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One-loop–induced fermion masses and exotic interactions in a standard-model context

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The standard $SU(2)\times U(1)$ electroweak gauge model is extended to include a fourth generation of quarks and leptons as well as four extra Higgs-boson doublets. With the implementation of a Z_4 discrete symmetry, which is softly broken down to Z_2, realistic radiative quark and lepton masses for the first two generations can be obtained. Assuming that the third and fourth *lepton* generations are odd under the unbroken Z_2, we discuss a number of interesting phenomenological consequences such as a longer τ lifetime and some possible anomalous τ interactions. We also single out some processes involving fourth-generation quarks and leptons which cannot be confused experimentally with those of the standard model.

I. INTRODUCTION

In the standard $SU(2)\times U(1)$ electroweak gauge model,[1] quark and lepton masses are proportional to their Yukawa couplings to the Higgs-boson doublet. The wide range of actual masses, from less than 18 eV for the electron neutrino to over 44 GeV for the yet undiscovered t quark,[2] is then attributed to the wide range of Yukawa couplings. On the other hand, the electroweak mass scale is of the order $v=(\sqrt{2}G_F)^{-1/2}\approx 246$ GeV, where G_F is the Fermi weak coupling, and one wonders why all masses should not be greater than at least a few GeV. Perhaps all Yukawa couplings are of the order 10^{-2} to unity, but the light quarks and leptons do not pick up their masses directly, but through radiative corrections. This can only be achieved if these fermions are prevented from coupling to the Higgs-boson doublet with the nonzero vacuum expectation value by some symmetry which is then softly or spontaneously broken. Recently, one of us proposed[3] exactly such a model, based only on the standard $SU(2)\times U(1)$ gauge group and using only the conventional representations of fermions and bosons. Other models[4–8] which obtain radiative masses all tend to be somewhat more complicated. Together with a recent proposal[9] for radiative Majorana-neutrino masses through double W exchange, we now have a simple specific renormalizable theory of radiative quark and lepton masses.

In Sec. II the model is described in detail. In particular, we choose to discuss it in the context of soft instead of spontaneous symmetry breaking, the latter having been formulated already in Ref. 3. In Sec. III the quark and lepton mass matrices of this model are analyzed to show how the observed masses and mixing angles are related to one another. In Sec. IV the induced off-diagonal fermion couplings of this model are derived and shown to be much greater than those of the standard model. In Secs. V and VI, respectively, we discuss the phenomenological consequences of this model on particles of the standard three-generation model as well as the proposed new particles. Finally in Sec. VII there are some concluding remarks.

II. DESCRIPTION OF MODEL

The proposed model[3] is based on the standard $SU(2)\times U(1)$ electroweak gauge group with all fermions and bosons in conventional representations. There are four generations of quarks and leptons, as well as five Higgs-boson doublets. There is also a discrete Z_4 symmetry which is softly broken down to Z_2 in the Higgs-boson sector by explicit gauge-invariant mass terms. Let the fourth-generation quarks be h and y; then, under Z_4, we have

$$\begin{aligned} &1:\ (u,d)_L,(c,s,)_L,(t,b)_L,t_R,b_R\ , \\ &\omega:\ (h,y)_L,h_R,y_R\ , \\ &\omega^2:\ u_R,d_R,s_R,c_R\ , \end{aligned} \tag{2.1}$$

where 1, ω, ω^2, and ω^3 are the elements of Z_4 with $\omega^4=1$. Parallel assignments are of course possible for the leptons, but a more interesting scenario is to group together the third and fourth generations as indicated below:

$$\begin{aligned} &1:\ (\nu_e,e)_L,(\nu_\mu,\mu)_L\ , \\ &\omega:\ (\nu_\tau,\tau)_L,(N,E)_L,\tau_R,N_R,E_R\ , \\ &\omega^2:\ e_R,\mu_R,\nu_R\ . \end{aligned} \tag{2.2}$$

The Higgs-boson sector consists of five doublets, each of the form (ϕ^+,ϕ^0). Under Z_4, we have

$$\begin{aligned} &1:\ \Phi_1\ , \\ &\omega^3:\ \Phi_2^{(1)},\Phi_2^{(2)}\ , \\ &\omega:\ \Phi_3^{(1)},\Phi_3^{(2)}\ . \end{aligned} \tag{2.3}$$

As a result, the allowed Yukawa interactions of this model are given by

$$
\begin{aligned}
-L_y = {} & f_1(\overline{h,y})_L h_R \tilde{\Phi}_1 + f_2(\overline{h,y})_L y_R \Phi_1 + f_3(\overline{t,b'})_L t_R \tilde{\Phi}_1 + f_4(\overline{t',b})_L b_R \Phi_1 \\
& + f_5(\overline{h,y})_L t_R \tilde{\Phi}_2 + f_6(\overline{h,y})_L b_R \Phi_3 + f_7 \bar{q}_L h_R \tilde{\Phi}_3 + f_8 \bar{q}_L y_R \Phi_2 \\
& + f_9^c(\overline{h,y})_L c_R \tilde{\Phi}_3 + f_9^u(\overline{h,y})_L u_R \tilde{\Phi}_3 + f_{10}^s(\overline{h,y})_L s_R \Phi_2 + f_{10}^d(\overline{h,y})_L d_R \Phi_2 \\
& + f_1^E(\overline{N,E})_L N_R \tilde{\Phi}_1 + f_1^\tau(\overline{\nu_\tau,\tau})_L N_R \tilde{\Phi}_1 + f_2^E(\overline{N,E})_L E_R \Phi_1 + f_2^\tau(\overline{\nu_\tau,\tau})_L \tau_R \Phi_1 \\
& + f_7' \bar{l}_L N_R \tilde{\Phi}_3 + f_8^E \bar{l}_L E_R \Phi_2 + f_8^\tau \bar{l}_L \tau_R \Phi_2 + f_9^E(\overline{N,E})_L \nu_R \tilde{\Phi}_3 + f_9^\tau(\overline{\nu_\tau,\tau})_L \nu_R \tilde{\Phi}_3 \\
& + f_{10}^E(\overline{N,E})_L l_R \Phi_2 + f_{10}^\tau(\overline{\nu_\tau,\tau})_L l_R \Phi_2 + \text{H.c.} \,, \qquad (2.4)
\end{aligned}
$$

