Radiative decays of light-quark S- and P-wave mesons in the covariant oscillator quark model

Shin Ishida and Kenji Yamada*
Atomic Energy Research Institute, College of Science and Technology, Nihon University, Tokyo 101, Japan

Masuho Oda
Faculty of Engineering, Kokushikan University, Tokyo 154, Japan
(Received 28 October 1988)

The radiative decays of the ground S-wave and the first excited P-wave mesons in light-quark $q\bar{q}$ systems are investigated in the covariant oscillator quark model. An important feature of this scheme is that the effective electromagnetic currents for respective processes, which are manifestly covariant and conserved, are given in a unified and systematic way. The results for the S-wave mesons seem to be in good agreement with experiment, and the results for the P-wave mesons are satisfactory, as far as the present poor experiments are concerned, except for the process $b_1^+ \to \pi^+\gamma$.

I. INTRODUCTION

Recently there seems to be no doubt that quantum chromodynamics (QCD) plays an important role as a theoretical basis for the strong interaction of hadrons. As a matter of fact the success of nonrelativistic (NR) quark models, being guided by QCD, has become more remarkable than before especially for heavy-quark systems, while various interesting models reflecting some physical pictures of QCD, such as the bag-model approach, seem to reproduce some qualitative features of respective physical phenomena.

On the other hand, from the viewpoint of practical quantitative application to the various low-energy hadron phenomena, QCD itself has not yet reached the stage of making definite numerical predictions, except for the lower-mass spectrum, and the NR quark potential models seem to be still keeping their important roles. However, their application should be, in principle, limited to static problems such as mass spectra, magnetic moments, and transition reactions with small threshold energy. The difficulty is most seriously seen[1] in treating the electromagnetic (EM) processes of hadrons, where it is indispensable to get the effective currents in terms of "observable" hadrons themselves, and in doing so the covariant description of the center-of-mass motion of hadrons is necessary. This difficulty still remains in recent semirelativistic attempts,[2] which can, by taking into account the effects of relativistic motion properly, describe the spectra of light- and heavy-quark systems in a unified way.

This point has been one of the most important motives for the covariant oscillator quark model[3] (COQM), where hadrons are defined as Feirz components of multilocal fields which satisfy a covariant equation heuristically obtained, and the conserved effective hadron currents are given explicitly in a covariant form. The COQM has[3] a long history of development and we have applied it to various problems[1,3-5] with satisfactory results.

In this paper we shall apply the COQM to analyze the radiative decays of light-quark $q\bar{q}$ mesons. From the above-mentioned point of covariance and of current conservation the radiative decay processes of the P-wave mesons are especially interesting, since the final particles have generally large kinetic energy. Reflecting this difficult situation there have been published only a few works[6,7] treating the radiative decays of the P-wave mesons from a different viewpoint.

The contents of this paper are as follows. In Sec. II we give a brief review of the COQM and give a general form of the conserved EM currents. In Sec. III the effective meson currents for the respective processes are derived and the corresponding formulas for the decay width are given. In Sec. IV our predicted decay width for the S-wave mesons is compared with experiment. The main contents of this paper, the analysis of the decay of the P-wave mesons, is described in detail in Sec. V. Finally in Sec. VI are given some concluding remarks.

II. CONSERVED CURRENT IN COQM

A general expression for conserved minimal electromagnetic currents in the COQM has been derived in our previous works. In this section we shall collect the necessary formulas for our present application and recapitulate the derivation of them.

In this scheme the mesons of quark-antiquark systems are generally described as a bilocal field (wave function)

$$\Phi_\alpha^\beta(x_1,x_2)\ , \tag{2.1}$$

where x_1 (x_2) is the Lorentz four-vector representing the space-time coordinate of a constituent quark (antiquark), α (β) $(\alpha,\beta=1-4)$ denotes the index of the Dirac spinor of a quark (antiquark), and the flavor and the color indices are omitted for simplicity. Here the respective parts of the wave function, the spin and the space-time wave function, are covariantly extended, separately, from the corresponding part of the NR one. Concerning the spin part we choose the Bargmann-Wigner (BW) scheme[8] out of the two thus far proposed, and the wave function (2.1) is required to satisfy the BW equation.

