bars of the experimental ones. We predict charged-kaon electromagnetic polarizabilities of the order of 10^{-4} fm^3, whereas the ones of the neutral pion and kaon are an order of magnitude smaller.

I. INTRODUCTION

Chiral symmetry and its dynamical breaking are very important concepts in low-energy hadron physics. They have been explored in the framework of various models.[1] A particularly attractive one is the model based on analogies with superconductivity proposed by Nambu and Jona-Lasinio[2] (NJL) long before the advent of QCD. There, the particles acquire a mass due to dynamical breaking of chiral symmetry. This same mechanism is supposed to take place in QCD. The NJL model has the advantage of great simplicity, thus enabling one to address relevant problems such as the strange-quark content of the proton,[3] the aspects related to the chiral phase transition, and the properties of mesons in hot and dense media.[4] In this model the eight pseudoscalar mesons (π,K,η) can be identified as the Goldstone bosons of the spontaneously broken chiral $SU(3)_L \otimes SU(3)_R$ symmetry. Their nonzero masses arise from the finite values of the (u,d,s) current-quark masses. The ninth pseudoscalar meson, the η', which does not appear as a Goldstone boson in the hadron spectrum is related to the breaking of the $U(1)_A$ symmetry. All these mesons are well described as collective $q\bar{q}$ excitation of the nonperturbative vacuum and their properties are completely determined by the few parameters of the model. Several quantities, namely, meson decay constants, quark-meson coupling constants, and meson radii,[5] have already been investigated. Additional fundamental meson structure parameters, the electric and magnetic polarizabilities α_M and β_M which determine the low-energy amplitude for Compton scattering,[6] remain to be explored. A definition of these quantities in terms of the gauge-invariant tensors of the Compton amplitude can be found in Ref. 7.

The polarizabilities α_M and β_M measure the induced meson dipole moment $\mathbf{d}=\alpha_M \mathbf{E}$ and magnetic moment $\mathbf{m}=\beta_M \mathbf{B}$ in an external electric and magnetic field, respectively. They characterize the deformation of the particle in such fields. It will be the purpose of this paper to study these quantities for the pion and kaon. The electric polarizability α_M can be decomposed as[8]

$$\alpha_M = \alpha^{\rm int} + \Delta\alpha_M \ . \tag{1}$$

$\alpha^{\rm int}$ is the so-called intrinsic polarizability:

$$\alpha^{\rm int} = 2 \sum_{n\neq 0} \frac{|\langle 0|D_z|n\rangle|^2}{E_n - E_0} \ , \tag{2}$$

where the summation is over all possible intermediate states $|n\rangle$ of the system (bound and unbound) with energies E_n, and D_z is the projection of the dipole operator onto the z axis. The correction term $\Delta\alpha_M$ comes from finite-size and recoil effects. In the classical limit it is given by[8]

$$\Delta\alpha_M \equiv \alpha^{\rm cl} = \frac{e^2 r^2}{3m_M} \ . \tag{3}$$

Here, m_M is the mass of the particle under consideration and r is its charge radius. This radius is well known experimentally for the pion and the kaon. Recent measurements give[9]

$$r^2_{\pi^+} = (0.44\pm0.02)\ {\rm fm}^2, \quad r^2_{K^+} = (0.34\pm0.05)\ {\rm fm}^2 \tag{4}$$

leading to the classical values (we use Gaussian units with $e^2=\frac{1}{137}$)

$$\alpha^{\rm cl}_{\pi^\pm} = 15\times10^{-4}\ {\rm fm}^3 \ , \tag{5a}$$

$$\alpha^{\rm cl}_{K^+} = 3.3\times10^{-4}\ {\rm fm}^3 \ . \tag{5b}$$

For nuclei and nucleons $\alpha^{\rm cl}$ is small compared to $\alpha^{\rm int}$ essentially because the masses of these objects are large. We will see that the situation is opposite for the π and K mesons as was already noticed in Ref. 10, where the pion polarizability was studied in different chiral models.

A decomposition similar to (1) can be made for β_M. Its

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intrinsic part $\beta^{\rm int}$ obeys an equation similar to (2) with $\mathbf{D}$ replaced by $\boldsymbol{\mu}$, the magnetic dipole operator. The correction term $\Delta\beta_M$ is generally written in nonrelativistic constituent models as

$$\Delta\beta_M = -\sum_{i=1}^{N} \frac{e_i^2}{6m_i}\langle \mathbf{r}_i^2\rangle - \frac{1}{2m}\langle 0|D^2|0\rangle \; , \qquad (6)$$

where $e=\sum_i e_i$, $m=\sum_i m_i$, and $\mathbf{r}_i$ is the radius vector of particle relative to the center of mass.

In what follows we will consider the response of the pion and kaon when the corresponding systems of quarks, described by a generalized Nambu–Jona-Lasinio model[3] are placed in an external electromagnetic field. We will see that chiral symmetry imposes constraints on the values of α_M and β_M resulting in a suppression of $\alpha^{\rm int}$ compared to $\Delta\alpha_M$. For a review of theoretical studies up to 1981, the reader is referred to the article by Petrun'kin.[11] Since then a few calculations have been performed in particular by Volkov and co-workers[12] and Fil'kov and co-workers.[13]

Unfortunately, little is known experimentally about the electromagnetic polarizabilities of the pion and the kaon. Two methods have been used to determine α_M and β_M. The first one consists in a measurement of the level shifts in mesonic atoms. This method has been used long ago in the case of K-mesonic atoms leading to a very weak bound on the polarizability of the K^- (Ref. 14):

$$-15\times10^{-3}\ {\rm fm}^3 \le \alpha^{K^-} \le 7\times10^{-3}\ {\rm fm}^3 \; . \qquad (7)$$

The second method follows from the Primakoff effect[15] and leads to the determination of the pion polarizability. High-energy charged pions are scattered on a nucleus target and the effect of the nuclear Coulomb field on the pion is studied. The most recent analysis[16] gives

$$\alpha_\pi+\beta_\pi=(1.4\pm5.5)\times10^{-4}\ {\rm fm}^3 \qquad (8a)$$

together with

$$\beta_\pi=(-7.1\pm4.5)\times10^{-4}\ {\rm fm}^3 \; . \qquad (8b)$$

Nothing is known about the polarizability of the neutral π^0 and K^0 mesons.

Our paper is organized as follows. We present the generalized NJL model in the presence of an external electromagnetic field in Sec. II and devote Sec. III to results and discussions.

II. GENERALIZED NJL MODEL IN AN ELECTROMAGNETIC FIELD

A. General formalism

An SU(3) version of the NJL model which respects the known symmetries of QCD,

$$\tilde G = {\rm SU}(3)_L\otimes{\rm SU}(3)_R\otimes{\rm SU}(N_c)_V\otimes{\rm U}(1)_V\otimes F \; , \qquad (9)$$

where $\mathrm{SU}(3)_L\otimes\mathrm{SU}(3)_R$ is the chiral-flavor symmetry of the light quarks, $\mathrm{SU}(N_c)_V$ is the vectorial global color symmetry, $\mathrm{U}(1)_V$ is the quark baryon number, and F is the usual set of discrete symmetries P,C,T, has been derived in Ref. 3. Since we are interested in properties of the pion and kaon we will adopt here the combination of four-quark interactions invariant under $\tilde G$ used in that reference. As argued there this choice is dictated by simplicity any other choices would lead to similar results. In the presence of an electromagnetic field the Lagrangian density of Ref. 3 becomes

$$\mathcal{L} = \bar q(i\not{D}-m)q + G\sum_{a=0}^{8}[(\bar q\lambda_a q)^2-(\bar q\gamma_5\lambda_a q)^2] - K[\det\bar q(1+\gamma_5)q+\det\bar q(1-\gamma_5)q] \; , \qquad (10)$$

where q denotes quark (spinor) fields, $m={\rm diag}(m_u, m_d, m_s)$ is the bare or current-quark mass matrix, and the det is over flavor indices. In what follows we will work in the SU(2)-isospin-symmetric case $m_u=m_d$. We denote by G and K the coupling constants of dimension $({\rm mass})^{-2}$ and $({\rm mass})^{-5}$, respectively. The term proportional to K is a six-fermion interaction which must be added to the Lagrangian in order to break the unwanted $\mathrm{U}(1)_A$ symmetry. This term is responsible for the η-η' mass splitting. The matrices λ_a $(a=0,\dots,8)$ are the generators of $\mathrm{SU}(3)_f$ with $\lambda_0=\sqrt{2/3}I$, while D_μ is the usual covariant derivative:

$$D_\mu=\partial_\mu-\frac{e}{2}\left[\lambda_3+\frac{1}{\sqrt3}\lambda_8\right]A_\mu \; . \qquad (11)$$

Note that the Lagrangian Eq. (10) is invariant under $\mathrm{SU}(3)_L\otimes\mathrm{SU}(3)_R$ only in the absence of an electromagnetic field. Indeed the quantity $\gamma_\mu\lambda_j$ $(j=1,\dots,8)$ breaks chiral symmetry explicitly.