where q_L denotes any one of the left-handed quark doublets of the first three generations, l_L is short for $(\nu_e,e)_L$ and $(\nu_\mu,\mu)_L$, l_R for e_R, and μ_R, $t_L(b_L)$ is defined as that which couples to $t_R(b_R)$ through $\tilde{\Phi}_1(\Phi_1)$, $b_L'(t_L')$ as that which couples to $t_L(b_L)$ through the W boson and thus not necessarily the same as $b_L(t_L)$, $\tilde{\Phi} \equiv i\sigma_2\Phi^* = (\bar{\phi}^0, -\phi^-)$, and the superscripts on $\Phi_{2,3}$ are omitted.

As ϕ_1^0 acquires a nonzero vacuum expectation value $v/\sqrt{2}$, thereby breaking SU(2)×U(1) but not Z_4, the h, y, t, and b quarks as well as the N, E, and τ leptons will become massive. The other quarks and leptons are massless at this stage, but will pick up radiative masses if we break Z_4 softly down to Z_2 as described below. Consider the Higgs potential of this model

$$
\begin{aligned}
V = {} & \sum_i \mu_i^2 \Phi_i^\dagger \Phi_i + (\tilde{\mu}^2 \Phi_2^\dagger \Phi_3 + \text{H.c.}) \\
& + \sum_{i,j} \lambda_{ij} (\Phi_i^\dagger \Phi_i)(\Phi_j^\dagger \Phi_j) \\
& + \sum_{i \neq j} \eta_{ij} (\Phi_i^\dagger \Phi_j)(\Phi_j^\dagger \Phi_i) \\
& + [r(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_3) + t(\Phi_2^\dagger \Phi_3)^2 + \text{H.c.}] \,. \qquad (2.5)
\end{aligned}
$$

The $\tilde{\mu}^2$ term is a gauge-invariant mass term which transforms as ω^2 under Z_4. Let $\mu_1^2 < 0$ and $\mu_{2,3}^2 > 0$, then

$$\mu_1^2 + \lambda_{11} v^2 = 0 \,. \qquad (2.6)$$

Let $\phi_1^0 \to (v + H_1 + i\chi_1)/\sqrt{2}$, $\phi_{2,3}^0 \to (H_{2,3} + i\chi_{2,3})/\sqrt{2}$, then the mass terms of V are given by

$$
\begin{aligned}
V^{(2)} = {} & \lambda_{11} v^2 H_1^2 + (\mu_2^2 + \lambda_{12} v^2)\phi_2^- \phi_2^+ + (\mu_3^2 + \lambda_{13} v^2)\phi_3^- \phi_3^+ + \tilde{\mu}^2(\phi_2^- \phi_3^+ + \phi_3^- \phi_2^+) \\
& + \tfrac{1}{2}(\mu_2^2 + \lambda_{12} v^2 + \eta_{12} v^2)(H_2^2 + \chi_2^2) + \tfrac{1}{2}(\mu_3^2 + \lambda_{13} v^2 + \eta_{13} v^2)(H_3^2 + \chi_3^2) + \tilde{\mu}^2(H_2 H_3 + \chi_2 \chi_3) + \tfrac{1}{2} r v^2 (H_2 H_3 - \chi_2 \chi_3) \,, \qquad (2.7)
\end{aligned}
$$

where v, $\tilde{\mu}^2$, and r have been assumed real for simplicity. Therefore, mixing occurs between $\phi_2^\pm$ and $\phi_3^\pm$, H_2 and H_3, as well as χ_2 and χ_3. Specifically, we show in Fig. 1 the diagrams leading to radiative masses in the (u,c,t) sector. Let (θ, m_1, m_2), $(\theta_R, m_{R1}, m_{R2})$, and $(\theta_I, m_{I1}, m_{I2})$ be the mixing angle and mass eigenvalues of the $(\phi_2^\pm, \phi_3^\pm)$, (H_2, H_3), and (χ_2, χ_3) matrices, respectively, then each radiative mass is of the form

$$
\begin{aligned}
m_{\text{rad}} = {} & \left[\frac{f_8 f_9 \sin 2\theta}{32\pi^2} \right] m_y [F(m_1, m_y) - F(m_2, m_y)] \\
& + \left[\frac{f_7 f_9}{32\pi^2} \right] m_h [\sin^2\theta_R F(m_{R1}, m_h) + \cos^2\theta_R F(m_{R2}, m_h) - \sin^2\theta_I F(m_{I1}, m_h) - \cos^2\theta_I F(m_{I2}, m_h)] \,, \qquad (2.8)
\end{aligned}
$$

where the function F is given by

$$F(a,b) = \frac{a^2 \ln(a^2/b^2)}{a^2 - b^2} \,. \qquad (2.9)$$

From Eq. (2.7), it is easily seen that if $\tilde{\mu}^2 = 0$, then $\theta = 0$, $\theta_R = -\theta_I$, $m_{R1} = m_{I1}$, $m_{R2} = m_{I2}$, so $m_{\text{rad}} = 0$ in Eq. (2.8) as expected. Also, if $r = 0$, then $\theta_R = \theta_I$, $m_{R1} = m_{I1}$, $m_{R2} = m_{I2}$, so the m_h term vanishes. As a numerical exercise, let $f_8 f_9 / 4\pi = 0.2$, $\sin 2\theta = 0.5$, $m_y = 250$ GeV, and $F(m_1, m_y) - F(m_2, m_y) = 0.75$, then $m_{\text{rad}} = 1.5$ GeV$= m_c$ if we also assume that the m_h term contributes as much as the m_y term.