Concerning the space-time part, it is required[9] to satisfy the equation

$$\left[\sum_{i=1}^{2} \frac{1}{2m_i}\frac{\partial^2}{\partial x_{i\mu}^2} - U(x_1,x_2)\right]\Phi(x_1,x_2)=0 , \qquad (2.2)$$

where m_1 (m_2) is the parameter corresponding to quark (antiquark) mass, and the potential term consists of two parts:

$$U=U_0+\lambda U_G,\quad U_0=\tfrac{1}{2}K(x_{1\mu}-x_{2\mu})^2 . \qquad (2.3)$$

Here the first term U_0 is a confining part corresponding to a four-dimensional harmonic oscillator (HO), and the second term is a perturbative potential representing the one-gluon-exchange effects. In this paper we shall apply the symmetrical current, which is conserved only up to the zeroth order of λ and represented in terms of unperturbed meson masses. The second term is needed to determine their values unambiguously through the analysis[5] of the actual meson spectrum. Equation (2.2) is rewritten in terms of the center-of-mass coordinates X_μ $[=(m_1x_{1\mu}+m_2x_{2\mu})/(m_1+m_2)]$ and the relative coordinates x_μ $(=x_{1\mu}-x_{2\mu})$ as

$$\left[\frac{\partial^2}{\partial X_\mu^2} - \mathcal{M}^2(x)\right]\Phi(X,x)=0 \qquad (2.4)$$

with the square-mass operator of the form

$$\mathcal{M}^2=dH=d\left[-\frac{1}{2\mu}\frac{\partial^2}{\partial x_\mu^2}+U(x)\right] , \qquad (2.5a)$$

$$\mu=\frac{m_1m_2}{m_1+m_2},\quad d=2(m_1+m_2) . \qquad (2.5b)$$

Here it is to be noted that H has a similar form to the energy of relative motion in the NR potential model except for now using Lorentz four-vectors instead of three-vectors. In order to freeze the redundant freedom of relative time, we suppose the definite-metric-type subsidiary condition[10] for the unperturbed wave function. As is well known, the wave function satisfying this condition is normalizable and gives[4,11] the desirable asymptotic behavior of EM form factors of hadrons.

Thus our scheme transforms the results of the NR quark model for linear mass of hadrons to results for masses squared, keeping Lorentz covariance. As an important result we get linear orbital Regge trajectories, as is desired, for square mass of hadrons in the unperturbed limit. The general form of conserved meson currents is most clearly derived by recourse to the Lagrangian method,[12] following the heuristic prescription by Feynman, Kislinger, and Ravndal[13] (FKR) with some generalization. We obtained[9] the action for EM interaction (see Fig. 1), up to the first order in the EM coupling constant, as

$$L_I=\int d^4x_1d^4x_2 \sum_{i=1}^{2} j_{i\mu}(x_1,x_2)A_\mu(x_i)$$
$$\equiv \int d^4X\, J_\mu(X)A_\mu(X) , \qquad (2.6a)$$

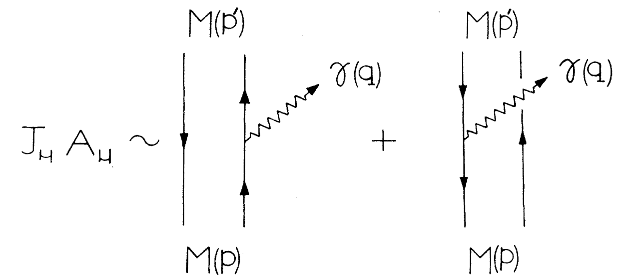

FIG. 1. Electromagnetic interaction of $q\bar{q}$-meson systems in the COQM.

$$j_{i\mu}=\bar{\Phi}e_i\left[-i\frac{d}{2m_i}\frac{\overleftrightarrow{\partial}}{\partial x_{i\mu}} + g_M\frac{d}{2m_i}\sigma_{\mu\nu}^{(i)}\left[\frac{\overrightarrow{\partial}}{\partial x_{i\nu}}+\frac{\overleftarrow{\partial}}{\partial x_{i\nu}}\right]\right]\Phi , \qquad (2.6b)$$

where e_i is the charge of the ith quark, g_M is the parameter related with the gyromagnetic ratio, $\sigma_{\mu\nu}^{(i)}$ are the spin matrices of the ith quark with the definition $\sigma_{\mu\nu}^{(1)}\equiv(\gamma_\mu^{(1)}\gamma_\nu^{(1)}-\gamma_\nu^{(1)}\gamma_\mu^{(1)})/2i$ ($\sigma_{\mu\nu}^{(2)}$ should be understood as $-\sigma_{\mu\nu}^{(2)T}$, A^T denoting the transpose of A), and $\bar{\Phi}=\Phi^\dagger\gamma_4^{(1)}\gamma_4^{(2)T}$ ($\Phi^\dagger$ is the Hermitian conjugate of Φ). Here it may be notable that the first and second terms in Eq. (2.6b) are just the generalization of the convection and the spin current, respectively, in NR nuclear physics.