As a result of self-interactions quarks of flavor i acquire a constituent mass M_i given in the Hartree approximation by the gap equation

$$M_i(A_\mu)=m_i+4n_cGi\,{\rm tr}[S_A^i(x,x)] + 2n_c^2Ki\,{\rm tr}[S_A^j(x,x)]i\,{\rm tr}[S_A^k(x,x)] \; , \quad i\ne j\ne k \; , \qquad (12)$$

where the trace is over Dirac matrices and color indices and S_A is a matrix in flavor space whose elements S_A^i are the propagators of free quarks of mass M_i in an external field:

$$S_A^i(x,x)=\int\frac{d^4p}{(2\pi)^4}\frac{1}{\not{p}-e_i\not{A}-M_i} \; . \qquad (13)$$

A more detailed expression for S_A^i will be given below in the case of a constant electric or magnetic field up to second order in the fields. We restrict ourselves to the Hartree approximation for simplicity. In the Hartree-Fock approximation the self-energy would contain a term proportional to $\sigma_{\mu\nu}$ corresponding to an additional spin magnetic moment. This would make the calculation more complicated but would not lead to qualitatively different results. When $m=0$, Eq. (12) has nontrivial solutions $(M_i\ne0)$ above some critical value of G and/or K corresponding, in the absence of an electromagnetic field, to a dynamical breaking of chiral symmetry. Note that due to self-consistency M_i depends on the electromagnetic field.

Many of the issues surrounding dynamical symmetry breaking can be related to the constituent mass M or equivalently to the properties of the quark propagator. Knowing M one can indeed determine the properties of the Goldstone bosons as well as the vacuum-expectation values of the quark condensates:

$$\langle \bar{q}_i q_i \rangle = \langle \Omega | \bar{q}_i q_i | \Omega \rangle - \langle 0 | \bar{q}_i q_i | 0 \rangle = i\,\mathrm{tr}[S_A^i(x,x)] - i\,\mathrm{tr}\int \frac{d^4p}{(2\pi)^4} \frac{1}{\not{p} - e_i \not{A} - m_i} , \tag{14}$$

where the expectation value of $\bar{q}_i q_i$ in the perturbative vacuum $|0\rangle$ has been subtracted from its expectation value in the true vacuum $|\Omega\rangle$ in order to compare with the vacuum expectation values extracted from QCD sum rules. All these quantities will thus depend on the strength of the electromagnetic field.

The response of the pion and of the kaon to an external field is determined by the Bethe-Salpeter equation[17]

$$\left[\not{p} + \frac{\not{q}}{2} - e_i \not{A} - M_i\right] \chi(p,q) \left[\not{p} - \frac{\not{q}}{2} - e_j \not{A} - M_j\right] = 2V_{\mathrm{eff}} \gamma_5 i \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}[\chi(p+k,q)\gamma_5] , \tag{15}$$

where e_i and e_j are the charges of the quark and of the antiquark bound into the meson, respectively, M_i and M_j their constituent-quark masses determined in Eq. (12), while $\chi(p,q)$ is the Bethe-Salpeter amplitude. In Eq. (15) V_{eff} is a flavor-dependent interaction to be specified later on, consisting of the trivial flavor-diagonal piece from the four-fermion part and an effective two-body interaction induced by the determinantal term of the form $KS_A^i(x,x)$ and one has made the minimal substitution $p \to p - eA$. One easily recognizes in Eq. (15) the operator S_A^{-1} [see Eq. (13)]. For a given value $\mathbf{q}$ of the pion momentum, this equation is an eigenvalue equation for the pion energy q_0. Owing to the special form of the Lagrangian density, Eq. (10), it can be simplified. Indeed since the right-hand side of Eq. (15) is proportional to the matrix γ_5 the following relation must hold:

$$\chi(p,q) = C(A_\mu) S_A^i \left[p + \frac{q}{2}\right] \gamma_5 S_A^j \left[p - \frac{q}{2}\right] , \tag{16}$$

where $C(A_\mu)$ can be obtained from the normalization condition of χ. Inserting Eq. (16) into Eq. (15) one finds the eigenvalue equation

$$J(q_0, \mathbf{q}) = 1 \tag{17a}$$

with

$$J(q_0,\mathbf{q}) = -2iV_{\mathrm{eff}} \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\left[S_A \left[p + \frac{q}{2}\right] \gamma_5 T \times S_A \left[p - \frac{q}{2}\right] \gamma_5 T \right] , \tag{17b}$$

where T is a flavor matrix:

$$T = \begin{cases} \lambda_3, & \pi^0 , \\ \frac{1}{\sqrt{2}}(\lambda_1 \pm i\lambda_2), & \pi^\pm , \\ \frac{1}{\sqrt{2}}(\lambda_6 \pm i\lambda_7), & K^0, \bar{K}^0 , \\ \frac{1}{\sqrt{2}}(\lambda_4 \pm i\lambda_5), & K^\pm . \end{cases} \tag{17c}$$

Once the solution q_0 of this equation is obtained, one can easily determine the electromagnetic polarizabilities as follows. Consider a meson at rest ($\mathbf{q}=0$) in the absence of an electric field. Its energy q_0 determined by solving Eq. (17) is its mass m_M. Let us now apply an external field. The energy of the meson will be shifted by an amount Δq_0 proportional to the intrinsic polarizability [see Eqs. (1) and (2)]. By definition one has

$$\Delta q_0 = -\tfrac{1}{2}\alpha^{\mathrm{int}} E^2 , \tag{18}$$

where E is the strength of the external field. In addition to this effect a force will be exerted on the meson (if it has a nonzero charge), leading to a change of its momentum $\Delta|\mathbf{q}|$. This momentum shift is related to the term $\Delta\alpha_M$ in the decomposition of the electric polarizability. In analogy with Eq. (18) we write

$$\frac{\Delta|\mathbf{q}|^2}{2m_M} = \tfrac{1}{2}\Delta\alpha_M E^2 . \tag{19}$$

Expanding Eq. (17) to second order in E one can determine the quantity $u - v$ where u and v are the changes in the square of the energy and momentum, respectively, due to the presence of the electric field:

$$\Delta q_0^2 = uE^2, \quad \Delta \mathbf{q}^2 = vE^2 \tag{20}$$

and thus calculate α_M which according to Eqs. (18) and (19) is given in the meson rest frame by

$$\alpha_M = \frac{v - u}{m_M} . \tag{21}$$

A similar expression holds for β_M where u and v are now changes to second order in the magnetic field.

