Neutralino mass matrix in the minimal supersymmetric model

A. Bartl
Institut für Theoretische Physik, Universität Wien, Vienna, Austria

H. Fraas
Physikalisches Institut, Universität Würzburg, Würzburg, West Germany

W. Majerotto and N. Oshimo
Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, Vienna, Austria
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We study systematically the dependence of the neutralino mass eigenvalues and eigenstates on the parameters of the mixing matrix. Starting from analytical solutions for special values of the parameters we work out various practical approximation formulas for the masses. We examine their applicability in the different regions of parameters accessible with the next generation of accelerators.

I. INTRODUCTION

In the minimal supersymmetric extension[1,2] of the electroweak standard model there are four neutral gauge and Higgs fermions $\tilde{\gamma}$, $\tilde{Z}$, $\tilde{H}_1^0$, $\tilde{H}_2^0$, the supersymmetric partners of the photon, Z^0 boson, and the neutral members of the two Higgs doublets, respectively. Their mass eigenstates are the four neutralinos χ_i^0, $i=1,\ldots,4$, whose properties are determined by the 4×4 neutral gaugino-Higgsino mixing matrix.[3,4]

Without further restrictions this matrix depends on four parameters M, M', μ, and $\tan\theta_v \equiv v_1/v_2$: M and M' are the SU(2) and U(1) gauge fermion masses, respectively; μ is the Higgsino mass parameter, and v_1 and v_2 are the vacuum expectation values of the two Higgs doublets with U(1) hypercharge $-\frac{1}{2}$ and $+\frac{1}{2}$, respectively. (Notice that in the literature also $\tan\beta \equiv v_2/v_1$ is used, i.e., $\tan\beta = \cot\theta_v$.) These parameters determine the phenomenology (i.e., masses, couplings) of the neutralinos,[2,5,6] in particular the nature of the lightest one, usually assumed to be the lightest supersymmetric particle.

In this paper we study systematically the dependence of the neutralino mass eigenvalues and eigenstates on the mixing parameters. In view of the experiments to be performed at the Fermilab Tevatron, KEK TRISTAN, SLAC Linear Collider (SLC), CERN LEP, and DESY HERA we work out all cases where neutralinos with masses accessible in this energy range could be produced. We draw attention to the fact that in addition to the scenarios most frequently studied, those with a light photino or with a light Higgsino, there are also other possibilities for light neutralinos.

Starting from simple analytical solutions of the eigenvalue problem we systematically give practical approximation formulas for the masses and states over the whole parameter space relevant for the energy range of accelerators of the next generation. Some of the formulas derived here can already be found in the literature,[5-7] which are, however, only applicable for the light-photino scenario with $v_1/v_2 \approx 1$ or the case of large M, M', and/or μ. They are naturally included in our systematics. It is the purpose of the experiments searching for supersymmetric particles to either fix these parameters or exclude certain regions of parameter space. Since four parameters are involved here this will in reality be a complex procedure. In the analysis of experimental data our considerations will help to indicate the domain where a systematic variation of parameters is meaningful. Moreover, the approximation formulas presented here show very clearly the physical properties of the neutralinos for all interesting cases and thus provide an easy understanding of the physics involved.

After describing in Sec. II the notation and conventions used we present in Sec. III all those cases where an analytic solution can easily be obtained. Section IV contains a systematic discussion of the general dependence of the neutralino mass eigenvalues on the parameters. In Sec. V we give four sets of approximation formulas for neutralino masses covering in this way all cases relevant for the energy range of the Tevatron, TRISTAN, SLC, LEP, and HERA. The range of validity of the approximations is discussed in Sec. VI.

II. THE NEUTRALINO MASS MATRIX

As basis of the neutral gaugino-Higgsino system we conveniently take[2]

$$\psi_j^0 = (-i\lambda_\gamma, -i\lambda_z, \psi_H^a, \psi_H^b), \quad j=1,\ldots,4 \tag{1}$$

with the Higgsino states

$$\psi_H^a = \psi_{H1}^1 \sin\theta_v - \psi_{H2}^2 \cos\theta_v \ , \tag{1a}$$

$$\psi_H^b = \psi_{H1}^1 \cos\theta_v + \psi_{H2}^2 \sin\theta_v \ . \tag{1b}$$

λ_γ, λ_z, ψ_{H1}^1, and ψ_{H2}^2 are the two-component spinors of the photino, Z-ino, and the two neutral Higgsinos $\tilde{H}_1^0$ and $\tilde{H}_2^0$, respectively. The mass term in the Lagrangian has the form

$$L_M^0 = -\tfrac{1}{2} m_Z \psi_i^0 Y_{ij} \psi_j^0 + \text{H.c.} \tag{2}$$

with the mass matrix

$$Y=\begin{pmatrix} \lambda(\sin^2\theta_W+\alpha\cos^2\theta_W) & (1-\alpha)\lambda\sin\theta_W\cos\theta_W & 0 & 0 \\ (1-\alpha)\lambda\sin\theta_W\cos\theta_W & \lambda(\cos^2\theta_W+\alpha\sin^2\theta_W) & 1 & 0 \\ 0 & 1 & \nu\sin2\theta_v & \nu\cos2\theta_v \\ 0 & 0 & \nu\cos2\theta_v & -\nu\sin2\theta_v \end{pmatrix} . \tag{3}$$

m_Z is the mass of the Z^0 boson and θ_W is the Weinberg angle. We have introduced

$$\lambda=\frac{M}{m_Z}, \quad \nu=\frac{\mu}{m_Z}, \quad \alpha=\frac{M'}{M} . \tag{4}$$

θ_v, M, M', and μ as defined in the Introduction are the parameters of the model. For convenience, in Eq. (2) we have taken out a factor m_Z to deal with dimensionless parameters.

Neglecting *CP* violation, Y is a real symmetric matrix which can be diagonalized by a unitary 4×4 matrix N:

$$N_{im}N_{kn}Y_{mn}=\xi_i\delta_{ik} \tag{5}$$

with

$$\xi_i=\frac{m_i}{m_Z} , \tag{6}$$

m_i being the mass eigenvalue of the neutralino state

$$\chi_i^0=N_{ij}\psi_j^0 . \tag{7}$$

We shall take N_{im} real and orthogonal. Then some of the mass eigenvalues m_i may be negative. In principle, by an appropriate choice of the phases of N_{im} all eigenvalues m_i could be made positive, but this does not concern us here. The sign of m_i is related to the *CP* quantum number of χ_1^0 (Refs. 8 and 9). For more details see Refs. 2 and 10.

The four eigenvalues ξ_i are the solutions of the eigenvalue equation

$$(\xi^2-\nu^2)(\xi-\lambda)(\xi-\alpha\lambda)-[\xi-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)]\times(\xi+\nu\sin2\theta_v)=0 . \tag{8}$$

Usually, the eigenvalues ξ_i are ordered by their absolute values. This will not be done here as we shall apply different approximation schemes to Eq. (8) where another ordering is more appropriate. Notice that $\xi(-\lambda,-\nu,\sin2\theta_v)=-\xi(\lambda,\nu,\sin2\theta_v)$ and $\xi(\lambda,-\nu,-\sin2\theta_v)=\xi(\lambda,\nu,\sin2\theta_v)$.

Given the eigenvalues ξ_i, the neutralino eigenstates can be obtained as

$$\chi_i^0=\frac{1}{N_i}\begin{pmatrix} (1-\alpha)\lambda\sin\theta_W\cos\theta_W(\xi_i^2-\nu^2) \\ [\xi_i-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)](\xi_i^2-\nu^2) \\ [\xi_i-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)](\xi_i+\nu\sin2\theta_v) \\ [\xi_i-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)]\nu\cos2\theta_v \end{pmatrix} \tag{9}$$

in the basis Eq. (1). The normalization factor is

$$N_i=\{(\xi_i^2-\nu^2)^2[\sin^2\theta_W(\xi_i-\lambda)^2+\cos^2\theta_W(\xi_i-\alpha\lambda)^2+(\xi_i-\lambda)^2(\xi_i-\alpha\lambda)^2]+[\xi_i-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)]^2(\nu\cos2\theta_v)^2\}^{1/2} . \tag{9a}$$

The cases $\alpha=1$ and $\sin2\theta_v=1$ have to be handled with care. We shall discuss them separately in the next section. The neutralino components given in Eq. (9) are the elements N_{ij} of the transformation matrix which diagonalizes the mass matrix Y. They determine the couplings to the other particles.[2-15]

III. SPECIAL SOLUTIONS OF THE MASS-EIGENVALUE EQUATION

In the following we shall list those special cases where the eigenvalue equation (8) factorizes so that a solution can easily be obtained.