III. EFFECTIVE MESON CURRENT AND RADIATIVE DECAY WIDTH

A. Unperturbed meson wave functions

The unperturbed meson wave function can be written as

$$\Phi_\alpha^\beta(x_1,x_2)=Ne^{ip\cdot X}\Psi_\alpha^\beta(x;p) , \qquad (3.1)$$

where N is a normalization constant for the plane wave and $\Psi_\alpha^\beta(x;p)$, p_μ being the four-momentum of the center-of-mass motion, is the internal HO and spin wave function. [In Eq. (3.1) we describe only the annihilation part of the meson bilocal field.] As was mentioned in Sec. II we take the BW scheme and the definite-metric-type HO for the spin and the internal space-time parts, respectively. The detailed forms of the internal wave functions for the relevant meson systems are given as follows. For ground S-wave mesons,

$$\Psi_\alpha^\beta(x;p)=\frac{1}{2\sqrt{2}}\left[\gamma_5\left[1+\frac{i\gamma_\nu p_\nu}{M_0}\right]\right]_\alpha^\beta P(p)f(x;p)$$
$$+\frac{1}{2\sqrt{2}}\left[\gamma_\mu\left[1+\frac{i\gamma_\nu p_\nu}{M_0}\right]\right]_\alpha^\beta V_\mu(p)f(x;p) , \qquad (3.2a)$$

where

$$f(x;p)=\frac{\beta}{\pi}\exp\left[-\frac{\beta}{2}\left\{\delta_{\rho\sigma}+2\frac{p_\rho p_\sigma}{M_0^2}\right\}x_\rho x_\sigma\right], \qquad (3.2b)$$

$P(p)$ is a local field representing the pseudoscalar-meson nonet, and $V_\mu(p)$ the vector-meson nonet satisfying

$$p_\mu V_\mu(p)=0 . \qquad (3.2c)$$

For first excited P-wave mesons,

$$\Psi_\alpha^\beta(x;p)=\frac{1}{2\sqrt{2}}\left[\gamma_5\left\{1+\frac{i\gamma_\nu p_\nu}{M_1}\right\}\right]_\alpha^\beta P_\mu(p)f_\mu(x;p)+\frac{1}{2\sqrt{2}}\left[\gamma_\mu\left\{1+\frac{i\gamma_\lambda p_\lambda}{M_1}\right\}\right]_\alpha^\beta V_{\mu\nu}(p)f_\nu(x;p), \qquad (3.3a)$$

where

$$f_\mu(x;p)=i\sqrt{2\beta}\left\{\delta_{\mu\tau}+\frac{p_\mu p_\tau}{M_1^2}\right\}x_\tau \times\frac{\beta}{\pi}\exp\left[-\frac{\beta}{2}\left\{\delta_{\rho\sigma}+2\frac{p_\rho p_\sigma}{M_1^2}\right\}x_\rho x_\sigma\right], \qquad (3.3b)$$

$P_\mu(p)$ and $V_{\mu\nu}(p)$ are local fields representing the axial-vector nonet ($^1P_1$ state) and the scalar, axial-vector, and tensor nonets ($^3P_0$, $^3P_1$, and $^3P_2$ states), respectively, which satisfy the subsidiary conditions

$$p_\mu P_\mu(p)=0 , \qquad (3.3c)$$

$$p_\mu V_{\mu\nu}(p)=p_\nu V_{\mu\nu}(p)=0 , \qquad (3.3d)$$

and

$$V_{\mu\nu}(p)=V_{\nu\mu}(p),\quad V_{\mu\mu}(p)\neq 0 \quad \text{for scalar mesons},$$
$$V_{\mu\nu}(p)=-V_{\nu\mu}(p) \quad \text{for axial-vector mesons}, \qquad (3.3e)$$
$$V_{\mu\nu}(p)=V_{\nu\mu}(p),\quad V_{\mu\mu}(p)=0 \quad \text{for tensor mesons}.$$