B. Second-order perturbation theory in the electric field

In this section we will consider the response of a meson to a small constant electric field. We choose the gauge where

$$A_0(x) = Ex_3, \quad A_1 = A_2 = A_3 = 0 . \tag{22}$$

In order to perform the calculations it will be convenient to bring $A_0(x)$ into the equivalent form

$$A_0(x) = -iE \lim_{a\to 0} \frac{d}{da_3} \exp(iax) . \tag{23}$$

E being small, one can expand all quantities to second order in the field. The constituent mass of a quark of flavor i reads

$$M_i(E)=M_0^i+e_i^2E^2M_2^i \tag{24}$$

and the propagator S_A^i in momentum space is given by

$$S_A^i(p,X)=S_0^i(p)+e_iS_1^i(p,X)+e_i^2E^2\left[S_2^i(p,X)+M_2^i\frac{\partial S_0^i}{\partial M_0^i}\right], \tag{25}$$

where p and X denote, respectively, the relative momentum and center-of-mass variable. The free propagator in the absence of electric field S_0^i does not depend on X due to translational invariance. Note that in Eq. (24) there is no term linear in the field. Indeed, one can show that the gap equation (12) does not allow for such a term as long as gauge invariance is respected. The coefficients S_0^i,S_1^i,S_2^i in (25) are obtained by expanding the integral equation obeyed by S_A^i to second order in E. In coordinate space this equation reads

$$S_A^i(x_2,x_1)=S^i(x_2,x_1)+e_i\int d^4x_3S^i(x_2,x_3)A\!\!\!/(x_3)S_A^i(x_3,x_1), \tag{26}$$

where the propagator S^i satisfies the equation

$$S^i(x_2,x_1)[-i\partial\!\!\!/_1-M_i(E)]=\delta^4(x_2-x_1). \tag{27}$$

This yields, to second order in the electric field,

$$S_A^i(x_2,x_1)=S^i(x_2,x_1)+\int d^4x_3S^i(x_2,x_3)e_iA\!\!\!/(x_3)S^i(x_3,x_1)+\int d^4x_4d^4x_3S^i(x_2,x_4)e_iA\!\!\!/(x_4)\times S^i(x_4,x_3)e_iA\!\!\!/(x_3)S^i(x_3,x_1), \tag{28}$$

where S has to be expanded consequently leading to

$$S_0^i(p)=\frac{p\!\!\!/+M_0^i}{p^2-(M_0^i)^2}, \tag{29a}$$

$$S_1^i(p,X=0)\equiv S_1^i(p)=\frac{\{p\!\!\!/+M_0^i,\sigma_{03}\}}{2[p^2-(M_0^i)^2]^2}, \tag{29b}$$

$$S_2^i(p,X=0)\equiv S_2^i(p)=\frac{2[p_0\gamma_0-p_3\gamma_3-(p_0^2-p_3^2)S_0^i]}{[p^2-(M_0^i)^2]^3}. \tag{29c}$$

The curly brackets in Eq. (29b) denote an anticommutator. We only give here the propagator at $X=0$ since terms proportional to X contribute neither to the constituent mass nor to the Bethe-Salpeter equation as a result of gauge invariance.

Inserting expression(28) into the gap equation (12) and using the decomposition Eq. (24), the coefficients M_0^i and M_2^i obey the equation

$$M_0^i=m_i+16n_cGA_\Lambda^i+32n_c^2KA_\Lambda^jA_\Lambda^k,\quad i\neq j\neq k, \tag{30}$$

$$M_2^i=16n_cGB_\Lambda^i+32n_c^2K\left[\frac{e_k^2}{e_i^2}A_\Lambda^jB_\Lambda^k+\frac{e_j^2}{e_i^2}A_\Lambda^kB_\Lambda^j\right], \tag{31}$$

where A_Λ^i and B_Λ^i are given by

$$4A_\Lambda^i=i\,\mathrm{tr}_\Lambda S_0^i(x,x)=4i\int_\Lambda\frac{d^4p}{(2\pi)^4}\frac{M_0^i}{p^2-(M_0^i)^2}, \tag{32}$$

$$4B_\Lambda^i=i\,\mathrm{tr}_\Lambda S_2^i(x,x)=4i\int_\Lambda\frac{d^4p}{(2\pi)^4}\left[\frac{M_2^i}{p^2-(M_0^i)^2}\left[1+\frac{2(M_0^i)^2}{p^2-(M_0^i)^2}\right]-\frac{2M_0^i(p_0^2-p_3^2)}{[p^2-(M_0^i)^2]^4}\right]. \tag{33}$$

In Eqs. (30) and (31) one has introduced a subscript Λ on different quantities. Indeed the integrals, Eqs. (32) and (33), need to be regularized. Here we will not use the commonly adopted covariant cutoff but rather a regularization motivated by the one of Pauli and Villars[18] in order to maintain gauge invariance. More details can be found in Appendix A. Working in the isospin symmetric case $m_u=m_d$, the quantities M_0^d and M_0^u are equal. However, because of flavor mixing ($K\neq 0$) together with the different charges of the up and down quarks M_2^d is different from M_2^u. In the limit of vanishing current-quark masses the following equalities hold: $M_0^u=M_0^d=M_0^s$ and $M_2^d=M_2^s$ but $M_2^d\neq M_2^u$. Consequently, in the presence of an electric field one must differentiate between the propagator of an up quark and that of a down quark.

Expanding in Eq. (17b) the propagator S_A^i as given by Eq. (28) the Bethe-Salpeter Eq. (17a) can be written as the set of two equations:

$$1=J_0(q_0^2,0)=-4n_cV_{\mathrm{eff}}i\,\mathrm{Tr}_\Lambda[S_0^j(p+q)\gamma_5S_0^k(p)\gamma_5], \tag{34a}$$

$$0=J_2(q_0^2,0)+(u-v)\frac{\partial J_0}{\partial q_0^2}(q_0^2,0) \tag{34b}$$

with

$$J_2(q_0^2,0)=-4n_cV_{\mathrm{eff}}i\left[e_je_k\mathrm{Tr}_\Lambda[S_1^j(p+q)\gamma_5S_1^k(p)]+e_k^2\mathrm{Tr}_\Lambda[S_0^j(p+q)\gamma_5S_2^k(p)\gamma_5]+e_j^2\mathrm{Tr}_\Lambda[S_0^k(p+q)\gamma_5S_2^j(p)\gamma_5]+\frac{3K}{8\pi^2}e_i^2B_\Lambda^i\frac{i}{4n_cV_{\mathrm{eff}}^{i2}}\right]\equiv-4e^2n_cV_{\mathrm{eff}}\tilde{J}_2(q_0^2,0), \tag{35}$$

where u and v are the same as in Eq. (20) and e_j is the charge of quarks of flavor j. Tr_Λ means a trace over Dirac and color matrices and a four-momentum integration. This integration needs to be regularized which we indicated by the symbol Λ on Tr. More detailed expressions for J_0 and J_2 can be found in Appendix B. For the pion and kaon systems studied below, the flavor indices i,j,k are

$$\begin{aligned} &i=s, \quad j=d, \quad k=u, \quad \pi^{\pm}, \\ &i=d, \quad j=s, \quad k=u, \quad K^{\pm}, \\ &i=u, \quad j=s, \quad k=d, \quad K^0,\bar{K}^0 \; . \end{aligned} \tag{36}$$

In the case of the neutral pion one must add the two sets:

$$i=s, \quad j=k=u \tag{37a}$$

and

$$i=s, \quad j=k=d \; . \tag{37b}$$

The effective interaction to zeroth order in E is given by[3]

$$V^i_{\mathrm{eff}}=G\left[1+\frac{3K}{8\pi^2 G}A^i_\Lambda\right] . \tag{38}$$

When deriving the two-body interaction induced by the determinantal term in the π^0 channel we have assumed for simplicity that $S^u_A=S^d_A$. We expect this approximation to be good since we are dealing with a small electric field. From the definition of the polarizability, Eq. (21) one obtains the equation

$$\alpha_M=\frac{6e^2 g^2_{Mqq}\tilde{J}_2(m^2_M,0)}{m_M} , \tag{39}$$

where use has been made of the relation between $\partial J_0/\partial q_0^2$ and g_{Mqq} the meson quark coupling constant:

$$g^2_{Mqq}=2V_{\mathrm{eff}}\left[\frac{\partial J_0}{\partial q_0^2}\right]^{-1} . \tag{40}$$