A. Partial solutions

Most of the following special cases are already known in the literature.

(a) $\sin2\theta_v=1$. Here one has as solution a Higgsino

$$\chi_4^0=\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{10}$$

with mass

$$\xi_4=-\nu \tag{11}$$

together with the three other states in general being photino–*Z*-ino–Higgsino mixtures

$$\chi_i^0=\frac{1}{P_i}\begin{pmatrix} (1-\alpha)\lambda\sin\theta_W\cos\theta_W \\ \xi_i-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W) \\ (\xi_i-\lambda)(\xi_i-\alpha\lambda) \\ 0 \end{pmatrix}, \quad i=1,2,3 \tag{12}$$

with

$$P_i=[\sin^2\theta_W(\xi_i-\lambda)^2+\cos^2\theta_W(\xi_i-\alpha\lambda)^2 +(\xi_i-\lambda)^2(\xi_i-\alpha\lambda)^2]^{1/2}\ . \tag{12a}$$

The analogous case $\sin 2\theta_v=-1$ is obtained by substituting $\nu\to-\nu$. For more details see Ref. 16.

(b) $\nu=0$. This special solution is similar to case (a). It contains a *massless* Higgsino

$$\chi_4^0=\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{13}$$

and the three other states as given in Eq. (12). This case is usually called the "light-Higgsino scenario."

(c) $\alpha=1$. In this case one obtains a photino

$$\chi_1^0=\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \tag{14}$$

with mass

$$\xi_1=\lambda \tag{15}$$

together with the three other states which in general are Z-ino–Higgsino mixtures

$$\chi_i^0=\frac{1}{Q_i}\begin{pmatrix}0\\ \xi_i^2-\nu^2\\ \xi_i+\nu\sin 2\theta_v\\ \nu\cos 2\theta_v\end{pmatrix},\quad i=2,3,4 \tag{16}$$

with

$$Q_i=[(\xi_i^2-\nu^2)^2+(\xi_i+\nu\sin 2\theta_v)^2+(\nu\cos 2\theta_v)^2]^{1/2}\ . \tag{16a}$$

(d) $\lambda=0$. This special solution is similar to case (c). It contains a *massless* photino

$$\chi_1^0=\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \tag{17}$$

and the three other states as given in Eq. (16) for $\alpha=1$. This case corresponds to the "light-photino scenario."

(e) $\sin^2\theta_W=0$. As it will turn out, it is useful to also consider the limiting case $\sin^2\theta_W=0$ (Ref. 17). Then the solution is again similar to case (c). One has a photino

$$\chi_1^0=\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \tag{18}$$

with mass

$$\xi_1=\alpha\lambda \tag{19}$$

and the three other states as given in Eq. (16).

(f) Inspection of the eigenvalue equation (8) shows that, if the parameters obey the condition[10,18]

$$\alpha\lambda\nu=(\sin^2\theta_W+\alpha\cos^2\theta_W)\sin 2\theta_v \tag{20}$$

there also exists a *massless* neutralino state χ_3^0, which is a mixture of all four interaction eigenstates

$$\chi_3^0=\frac{1}{N_3}\begin{pmatrix}-(1-\alpha)\sin\theta_W\cos\theta_W\\ \sin^2\theta_W+\alpha\cos^2\theta_W\\ -\alpha\lambda\\ -\alpha\lambda\cot 2\theta_v\end{pmatrix} \tag{21}$$

with

$$N_3=\left[\sin^2\theta_W+\alpha^2\cos^2\theta_W+\left[\frac{\alpha\lambda}{\sin 2\theta_v}\right]^2\right]^{1/2}\ . \tag{21a}$$

For $\alpha=1$, χ_3^0 has no photino component, for $\sin 2\theta_v=1$ the ψ_H^b component vanishes. If the parameters M and M' are related through[3] $M'/M=\frac{5}{3}\tan^2\theta_W$ (i.e., $\alpha=0.47$ for $\sin^2\theta_W=0.22$), then the massless state has the form

$$\chi_3^0=\frac{1}{N_3}\begin{pmatrix}-0.22\\ 0.59\\ -0.47\lambda\\ -0.47\lambda\cot 2\theta_v\end{pmatrix},$$

$$N_3=\left[0.39+\left[\frac{0.47\lambda}{\sin 2\theta_v}\right]^2\right]^{1/2}\ .$$

As one can see, for $\lambda<1$, χ_3^0 is mainly a Z-ino, whereas for $\lambda>1$ it is mainly a Higgsino. Therefore, one could call this case the "light Z-ino–Higgsino scenario."

B. Complete solutions

By suitably combining two of the conditions (a)–(f) the eigenvalue equation (8) can be reduced to a quadratic equation giving the following simple solutions.

(i) $\sin^2\theta_W=0$ and $\sin 2\theta_v=1$. Here we obtain the following neutralino states:

$$\chi_1^0=\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\quad \chi_2^0=\begin{pmatrix}0\\ \cos\phi\\ \sin\phi\\ 0\end{pmatrix},\quad \chi_3^0=\begin{pmatrix}0\\ \sin\phi\\ -\cos\phi\\ 0\end{pmatrix},\quad \chi_4^0=\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{22}$$

with the mixing angle

$$\sin\phi=\frac{1}{\sqrt{2}}\left[1-\frac{\lambda-\nu}{\sqrt{(\lambda-\nu)^2+4}}\right]^{1/2},$$

$$\cos\phi=\frac{1}{\sqrt{2}}\left[1+\frac{\lambda-\nu}{\sqrt{(\lambda-\nu)^2+4}}\right]^{1/2}.$$

The masses are[17]

$$\begin{aligned} \xi_1^0 &= \alpha\lambda\ , \\ \xi_2^0 &= \tfrac{1}{2}[\lambda+\nu+\sqrt{(\lambda-\nu)^2+4}]\ , \\ \xi_3^0 &= \tfrac{1}{2}[\lambda+\nu-\sqrt{(\lambda-\nu)^2+4}]\ , \\ \xi_4^0 &= -\nu\ . \end{aligned} \tag{23}$$

It is interesting to note that in this case the chargino masses also take the values ξ_2^0 and ξ_3^0 of Eq. (23).

In Sec. V we shall use this solution as a starting point for perturbation theory. For illustration Fig. 1 exhibits the dependence of the neutralino masses, Eq. (23), as a function of ν for $\lambda=2$ and $\alpha=0.47$.

(ii) $\sin^2\theta_W=0$ and $\nu=0$. This gives the same eigenstates and masses as in Eqs. (22) and (23) with $\nu=0$.

(iii) $\lambda=0$ and $\sin 2\theta_v=1$. Here again the eigenstates and eigenvalues are given by Eqs. (22) and (23) with $\lambda=0$ (Ref. 5).

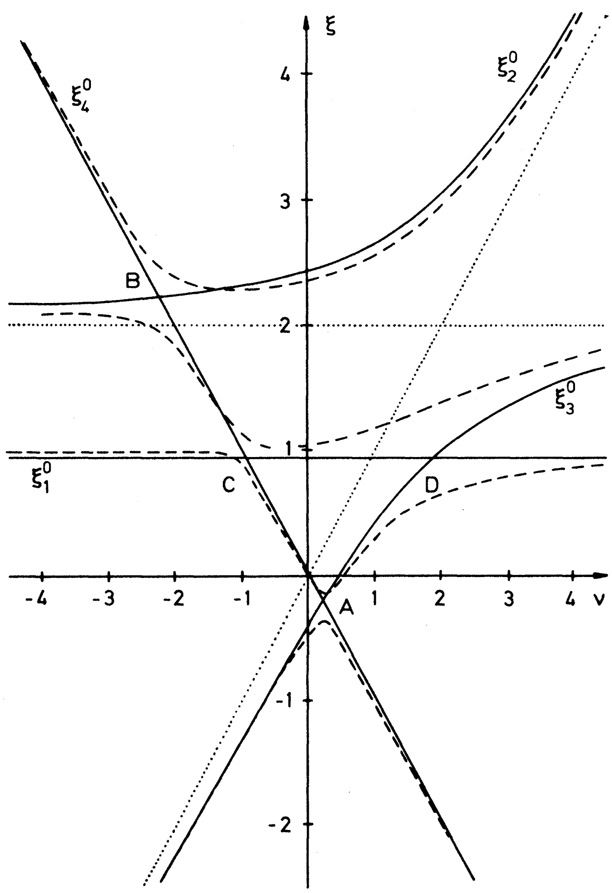

FIG. 1. Solutions ξ of the mass eigenvalue Eq. (8) as a function of ν (dashed line) for $\lambda=2$, $\alpha=0.47$, $\sin^2\theta_W=0.22$, $\sin 2\theta_v=\sqrt{3}/2$. For comparison the solutions ξ (solid line) for $\lambda=2$, $\alpha=0.47$, $\sin^2\theta_W=0$, $\sin 2\theta_v=1$ according to Eq. (23) are also shown. Also shown are the points of degeneracy A, B, C, and D. Dotted lines correspond to the asymptotes $\xi=\lambda$ and $\xi=\nu$.