The same mechanism is of course also applicable to the (d,s,b) sector. We note in particular that if there is only one Φ_3, then only a linear combination of u_R and c_R will enter in Fig. 1, and we can simply redefine that as being c_R. Hence we need two Φ_3's and two Φ_2's as stated in Eq. (2.3) if we want m_u and m_d to be nonzero as well. On the other hand, since both τ and E transform as ω under Z_4, the Yukawa couplings of e_R to τ_L and E_L cannot both be rotated away even if there is only one Φ_2. This leads us to speculate on the intriguing possibility of keeping only one Φ_2 and one Φ_3, and having $m_u = m_d = 0$ in the weak-interaction Lagrangian, but $m_u \neq 0$ and $m_d \neq 0$ through nonperturbative mass renormalization by instantons in quantum chromodynamics.[10] In the leptonic sector, e and μ have nonzero radiative masses through their Yukawa couplings to τ, E, and N. One linear combination of ν_τ and N_L pairs up with N_R and gets a tree-level Dirac mass, whereas N_R itself can have a gauge-invariant

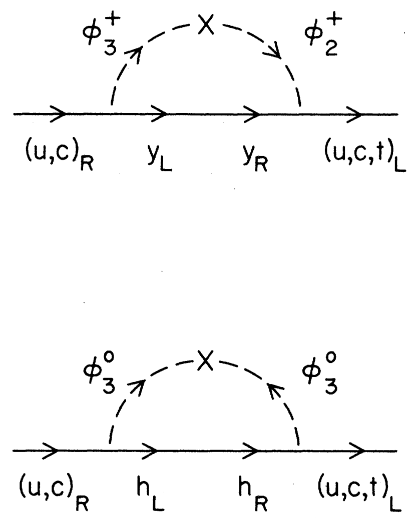

FIG. 1. Mechanism for radiative masses in the (u,c,t) sector.

Majorana mass [which transforms as ω^2 under Z_4, but it is an allowed soft term just as $\tilde{\mu}^2$ in Eq. (2.5)]. The other linear combination of ν_τ and N_L is massless at this stage, but will pick up a small Majorana mass through double W exchange.[9] Similarly, one linear combination of ν_e and ν_μ pairs up with ν_R and gets a *radiative* Dirac mass, whereas ν_R itself can have a Z_4-allowed gauge-invariant Majorana mass. The other linear combination then also picks up a mass through double W exchange.

III. QUARK AND LEPTON MASS MATRICES

Consider the doublet $(t,b')_L$. It is defined to be that which couples to t_R through $\tilde{\Phi}_1$ in Eq. (2.4). Of the two remaining q_L doublets $(c,s')_L$ and $(u,d')_L$, we arbitrarily define $(u,d')_L$ to be that which does not couple to h_R through $\tilde{\Phi}_3^{(1)}$. Similarly, $(u',d)_L$ is defined to be that which does not couple to y_R through $\Phi_2^{(1)}$. The charged current of this model is then of the form

$$\begin{aligned} J &= \bar{u}_L\gamma d'_L + \bar{c}_L\gamma s'_L + \bar{t}_L\gamma b'_L \\ &= \bar{u}'_L\gamma d_L + \bar{c}'_L\gamma s_L + \bar{t}'_L\gamma b_L \\ &= (\overline{u,c,t})_L \gamma U_0 \begin{bmatrix} d \\ s \\ b \end{bmatrix}_L , \end{aligned} \tag{3.1}$$

where U_0 is a 3×3 unitary matrix. We also arbitrarily define u_R and d_R to be those which do not couple to Q_L through $\tilde{\Phi}_3^{(1)}$ and $\Phi_2^{(1)}$, respectively. In this basis, we have

$$-L_M = (\overline{u,c,t})_L M_1 \begin{bmatrix} u \\ c \\ t \end{bmatrix}_R + (\overline{d,s,b})_L M_2 \begin{bmatrix} d \\ s \\ b \end{bmatrix}_R + \text{H.c.} , \tag{3.2}$$

where $M_{1,2}$ are 3×3 mass matrices to be discussed in detail later. Let

$$(U_L^{(1,2)})^{-1} M_{1,2} U_R^{(1,2)} = M_{1,2}^{(\text{diag})} , \tag{3.3}$$

where U_L and U_R are unitary matrices, then the usual Kobayashi-Maskawa matrix[11] is given in this model by

$$U_{\text{KM}} = (U_L^{(1)})^{-1} U_0 U_L^{(2)} . \tag{3.4}$$

Let us consider in detail the (d,s,b) mass matrix M_2. With $\langle \phi_1^0 \rangle = v/\sqrt{2}$ alone,

$$M_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_4 v/\sqrt{2} \end{bmatrix} . \tag{3.5}$$