C. Second-order perturbation theory in the magnetic field

A similar analysis can be made for the magnetic polarizability. Choosing the gauge where

$$A_2=Bx_1, \quad A_1=A_3=A_0=0 \tag{41}$$

one obtains the expressions for the propagator S_A:

$$S^i_A(p,X)=S^i_0(p)+e_iBS^i_1(p,X)+e_i^2B^2S^i_2(p,X) \tag{42}$$

with

$$S^i_0(p)=\frac{\not{p}+M^i_0}{p^2-(M^i_0)^2} , \tag{43a}$$

$$S^i_1(p,X=0)\equiv S^i_1(p)=\frac{\{\not{p}+M^i_0,\sigma_{21}\}}{2[p^2-(M^i_0)^2]^2} , \tag{43b}$$

$$S^i_2(p,X=0)\equiv S^i_2(p)=\frac{2[p_2\gamma_2+p_1\gamma_1-(p_1^2+p_2^2)S^i_0]}{[p^2-(M^i_0)^2]^3} . \tag{43c}$$

The constituent mass is

$$M_i(B)=M^i_0+e_i^2B^2M^i_2 \; . \tag{44}$$

where M^i_0 satisfies Eq. (30) and M^i_2 is given by an expression similar to (31) but with B^i_Λ now defined by

$$\begin{aligned} 4B^i_\Lambda &= i\,\mathrm{tr}_\Lambda[S^i_2(x,x)] \\ &=4i\int_\Lambda \frac{d^4p}{(2\pi)^4}\left[\frac{M^i_2}{p^2-(M^i_0)^2}\left[1+\frac{2(M^i_0)^2}{p^2-(M^i_0)^2}\right] -\frac{2M^i_0(p_1^2+p_2^2)}{[p^2-(M^i_0)^2]^4}\right] . \end{aligned} \tag{45}$$

From these equations it may be noted that replacing the external electric field by a magnetic one amounts to replacing in the starting expressions the indices 0 and 3 of the Dirac matrices by 2 and 1, respectively, and the three-momentum $\mathbf{p}$ by $-\mathbf{p}$. This means for the coefficient M_2 an opposite sign when dealing with a magnetic field instead of an electric one. Thus if the constituent mass decreases as a response to an electric field it increases as a response to a magnetic one.

The magnetic polarizability is given by an expression similar to Eq. (39):

$$\beta_M=\frac{6e^2 g^2_{Mqq}\tilde{J}_2(m^2_M,0)}{m_M} , \tag{46}$$

where $\tilde{J}_2$ is defined in Eq. (35) with S^i_0, S^i_1, S^i_2, and M^2_i given by Eqs. (42)–(44).

III. RESULTS AND DISCUSSIONS

Let us first consider the limit where all the current-quark masses are zero. In this case, the calculations can be performed analytically and the results turn out to be parameter independent. In the absence of electromagnetic field the Lagrangian is invariant under chiral symmetry and one has the following well-known properties: (i) The masses of the pseudoscalar mesons π and K vanish as dictated by the Goldstone theorem; (ii) the Goldberger-Treiman relation holds at the quark level ($g_A^{\mathrm{quark}}=1$):

$$g_{Mqq}=\frac{M}{f_M} , \tag{47}$$

where M is the constituent mass of quarks of any flavor and f_M is the meson decay constant defined by

$$\langle \Omega | A^i_\mu(x) | M^j_p \rangle = -if_M p_\mu e^{ipx}\delta_{ij} \; . \tag{48}$$

A more detailed expression for f_M can be found in Appendix B.

Evaluating the traces in Eq. (35) one can bring $\tilde{J}_2(0,0)$ into the form

$$\tilde{J}_2(0,0)=B+C \; . \tag{49}$$

The quantity B depends on the charge of the meson:

$$\begin{aligned} B &= \text{const}\times(-B_{22}-2B_{21}+D_{12}) \\ &= 0 \quad \text{for a neutral meson} , \\ B &= \text{const}\times(2B_{22}-5B_{21}-2D_{12}) \\ &= -3\times\text{const}/2M^2 \quad \text{for a charged meson} , \end{aligned} \tag{50}$$

where the constant has the same absolute value but an opposite sign for an electric or a magnetic field. The quantity C has the form

$$C=\text{const}\times\left[M_0 D_{20}(e_k^2 M_2^k + e_j^2 M_2^j) + (e_k^2+e_j^2)B_{40} + \frac{aye_i^2 B_\Lambda^i}{x {V_{\text{eff}}^i}^2} \right] . \tag{51}$$

The coefficients D_{mn} and B_{mn} are defined in Appendix B, and $x=3G/\pi^2$, $y=3K/8\pi^2 G$, $a=1$ for all mesons except the neutral pion for which $a=2$. It is easy to show, using the gap equations [Eqs. (30) and (31)], that $C=0$. Hence we find $\tilde{J}_2=0$ for a neutral meson. Stated in another way, the Bethe-Salpeter equation reduces (as in the case $E=0$) to the self-consistency condition because of the invariance of the Lagrangian under chiral symmetry in the absence of electromagnetic field. Indeed a neutral Goldstone boson is a pointlike particle. This can be seen from Eq. (C7) in Appendix C where we give for completeness the calculations of the pion and kaon radii performed with the Bethe-Salpeter amplitude Eq. (16) and with a Pauli-Villars-type regularization. It was shown in Ref. 8 that a pointlike particle has no excited states and thus cannot be polarized [see Eq. (2)]. The electromagnetic polarizabilities in the limit of vanishing current-quark masses thus satisfy

$$\begin{aligned} &\alpha_M+\beta_M=0 , \\ &\alpha_{M^0}=-\beta_{M^0}=0 \quad \text{neutral meson} , \\ &\alpha_{M^\pm}=-\beta_{M^\pm}=\frac{e^2}{4\pi^2 m_M f_M^2} \quad \text{charged meson} . \end{aligned} \tag{52}$$

The up and down quarks have small finite current masses (of the order of 7 MeV). SU(2) chiral symmetry is softly broken and as a consequence the pion acquires a finite mass small compared to the constituent masses of the quarks. We thus do not expect strong deviations from the result Eq. (52). We will in fact confirm this assertion in the following. Hence using the physical pion mass $m_\pi=140$ MeV and the physical pion decay constant $f_\pi=93$ MeV one gets

$$\alpha_{\pi^\pm}=-\beta_{\pi^\pm}=1.2\times10^{-3}\ \text{fm}^3 . \tag{53}$$

This result is in agreement with the calculation of the pion polarizability in a linear σ model with quarks or a nonlinear σ model.[10] One finds a somewhat smaller value than the classical one Eq. (5a). However, one also finds in the mean-field approximation in the NJL model a smaller radius than the experimental one. In fact using Eq. (C9) one sees that the result Eq. (53) is nothing but the classical result. Thus in the Hartree approximation the sum of α^{int} and relativistic corrections vanishes.