(iv) $\alpha=1$ and $\sin 2\theta_v=1$ or $\nu=0$. Again, the masses and states can be read off from Eqs. (22) and (23).

(v) $\alpha\lambda\nu=(\sin^2\theta_W+\alpha\cos^2\theta_W)\sin 2\theta_v$. If the condition (20) is fulfilled ("light Z-ino–Higgsino scenario"), complete solutions of the eigenvalue equation (8) are possible for the following.

(a) $\sin^2\theta_W=0$. Here the mass eigenvalues are

$$\begin{aligned} \xi_1^0 &= \alpha\lambda\ , \\ \xi_2^0 &= \tfrac{1}{2}[\lambda+\sqrt{\lambda^2+4(\nu^2+1)}]\ , \\ \xi_3^0 &= 0\ , \\ \xi_4^0 &= \tfrac{1}{2}[\lambda-\sqrt{\lambda^2+4(\nu^2+1)}]\ . \end{aligned} \tag{24}$$

The state χ_1^0 is a photino

$$\chi_1^0=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}. \tag{25a}$$

χ_3^0 is given by Eq. (21) with $\sin\theta_W=0$ and the other two states are

$$\chi_i^0=\frac{1}{N_i}\begin{bmatrix}0\\(\xi_i^0)^2-\nu^2\\ \xi_i^0+\nu\sin 2\theta_v\\ \nu\cos 2\theta_v\end{bmatrix},\quad i=2,4 \tag{25b}$$

with

$$N_i=\{[(\xi_i^0)^2-\nu^2]^2+(\xi_i^0)^2+\nu^2+2\xi_i^0\nu\sin 2\theta_v\}^{1/2}\ . \tag{25c}$$

(b) $\alpha=1$. In this case the results can be obtained from Eqs. (24) and (25), in particular $\xi_1^0=\lambda$.

(c) $\sin 2\theta_v=1$. The eigenvalues are

$$\begin{aligned} \xi_{1,2}^0 &= \alpha\lambda+\tfrac{1}{2}\{\lambda(1-\alpha) \\ &\quad +\nu\mp\sqrt{[\lambda(1-\alpha)+\nu]^2+4(1-\lambda\nu)}\}\ , \\ \xi_3^0 &= 0,\quad \xi_4^0=-\nu\ . \end{aligned} \tag{26}$$

The eigenstates χ_3^0 and χ_4^0 are given by Eqs. (21) and (10), respectively, whereas χ_1^0 and χ_2^0 can be obtained from Eq. (12).

(vi) $|\lambda|\gg 1$ and/or $|\nu|\gg 1$. Also in this case a complete solution can be derived.[7] Asymptotically one has the following mass eigenvalues and neutralino states:

$$\xi_1^0=\alpha\lambda,\quad \xi_2^0=\lambda,\quad \xi_3^0=\nu,\quad \xi_4^0=-\nu\ , \tag{27}$$

$$\chi_1^0=\begin{bmatrix}\cos\theta_W\\-\sin\theta_W\\0\\0\end{bmatrix},\quad \chi_3^0=\begin{bmatrix}0\\0\\ \cos\left[\theta_v-\frac{\pi}{4}\right]\\ -\sin\left[\theta_v-\frac{\pi}{4}\right]\end{bmatrix}, \tag{28}$$

$$\chi_2^0 = \begin{pmatrix} \sin\theta_W \\ \cos\theta_W \\ 0 \\ 0 \end{pmatrix}, \quad \chi_4^0 = \begin{pmatrix} 0 \\ 0 \\ \sin\left(\theta_v - \frac{\pi}{4}\right) \\ \cos\left(\theta_v - \frac{\pi}{4}\right) \end{pmatrix} .$$

Clearly, χ_1^0 is a b-ino, χ_2^0 a W^3-ino, $\chi_3^0=(1/\sqrt{2})(\psi_{H1}^1-\psi_{H2}^2)$, and $\chi_4^0=(1/\sqrt{2})(\psi_{H1}^1+\psi_{H2}^2)$.

C. Degeneracy

Before discussing perturbation theory one has to find out whether two of the eigenvalues are degenerate. For the special case (i), $\sin^2\theta_W=0$ and $\sin 2\theta_v=1$, there are four points of degeneracy, A–D, as shown in Fig. 1. At these points the following eigenvalues of Eq. (23) coincide (assuming $\lambda>0$):

$$\begin{aligned} A&: \ \xi_3^0=\xi_4^0=-\nu_A=\tfrac{1}{2}(\lambda-\sqrt{\lambda^2+2}) , \\ B&: \ \xi_2^0=\xi_4^0=-\nu_B=\tfrac{1}{2}(\lambda+\sqrt{\lambda^2+2}) , \\ C&: \ \xi_1^0=\xi_4^0=-\nu_C=\alpha\lambda , \\ D&: \ \xi_3^0=\xi_1^0=\alpha\lambda \ \text{ for } \nu_D=\alpha\lambda+\frac{1}{(1-\alpha)\lambda} . \end{aligned} \tag{29}$$

The degenerate states will split for $\sin 2\theta_v\neq 1$ in points A and B, or for $\sin^2\theta_W\neq 0$ in point D. The degeneracy in point C, however, is removed only if both $\sin 2\theta_v \neq 1$ and $\sin^2\theta_W\neq 0$ (see Fig. 1).

In cases (ii)–(vi) one encounters a similar situation.

IV. GENERAL STRUCTURE OF THE NEUTRALINO MASS SPECTRUM

In order to get more insight into the structure of the mass spectrum we shall discuss the dependence of the solutions of the eigenvalue equation (8) on the parameters λ, ν, and $\sin 2\theta_v$. For this purpose we solve Eq. (8) for λ, ν, and $\sin 2\theta_v$, respectively, as a function of ξ. This procedure allows us to give a complete survey of the mass spectrum in analytic form. Furthermore, it will show in which regions of the parameter space the different approximation schemes are applicable.

A. λ dependence of the neutralino masses

Solving the eigenvalue equation (8) for λ one obtains the two branches

$$\lambda_\pm = \frac{1+\alpha}{2\alpha}\xi - \frac{\alpha+(1-\alpha)\sin^2\theta_W}{2\alpha}\,\frac{\xi+\nu\sin 2\theta_v}{\xi^2-\nu^2} \pm \frac{1}{2\alpha(\xi^2-\nu^2)}\{[(1-\alpha)\xi(\xi^2-\nu^2)+(\alpha\cos^2\theta_W-\sin^2\theta_W)(\xi+\nu\sin 2\theta_v)]^2 + \alpha[2\sin\theta_W\cos\theta_W(\xi+\nu\sin 2\theta_v)]^2\}^{1/2} . \tag{30}$$

Figure 2 shows ξ as a function of λ for $\nu=1$, $\alpha=0.47$ (corresponding to $M'/M=\frac{5}{3}\tan^2\theta_W$) and the two values $\sin 2\theta_v=0$ and $\sin 2\theta_v=1$. The crossing at the points A and C occurring for $\sin 2\theta_v=1$ splits for $\sin 2\theta_v\neq 1$. The mass eigenvalues depend sensitively on $\sin 2\theta_v$ only in the neighborhood of these crossing points. This will be important in our treatment of the various approximation methods. For $|\lambda|>4$ the solutions rather quickly approach their asymptotic values $\xi=\lambda$, $\xi=\alpha\lambda$, and $\xi=\pm\nu$.