In this approximation, only the b quark has a nonzero mass. With the addition of $\Phi_2^{(1)}$ and $\Phi_3^{(1)}$, the second column is filled with nonzero radiative masses analogous to that of Eq. (2.8), but the first column will still be empty. To fill up both the first and second columns, we need to add $\Phi_2^{(2)}$ and $\Phi_3^{(2)}$. We now have

$$M_2 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ x & x & f_4 v/\sqrt{2} \end{bmatrix} , \tag{3.6}$$

where x denotes a nonzero radiative mass. Without loss of generality, we can first rotate away the $\bar{b}_L d_R$ entry by redefining d_R and s_R, and then rotate away the $\bar{d}_L s_R$ entry by redefining d_L and s_L. Hence we obtain

$$M'_2 = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ 0 & d & f \end{bmatrix} , \tag{3.7}$$

where $f \equiv f_4 v/\sqrt{2}$, and a,b,c,d are radiative mass terms. Assuming that M'_2 is real for simplicity, we first diagonalize the 2×2 submatrix spanning s and b. Let

$$\begin{bmatrix} c_L & s_L \\ -s_L & c_L \end{bmatrix} \begin{bmatrix} c & 0 \\ d & f \end{bmatrix} \begin{bmatrix} c_R & -s_R \\ s_R & c_R \end{bmatrix} = \begin{bmatrix} m'_2 & 0 \\ 0 & m_3 \end{bmatrix} , \tag{3.8}$$

where $c_{L,R} \equiv \cos\theta_{L,R}$ and $s_{L,R} \equiv \sin\theta_{L,R}$. Assuming that $c^2 \ll d^2, f^2$, we find $\tan\theta_L \approx cd/(f^2+d^2)$, $\tan\theta_R \approx d/f$, $m'_2 \approx cf/\sqrt{f^2+d^2}$, and $m_3 \approx \sqrt{f^2+d^2}$. We now have

$$M''_2 = \begin{bmatrix} a & 0 & 0 \\ bc_L & m'_2 & 0 \\ -bs_L & 0 & m_3 \end{bmatrix} \tag{3.9}$$

and we can diagonalize the 2×2 submatrix spanning d and s in the same way. Let

$$\begin{bmatrix} c'_L & s'_L \\ -s'_L & c'_L \end{bmatrix} \begin{bmatrix} a & 0 \\ bc_L & m'_2 \end{bmatrix} \begin{bmatrix} c'_R & -s'_R \\ s'_R & c'_R \end{bmatrix} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} , \tag{3.10}$$

where $c'_{L,R} \equiv \cos\theta'_{L,R}$ and $s'_{L,R} \equiv \sin\theta'_{L,R}$. Assuming that $a^2 \ll b^2c_L^2, m_2'^2$, we find $\tan\theta'_L \approx abc_L/(m_2'^2 + b^2c_L^2)$, $\tan\theta'_R \approx bc_L/m'_2$, $m_1 \approx am'_2/(m_2'^2+b^2c_L^2)^{1/2}$, and $m_2 \approx (m_2'^2+b^2c_L^2)^{1/2}$. Hence

$$M_2''' = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ -bs_L c_R' & bs_L s_R' & m_3 \end{bmatrix} . \tag{3.11}$$

The two nonzero off-diagonal entries are now negligible in the approximation that we have taken. Therefore,

$$m_b^2 \approx f^2 + d^2 , \tag{3.12}$$

$$m_s^2 \approx b^2 + \frac{c^2 f^2}{f^2+d^2} , \tag{3.13}$$

$$m_d^2 \approx \frac{a^2}{1+\frac{b^2(f^2+d^2)}{c^2 f^2}} , \tag{3.14}$$

and the mixing between s and b from M_2' [Eq. (3.7)] is of the order

$$s_L \approx \frac{cd}{f^2+d^2} \le \left[\frac{d}{f}\right]\left[\frac{m_s}{m_b}\right] . \tag{3.15}$$

Assuming that s_L is of the order $(U_{\rm KM})_{cb}$ [Eq. (3.4)], which is determined by the measured b lifetime[2] to be about 0.046, and using $m_s \approx 0.15$ GeV, $m_b \approx 4.7$ GeV, we find $d/f \gtrsim 1$. This means that m_b [Eq. (3.12)] has comparable contributions from both the tree-level mass term f and the radiative mass term d. This is certainly acceptable for a mass of a few GeV. As for the corresponding mixing angle between c and t, it must be much smaller than m_c/m_t, because m_t is expected to be of the order 100 GeV, so the corresponding d/f ratio has to be small.

In the lepton sector, because of Eq. (2.2), only the first two generations are even under the unbroken Z_2, whereas both the third and fourth generations are odd. This is certainly allowed because there is no experimental evidence at present for any nonzero mixing between the third and the first or second lepton generations. From Eq. (2.4) it is clear that $m_\tau = f_2^\tau v/\sqrt{2}$, $m_E = f_2^E v/\sqrt{2}$, and the linear combination

$$\nu_4 = N_L \cos\theta_\tau + \nu_\tau \sin\theta_\tau , \tag{3.16}$$

where $\tan\theta_\tau = f_1^\tau / f_1^E$, pairs up with N_R to acquire a tree-level Dirac mass m_D. Together with a gauge-invariant Majorana mass m_R for N_R, a seesaw Majorana mass $\approx m_D^2/m_R$ for ν_4 is obtained. The orthogonal linear combination

$$\nu_3 = \nu_\tau \cos\theta_\tau - N_L \sin\theta_\tau \tag{3.17}$$

now picks up a small radiative Majorana mass through double W exchange.[9] If we assume that ν_4 is heavier than τ, then τ decay involves only ν_3, with a reduced coupling and thus a longer lifetime.[12] The current world average[2] of the τ lifetime is $(3.04\pm0.09)\times10^{-13}$ s, which is indeed slightly greater than the predicted value of $(2.80\pm0.07)\times10^{-13}$ s, based on a branching fraction[2] of $(17.5\pm0.4)\%$ for $\tau\to e\nu\bar\nu$ and using only the three-generation standard model.

For the first two generations, e and μ have radiative masses which are induced by m_τ, m_E, and m_D through the exchange and mixing of Φ_2 and Φ_3. One linear combination (ν_2) of ν_e and ν_μ pairs up with ν_R to acquire a radiative Dirac mass, and since there is also a gauge-invariant Majorana mass for ν_R, we have again a seesaw mass for ν_2. The orthogonal linear combination ν_1 then picks up a small radiative Majorana mass through double W exchange, in parallel with the case of ν_3 and ν_4. Neutrino oscillations are possible between ν_e and ν_μ, but not ν_τ.