Let us consider the effect on the polarizabilities of explicit $SU(3)_L\times SU(3)_R$ symmetry breaking (m_u, $m_d,m_s,\neq0$). Equation (49) must be corrected for mass terms. A detailed expression of $J_2(m_K^2,0)$ is given in Appendix B. Since the kaon and the quark masses are of the same order of magnitude one cannot simplify by expanding in the mass. We thus have performed the integrations numerically. The results depend on three parameters G, K, and the regularization parameter Λ. We determine them as in Ref. 3, i.e., by requiring an overall good fit for $\mathbf{E}=\mathbf{B}=0$ to the quark condensates $\langle\bar{u}u\rangle=\langle\bar{d}d\rangle,\langle\bar{s}s\rangle$, the meson masses ($m_\pi,m_K$, $m_\eta,m_{\eta'}$), and decay constants (f_π,f_K). As further constraints we impose $M_u=M_d\simeq250-400$ MeV and $\langle\bar{s}s\rangle<\langle\bar{u}u\rangle$. In Tables I and II we give the results obtained for $m_u=m_d\simeq7$ MeV and $m_s=175$ MeV using two different sets of parameters:

$$\begin{aligned} &\text{(I)}\quad \pi^2/3G\Lambda^2=1.4,\quad 3K\Lambda^3/8\pi^2G=0.45 , \\ &\qquad \Lambda=795\ \text{MeV} , \\ &\text{(II)}\quad \pi^2/3G\Lambda^2=1.2,\quad 3K\Lambda^3/8\pi^2G=0.6 , \\ &\qquad \Lambda=700\ \text{MeV} . \end{aligned}$$

Set (I) corresponds to the barely broken regime while set (II) corresponds to the firmly broken one according to the definition of Ref. 3. These two sets illustrate the predicted range of values of the electromagnetic polarizabilities with the constraints given above to determine the parameters. Table I shows the dynamical masses $\hat{M}^i=M_0^i-m_i$, the vacuum and meson properties in the absence of electromagnetic field. There is a quite good overall agreement between theory and experiment as can be seen by comparing the first two lines [sets (I) and (II)] with the last one (Expt.). Table II gives the corrections M_2^i to the constituent masses in the presence of a small constant electromagnetic field as well as the polarizabilities α_M and β_M of the kaon and pion. As was already mentioned

TABLE I. Dynamical quark masses, quark condensates, and meson properties in the absence of electromagnetic fields for the two different sets of parameters (I) and (II) defined in the text. For comparison, the empirical values are also given.

	m_u (MeV)	$\hat{M}_u$ (MeV)	$\hat{M}_s$ (MeV)	$-\langle\bar{u}u\rangle_0^{1/3}$ (MeV)	$\frac{\langle\bar{s}s\rangle_0}{\langle\bar{u}u\rangle_0}$ (MeV)	m_π (MeV)	f_π (MeV)	m_K (MeV)	f_K (MeV)	m_η (MeV)	$m_{\eta'}$ (MeV)
(I)	6.2	247	315	236	0.66	141	92	512	105	462	920
(II)	7.8	360	383	228	0.51	140	99	470	104	479	1084
Expt.	6.2±1.9			225±35	0.8±0.2	139	93.3	494	120±10	549	958

TABLE II. Meson properties in the presence of an electromagnetic field. The corrections M_2 to the dynamical masses to second order in the field are given in units of Λ^3. The lower (upper) signs correspond to the electric (magnetic) case. The electromagnetic polarizabilities of the charged (neutral) particles are in units of 10^{-4} (10^{-5}). The sets of parameters (I) and (II) are the same as in Table I.

	M_2^u	M_2^d	M_2^s	$\alpha_{\pi^\pm}$ (fm^3)	$\beta_{\pi^\pm}$ (fm^3)	$\alpha_{K^\pm}$ (fm^3)	$\beta_{K^\pm}$ (fm^3)	α_{π^0} (fm^3)	β_{π^0} (fm^3)	α_{K^0} (fm^3)	β_{K^0} (fm^3)
(I)	±2.2	±5.78	±2.73	12.5	−11.8	3.88	−1.74	1.24	9.97	3.50	6.35
(II)	±0.95	±2.42	±1.72	10.5	−10.3	3.14	−2.45	2.30	1.93	3.42	0.14

before, the changes in the constituent masses are opposite for external electric or magnetic fields, the change being negative for an electric field. A recent paper[19] investigating the effect of a constant electromagnetic field in the original version of the Nambu–Jona-Lasinio model indicates that such a trend persists for stronger field leading to a zero value of M at a certain critical value of E and prohibiting it in the case of a magnetic field. The electric field has the effect of preventing the formation of a quark condensate in the vacuum while the magnetic one enforces this formation. Let us make a final remark about the quantities M_2^i by returning to the zero current-quark mass limit. One obtains, in that case, for our two sets of parameters,

$$\text{(I)}\quad M_2^u=-18.86\Lambda^{-3},\quad M_2^d=-61.65\Lambda^{-3},$$
$$M_2^s=-61.65\Lambda^{-3},$$
$$\text{(II)}\quad M_2^u=-1.36\Lambda^{-3},\quad M_2^d=-3.67\Lambda^{-3},$$
$$M_2^s=-3.67\Lambda^{-3}.$$

Comparing with Table II one observes a strong quenching effect with the strange-quark mass in the barely broken regime [set (I)] similar to the one obtained for the strange-quark content of the proton. Strong nonlinearities in the mass persist when applying an electromagnetic field.

The results of the electromagnetic polarizabilities in the case of charged mesons are rather independent of the parameters (as long as the properties of these mesons—masses and decay constants—are well reproduced) whereas those concerning the neutral particles are more sensitive as can be seen by comparing the first two lines in Table II. This essentially comes from the fact that recoil effects do not contribute for neutral mesons. In this case one is left with the intrinsic part, Eqs. (1) and (2), which receives both positive contributions from possible excited states reached by dipole transitions and negative contributions from vacuum-polarization effects. As a result of these delicate cancellations one obtains a small number which is very sensitive to possible nonlinearities.

The comparison of the results of the pion polarizabilities with the ones obtained in the zero current-quark mass limit shows few deviations. The main effects are to relax the constraint $\alpha_M=-\beta_M$ and to give small but nonvanishing values to the electromagnetic polarizabilities of the neutral mesons. These values are still very small, 1–2 orders of magnitude smaller than the charged ones. The results concerning the charged pion are within the error bars of the experimental ones Eq. (8): $\beta_{\pi^\pm}$ is at the upper end of the allowed values whereas $\alpha_{\pi^\pm}+\beta_{\pi^\pm}$ whose value is of order $(0.3-0.7)\times 10^{-4}$ fm^3 is somewhat smaller than the central value.

In the chiral limit the pion and kaon are degenerate in energy thus leading to the same polarizabilities. However, the explicit breaking of SU(3) by mass terms is larger than in SU(2). The mass of the strange quark is of the same order than Λ_{QCD} and the kaon acquires a mass which is comparable to the constituent-quark masses. One thus expects large corrections to the zero current-quark mass result Eq. (53) for the kaon. Indeed as shown in Table II the charged-kaon polarizabilities are about three to four times smaller than the pion which is roughly the ratio of m_π to m_K. One notices that the effect of the breaking of chiral symmetry by mass terms is stronger in the magnetic case than in the electric case leading to a bigger value of $\alpha_K+\beta_K$. As was just pointed out the case of the neutral particles is more delicate. α_{K^0} is slightly bigger than α_{π^0} while β_{K^0} turns out to be somewhat smaller. Let us return to the charged kaon. The value of the electric polarizability of the charged kaon lies within the large error bars of the experimental result, Eq. (7). The result for $\alpha_{k^\pm}$ is somewhat bigger than its classical value as can be seen by comparing Tables II and III.

TABLE III. Meson radii and classical electric polarizabilities (in units of 10^{-4}) as defined in Eq. (3) for the same set of parameters as in Table I. The experimental values of the radii are given for comparison. The value for the neutral kaon is taken from Ref. 20.

	$r_{\pi^\pm}$ (fm)	$r_{K^\pm}$ (fm)	$r_{K^0}^2$ (fm^2)	$\alpha^{\text{cl}}_{\pi^\pm}$ (fm^3)	$\alpha^{\text{cl}}_{K^\pm}$ (fm^3)
(I)	0.59	0.54	−0.062	12.0	2.72
(II)	0.54	0.49	−0.033	10.2	2.18
Expt.	0.66±0.01	0.58±0.025	−(0.054±0.026)		

Thus, if the pion electric polarizability is essentially given by its classical parts, the kaon receives some contribution from the intrinsic part (in the case of the kaon, possible relativistic corrections are expected to be small). The value of the electric polarizability seems to increase slightly when the nonlinearities in m_s become bigger while the magnetic one is decreasing.