B. ν dependence of the neutralino masses

Solving the eigenvalue equation (8) for the parameter ν one again obtains two branches:

$$\begin{aligned} \nu_\pm = \frac{1}{(\xi-\lambda)(\xi-\alpha\lambda)} [& -\tfrac{1}{2}\sin 2\theta_v[\xi-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)] \\ & \pm(\{\xi(\xi-\lambda)(\xi-\alpha\lambda)-[\xi-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)]\cos^2\theta_v\} \\ & \times\{\xi(\xi-\lambda)(\xi-\alpha\lambda)-[\xi-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)]\sin^2\theta_v\})^{1/2}] . \end{aligned} \tag{31}$$

For $\sin 2\theta_v=1$ this simplifies to

$$\nu_+ = \xi - \frac{\xi-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)}{(\xi-\lambda)(\xi-\alpha\lambda)}, \quad \nu_-=-\xi .$$

In Fig. 3 we show ξ as a function of ν for $\alpha=0.47$, $\lambda=2$, and $\sin 2\theta_v=0$ and 1. One observes that for $\nu<0$ the mass spectrum is rather different from that for $\nu>0$. One can also see that there are three regions showing different behavior of ξ with ν: (i) $|\nu|<0.5$; (ii) $0.5<|\nu|<2$; (iii) $|\nu|>2$. As we shall see, three different types of perturbation formulas correspond to these regions.

The square root in Eq. (31) gives rise to three excluded mass regions, the boundaries being determined by the zeros of the two factors in the square root. Figure 4 shows the λ dependence of the excluded regions for $\alpha=0.47$, $\sin 2\theta_v=\sqrt{3}/2$, and $\sin 2\theta_v=0$. They are largest for $\sin 2\theta_v=0$. The widths of the excluded mass regions depend on the parameters α, λ, and $\sin 2\theta_v$. For $\sin 2\theta_v\to 1$ these three excluded neutralino mass intervals

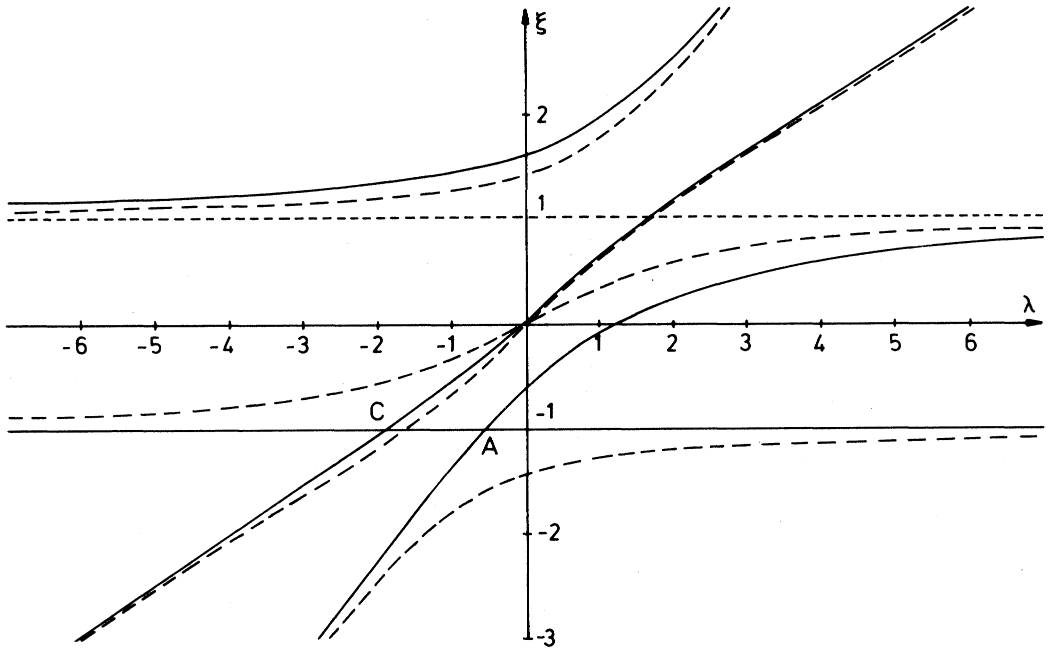

FIG. 2. λ dependence of solutions ξ of the mass eigenvalue Eq. (8) for $\nu=1$, $\alpha=0.47$, $\sin^2\theta_W=0.22$, $\sin 2\theta_v=1$ (solid line), and $\sin 2\theta_v=0$ (long-dashed line). Also shown are the crossing points A and C, and the asymptote $\xi=\nu$ (short-dashed line).

will shrink to the three crossing points $A-C$ of Fig. 1, corresponding to degenerate mass states. [The crossing point D of Fig. 1 is already removed since we have $\sin^2\theta_W\neq 0$ in Eq. (31).] For $\lambda\to 0$ the excluded region near $\xi=\alpha\lambda$ shrinks whereas the others extend up to their maxima. Also, for $\alpha=1$, i.e., $M'=M$, only two excluded mass regions appear.

C. θ_v dependence of the neutralino masses

Proceeding as before we show in Fig. 5 the dependence of ξ on $\sin 2\theta_v$ for $\alpha=0.47$ and two pairs of parameter values, $\lambda=0.3$, $\nu=0.5$, and $\lambda=0.3$, $\nu=1$, respectively. Apart from the regions near $\sin 2\theta_v=\pm 1$ the dependence of ξ on $\sin 2\theta_v$ is rather weak. Furthermore for $\alpha=1$ the upper and lower parts of the curve would merge at $\sin 2\theta_v=-\lambda/\nu$ and cross the solution $\xi=\lambda$. A similar crossing would occur for $\sin^2\theta_W=0$.

In the case of no degeneracy of two eigenvalues the curves $\xi(\sin 2\theta_v)$ extend into the unphysical regions $|\sin 2\theta_v|>1$. Then there are excluded mass regions at $|\sin 2\theta_v|=1$. If there is degeneracy the curve becomes tangent to the line $\sin 2\theta_v=1$ or $\sin 2\theta_v=-1$. This corresponds to the degeneracy of the mass eigenstates at the crossing point A of Fig. 1.

Generally, if $\lambda>0$ and $\nu>0$, the lowest branch of the curve $\xi(\sin 2\theta_v)$ corresponds to the solution ξ_4 for $\nu>\nu_A$, and to ξ_3 for $\nu<\nu_A$, ξ_3 and ξ_4 as defined in Sec. V.

D. The lightest and the second lightest neutralinos

Because of its importance for phenomenology we show in Fig. 6(a) a contour plot[13] of the mass eigenvalue (including the sign) of the lightest neutralino as a function of ν and λ, following from Eq. (8), for $\sin^2\theta_W=0.22$, $\alpha=0.47$, and $\sin 2\theta_v=0.4$. In accordance with the solutions (b), (d), and (f) of Sec. III A there are zero-mass contour lines for $\lambda=0$, $\nu=0$, and the hyperbola corresponding to Eq. (20). In Fig. 6(a) there appears another zero line at $\lambda\approx 0.1$ and $\nu>0$ which in reality is a discontinuity where the eigenvalue changes from positive to negative values, jumping from one solution to the other. Only in the domain inside these zero lines the lowest mass eigenvalue is negative. Furthermore one can see from Fig. 6(a) that the lightest neutralino has a mass smaller than $\frac{1}{2}m_Z$ for $\lambda\leq 1$ and all ν, and for $|\nu|<0.8$ and all λ (for $\sin 2\theta_v=0.4$). For illustration, we show in Fig. 6(b) the corresponding three-dimensional plot.[19]

Figure 7(a) is a contour plot in λ and ν for the mass eigenvalue (including the sign) of the second lightest neutralino for $\alpha=0.47$ and $\sin 2\theta_v=0.4$. There is a line of discontinuity between positive and negative values reflecting the fact that the second lightest mass eigenvalue jumps from one branch of the solution to the other. The zero line near $\lambda\approx 0.1$ and $\nu>0$ is the same as that in Fig. 6(a) corresponding to a degeneracy of the two lightest states. As one can see the second lightest neutralino has a mass smaller than m_Z for $\lambda<1$ and all ν, and for $|\nu|<0.8$ and all λ (for $\sin 2\theta_v=0.4$). Again we also show the corresponding three-dimensional plot in Fig. 7(b).

V. APPROXIMATE SOLUTIONS

In the following we shall present approximation formulas for the neutralino masses for any value of the parameters α, λ, ν, and $\tan\theta_v$. Depending on the region of parameter space we shall start from the complete solutions given in Sec. III B and then apply perturbation theory. We shall first give four different sets of approximation formulas and then indicate their range of applicability.

