Glueball components of the meson $f_2(1270)$

Qi-Xing Shen and Hong Yu
Center of Theoretical Physics, Chinese Center of Advanced Science and Technology (World Laboratory) and Institute of High Energy Physics, Academia Sinica, P.O. Box 918, Beijing, Republic of China
(Received 18 April 1989)

It is shown that in order to explain the ratios x and y of the helicity amplitudes for the process $J/\psi \to \gamma + f_2(1270)$ and the experimental datum $R = B(\psi'(3685) \to \gamma + f_2(1270))/B(J/\psi \to \gamma + f_2(1270)) = (9\pm3)\%$ in terms of the mixture between the meson $f_2(1270)$ and a 2^{++} glueball, two D-wave components have to be considered in addition to the S-wave component of the 2^{++} glueball.

It is expected from quantum chromodynamics (QCD) that gluonium states, or glueballs, bound states consisting of gluons only, should exist because of the self-coupling between the gauge bosons (gluons).[1] Therefore, the identification of glueballs is an essential proof for the validity of QCD.

The radiative decay of J/ψ to states other than η_c is believed to proceed mainly through a two-gluon intermediate state in the lowest order of perturbative QCD (Ref. 2). Therefore, radiative J/ψ decays are considered to be an excellent place to search for glueballs.

A number of candidates for glueballs have been reported. $\iota/\eta(1440)$ and $\theta/f_2(1720)$ are two of these candidates. $\eta(1440)$ was first observed in 1980 by the Mark II Collaboration in the radiative J/ψ decay $J/\psi \to \gamma\eta(1440) \to \gamma K_S^0 K^\pm \pi^\mp$ (Ref. 3). The spin and parity of $\eta(1440)$ were determined to be $J^{PC}=0^{-+}$ (Ref. 4). The tensor state $f_2(1720)$ was discovered in 1982 by the Crystal Ball Collaboration in the reaction $J/\psi \to \gamma f_2(1720) \to \gamma\eta\eta$ (Ref. 5). The $J^{PC}=2^{++}$ assignment of $f_2(1720)$ is supported by the Mark III Collaboration.[6] These states found in J/ψ radiative decay do not seem to fit into $q\bar{q}$ nonets of the quark model and hence are viewed as glueball candidates.[4,7] But an analysis of $\Gamma(J/\psi \to \{\gamma,\omega,\phi\} + f_2(1720))$ and $\Gamma(f_2(1720) \to \gamma\gamma)$ shows that $f_2(1720)$ has some type of quarkonium admixture in addition to the major glueball component.[8] Because there is no indication of $f_2(1720)$ from LASS data,[9] it can be expected that the $s\bar{s}$ component included in $f_2(1720)$ is very small. A component $(u\bar{u}+d\bar{d})/\sqrt{2}$ is included in $f_2(1720)$; hence, there should be a glueball component (gg) in the meson $f_2(1270)$. Many arguments support that there is a glueball component in the meson $f_2(1270)$ (Ref. 10).

In Ref. 11 the meson $f_2(1270)$ is considered as a bound state of a pure quark-antiquark $q\bar{q}$ pair and the calculation of the helicity amplitudes shows a 4σ discrepancy compared to the experimental data (Table I). In order to explain the experimental data a mixture between the $(q\bar{q})$ component and the glueball component (gg) was introduced in the meson $f_2(1270)$ in Ref. 12.

The ratio

$$R = \frac{B(\psi'(3685) \to \gamma + f_2(1270))}{B(J/\psi \to \gamma + f_2(1270))} = (9\pm3)\% \qquad (1)$$

has been measured by the Crystal Ball Collaboration.[13] In this paper it is shown that in order to explain the ratios x and y in Table I and R in Eq. (1) not only an S-wave component but also two D-wave components should be included in the glueball component (gg) of the meson $f_2(1270)$.

In perturbative QCD the diagrams for the processes $J/\psi \to \gamma + (q\bar{q})$ and $J/\psi \to \gamma + (gg)$ can be shown by Figs. 1(a) and 1(b), respectively. As mentioned in Ref. 12 the glueball component of the meson $f_2(1270)$ gives the main contribution to $J/\psi \to \gamma + f_2(1270)$. Therefore, we shall calculate the helicity amplitudes contributing only from Fig. 1(b).