IV. SUMMARY AND CONCLUSIONS

In this paper we have investigated two structure parameters of the pion and of the kaon: namely, their electric and magnetic polarizabilities. These parameters appear as coefficients in the low-energy expansion of the amplitude for Compton scattering. We have used the generalized NJL model which was extended in Ref. 3 to N_f flavors preserving the known symmetries of QCD and have considered the case of three flavors. The only degrees of freedom of the model are quarks. We have minimally coupled them to a small constant electromagnetic field. We have studied the effect of such a field on the gap equation and on the Bethe-Salpeter equation self-consistently in the Hartree approximation. We have seen that the solution of the latter to second order in the field is intimately related to the structure parameters α_M and β_M. The model has three parameters in addition to the quark masses: namely, two coupling constants G and K which give the strengths of the interactions, and a regularizing parameter Λ. Indeed the NJL model is not renormalizable and one needs to render loop integrals finite. Here we did not use the common method which can be traced back to the original paper by Nambu and Jona-Lasinio consisting of introducing a cutoff function in the loop integrals. We have rather implemented a gauge-invariant regularization motivated by the method of Pauli and Villars.

We have calculated α_M and β_M in the barely and firmly broken regimes as defined in Ref. 3 and obtained strong constraints on our results from chiral symmetry. We have seen that in the zero current-quark mass limit the electromagnetic polarizabilities of the neutral pseudoscalar mesons vanish because of their pointlike nature. In the case of charged mesons it is the sum $\alpha_M+\beta_M$ which vanishes. Deviations from these results away from the chiral limit come from mass terms. To analyze our results we make a standard decomposition of α_M into a part which takes into account recoil effects and is related up to some relativistic corrections to the electromagnetic size of the object (classical part) and an intrinsic part associated with possible excited states reached by dipole transitions and with vacuum-polarization effects. It turns out that the electric polarizability of the charged pion (kaon) is to a very good approximation (for a major part) given by the classical term while the ones of the neutral pion and kaon are necessarily only given by the latter. The value of the electric (magnetic) polarizability of the kaon increases (decreases) slightly as the nonlinearities in the strange current-quark mass become stronger. The values we have obtained were within the error bars of the experimental data; however, the lack of accuracy in the measurements did not allow us to draw any decisive conclusion. Better experimental results are thus highly desirable.

Since our approach involves the mean-field approximation together with a strong-coupling theory it is necessary to discuss the following question: What would happen if one would include higher-order effects? These would naturally affect the intrinsic part since one would include more possible excited states and more vacuum-polarization effects. However, as we have seen these two effects are of opposite signs having a tendency of canceling each other. Furthermore, as we have emphasized in this paper, the electromagnetic polarizabilities are very much constrained by chiral symmetry. We thus do not expect significant changes in the results concerning the pion for which the soft-pion limit should be a good approximation. The kaon polarizabilities could be more sensitive to higher orders.

We would like to conclude by stressing the advantage of using the NJL model compared to other models. First of all, of course, it incorporates chiral symmetry and its dynamical breaking in the same way as one believes it happens in QCD which was seen to be crucial. Second, it includes in a consistent way within the approximation scheme all possible excited states as well as vacuum-polarization effects. In constrast, most other models select some intermediate mesonic states which can be coupled to pions and photons usually through quark loops.[11] In this case, one can always add some direct couplings of these mesons as has been done in Ref. 10. There, a non-negligible increase in the pion electric polarizability was observed as a consequence.

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APPENDIX A: REGULARIZATION

The gap equations, Eqs. (30) and (31) as well as the Bethe-Salpeter equations, Eqs. (34) and (35) involve divergent integrals, for which a regularization procedure has to be defined. In order to preserve the symmetries of the system, namely, chiral symmetry, gauge invariance, and Lorentz invariance, we follow the regularization method of Pauli and Villars.[17] In this method, integrals of the type

$$I_{\alpha\beta\gamma}(m_i,m_j;q)=i\int\frac{d^4p}{(2\pi)^4}\frac{p_\mu^\alpha}{(p^2-m_i^2)^\beta[(p-q)^2-m_j^2]^\gamma} \tag{A1}$$

are replaced by a sum of contributions corresponding to different masses,

$$I=\sum_{s=0}^{N} c_s I(M_i^s,M_j^s,q) \ , \tag{A2}$$

with the convention $c_0=1$, $M_i^0=m_i$, and $M_j^0=m_j$, and the usual conditions

$$\sum_{s=0}^{N} c_s=0, \quad \sum_{s=0}^{N} c_s(M_i^s)^2=0 \ . \tag{A3}$$

Equations (A3) can be satisfied through the introduction of two auxiliary masses for each flavor index i. We use the standard definitions

$$(M_i^1)^2=m_i^2+2\Lambda^2, \quad (M_i^2)^2=m_i^2+\Lambda^2 \ , \tag{A4}$$

where Λ is a cutoff parameter. These equations lead to

$$c_1=\frac{(M_i^2)^2-m_i^2}{(M_i^1)^2-(M_i^2)^2}=1, \quad c_2=\frac{(m_i^1)^2-m_i^2}{(M_i^2)^2-(M_i^1)^2}=-2 \ . \tag{A5}$$

The gap and Bethe-Salpeter equations involve the functions I_{010} and I_{011}. For example, the quantities A and B [Eqs. (32) and (34)] which appear in the gap equation can be written as

$$4A_\Lambda^i=4M_0^i I_{010}(M_0^i,M_0^i;0) \ , \tag{A6a}$$

$$4B_\Lambda^i=4\{M_2^i[I_{010}(M_0^i,M_0^i;0)+2(M_0^i)^2 I_{010}(M_0^i,M_0^i;0)] + \text{nondivergent quantity}\} \ . \tag{A6b}$$

According to our prescription one finds

$$I_{010}(M_0^i,M_0^i;0)=\frac{(M_0^i)^2}{16\pi^2}[(1+2x_i)\ln(1+2x_i)-2(1+x_i)\ln(1+x_i)] \ , \tag{A7}$$

$$\begin{aligned} I_{011}(M_0^i,M_0^j;q^2)=\frac{1}{32\pi^2}\Bigg\{&\frac{M^2}{m^2}[(a^++2x)\ln(a^++2x)-(a^-+2x)\ln(a^-+2x)\\ &-(a^++x)\ln(a^++x)+2(a^-+x)\ln(a^-+x)]\\ &+b^+\ln a^++b^-\ln b^--2\left[1+c\left\{\arctan\frac{b^+}{c}+\arctan\frac{b^-}{c}\right\}\right]\Bigg\} \ , \end{aligned} \tag{A8}$$

where the following notation has been used:

$$x_i=\frac{\Lambda^2}{M_0^{i2}}, \quad x=\frac{\Lambda^2}{M^2}, \quad a^\pm=1\pm\frac{m^2}{M^2}, \quad b^\pm=1\pm\frac{2m^2}{q^2}, \quad c=\left[\frac{4M^2}{q^2}-1-\frac{4m^4}{q^4}\right]^{1/2} \ . \tag{A9}$$

In Eq. (A9) we have also introduced the average square mass M^2 and the square mass difference m^2:

$$M^2=\tfrac{1}{2}[(M_0^i)^2+(M_0^j)^2], \quad m^2=\tfrac{1}{2}[(M_0^i)^2-(M_0^j)^2] \ . \tag{A10}$$

For equal masses, expression (A8) simplifies to (for $q^2=0$)

$$I_{011}(M_0^i,M_0^i;0)=\ln(1+2x_i)-2\ln(1+x_i) \ . \tag{A11}$$

APPENDIX B: BETHE-SALPETER EQUATIONS AND DECAY CONSTANT

1. Bethe-Salpeter equations

a. In the absence of an electromagnetic field

Performing the traces over the Dirac and color matrices, Eq. (34a) becomes

$$J_0(q^2)=-4N_c i\int\frac{d^4p}{(2\pi)^4}\frac{M_0^iM_0^j-p(p-q)}{[p^2-(M_0^i)^2][(p-q)^2-(M_0^j)^2]} \ . \tag{B1}$$