A. Expansion in $\sin^2\theta_W$ and $\sin 2(\theta_v - \pi/4)$

Here we start from the solution (i) of Sec. III B, Eqs. (22) and (23). It turns out that expanding in $\sin^2\theta_W$ works numerically better within a large range of λ ($\lambda < 10$) than expanding around $\lambda = 0$ (Ref. 5) or $\alpha = 1$. Similarly, one finds that the expansion in $\sin 2(\theta_v - \pi/4)$ to first order is applicable in a rather large range of ν and $\tan\theta_v$ ($|\nu| \le 10$, $0.1 \le \tan\theta_v \le 10$).

The mass matrix Y, Eq. (3), can be written as

$$Y = Y_0 - 2\nu \sin\epsilon \cos\epsilon \Sigma_1 - 2\nu \sin^2\epsilon \Sigma_3 + \lambda \sin\theta_W \cos\theta_W (1-\alpha)\Sigma_1' + \lambda \sin^2\theta_W (1-\alpha)\Sigma_3' , \tag{32}$$

where $\epsilon = \theta_v - \pi/4$ and Y_0 is the mass matrix for $\sin\theta_W = 0$ and $\sin 2\theta_v = 1$ ($\sin\epsilon = 0$), and the 4×4 matrices

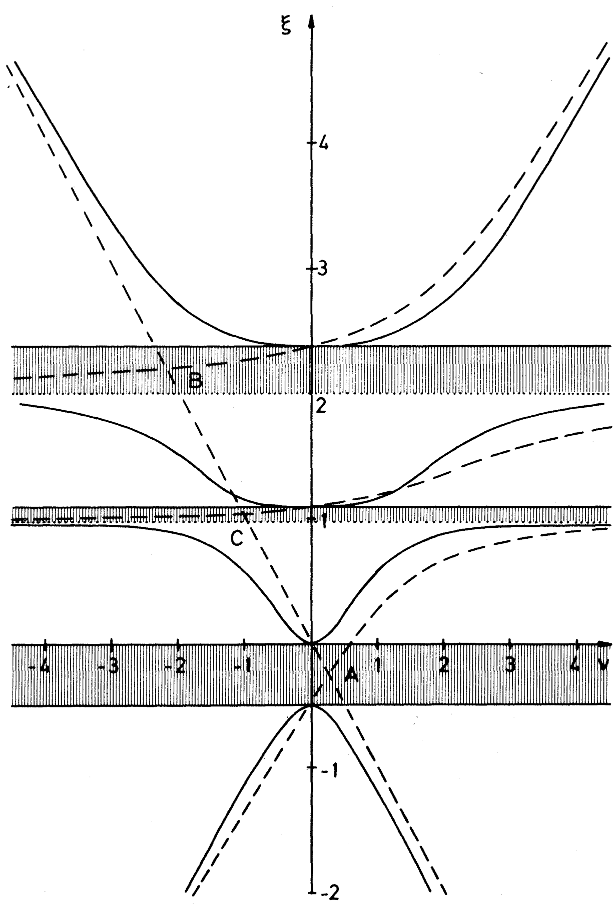

FIG. 3. ν dependence of solutions ξ of the mass eigenvalue Eq. (8) for $\lambda = 2$, $\alpha = 0.47$, $\sin^2\theta_W = 0.22$, $\sin 2\theta_v = 1$ (dashed line), and $\sin 2\theta_v = 0$ (solid line). Shaded areas indicate excluded mass regions. Crossing points A, B, C correspond to points of degeneracy.

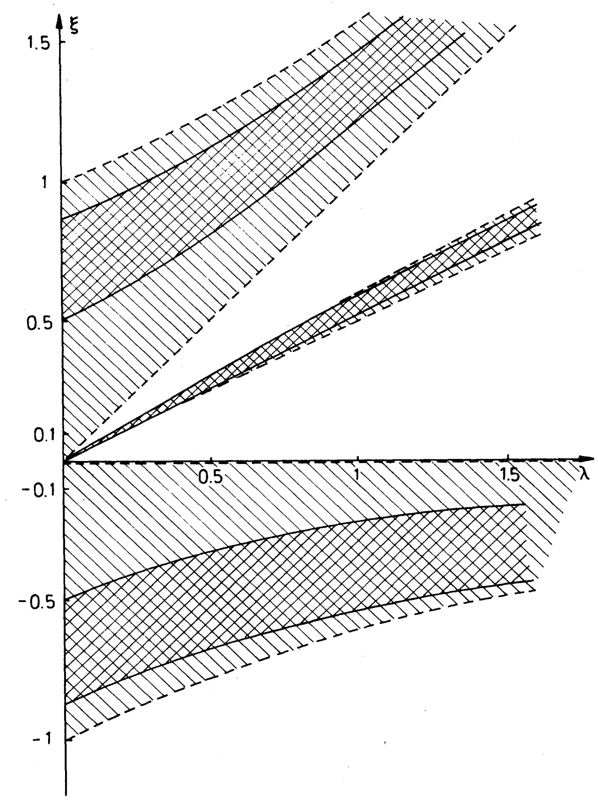

FIG. 4. λ dependence of excluded mass regions for $\alpha = 0.47$, $\sin^2\theta_W = 0.22$, $\sin 2\theta_v = \sqrt{3}/2$ (///), and $\sin 2\theta_v = 0$ (\\\\\\).

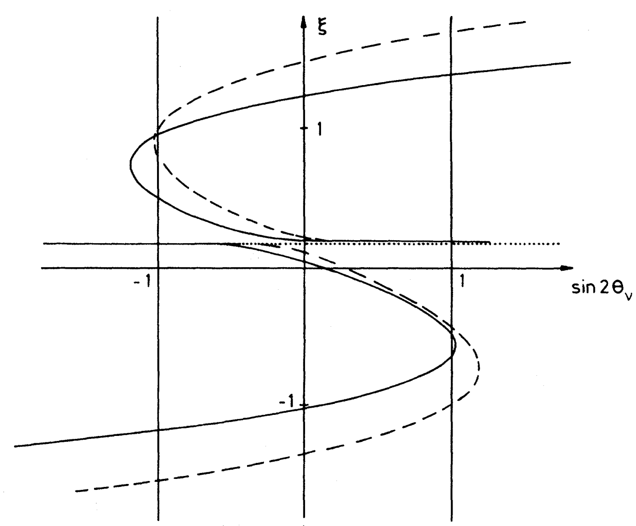

FIG. 5. $\sin 2\theta_v$ dependence of solution ξ of the mass eigenvalue Eq. (8) for $\alpha = 0.47$, $\sin^2\theta_W = 0.22$, $\lambda = 0.3$, $\nu = 0.5$ (solid line), and $\nu = 1$ (dashed line). The dotted line corresponds to $\xi = \lambda(\sin^2\theta_W + \alpha \cos^2\theta_W)$.

Σ_i and Σ_i' are

$$\Sigma_i = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_i \end{bmatrix}, \quad \Sigma_i' = \begin{bmatrix} \sigma_i & 0 \\ 0 & 0 \end{bmatrix},$$

where σ_i are the Pauli spin matrices. Applying perturbation theory to Eq. (32) yields

$$\begin{aligned}
\xi_1 &= \lambda \left[\alpha + \frac{(1-\alpha)\sin^2\theta_W}{1+\lambda(1-\alpha)(\alpha\lambda-\nu)} \right], \\
\xi_2 &= \xi_2^0 - \frac{1}{\sqrt{(\lambda-\nu)^2+4}} \left[\frac{(1-\alpha)\lambda\sin^2\theta_W}{\xi_2^0-\xi_1^0} + \frac{\nu(1-\sin 2\theta_v)}{\xi_2^0-\xi_4^0} \right], \\
\xi_3 &= \xi_3^0 + \frac{1}{\sqrt{(\lambda-\nu)^2+4}} \left[\frac{(1-\alpha)\lambda\sin^2\theta_W}{\xi_3^0-\xi_1^0} + \frac{\nu(1-\sin 2\theta_v)}{\xi_3^0-\xi_4^0} \right], \\
\xi_4 &= -\nu \frac{2\nu(\lambda+\nu)-\sin 2\theta_v}{2\nu(\lambda+\nu)-1} .
\end{aligned} \tag{33}$$

Here ξ_i^0, $i=1,\ldots,4$, are the eigenvalues of Y_0 and are given by $\xi_1^0=\alpha\lambda$, $\xi_{2,3}^0=\frac{1}{2}[\lambda+\nu\pm\sqrt{(\lambda-\nu)^2+4}]$, $\xi_4^0=-\nu$, see Eq. (23). In the case of degeneracy there appear singularities in Eq. (33), corresponding to the crossing points A–D in Fig. 1. Here the formulas Eq. (33) are obviously not applicable. One has to apply degenerate perturbation theory. For the present case this is worked out in the Appendix.