Here $f(x;p)$ in Eq. (3.2b) [$f_\mu(x;p)$ in Eq. (3.3b)] is the ground- (first-excited-) state wave function of the unperturbed HO, and the parameter β, being originally defined by $\beta\equiv\sqrt{\mu K}$, is represented by, assuming the experimental fact of the universal orbital Regge trajectories,

$$\beta=\frac{m_1 m_2}{2(m_1+m_2)^2}\Omega , \qquad (3.4)$$

Ω^{-1} being the slope of the trajectories; and the symmetric masses M_0 and M_1, appearing in Eqs. (3.2) and (3.3), respectively, for the ground- and first-excited-state mesons are related by

$$M_1^2=M_0^2+\Omega . \qquad (3.5)$$

B. Effective meson transition current

The interaction (2.6) describes systematically all EM interactions of $q\bar{q}$-meson systems, that is, the ground- and/or excited-state mesons. The effective meson currents $J_\mu(X)$ for respective processes are obtained merely by substituting the corresponding wave functions given in Eqs. (3.1)–(3.3) into Eq. (2.6) and performing the calculation on redundant variables in Eq. (2.6a).

In Table I we have collected the covariant expressions of the respective effective meson currents $J_\mu(X)$ (in the momentum representation) thus obtained. In this table $P(p)$, $V_\mu(p)$, $P_\mu(p)$, and $V_{\mu\nu}(p)$ [$\bar{P}(p')$ and $\bar{V}_\mu(p')$] denote the initial [final] pseudoscalar, vector, $^1P_1$ axial vector, and $^3P_{0,1,2}$ scalar, axial-vector, tensor mesons [pseudoscalar and vector mesons] with the four-momentum p_μ [p'_μ] (in the actual application we assume it is on the physical mass shell), and F ($\bar{F}$) is the overlap integral between the initial and the final HO wave functions of the EM interaction with the constituent quark (antiquark). Here it is to be noted that in the expressions following the arrow the terms proportional to p_μ or p'_μ are omitted, considering that they give no contribution to the present real-photon processes.

By inspecting this table we can easily confirm that the respective magnetic parts with the coupling coefficients μ's of our effective currents are trivially conserved. In the following we shall show explicitly the conservation of our effective electric (convection) currents with the coefficients ϵ for the transitions $^3P_J\to{}^3S_1+\gamma$ and $^1P_1\to{}^1S_0+\gamma$:

$$\begin{aligned} q_\mu J_\mu &\propto \frac{1}{\sqrt{2\beta}}\frac{m_2}{m_1+m_2}\left\{q_\rho-\frac{qp}{pp'}p'_\rho\right\}[(p'+p)q]+\sqrt{2\beta}\left\{1+\frac{m_2}{m_1}\right\}\left\{q_\rho-\frac{qp}{pp'}p'_\rho\right\}+(m_1\leftrightarrow m_2)\\ &=\left\{q_\rho-\frac{qp}{pp'}p'_\rho\right\}\frac{1}{\sqrt{2\beta}}\frac{m_2}{m_1+m_2}\left\{(p^2-p'^2)+2\beta\frac{(m_1+m_2)^2}{m_1m_2}\right\}+(m_1\leftrightarrow m_2)\\ &\propto m_2(-M_1^2+M_0^2+\Omega)+(m_1\leftrightarrow m_2)\ =0 . \end{aligned} \qquad (3.6)$$

In the first line of Eq. (3.6) we have written down only the common relevant factor of the currents to both transitions which comes from the overlapping of the internal space-time wave function. In the third [final] step is used the relation (3.4) [the relation (3.5)].

C. Radiative decay width

From the effective meson transition currents given in Table I we can derive straightforwardly the formulas for radiative decay width following the usual procedure.

TABLE I. Effective transition currents for the radiative decays of S- and P-wave mesons.