Using the identity

$$2[M_0^iM_0^j-p(p-q)]=[(M_0^i)^2-p^2]+[(M_0^j)^2-(p-q)^2]+[q^2-(M_0^i-M_0^j)^2] \tag{B2}$$

one can write $J_0(q^2)$ in terms of the quantities I_{011} and I_{010} introduced in Appendix A [see Eqs. (A1), (A7), and (A8)] as

$$J_0(q^2)=2N_c\{I_{010}(M_0^i,M_0^j;0)+I_{010}(M_0^j,M_0^i;0)-[q^2-(M_0^i-M_0^j)^2]I_{011}(M_0^i,M_0^j;q^2)\} \ . \tag{B3}$$

b. In the presence of an electromagnetic field

Let us first define the quantities

$$\begin{aligned} D_{nk}^{ij}(q^2)&=\frac{\Gamma(n+k-2)}{\Gamma(n)\Gamma(k)}\int_0^1 d\alpha \frac{\alpha^{k-1}(1-\alpha)^{n-1}}{[m_j^2\alpha+m_i^2(1-\alpha)-\alpha(1-\alpha)q^2]^{k+n-2}} ,\\ B_{nk}^{ij}(q^2)&=\frac{\Gamma(n+k-3)}{\Gamma(n)\Gamma(k)}\int_0^1 d\alpha \frac{\alpha^{k-1}(1-\alpha)^{n-1}}{[m_j^2\alpha+m_i^2(1-\alpha)-\alpha(1-\alpha)q^2]^{k+n-3}} ,\\ A_{nk}^{ij}(q^2)&=\frac{\Gamma(n+k-2)}{\Gamma(n)\Gamma(k)}\int_0^1 d\alpha \frac{\alpha^{k+1}(1-\alpha)^{n-1}}{[m_j^2\alpha+m_i^2(1-\alpha)-\alpha(1-\alpha)q^2]^{k+n-2}} ,\\ C_{nk}^{ij}(q^2)&=\frac{\Gamma(n+k-2)}{\Gamma(n)\Gamma(k)}\int_0^1 d\alpha \frac{\alpha^{k}(1-\alpha)^{n-1}}{[m_j^2\alpha+m_i^2(1-\alpha)-\alpha(1-\alpha)q^2]^{k+n-2}} . \end{aligned} \tag{B4}$$

Note that the divergent quantities $D_{10}^{ij}, D_{11}^{ij}, D_{20}^{ij}\equiv D_{11}^{ii}$ are related to I_{010} and I_{011}, respectively, by

$$D_{10}^{ij}(q^2)=I_{010}(M_0^i,M_0^j;q^2),\quad D_{11}^{ij}(q^2)=-I_{011}(M_0^i,M_0^j;q^2) . \tag{B5}$$

The traces appearing in the expression of J_2, Eq. (35) can be expressed in terms of these quantities as

$$\begin{aligned} \mathrm{Tr}[\gamma_5 S_1^i(p+q)\gamma_5 S_1^j(p)]=&\mp 4n_c i\{\tfrac{1}{2}D_{12}^{ij}(q^2)+\tfrac{1}{2}D_{21}^{ij}(q^2)-B_{22}^{ij}(q^2)\\ &-(q_{3_1}^2\mp q_{0_2}^2)[A_{22}^{ij}(q^2)-C_{22}^{ij}(q^2)]+\tfrac{1}{2}[q^2-(M_0^i-M_0^j)^2]D_{22}(q^2)\} ,\\ \mathrm{Tr}[\gamma_5 S_2^i(p+q)\gamma_5 S_0^j(p)]=&\mp 4n_c i(-B_{31}^{ij}(q^2)+B_{40}^{ij}(0)\pm M_2^i[M_0^j D_{20}^{ij}(0)+D_{11}^{ij}(q^2)(M_0^i-M_0^j)]\\ &-(q_{3_1}^2\mp q_{0_2}^2)\{A_{31}^{ij}(q^2)-2C_{31}^{ij}(q^2)+[q^2-(M_0^i-M_0^j)^2]A_{41}^{ij}(q^2)\}\\ &+[q^2-(M_0^i-M_0^j)^2][B_{41}^{ij}(q^2)+M_0^i M_0^j D_{21}^{ij}(q^2)]) , \end{aligned} \tag{B6}$$

where the upper (lower) signs and indices correspond to the electric (magnetic) case, and

$$\mathrm{Tr}[\gamma_5 S_0^i(p+q)\gamma_5 S_2^j(p)]=\mathrm{Tr}[\gamma_5 S_2^j(p+q)\gamma_5 S_0^i(p)] . \tag{B7}$$

From these equations it is straightforward to calculate J_2. As an example, let us derive the expression of $J_2(m_K^2,0)$ in the case of the charged kaon. Let us introduce the quantities G and F defined as

$$\begin{aligned} G=&\begin{cases}(2e^2 n_c V_{\mathrm{eff}}/18\pi^2)[2A_{22}^{us}-4A_{31}^{us}-A_{31}^{su}-(M_0^s-M_0^u)^2(4A_{41}^{us}+A_{41}^{su})+m_K^2(4A_{41}^{us}+A_{41}^{su})] & \text{for an electric field} ,\\ 0 \quad \text{for a magnetic field} , & \end{cases}\\ F=&2e^2 n_c V_{\mathrm{eff}}\left[\pm\frac{1}{18\pi^2}(D_{12}^{us}+D_{21}^{su}-(M_0^u-M_0^s)^2 D_{22}^{us}\right.\\ &\pm 4M_2^u\{D_{11}^{us}(M_0^s-M_0^u)-M_0^u[D_{20}^{us}(0)-(M_0^u-M_0^s)^2 D_{21}^{us}]\}\\ &\pm M_2^s\{D_{11}^{su}(M_0^s-M_0^u)-M_0^s[D_{20}^{su}(0)-(M_0^u-M_0^s)^2 D_{21}^{su}]\}\\ &-2B_{22}^{us}+4B_{31}^{us}+B_{31}^{su}-B_{40}^{us}(0)-B_{40}^{su}(0)+(M_0^s-M_0^u)^2(4B_{41}^{us}+B_{41}^{su})\\ &\left.+m_K^2(D_{22}^{us}-4M_0^u M_2^u D_{21}^{us}-M_0^s M_2^s D_{21}^{su}-4B_{41}^{us}-B_{41}^{su}-2C_{22}^{us}+8C_{31}^{us}+2C_{31}^{su}))+\frac{3K}{8\pi^2 G}e_d^2\frac{B_\Lambda^d(\frac{E}{B})}{2n_c V_{\mathrm{eff}}^{d2}}\right] , \end{aligned} \tag{B8}$$

where $B_\Lambda^d(\frac{E}{B})$ is the function defined by Eq. (33) [see also Eq. (A6a)] which depends on the nature of the field (electric or magnetic) through the quantity M_2. All the functions A,B,C,D have to be taken at $q^2=m_K^2$ unless otherwise specified. The resulting expression of $J_2(m_K^2,0)$ reads

$$J_2(m_K^2,0)=F+G . \tag{B9}$$

A similar expression can be obtained for the case of the pion. By using Eq. (39) one can then calculate the electromagnetic mesonic polarizabilities. As an illustration let us give an approximate analytical expression for the neutral-pion electric polarizability as obtained using the original SU(2) NJL model:

$$\alpha_{\pi^0}\approx\frac{5}{96}\frac{e^2}{f_\pi^4}m_\pi\left[1-\frac{\pi^2}{3G}\frac{m}{Mm_\pi^2}\right] .$$