B. Expansion in $\sin^2\theta_W$ and ν

Expanding around $\nu=0$ provides approximation formulas for $|\nu|<1$ ("light-Higgsino scenario") valid for all values of $\tan\theta_v$. Starting from the solution (ii) of Sec. III B and expanding in $\sin^2\theta_W$ and ν we get

$$\begin{aligned}
\xi_1 &= \lambda(\sin^2\theta_W+\alpha\cos^2\theta_W) - \alpha\lambda\frac{[\lambda(1-\alpha)\sin\theta_W\cos\theta_W]^2}{\alpha(1-\alpha)\lambda^2+1}, \\
\xi_2 &= \xi_2^0 - \lambda(1-\alpha)\cos^2\phi\,\sin^2\theta_W + \nu\sin 2\theta_v \sin^2\phi + \frac{[\lambda(1-\alpha)\cos\phi\,\sin\theta_W\cos\theta_W]^2}{\xi_2^0-\xi_1^0} \\
&\quad + \frac{(\nu\sin 2\theta_v\sin\phi\,\cos\phi)^2}{\xi_2^0-\xi_3^0} + \frac{(\nu\cos 2\theta_v\sin\phi)^2}{\xi_2^0-\xi_4^0}, \\
\xi_3 &= \xi_3^0 - \lambda(1-\alpha)\sin^2\phi\,\sin^2\theta_W + \nu\sin 2\theta_v\cos^2\phi + \frac{[\lambda(1-\alpha)\sin\phi\,\sin\theta_W\cos\theta_W]^2}{\xi_3^0-\xi_1^0} \\
&\quad + \frac{(\nu\sin 2\theta_v\sin\phi\,\cos\phi)^2}{\xi_3^0-\xi_2^0} + \frac{(\nu\cos 2\theta_v\cos\phi)^2}{\xi_3^0-\xi_4^0}, \\
\xi_4 &= -\nu\sin 2\theta_v + \lambda(\nu\cos 2\theta_v)^2 ,
\end{aligned} \tag{34}$$

where $\xi_1^0=\alpha\lambda$, $\xi_{2,3}^0=\frac{1}{2}(\lambda\pm\sqrt{\lambda^2+4})$, $\xi_4^0=0$, and

$$\sin\phi = \frac{1}{\sqrt{2}}\left[1-\frac{\lambda}{\sqrt{\lambda^2+4}}\right]^{1/2},$$

$$\cos\phi = \frac{1}{\sqrt{2}}\left[1+\frac{\lambda}{\sqrt{\lambda^2+4}}\right]^{1/2}.$$

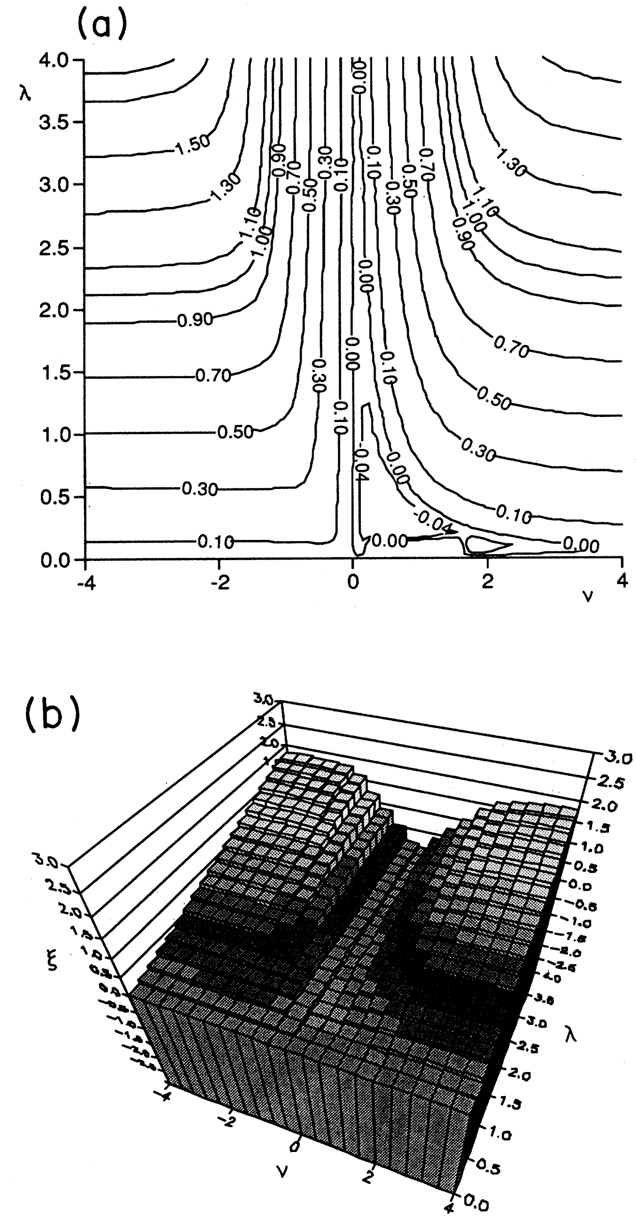

FIG. 6. Mass eigenvalue ξ of the lightest neutralino in units of m_Z (including the sign) as a function of ν and λ, following from Eq. (8), for $\alpha=0.47$, $\sin^2\theta_W=0.22$, $\sin 2\theta_v=0.4$. (a) Contour plot and (b) three-dimensional plot.

C. Expansion in $1/\lambda$ or $1/\nu$

Taking solution (vi) of Sec. III B, Eqs. (27) and (28), as the zeroth approximation perturbation theory yields[7]

$$\begin{aligned}
\xi_1&=\alpha\lambda+\sin^2\theta_W\frac{\alpha\lambda+\nu\sin2\theta_v}{(\alpha\lambda)^2-\nu^2}\ ,\\
\xi_2&=\lambda+\cos^2\theta_W\frac{\lambda+\nu\sin2\theta_v}{\lambda^2-\nu^2}\ ,\\
\xi_3&=\nu+(1+\sin2\theta_v)\frac{\nu-\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)}{2(\alpha\lambda-\nu)(\lambda-\nu)}\ ,\\
\xi_4&=-\nu-(1-\sin2\theta_v)\frac{\nu+\lambda(\sin^2\theta_W+\alpha\cos^2\theta_W)}{2(\alpha\lambda+\nu)(\lambda+\nu)}\ .
\end{aligned}\tag{35}$$

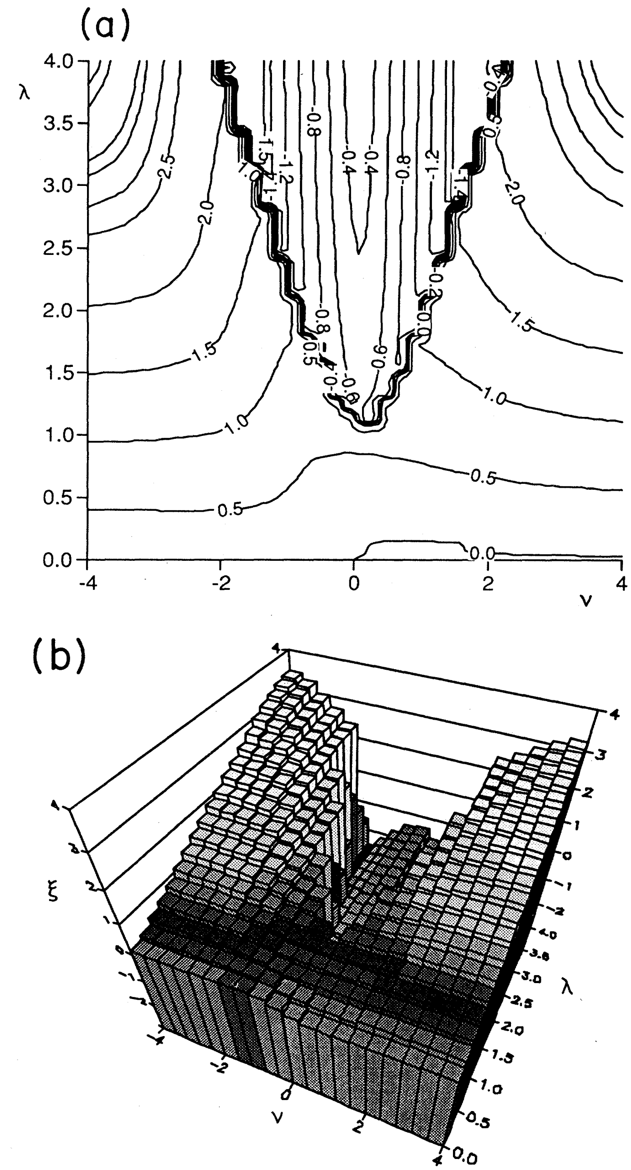

FIG. 7. Mass eigenvalue of the second lightest neutralino in units of m_Z (including the sign) as a function of ν and λ, following from Eq. (8), for $\alpha=0.47$, $\sin^2\theta_W=0.22$, $\sin2\theta_v=0.4$. (a) contour plot and (b) three-dimensional plot.