The S-matrix element corresponding to the diagrams in Fig. 1(b) can be written as

$$\langle (gg)_{\lambda_2}\gamma_{\lambda_1}|S|J_\lambda\rangle = (2\pi)^4\delta^4(p_J - p_\gamma - p_G)\frac{eg^2}{3\sqrt{6\omega_\gamma}}\delta_{ab}e_\mu^{\lambda_1 *}(p_\gamma)$$
$$\times \int d^4x_1 d^4x_2 \mathrm{Tr}[\chi_\lambda(0,x_1)\gamma^\alpha S_F(x_1-x_2)\gamma^\beta S_F(x_2)\gamma^\mu + \chi_\lambda(x_1,x_2)\gamma^\alpha S_F(x_2)\gamma^\mu S_F(-x_1)\gamma^\beta$$
$$+ \chi_\lambda(x_2,0)\gamma^\mu S_F(-x_1)\gamma^\alpha S_F(x_1-x_2)\gamma^\beta]G_{\alpha\beta}^{ab}(x_1,x_2)_{\lambda_2} , \qquad (2)$$

TABLE I. The experimental data (Ref. 8) of the ratios x and y of the helicity amplitudes for the meson $f_2(1270)$.

	x	y
Crystal Ball	0.88±0.13	0.04±0.19
Mark II	0.81±0.16	0.02±0.15
PLUTO	0.6 ±0.3	$0.3^{+0.6}_{-1.6}$
Mark III	0.94±0.10	0.06±0.11
DM2	0.83±0.06	0.01±0.06

where

$$\chi_\lambda(x_1,x_2)=\frac{1}{2\sqrt{2}}\left[\frac{m_J}{E_J}\right]^{1/2}\exp\left[\frac{i}{2}p_J\cdot(x_1+x_2)\right]\times\left[1+\frac{\not p_J}{m_J}\right]\not e^{\lambda}(p_J)\psi_J(x) \quad (3)$$

is the wave function of J/ψ. $\psi_J(x)$ is its internal wave function and

$$G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2}=\langle (gg)_{\lambda_2}|T[A^a_\alpha(x_1)A^b_\beta(x_2)]|0\rangle \quad (4)$$

is the wave function of the glueball component (gg). $\lambda=\pm1,0$, $\lambda_1=\pm1$, and $\lambda_2=\pm2,\pm1,0$ are the helicities of J/ψ, the photon, and the glueball, respectively. $e^{\lambda_1}(p_\gamma)$ and $e^{\lambda}(p_J)$ are the polarization vectors for the photon and the J/ψ particle, respectively. For a $J^{PC}=2^{++}$ glueball, the bound state of two vector gluons, the relative orbital angular momentum can be 0, 2, or 4. For simplicity, here, we only consider the S wave and two D waves. There is one wave function in the S wave. For a 2^{++} glueball in the D wave, the total spin s of two gluons can be taken as 0 or 2; hence, there are two D-wave functions. They have $S=0$, $l=2$ and $S=2$, $l=2$, respectively. The wave function of the S wave ($l=0$, $S=2$) is

$$G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2}=\frac{\delta_{ab}}{\sqrt{2m_G}}e^{ip_G\cdot X}G_S(x)\times\sum_{m_1,m_2}C^{2\ \ \lambda_2}_{1m_1\,1m_2}e^{m_1*}_\alpha e^{m_2*}_\beta , \quad (5)$$

where $C^{2\ \ \lambda_2}_{1m_1\,1m_2}$ is a Clebsch-Gordan coefficient, $e^{m_1*}_\alpha$ and $e^{m_2*}_\beta$ are spherical polarization vectors, and

$$X=\tfrac{1}{2}(x_1+x_2),\quad x=x_1-x_2 .$$

The wave functions of the D wave ($l=2$, $S=0$ and $l=2$, $S=2$) are, respectively,

$$G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2}=\frac{\delta_{ab}}{\sqrt{2m_G}}e^{ip_G\cdot X}G_d(x)\times\sum_{m_1,\ldots,m_4}C^{0\ \ 0}_{1m_1\,1m_2}e^{m_1*}_\alpha e^{m_2*}_\beta C^{2\ \ \lambda_2}_{1m_3\,1m_4}\times m_G^2(x\cdot e^{m_3*})(x\cdot e^{m_4*}) \quad (6)$$

and

$$G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2}=\frac{\delta_{ab}}{\sqrt{2m_G}}e^{ip_G\cdot X}G_{d'}(x)\times\sum_{m_1,\ldots,m_6}C^{2\ \ \lambda_2}_{2m_5\,2m_6}C^{2\ \ m_6}_{1m_1\,1m_2}e^{m_1*}_\alpha e^{m_2*}_\beta\times C^{2\ \ m_5}_{1m_3\,1m_4}m_G^2(x\cdot e^{m_3*})\times(x\cdot e^{m_4*}) . \quad (7)$$