2. Meson decay constant ($E=B=0$)

The meson decay constant defined by Eq. (48) obeys the equation

$$f_M q_0 = -ig_{Mqq}\mathrm{Tr}_\Lambda[\gamma_5\gamma_0 S_0^i(p)\gamma_5 S_0^j(p-q)] , \tag{B10}$$

where $i=j=u$ for the pion and $i=u, j=s$ for the kaon, and where $g_{Mqq}\gamma_5$ is the meson quark vertex [g_{Mqq} being the quark-meson coupling given in Eq. (40)]. The integration over p in (B10) leads to a divergent expression. After calculating the traces over Dirac and color matrices one can write f_π in terms of the quantities $I_{\alpha\beta\gamma}$ (Appendix A):

$$f_K = 12g_{Mqq}[-M_0^i I_{011}(M_0^i, M_0^j; q^2) + (M_0^i - M_0^j)I_{111}(M_0^i, M_0^j; q^2)] , \tag{B11}$$

where I_{011} is given in Eq. (A8). Using the notation of Eq. (A9), I_{111} can be written as

$$I_{111}(M_0^i, M_0^j; q^2) = \tfrac{1}{2}I_{011} - \frac{1}{64\pi^2}\left[\frac{1}{2}\left\{1-\frac{M^4}{m^4}(1-2x)\right\}\ln\left\{\frac{a^+ + 2x}{a^- + 2x}\right\} - \left\{1-\frac{M^4}{m^4}(1+x)\right\}\ln\left\{\frac{a^+ + x}{a^- + x}\right\} - \frac{M^2}{m^2} + \frac{2M^2}{q^2}\left\{1-\frac{2m^4}{M^2q^2}\right\}\ln\frac{a^+}{a^-} + \frac{2m^2}{M^2} - \frac{4m^2}{q^2}c\left\{\arctan\frac{a^+}{c} - \arctan\frac{a^-}{c}\right\}\right] . \tag{B12}$$

APPENDIX C: CALCULATION OF MESON RADII

From the knowledge of the Bethe-Salpeter amplitude $\chi(k,p)$ of a meson in the absence of an electromagnetic field [see Eqs. (15) and (16) with $A_\mu = 0$] one can determine its form factor $F(q^2)$ and consequently its radius through the expressions (see, for example, Ref. 17)

$$\int d^4x\, e^{iqx}\langle M_p | j_\mu(x) | M_{p'}\rangle = e(p_\mu + p'_\mu)F(q^2) \tag{C1}$$

with

$$\langle M_p | j_\mu(x) | M_{p'}\rangle = -ie_i\int d^4x_j \mathrm{tr}[\bar\chi_p(x_i,x_j)\gamma_\mu^{(i)}\chi_{p'}(x_i,x_j)](i\overleftarrow{\partial}^{(j)} - m_j) - ie_j\int d^4x_i \mathrm{tr}[\bar\chi_p(x_i,x_j)(i\partial^{(i)} - m_i)\bar\chi_{p'}(x_i,x_j)]\gamma_\mu^{(j)} \tag{C2}$$

and

$$\langle r^2\rangle = -6\left.\frac{dF(q^2)}{dq^2}\right|_{q^2=0} . \tag{C3}$$

In (C1) $j_\mu(x)$ is the electromagnetic current, $q_\mu = (p'_\mu - p_\mu)$ is the momentum transfer, e is the charge of the meson, the indices (i) and (j) refer to particles i and j bound into the meson, and $\chi(p,q)$ and $\chi_p(x_i,x_j)$ are related by the transformation

$$\chi_p(x_i,x_j) = e^{-ipX}\int d^4k\, e^{-ikx}\chi(k,p) . \tag{C4}$$

The variables X and x are the center-of-mass and relative coordinates, respectively:

$$X = \mu_i x_i + \mu_j x_j, \quad x = x_i - x_j, \quad \mu_i = \frac{m_i}{m_i + m_j}, \quad \mu_j = \frac{m_j}{m_i + m_j} .$$

Inserting Eq. (C2) into Eq. (C1), one obtains the expression for $F(q^2)$:

$$e(\rho_\mu + \rho'_\mu)F(q^2) = -ie_i\mathrm{Tr}[\bar\chi(k,p)\gamma_\mu^{(i)}S_0^{(i)}(k+q+\mu_i p)\gamma_5] - ie_j\mathrm{Tr}[\bar\chi(k,p)\gamma_5 S_0^{(j)}(k-q-\mu_j p)\gamma_\mu^{(j)}] \tag{C5}$$

with

$$\bar\chi(k,p) \equiv CS_0^{(j)}(k-\mu_j p)\gamma_5 S_0^{(i)}(k-\mu_i p) . \tag{C6}$$

In order to determine the radius one can expand $F(q^2)$ in powers of q^2. Performing the traces, and using the identity

$$\frac{i}{p^2 - m^2} = \int_0^\infty d\alpha\, e^{i\alpha(p^2 - m^2)}$$

one obtains, to first order in q^2 ($q^2 > 0$),

$$F(q^2)=C^2N+\frac{q^2}{6}\left[\frac{3}{2\pi^2}C^2e_j\int_0^1 d\alpha\frac{(1-\alpha)^2(1+2\alpha)}{\alpha(M_0^j)^2+(1+\alpha)(M_0^i)^2-m_M^2\alpha(1-\alpha)}\right.$$
$$\left.+\frac{\alpha(1-\alpha)^3[\alpha m_M^2+M_i(M_j-M_i)]}{[\alpha(M_0^j)^2+(1-\alpha)(M_0^i)^2-m_M^2\alpha(1-\alpha)]^2}+(i\leftrightarrow j)\right], \tag{C7}$$

where i and j are defined by Eq. (36) for the kaon and charged pion. For the neutral pion one adds the four sets $i=j=u, i=j=d, i=j=\bar{u}, i=j=\bar{d}$. In Eq. (C7) N is a divergent quantity which can be brought into the form

$$N=24e_i\left[-I_{111}(M_0^i,M_0^j,m_M^2)+\frac{1}{16\pi^2}\int d\alpha\frac{\alpha(1-\alpha)[\alpha m_M^2+M_i(M_j-M_i)]}{\alpha(M_0^j)^2+(1-\alpha)(M_0^i)^2-m_M^2\alpha(1-\alpha)}\right]+(i\leftrightarrow j). \tag{C8}$$

The normalization factor C is usually chosen to give the correct total charge:

$$\int d^3x\langle M_p|j_0(x)|M_p\rangle=(e_i+e_j)\quad\text{(charged meson)}; \tag{C9}$$

i.e., $F(q^2=0)=C^2N=1$ for the charged pion and kaon. C can also be identified with the quark-meson coupling constant [see Eq. (40)] since the Bethe-Salpeter vertex $\Gamma=C\gamma_5$ describes the coupling of a $q\bar{q}$ pair to a meson. We have used this convention to calculate the meson decay constant [Eq. (B10)]. With $C\equiv g_{Mqq}$ we find that the relation $C^2N=1$ holds for the case of the pion. Slight deviations are observed for the kaon. We get $C^2N=0.99$ for set (I) and $C^2N=1.13$ for set (II). Note that it would be possible to adjust C^2N to unity by introducing different cutoffs Λ in the strange and nonstrange sector. Indeed, there is no *a priori* reason to choose them to be identical. However this has been done in order to minimize the number of parameters.

In the chiral limit $[(M_0^s)^2=(M_0^u)^2; m_M^2=0]$ one finds, from (C7) and (B11),

$$r_M^2=\frac{3}{4\pi^2 f_M^2}\quad\text{(charged pion)},$$
$$r_M^2=0\quad\text{(neutral pion)}. \tag{C10}$$

The results for sets (I) and (II) are summarized in Table III. They are in quite good agreement with experiment. The somewhat too small value obtained for the pion radius with the set of parameters (II) comes from the somewhat too large value of f_π (see Table I). It is interesting to note that the K^0 square radius is quite sensitive to the set of parameters used. The stronger the nonlinearities in m_s are, the larger is the absolute value of the square radius. The error bars in the experimental value are in any case too big to draw any decisive conclusion. The radius obtained for the pion is the same as the one derived in the linear σ model with quarks. Results of both pion and kaon radii correspond to the ones calculated[5] with a covariant cutoff whose value is taken to infinity

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