These formulas are valid for all values of $\tan\theta_v$. They give a good approximation for $\lambda\geq2$ ($|\nu|>0.5$) and $\nu<-2$ (λ arbitrary) and $\nu>2$ ($\lambda>0.5$) if there is no degeneracy. For the latter case we refer to the Appendix.

D. The "light Z-ino–Higgsino scenario"

If the parameters α, λ, ν, and $\sin^2\theta_v$ are such that the condition Eq. (20) is approximately satisfied it is useful to introduce the quantity

$$\delta=\alpha\lambda\nu-\sin2\theta_v[\alpha+(1-\alpha)\sin^2\theta_W]\ . \tag{36}$$

Starting from the solution (v)(a) of Sec. III B, Eqs. (24) and (25a)–(25c), an expansion in $\sin^2\theta_W$ and δ yields numerically satisfactory mass eigenvalues if $|\delta|<0.3$. They are given by

$$\begin{aligned}
\xi_1&=\alpha\lambda+\frac{(1-\alpha)\lambda(\alpha+\nu^2)}{\alpha[1+\nu^2+\alpha(1-\alpha)\lambda^2]}\sin^2\theta_W\ ,\\
\xi_3&=\frac{\nu\delta}{\alpha(1+\nu^2)}\left[1-\frac{(1-\alpha)(\alpha+\nu^2)}{\alpha^2(1+\nu^2)}\sin^2\theta_W-\frac{\nu\lambda\delta}{\alpha(1+\nu^2)^2}\right]\ ,\\
\xi_i&=\xi_i^{(0)}+a_i\left[\sin^2\theta_W+\frac{\nu(\xi_i^{(0)}-\alpha\lambda)\delta}{(1-\alpha)\lambda(\alpha+\nu^2)\xi_i^{(0)}}\right]\ ,\quad i=2,4
\end{aligned}\tag{37}$$

with

$$a_i=\frac{(1-\alpha)\lambda(\alpha+\nu^2)\xi_i^{(0)}}{\alpha\{(1-2\alpha)\lambda(1+\nu^2)+[(1-\alpha)\lambda^2+2(1+\nu^2)]\xi_i^{(0)}\}}\ ,\quad i=2,4\ ,$$

and $\xi_{2,4}^{(0)}=\frac{1}{2}[\lambda\pm\sqrt{\lambda^2+4\nu^2+4}]$, see Eq. (24).

Again these formulas are only valid if there is no degeneracy. In the degenerate case one has to proceed as outlined in the Appendix.

VI. RANGE OF VALIDITY OF THE APPROXIMATIONS

In Table I we show which of the approximation formulas of Sec. V can be appropriately used in the various ranges of λ and ν. As a criterion we have required a numerical accuracy of 20% for the mass eigenvalues, only for eigenvalues $|\xi_i|<0.2$ we allow bigger errors. Without loss of generality one can take $\lambda\geq0$ and $0\leq\tan\theta_v\leq1$.

If $0.1<\tan\theta_v\leq1$, then Eq. (33) is actually applicable in a larger range of λ and ν than indicated in Table I, except in the vicinity of points of degeneracy. How to proceed in this case we shall discuss below.

The other formulas, Eqs. (34), (35), and (37), are valid for all $\tan\theta_v$. The applicability of Eq. (37) depends on how well condition (20) is satisfied. Acceptable results are obtained for $|\delta|\leq0.3$, with δ defined in Eq. (36). In the domain $\nu>2$ and $0<\lambda<0.7$ Eq. (37) gives better results than Eq. (35), because here the condition (20) is nearly satisfied. On the contrary, for $\nu<-2$ Eq. (35) works in the whole range $\lambda>0$.

The quality of the approximation formulas (33), (34),

TABLE I. Approximation formulas as given in Sec. V for the neutralino mass eigenvalues in the appropriate regions of $\tan\theta_v$, λ, and v [defined in Eq. (4)]. Equation (33) is applicable for $\tan\theta_v > 0.1$. Equation (37) is applicable if $|\delta| \leq 0.3$. δ is defined in Eq. (36).

	$v < -2$	$-3 < v < -0.5$	$\|v\| < 0.5$	$0.5 < v < 3$	$v > 2$
$0 < \lambda < 0.7$	Eq. (33) Eq. (35)	Eq. (33) Eq. (37)	Eq. (33) Eq. (34) Eq. (37)	Eq. (33) Eq. (37)	Eq. (33) Eq. (37)
$0.5 < \lambda < 2$	Eq. (33) Eq. (35)	Eq. (33)	Eq. (33) Eq. (34) Eq. (37)	Eq. (33) Eq. (37)	Eq. (33) Eq. (35) Eq. (37)
$\lambda > 2$	Eq. (35)	Eq. (33) Eq. (35)	Eq. (33) Eq. (37)	Eq. (33) Eq. (35)	Eq. (35)

(35), and (37) is illustrated in Figs. 8(a)–8(c), respectively, in the appropriate regions of v for $\lambda=1$, $\alpha=0.47$, $\sin2\theta_v=0.4$. As one can see the application of these formulas covers the whole range of v except near the points of degeneracy corresponding to the crossing points A,B,D in Fig. 1. At point C Eq. (33) works because it is not singular.

The intervals in v at the crossing points A,B,D where Eq. (33) is not applicable can be specified as follows:

$$A:\quad \left|\frac{\xi_3^0-\xi_4^0}{\xi_A}\right| \leq \frac{0.1}{\sin2\theta_v},\quad \sin2\theta_v \geq 0.2\ ,$$

$$B:\quad \left|\frac{\xi_2^0-\xi_4^0}{\xi_B}\right| \leq \frac{0.1}{\sin2\theta_v},\quad \sin2\theta_v \geq 0.2\ , \qquad (38)$$

$$D:\quad \left|\frac{\xi_1^0-\xi_3^0}{\xi_D}\right| \leq 0.4\ .$$

Here ξ_i^0, $i=1,\ldots,4$, is the zeroth approximation given in Eqs. (23), and $\xi_A=\frac{1}{2}(\lambda-\sqrt{\lambda^2+2})$, $\xi_B=\frac{1}{2}(\lambda+\sqrt{\lambda^2+2})$, $\xi_D=\alpha\lambda$. Within these intervals one can use formula (A1) of the Appendix [for illustration see the dotted lines in Fig. 8(a)]. A rough estimate, however, can also be obtained by the zeroth approximation. Furthermore, at the points A and D the variables λ and v may be such that formula (34) or (37) can be applied [see Figs. 8(b) and 8(c)]. It may also happen that point C lies within the interval of point B as given by Eqs. (38). Then one can use Eq. (A1) for point C.