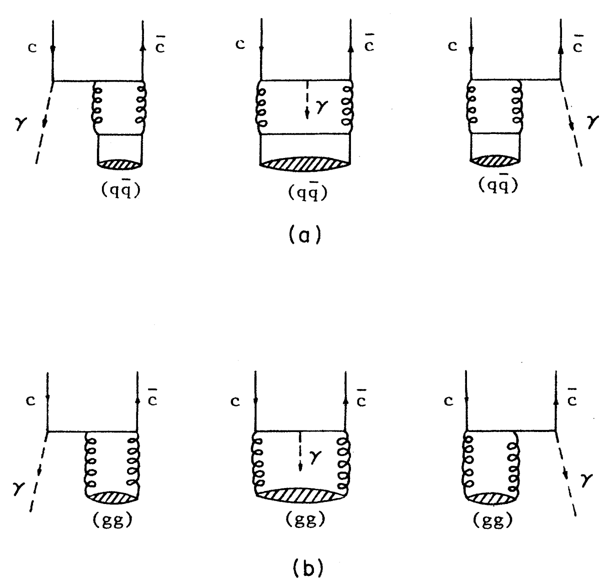

FIG. 1. Diagrams for the process $J/\psi\rightarrow\gamma+f_2(1270)$. $(q\bar{q})$ and (gg) are the quark component and glueball component of the meson $f_2(1270)$.

Substituting Eq. (3) and each of Eqs. (5)–(7) into Eq. (2) and comparing with the definition of the helicity amplitudes T_{λ_2},

$$\langle (gg)_{\lambda_2}\gamma_{\lambda_1}|S|J_\lambda\rangle=(2\pi)^4\delta^4(P_J-P_\gamma-P_G)\times\frac{e}{\sqrt{8\omega_\gamma E_J E_G}}T_{\lambda_2} , \quad (8)$$

we can obtain the formulas of the helicity amplitudes for each glueball orbital component. For the S-wave glueball wave function (5) we have

$$T_2=-\frac{32}{3\sqrt{3}}g^2G_S(0)\psi_J(0)\frac{\sqrt{m_J}}{m_c^2}\left[1-\frac{1}{2m_cm_J}(m_J^2-m_G^2)\right],$$

$$T_1=-\frac{16\sqrt{2}}{3\sqrt{3}}g^2G_S(0)\psi_J(0)\frac{1}{\sqrt{m_J}m_c^2}\left[E_J\left[1-\frac{m_Gp_J}{m_cm_J}\right]+m_Gp_J^2\left[\frac{1}{m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_G^2}-\frac{1}{m_cm_J}\right]\right], \tag{9}$$

$$T_0=-\frac{32}{9\sqrt{2}}g^2G_S(0)\psi_J(0)\frac{\sqrt{m_J}}{m_c^2}\left[1-\frac{m_GP_J}{m_Cm_J}+p_J^2\left[\frac{1+2m_c/m_J}{m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_G^2}-\frac{2}{m_cm_J}\right]\right],$$

where

$$E_J=\frac{m_J^2+m_G^2}{2m_G},\quad p_J=\frac{m_J^2-m_G^2}{2m_G}.$$

For the D-wave glueball wave functions (6), the helicity amplitudes T_{λ_2} can be written as

$$T_2=\tfrac{64}{9}g^2G_d(0)\psi_J(0)\frac{\sqrt{m_J}m_G^2}{m_c^4},\quad T_1=\frac{32\sqrt{2}}{9}g^2G_d(0)\psi_J(0)\frac{m_G^2}{m_c^4\sqrt{m_J}}\left[E_J+\frac{m_Gp_J^2}{m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_G^2}\right],$$

$$T_0=\frac{64}{9\sqrt{6}}g^2G_d(0)\psi_J(0)\frac{\sqrt{m_J}m_G^2}{m_c^4}\left\{1-\frac{2p_J^2}{m_c^2+\frac{1}{4}m_J^2-\frac{1}{2}m_G^2}\left[3+\frac{3}{8m_c^2}(m_J^2-m_G^2)-\frac{3m_GE_J}{2m_cm_J}-\left[\frac{1}{2}+\frac{m_c}{m_J}\right]\left[1-\frac{p_J^2}{m_c^2}\right]\right]\right.$$
$$\left.+\frac{p_J^2}{m_c^4}\left[m_c^2-\frac{m_c}{2m_J}(4p_J^2+m_J^2-m_G^2)\right]\right\}. \tag{10}$$