IV. INDUCED OFF-DIAGONAL FERMION COUPLINGS

In the standard model, induced off-diagonal fermion couplings to all neutral bosons (γ, Z, gluon, and the single Higgs boson H) are all highly suppressed relative to the diagonal couplings. In this model, because the light quarks and leptons have radiative masses, these off-diagonal couplings are less suppressed and may have important phenomenological consequences. To see how this comes about, consider the tree-level mass term $(f_4 v/\sqrt{2})\bar b_L b_R$ in Eq. (3.5). If we go to the mass-eigenstate basis, then this term contributes to off-diagonal mass terms such as $\bar d_L s_R$, $\bar d_L b_R$, etc. However, since these terms should be zero by definition, they must be exactly canceled by the corresponding radiative terms. In this basis, the mass matrix connecting $(\bar d, \bar s, \bar b)_L$ to $(d,s,b)_R$ is given by

$$m_{ij} = \begin{cases} m_i, & i=j=d,s,b , \\ 0, & i\ne j \end{cases}$$
$$= (f_4 v/\sqrt{2})(U_L)^*_{3i}(U_R)_{3j} + \tfrac{1}{2} i m_h f_7^i f_{10}^j \sin 2\theta \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_h^2}\left[\frac{1}{k^2-m_1^2}-\frac{1}{k^2-m_2^2}\right]$$
$$+\tfrac{1}{2} i m_y f_8^i f_{10}^j \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_y^2}\left[\frac{\cos^2\theta_R}{k^2-m_{R1}^2}+\frac{\sin^2\theta_R}{k^2-m_{R2}^2}-\frac{\cos^2\theta_I}{k^2-m_{I1}^2}-\frac{\sin^2\theta_I}{k^2-m_{I2}^2}\right] , \tag{4.1}$$

where the radiative contribution is of the same form as Eq. (2.8), but we have left the loop-momentum integration undone for comparison with another expression to be discussed below.

Consider the couplings of H_1 to the d, s, and b quarks. In the standard model, the diagonal couplings are given by $2^{-1}gm_i/M_W$, and the off-diagonal couplings are one-loop induced by W exchange. For example, the $\bar s_L b_R$ coupling is given by[13]

$$g_{sb}=\frac{g^3}{64\pi^2}\frac{m_b}{M_W}\sum_i U_{is}^* U_{ib} x_i \left[\frac{3}{2}+\frac{m_H^2}{M_W^2}f(x_i)\right] , \tag{4.2}$$

where $i=u,c,t$; U is the Kobayashi-Maskawa matrix, $x_i=m_i^2/M_W^2$, and

$$f(x)=\frac{-(3-x)}{4(1-x)^2}+\frac{(2-x)x\ln x}{2(1-x)^3} . \tag{4.3}$$

In this model, there is an additional contribution due to scalar exchange given by

$$g'_{sb}=\frac{igm_h}{4M_W}f_7^s f_{10}^b \sin 2\theta \int \frac{d^4k}{(2\pi)^4}\frac{k^2+m_h^2}{(k^2-m_h^2)^2}\left[\frac{1}{k^2-m_1^2}-\frac{1}{k^2-m_2^2}\right]$$
$$+\frac{igm_y}{4M_W}f_8^s f_{10}^b \int \frac{d^4k}{(2\pi)^4}\frac{k^2+m_y^2}{(k^2-m_y^2)^2}\left[\frac{\cos^2\theta_R}{k^2-m_{R1}^2}+\frac{\sin^2\theta_R}{k^2-m_{R2}^2}-\frac{\cos^2\theta_I}{k^2-m_{I1}^2}-\frac{\sin^2\theta_I}{k^2-m_{I2}^2}\right]$$
$$+\text{terms due to cubic scalar couplings} . \tag{4.4}$$

Comparing against Eq. (4.1), we see that g'_{sb} is of the order $2^{-1}gm'_{sb}/M_W$, where

$$m'_{sb}=(f_4 v/\sqrt{2})(U_L)_{32}^*(U_R)_{33}\le O(m_s) . \tag{4.5}$$

Hence the additional effective $H_1\bar{q}_{iL}q_{jR}$ coupling is given by

$$g'_{ij}=\frac{gm_i}{2M_W}\zeta_{ij} , \tag{4.6}$$

where $\zeta_{ij}\le 0(1)$, and may well be greater than the standard-model contribution. The parameters ζ_{ij} depend mostly on the unitary matrices which diagonalize the quark mass matrices according to Eq. (3.3). For $i\ne j$, they can be very much less than one. For example, if $d\ll f$ in Eq. (3.7), then $\zeta_{23}\ll 1$, and if $b\ll c$, then $\zeta_{12}\ll 1$.