Concerning the light Z-ino–Higgsino case, Eq. (37), it is noticeable that there is a rather stringent upper bound for the mass of the lightest chargino m_{C_1}. Assuming $\alpha=\frac{5}{3}\tan^2\theta_W$, we obtain

$$m_{C_1}^2 \leq m_W^2\{1+\sin2\theta_v+F(\delta) -[(1+\sin2\theta_v)^2+2(1+\sin2\theta_v)F(\delta)]^{1/2}\} \quad \text{for } F(\delta)\geq -\sin2\theta_v\ , \qquad (39)$$

$$m_{C_1}^2 \leq m_W^2\{1-\sin2\theta_v-F(\delta) -[(1-\sin2\theta_v)^2-2(1-\sin2\theta_v)F(\delta)]^{1/2}\} \quad \text{for } F(\delta)< -\sin2\theta_v\ ,$$

where

$$F(\delta)=\frac{3}{5}\left[\sin2\theta_v+\frac{\delta}{\sin^2\theta_W}\right]\ .$$

For example, for $\tan\theta_v=0.5$ and $|\delta|<0.2$ this gives $|m_{C_1}|<35$ GeV. Therefore, the corresponding region of parameter space may be experimentally excluded in the near future.

APPENDIX

As we have seen in Sec. III C in the case of solution (i), Eq. (23), there are four points of degeneracy $A-D$ (see Fig. 1). Near these points one has to apply degenerate perturbation theory. Starting from Eq. (23) as zeroth approximation one obtains

$$
\begin{aligned}
&\text{Point } A: \ \xi_{3,4}=-\nu_A\left[1\pm\frac{\cos2\theta_v}{(1+4\nu_A^2)^{1/2}}\right],\\
&\text{Point } B: \ \xi_{2,4}=-\nu_B\left[1\pm\frac{\cos2\theta_v}{(1+4\nu_B^2)^{1/2}}\right],\\
&\text{Point } C: \ \xi_{1,4}=-\nu_C-\frac{1}{2[2(1-\alpha)\lambda\nu-1]}((1-\alpha)\lambda\sin^2\theta_W+\nu(1-\sin2\theta_v)\\
&\qquad\qquad \mp\{[(1-\alpha)\lambda\sin^2\theta_W-\nu(1-\sin2\theta_v)]^2+8(1-\alpha)^2\lambda^2\nu^2\sin^2\theta_W\\
&\qquad\qquad \times(1-\sin2\theta_v)\})^{1/2},\\
&\text{Point } D: \ \xi_{1,3}=\alpha\lambda\pm\frac{(1-\alpha)\lambda\sin\theta_W\cos\theta_W}{\sqrt{1+(1-\alpha)^2\lambda^2}},
\end{aligned}
\tag{A1}
$$

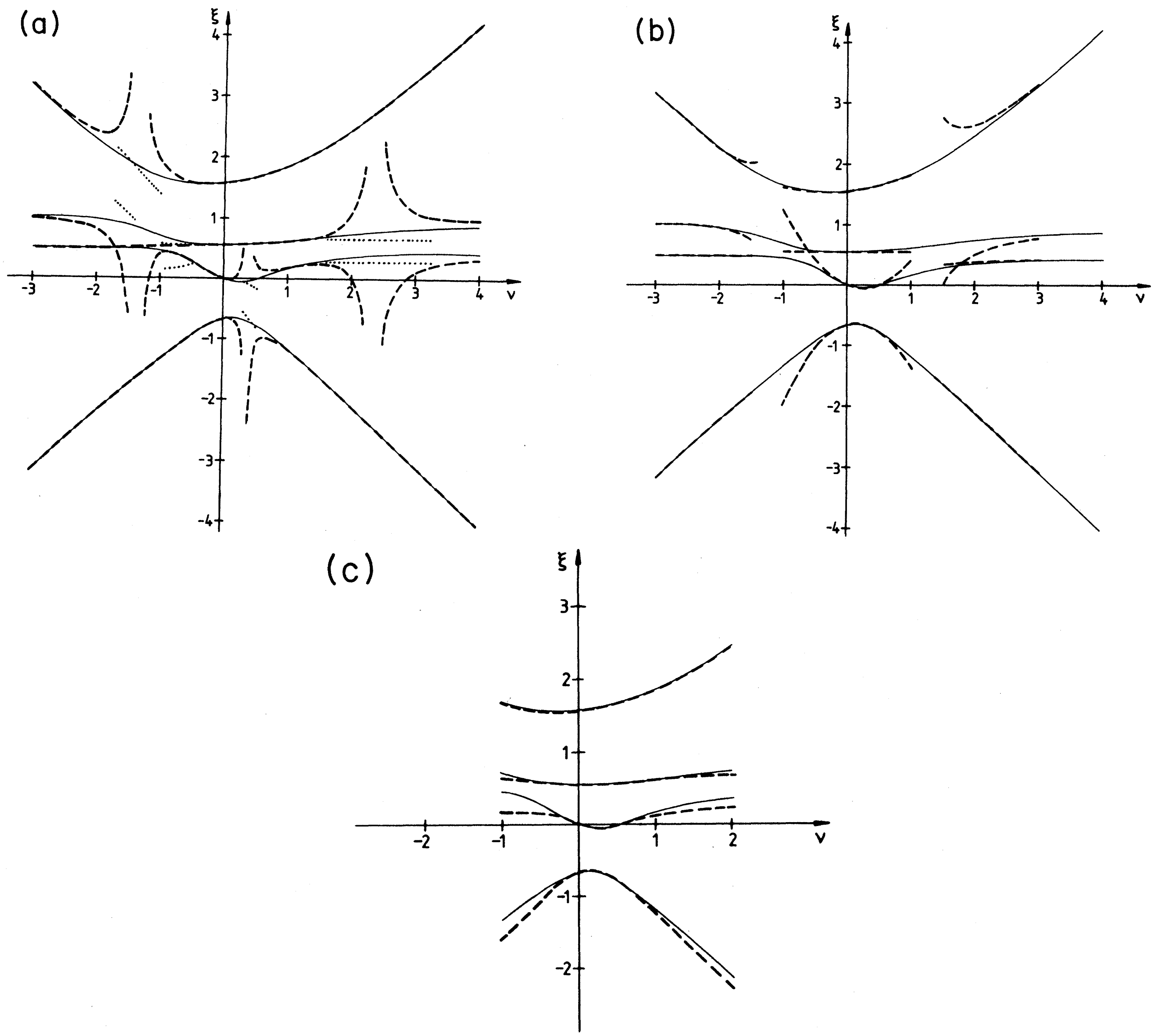

FIG. 8. Comparison of approximation formulas for the neutralino mass eigenvalues ξ (dashed line) with the exact solutions of the eigenvalue Eq. (8) (solid line) as a function of ν for $\alpha=0.47$, $\sin^2\theta_W=0.22$, $\lambda=1$, $\sin2\theta_v=0.4$. (a) Approximation formulas (33). In the vicinity of the points of degeneracy approximation formula (A1) (dotted line). (b) Approximation formulas (34) for $|\nu|<1$, and (35) for $|\nu|>1.5$. (c) Approximation formula (37).

where ν_A, ν_B, and ν_C are given in Eq. (29). We want to emphasize that Eq. (A1) gives the perturbed eigenvalues at $\nu=\nu_A$, ν_B, ν_C, and ν_D, respectively. This also holds for point C, although the situation here is more subtle. Equation (A1) gives the splitting of the two eigenvalues at $\nu=\nu_C$, if either $\sin\theta_W\neq 0$ or $\sin 2\theta_v\neq 1$. There is, however, still a crossing point of the two solutions ξ_1 and ξ_4, but at a position $\nu\neq\nu_C$. The degeneracy is completely removed only if both $\sin\theta_W\neq 0$ and $\sin 2\theta_v\neq 1$.

Similarly, in the case of solution (vi), Eq. (27), one has four points of degeneracy at $\nu=\pm\lambda$ and $\nu=\pm\alpha\lambda$. Degenerate perturbation theory yields the following splitting of the degenerate mass eigenvalues:

$$\begin{aligned}
\xi_{1,3}&=\alpha\lambda\pm\frac{1}{\sqrt{2}}\sin\theta_W\sqrt{1+\sin 2\theta_v}\quad\text{for } \nu=\alpha\lambda\ ,\\
\xi_{1,4}&=\alpha\lambda\pm\frac{1}{\sqrt{2}}\sin\theta_W\sqrt{1-\sin 2\theta_v}\quad\text{for } \nu=-\alpha\lambda\ ,\\
\xi_{2,4}&=\lambda\pm\frac{1}{\sqrt{2}}\cos\theta_W\sqrt{1-\sin 2\theta_v}\quad\text{for } \nu=-\lambda\ ,\\
\xi_{3,4}&=\lambda\pm\frac{1}{\sqrt{2}}\cos\theta_W\sqrt{1+\sin 2\theta_v}\quad\text{for } \nu=\lambda\ .
\end{aligned}\tag{A2}$$

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