Finally, if the D-wave glueball wave function (7) is taken, we obtain

$$T_2=\frac{128}{9\sqrt{7}}g^2G_{d'}(0)\psi_J(0)\frac{\sqrt{m_J}m_G^2}{m_c^4}\left[\frac{7}{2}+\frac{p_J^2}{m_J^2}\left[1-\frac{m_Gp_J}{m_cm_J}\right]\right],$$

$$T_1=\frac{64}{9\sqrt{14}}g^2G_{d'}(0)\psi_J(0)\frac{m_G^2}{m_c^4\sqrt{m_J}}\left\{7E_J-\frac{4m_Gp_J^2}{m_J^2-2m_G^2+4m_c^2}\left[7-\frac{p_J^2}{m_c^2}\right]+\frac{p_J^2}{m_c^2}\left[E_J\left[\frac{m_Gp_J}{m_Jm_c}-1\right]+\frac{m_Gp_J^2}{m_cm_J}\right]\right\}, \tag{11}$$

$$T_0=\tfrac{64}{9}\sqrt{2/21}g^2G_{d'}(0)\psi_J(0)\frac{\sqrt{m_J}m_G^2}{m_c^4}\left[\frac{7}{2}-\frac{4p_J^2}{m_J^2-2m_G^2+4m_c^2}\left[\frac{1}{2}+\frac{m_c}{m_J}\right]\left[7-\frac{2p_J^2}{m_c^2}\right]-\frac{p_J^2}{m_c^2}\left[1-\frac{m_Gp_J}{m_cm_J}-\frac{2p_J^2}{m_cm_J}\right]\right].$$

In formulas (9)–(11), m_c is the mass of charm quark. Here, instead of the internal wave function $\psi_J(x)$ and $G(x)$ we have taken $\psi_J(0)$ and $G(0)$, respectively. It is a reasonable approximation as mentioned in Ref. 12.

From Eqs. (9)–(11) the helicity amplitude ratios

$$x=T_1/T_0,\quad y=T_2/T_0 \tag{12}$$

can be obtained for each glueball orbital component. The only parameter in x and y is the charm-quark mass m_c.

The width for the process $J/\psi\to\gamma+f_2(1270)$ can be approximately written as

$$\Gamma(J/\psi\to\gamma+f_2(1270))=\frac{\alpha}{6m_J}\left[1-\frac{m_G^2}{m_J^2}\right]\times(|T_2|^2+|T_1|^2+|T_0|^2). \tag{13}$$

It is worthwhile to point out that formula (13) is also correct for the process $\psi'(3685)\to\gamma+f_2(1270)$ provided we take the mass $m_{\psi'}$ of ψ', the running coupling constant $\alpha_s(\psi')$, and the wave function at the origin $\psi_{\psi'}(0)$ instead of the m_J, $\alpha_s(J/\psi)$, and $\psi_J(0)$ in Eq. (13), respectively. From the calculation of perturbative QCD the running coupling constant is about $\alpha_s=0.21$ (0.20) for J/ψ (ψ'). The value of $|\psi_J(0)|^2$ can be obtained from the formula

$$\Gamma(J/\psi\to e^+e^-)=\frac{16\pi\alpha^2}{m_J^2}\frac{4}{9}|\psi_J(0)|^2 \tag{14}$$

and the experimental data about the width $\Gamma(J/\psi\to e^+e^-)$. Similarly, we can get the value of $|\psi_{\psi'}(0)|^2$. Therefore, after substituting Eq. (13) and the corresponding equation for ψ' into the ratio R in Eq. (1), the charm-quark mass m_c is only a parameter. Our numerical calculation shows that in the m_c range 1.1–1.6 GeV we cannot fit the data in Eq. (1) and Table I simultaneously for each glueball orbital component. For example, if we consider the S-wave glueball in the meson $f_2(1270)$ and take $m_c=1.3$ GeV as mentioned in Ref. 12, we have $x=0.66$, $y=0.04$, and $R=31\%$. It is clear that the value of R is not consistent with the datum in Eq. (1). It is possible that both the S and two D waves are included in the glueball component of the meson $f_2(1270)$. The mixed glueball wave function can be written as