Consider B^0-$\bar{B}^0$ mixing. The most important contribution from H_1 exchange involves the effective $\bar{b}_L d_R$ coupling and we have

$$\frac{G_F f_B^2 B_B}{2\sqrt{2}}\frac{m_b^2}{m_H^2}\zeta_{bd}^2<\frac{\Delta m_B}{m_B} , \tag{4.7}$$

where f_B is the B-meson decay constant, and B_B is the value of the hadronic matrix element for B^0-$\bar{B}^0$ mixing normalized to unity in the limit of the vacuum-saturation approximation. Using[2] $\Delta m_B=(3.7\pm 1.0)\times 10^{-13}$ GeV, $m_B=5.28$ GeV, and $m_b=4.7$ GeV, we find

$$\left[\frac{f_B\sqrt{B_B}}{0.15\text{ GeV}}\right]^2\frac{\zeta_{bd}^2}{m_H^2}<4.3\times 10^{-8}\text{ GeV}^{-2} . \tag{4.8}$$

Hence $\zeta_{bd}<0.05$ if $m_H=250$ GeV. This is consistent with our analysis of the mass matrix in Sec. III, which requires only that ζ_{bs} be of the order 0.5. In the case of D^0-$\bar{D}^0$ mixing, using[2] $\Delta m_D<1.3\times 10^{-13}$ GeV and the same assumptions as before, we find $\zeta_{cu}<0.15$. Similarly, from K^0-$\bar{K}^0$ mixing, we obtain $\zeta_{sd}<0.7$. There are also box-diagram contributions to all of the above mass differences due to Φ_2 or Φ_3 exchange, as has already been pointed out in Ref. 3. Their typical values are all consistent with present experimental inputs.

The induced off-diagonal fermion couplings to the neutral gauge bosons γ, Z, and gluon are all constrained by the corresponding effective four-fermion interactions coming from the box diagrams. This means that they have natural upper bounds and in the present model, these can be saturated. In contrast, such couplings in the standard model are often very much smaller than the allowed upper bounds. Consider, for example, the $Z\bar{d}s$ coupling. In the limit of vanishing external momenta, it is given in the standard model by[14]

$$g_{Z\bar{d}s}=\frac{g^3}{64\pi^2\cos\theta_W}\sum_i U_{id}^* U_{is}x_i \times\left[\frac{-6+x_i}{1-x_i}-\frac{(2+3x_i)\ln x_i}{(1-x_i)^2}\right] , \tag{4.9}$$

where the notation follows that of Eq. (4.2). Here, there is an additional contribution of the order $gf^d f^s/(32\pi^2\cos\theta_W)$ which may be much greater than $g_{Z\bar{d}s}$ but is still small because $f^d f^s/4\pi$ is constrained by Δm_k to be less than about 10^{-3}. Similar statements are applicable in the case of $\bar{d}s$ couplings to the photon and the gluon.

As for the leptons, if we had allowed e and μ to mix with τ, then any $\bar{e}\mu$ coupling would be related to the corresponding off-diagonal tree-level mass, as discussed already for the quarks. This mass would be of the order $m_\tau(U_{L,R})_{31}^*(U_{R,L})_{32}$ which should not be less than about 10^{-5} GeV, but if we used that as an estimate, we would find a branching fraction for $\mu\to e\gamma$ some 3 orders of magnitude above its experimental upper limit of 5×10^{-11}. As it is, since τ is odd under Z_2, it cannot mix with e or μ. The mass matrix in the e-μ sector is purely radiative. Upon diagonalization, the radiative contributions by themselves sum up to zero for each off-diagonal term; hence, the $\mu\to e\gamma$ amplitude is naturally suppressed and agreement with experiment is easily obtained. Other rare processes such as $\mu\to eee$ and μ-e conversion in nuclei are affected in the same way.

V. PHENOMENOLOGICAL CONSEQUENCES: OLD PARTICLES

Of all the known particles at present, the ones affected most directly by this model are ν_τ and τ. As mentioned already in Sec. III,

$$\nu_\tau = \nu_3 \cos\theta_\tau + \nu_4 \sin\theta_\tau \ , \tag{5.1}$$

where ν_3 is a Majorana particle with a radiative mass m_3, and ν_4 is a linear combination of two Majorana particles with masses m_D^2/m_R and m_R. From double W exchange,[9] we find

$$m_3 \approx \frac{g^4 \sin^2\theta_\tau \cos^2\theta_\tau}{256\pi^4} \left[\frac{m_E}{m_W}\right]^4 \frac{m_D^2}{m_R} \left[\ln\frac{m_R^2}{m_E^2} - \frac{3}{4}\ln\frac{m_R^2}{M_W^2}\right] . \tag{5.2}$$

Let us assume that $m_D^2/m_R > m_\tau$ so that ν_4 can decay into τ, then experimentally,[15] m_D^2/m_R is either smaller than 2.5 GeV or greater than 5.4 GeV if $\sin^2\theta_\tau > 10^{-2}$. Also, $m_E > 41$ GeV from collider data,[16] and $m_3 < 10^2$ eV from cosmology.[17] Assuming that $m_R = 250$ GeV, we then obtain $\sin^2\theta_\tau < 0.079$ if m_D^2/m_R is between m_τ and 2.5 GeV, and $\sin^2\theta_\tau < 0.024$ if $m_D^2/m_R > 5.4$ GeV. On the other hand, from the τ lifetime discrepancy, we find $\sin^2\theta_\tau = 0.079 \pm 0.036$. Hence the fourth neutrino of this model is likely to have a mass between m_τ and 2.5 GeV. Its various decay modes will be discussed in Sec. VI.

In τ decay, all standard-model rates are reduced by $\cos^2\theta_\tau$ due to neutrino mixing, but at the same time, there are additional contributions from scalar exchange. Using Eq. (2.4) and noting that ν_R has a large Majorana mass, we see that there are only two relevant Yukawa terms:

$$-L_Y = f_8^\tau \bar{l}_L \tau_R \Phi_2 + f_{10}^{\tau'} \overline{(\nu_3, \tau')}_L l_R \Phi_2 + \text{H.c.} \ , \tag{5.3}$$