$$G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2}=\frac{\delta_{ab}}{\sqrt{2m_G}}e^{ip_G\cdot X}G(0)\left[\sum_{m_1,m_2}C^{2\ \ \lambda_2}_{1m_1\,1m_2}e_\alpha^{m_1 *}e_\beta^{m_2 *}+a\sum_{m_1,\ldots,m_4}C^{0\ \ 0}_{1m_1\,1m_2}e_\alpha^{m_1 *}e_\beta^{m_2 *}C^{2\ \ \lambda_2}_{1m_3\,1m_4}m_G^2(x\cdot e^{m_3 *})(x\cdot e^{m_4 *})+b\sum_{m_1,\ldots,m_6}C^{2\ \ \lambda_2}_{2m_5\,2m_6}C^{2\ \ m_6}_{1m_1\,1m_2}e_\alpha^{m_1 *}e_\beta^{m_2 *}C^{2\ \ m_5}_{1m_3\,1m_4}m_G^2(x\cdot e^{m_3 *})(x\cdot e^{m_4 *})\right], \tag{15}$$

where a and b are two mixing parameters which can be fixed by fitting the data in Eq. (1) and Table I. x, y, and R for this wave function can be obtained. Now there are three parameters: a, b, and m_c. In a reasonable range of m_c we can fit the data in Table I and Eq. (1) by choosing mixing parameters a and b. For example, if we take $m_c=1.21$ GeV, $a=-0.0034$, and $b=-0.028$ our results are

$$x=0.75,\quad y=0.06,\quad R=10\%\ . \tag{16}$$

These values are in excellent agreement with the data. From the numerical calculation it is learned that D waves with $S=0$, $l=2$ and $S=2$, $l=2$ are indispensable, in addition to the S wave, in order to fit the experimental data, although their mixture is small. We also note that only the two D waves cannot fit the data.

To conclude, it is shown that the ratios x and y of the helicity amplitudes for the process $J/\psi\to\gamma+f_2(1270)$ and the experimental datum in Eq. (1) cannot be explained simultaneously if the glueball orbital components of the meson $f_2(1270)$ is solely an S wave or two D waves. These data can be understood, however, if both the S wave and the D waves are included in the glueball orbital components of the meson $f_2(1270)$. This means that not only must a glueball S-wave component be included in the meson $f_2(1270)$ but also a D wave with $s=0,l=2$ and a D wave with $s=2,l=2$, in addition to the major quarkonium component.

[1]H. Fritszch and P. Minkowski, Nuovo Cimento **30A**, 393 (1975); P. G. O. Freund and Y. Nambu, Phys. Rev. Lett. **34**, 1645 (1975); J. Willemsen, Phys. Rev. D **13**, 1327 (1976); J. Kogut, D. Sinclair, and L. Susskind, Nucl. Phys. **B114**, 199 (1976); D. Robson, *ibid.* **B130**, 328 (1977); P. Roy and T. Walsh, *ibid.* **B140**, 499 (1978); K. Ishikawa, Phys. Rev. D **20**, 731 (1979); C. E. Carlson, J. Coyne, P. Fishbane, F. Gross, and S. Meshkov, Phys. Lett. **98B**, 110 (1981); J. J. Coyne, P. M. Fishbane, and S. Meshkov, *ibid.* **91B**, 259 (1980).

[2]T. Appelquist *et al.*, Phys. Rev. Lett. **34**, 365 (1975); M. Chanowitz, Phys. Rev. D **12**, 918 (1975).

[3]D. L. Scharre *et al.*, Phys. Lett. **97B**, 329 (1980).

[4]C. Edwards *et al.*, Phys. Rev. Lett. **49**, 259 (1982); J. D. Richman, Ph.D. thesis, Report No. CALT-68-1231, 1985; J. E. Augustin *et al.*, Report No. LAL/85-27, 1985 (unpublished).

[5]C. Edwards *et al.*, Phys. Rev. Lett. **48**, 458 (1982).

[6]D. M. Coffman *et al.*, Phys. Rev. D **35**, 2077 (1987).

[7]J. F. Donoghue, K. Johnson, and B. A. Li, Phys. Lett. **99B**, 416 (1981).

[8]L. Kopke and N. Wermes, Phys. Rep. **174**, 67 (1979).

[9]D. Aston *et al.*, Nucl. Phys. **B301**, 525 (1988).

[10]J. L. Rosner, Phys. Rev. D **24**, 1347 (1981); J. F. Donoghue, *ibid.* **25**, 1975 (1982).

[11]M. Krammer, Phys. Lett. **74B**, 361 (1978).

[12]B. A. Li and Q. X. Shen, Phys. Lett. **126B**, 125 (1983).

[13]Roger A. Lee, in *QCD and Beyond,* proceedings of the XXth Rencontre de Moriond, Les Arcs, France, 1985, edited by J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, France, 1985), p. 311.