where $\tau' = \tau\cos\theta_\tau - E\sin\theta_\tau$. Therefore, the only additional decay modes are $\tau \to e\nu_e\bar{\nu}_3$, $\tau \to e\nu_\mu\bar{\nu}_3$, $\tau \to \mu\nu_\mu\bar{\nu}_3$, and $\tau \to \mu\nu_e\bar{\nu}_3$, as shown in Fig. 2. Since the usual leptonic decay modes of the τ are $\tau \to e\bar{\nu}_e\nu_3$ and $\tau \to \mu\bar{\nu}_\mu\nu_3$, these amplitudes interfere with the new ones only to the extent that ν_e, ν_μ, and ν_3 have Majorana masses, but such masses are very small; hence, we need only add their separate rates together. The effective interaction for $\tau \to e\nu_e\bar{\nu}_3$ is given by

$$H_{\text{eff}} = [(f_8^\tau)_e (f_{10}^{\tau'})_e / \tilde{m}_2^2] \bar{e} \left[\frac{1-\gamma_5}{2}\right] \nu_3 \bar{\nu}_e \left[\frac{1+\gamma_5}{2}\right] \tau \ , \tag{5.4}$$

where $\tilde{m}_2$ is the effective mass of $\phi_2^\pm$, which is certainly not a mass eigenstate. Experimentally, the measured rate for $\tau \to e\nu\bar{\nu}$ is the sum of the rates for $\tau \to e\bar{\nu}_e\nu_3$, $\tau \to e\nu_e\bar{\nu}_3$, and $\tau \to e\nu_\mu\bar{\nu}_3$. Hence it differs from the prediction of the standard model by $(\cos^2\theta_\tau + r_e)$, where

$$r_e = \frac{[(f_8^\tau)_e^2 + (f_8^\tau)_\mu^2](f_{10}^{\tau'})_e^2}{32 G_F^2 \tilde{m}_2^4} \ . \tag{5.5}$$

If r_e is significant, it will affect our previous analysis where the τ lifetime depends only on θ_τ. The effective interaction of Eq. (5.4) does not have the usual $V-A$ structure, but as it turns out, the only differences with the standard model are in the parameters[2] ξ and ξ', both of which become -1 instead of 1.

From Eq. (5.3), we see that there are two other possible experimental consequences. The process $e^- e^- \to \tau^- \tau^-$ occurs through ϕ_2^0 exchange, but the amplitude is proportional to $(f_8^\tau)_e (f_{10}^\tau)_e$ divided by the ϕ_2^0 effective mass squared and is undoubtedly very small because it is closely related to r_e of Eq. (5.5). Similarly, the amplitude for $e^- e^+ \to \tau^- \tau^+$ through ϕ_2^0 exchange is proportional to $(f_8^\tau)_e^2 + (f_{10}^\tau)_e^2$, which is not likely to be much bigger.

VI. PHENOMENOLOGICAL CONSEQUENCES: NEW PARTICLES

Consider first the new leptons. Certainly, E will decay into ν_4 or ν_3 plus a W boson in the usual way, but it can also decay into $\bar{\nu}_3 l\nu$ through $\phi_2^\pm$. More interestingly, it can decay through ϕ_2^0 into the exotic channels $\tau e^- e^+$, $\tau e^- \mu^+$, $\tau\mu^-\mu^+$, and $\tau\mu^- e^+$. Branching fractions of the order 10^{-3} for the above are quite possible. Note that E is odd under the unbroken Z_2 discrete symmetry of this model, so its decay product must contain another particle which is odd under Z_2 such as τ or ν_3. In fact, ν_3 is the lightest such particle and must therefore be stable. As for ν_4, if it is heavier than τ, it will decay into τ plus a W boson in the usual way. In Sec. V we used a number of experimental and theoretical constraints to estimate the mass of ν_4 to be between m_τ and 2.5 GeV. If this is the case, all $\nu_4 \to \tau$ decays will be kinematically suppressed. On the other hand, the exotic decays of ν_4 into $\nu_3 e^- e^+$, $\nu_3 e^- \mu^+$, $\nu_3\mu^-\mu^+$, and $\nu_3\mu^- e^+$ through $\phi_2^\pm$ exchange as shown in Fig. 3 are not. They may thus even be experimentally observable at a Z^0 factory such as LEP at CERN, where Z^0 decay would be a copious source of $\nu_4\bar{\nu}_4$. If ν_4 is lighter than τ, then τ decays into ν_4 which must itself decay into $\nu_3 e^- e^+$, etc. Such modes have not been identified in experimental studies of τ decay, but

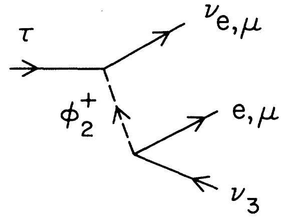

FIG. 2. Decays of τ via $\phi_2^\pm$.

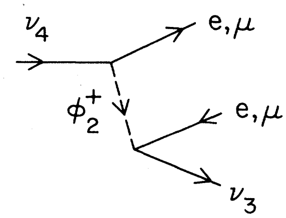

FIG. 3. Decays of ν_4 via $\phi_2^\pm$.

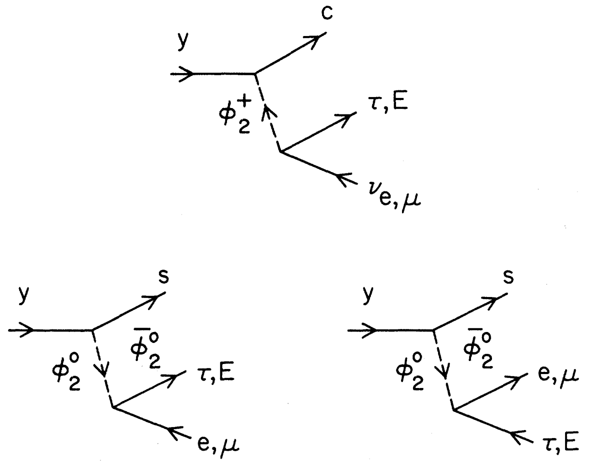

FIG. 4. Decays of y via $\phi_2^\pm$ and $\phi_2^0(\bar{\phi}_2^0)$.

should be detectable at a certain level depending on the acceptance and efficiency of the detectors involved.

Consider next the new quarks. Both h and y are presumably quite heavy, with masses of the order 10^2 GeV. They are odd under Z_2, so they cannot mix with the usual quarks and will not decay into them through the W boson. If $m_h > m_y$, then h decays into y plus a W boson in the usual way, but then y will have to decay by some other means. From Eq. (2.4), we see that $y \to cl\bar{\nu}_{3,4}$, $y \to cL\bar{\nu}_{e,\mu}$, $y \to sl\bar{L}$, and $y \to sL\bar{l}$, where l denotes e,μ, and L denotes τ, E, through $\phi_2^\pm$ and $\phi_2^0(\bar{\phi}_2^0)$ exchange as shown in Fig. 4 are probably the dominant decays. In other words, y will have only semileptonic decays and some of them, such as $y \to s\tau^+\mu^-$, are not even expected to be observable in the standard model. The experimental signature should be unmistakable. If $m_y > m_h$, then $h \to s\bar{l}\nu_{3,4}$ and $h \to s\bar{L}\nu_{e,\mu}$ through $\phi_2^\pm$ exchange are important decays. The processes $h \to c + 2$ neutral leptons through ϕ_3^0 exchange are suppressed by phase space because one of the leptons has to be ν_R or N_R, either of which has a Majorana mass of the order 10^2 GeV. The processes $h \to c + 2$ charged leptons require mixing between ϕ_2^0 and ϕ_3^0 which diminishes the amplitudes but may be compensated by the fact that $f_{7,9}^c$ are likely to be somewhat greater than $f_{8,10}^s$. If so, there would be again unmistakable decay modes such as $h \to c\tau^+\mu^-$.

As for the scalar bosons, H_1 acts very much like the standard-model Higgs boson, but its off-diagonal fermion couplings are enhanced, as discussed already in Sec. IV. The other scalar bosons are odd under Z_2, so they must decay eventually down to ν_3. For example, ϕ_2^0 or $\bar{\phi}_2^0$ (ϕ_3^0 or $\bar{\phi}_3^0$) can decay directly (through mixing) into $\mu^+\tau^-$, then τ^- decays into ν_3 plus a virtual W boson. The $\phi_{2,3}^\pm$ decay modes, such as $\phi_{2,3}^+ \to \nu_3\mu^+$, $\phi_{2,3}^- \to \nu_\mu\tau^-$, etc., are less spectacular, but there should be a finite probability for $\phi_{2,3}^+ \to \phi_{2,3}^0 W^+$, where the two final-state particles are not necessarily on shell, with subsequent conversion of $\phi_{2,3}^0$ into $\mu^+\tau^-$, etc.

VII. CONCLUSION

We have analyzed in this paper the model of Ref. 3 in some detail. This model has a Z_4 discrete symmetry which is softly broken down to Z_2. Under Z_2, the first three quark generations and the first two lepton generations are even, but the fourth quark generation and the third, fourth lepton generations are odd. There are two additional right-handed neutral-lepton singlets ν_R and N_R (even and odd, respectively, under Z_2) which have gauge-invariant Majorana masses. Upon spontaneous breaking of the gauge symmetry, the tree-level masses are obtained for all fermions odd under Z_2 as well as for one linear combination each of the (u,c,t) and (d,s,b) quarks. All other fermion masses and mixing are then one-loop induced by the exchange and mixing of scalar doublets odd under Z_2, or in the case of neutrinos, through double W exchange. Consequently, there is no need for very small couplings. The f's in Eq. (2.4) are all greater than 10^{-2}. There is also no need for very large masses, as would be the case if physics beyond the electroweak mass scale is required.

Because of mixing, ν_τ is a linear combination of ν_3 which is the lightest particle odd under Z_2 and ν_4 which is presumably heavier than the τ. Hence τ decays into only ν_3, with a reduced coupling and thus a longer lifetime. Using available experimental and theoretical constraints, we estimate the mass of ν_4 to be between m_τ and 2.5 GeV. In addition to decaying into τ plus a virtual W boson, ν_4 should also decay into $\nu_3 e^- e^+$, $\nu_3 e^- \mu^+$, $\nu_3 \mu^- \mu^+$, and $\nu_3 \mu^- e^+$ through $\phi_2^\pm$ exchange as shown in Fig. 3. Similarly, the fourth charged lepton E can decay into $\tau e^- e^+$, $\tau e^- \mu^+$, $\tau\mu^-\mu^+$, and $\tau\mu^- e^+$. Some of these modes should have unmistakable signatures.

The fourth-generation quarks h and y are odd under Z_2. If $m_h > m_y$, then y must decay into ν_3, ν_4, τ, or E as shown in Fig. 4. We expect decay modes such as $y \to s\tau^+\mu^-$ to stand well above background. Similarly, if $m_y > m_h$, then h must decay into ν_3, ν_4, τ, or E, and decay modes such as $h \to c\tau^+\mu^-$ will appear.

Flavor-changing neutral currents are absent at the tree level among the first three generations of quarks and among the first two of leptons. However, the one-loop-induced off-diagonal fermion couplings to H, γ, Z, and gluon receive additional contributions from scalar exchange and may be enhanced in some cases, such as, B^0-$\bar{B}^0$ mixing, and not in others. In fact, the prime motivation for separating the third and fourth lepton generations from the first and second is to make sure that $\mu \to e\gamma$ does not proceed at a rate higher than the experimental limit as discussed in Sec. IV. As a result, τ is odd under Z_2 and so is the neutrino that it decays into. The new particles of this model must then have decay products containing τ or ν_3. The observation of such a particle would be the key experimental evidence for it.

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