Process ${}^{2S+1}L_J \rightarrow {}^{2S+1}L_J+\gamma$	Transition current J_μ
${}^3S_1\rightarrow{}^1S_0+\gamma$, ${}^1S_0\rightarrow{}^3S_1+\gamma$	$J_\mu=ieg_M\left[1+\frac{m_2}{m_1}\right]\langle\bar{M}QM\rangle I\epsilon_{\mu\nu\kappa\lambda}q_\nu\frac{p'_\kappa+p_\kappa}{2M_0}[-\bar{P}(p')V_\lambda(p)+\bar{V}_\lambda(p')P(p)]+(m_1\leftrightarrow m_2,\bar{M}\leftrightarrow M)$ $\rightarrow ie\mu_0\epsilon_{\mu\nu\kappa\lambda}q_\nu p'_\kappa[-\bar{P}(p')V_\lambda(p)+\bar{V}_\lambda(p')P(p)]$
${}^3P_{2,1}\rightarrow{}^1S_0+\gamma$, ${}^1P_1\rightarrow{}^3S_1+\gamma$	$J_\mu=ie\frac{g_M}{\sqrt{2\beta}}\frac{m_2}{m_1}\langle\bar{M}QM\rangle\left[q_\rho-\frac{pq}{pp'}p'_\rho\right]I\epsilon_{\mu\nu\kappa\lambda}q_\nu\left[\frac{p'_\kappa}{2M_0}+\frac{p_\kappa}{2M_1}\right][-\bar{P}(p')V_{\lambda\rho}(p)+\bar{V}_\lambda(p')P_\rho(p)]$ $-(m_1\leftrightarrow m_2,\bar{M}\leftrightarrow M)$ $\rightarrow ie\mu\epsilon_{\mu\nu\kappa\lambda}q_\nu p'_\kappa p'_\rho[\bar{P}(p')V_{\lambda\rho}(p)-\bar{V}_\lambda(p')P_\rho(p)]$
${}^3P_{2,1,0}\rightarrow{}^3S_1+\gamma$	$J_\mu=e\langle\bar{M}QM\rangle I\Bigg[\Bigg\{\frac{1}{\sqrt{2\beta}}\frac{m_2}{m_1+m_2}\left[q_\rho-\frac{qp}{pp'}p'_\rho\right]\left[p'_\mu+p_\mu+\frac{m_2}{m_1}\left[\frac{qp}{pp'}p'_\mu-\frac{qp'}{pp'}p_\mu\right]\right]+\sqrt{2\beta}\left[1+\frac{m_2}{m_1}\right]\left[\delta_{\rho\mu}-\frac{p'_\rho p_\mu}{pp'}\right]\Bigg\}\left[\frac{1}{2}\left[1-\frac{pp'}{M_1M_0}\right]\delta_{\sigma\lambda}+\frac{p_\sigma p'_\lambda}{2M_1M_0}\right]$ $+\frac{g_M}{\sqrt{2\beta}}\frac{m_2}{m_1}\left[q_\rho-\frac{qp}{pp'}p'_\rho\right]\Bigg[\frac{1}{2}\left[q_\lambda-\frac{pp'}{M_1M_0}q_\lambda+\frac{qp}{M_1M_0}p'_\lambda\right]\delta_{\sigma\mu}$ $-\frac{1}{2}\left[q_\sigma-\frac{pp'}{M_1M_0}q_\sigma+\frac{qp'}{M_1M_0}p_\sigma\right]\delta_{\lambda\mu}$ $+\left[\frac{qp'}{2M_1M_0}p_\mu-\frac{qp}{2M_1M_0}p'_\mu\right]\delta_{\lambda\sigma}$ $-\frac{1}{2M_1M_0}(p_\mu q_\sigma p'_\lambda-p'_\mu q_\sigma p_\lambda)\Bigg]\Bigg]\bar{V}_\sigma(p')V_{\lambda\rho}(p)$ $+(m_1\leftrightarrow m_2,\bar{M}\leftrightarrow M)$ $\rightarrow e(\epsilon_1\delta_{\lambda\sigma}\delta_{\rho\mu}+\epsilon_2p_\sigma p'_\lambda\delta_{\rho\mu}+\mu_1p'_\rho q_\lambda\delta_{\sigma\mu}+\mu_2p'_\rho q_\sigma\delta_{\lambda\mu})\bar{V}_\sigma(p')V_{\lambda\rho}(p)$
${}^1P_1\rightarrow{}^1S_0+\gamma$	$J_\mu=e\langle\bar{M}QM\rangle I\Bigg[\Bigg\{\frac{1}{\sqrt{2\beta}}\frac{m_2}{m_1+m_2}\left[q_\rho-\frac{qp}{pp'}p'_\rho\right]\left[p'_\mu+p_\mu+\frac{m_2}{m_1}\left[\frac{qp}{pp'}p'_\mu-\frac{qp'}{pp'}p_\mu\right]\right]+\sqrt{2\beta}\left[1+\frac{m_2}{m_1}\right]\left[\delta_{\rho\mu}-\frac{p'_\rho p_\mu}{pp'}\right]\Bigg\}\frac{1}{2}\left[1-\frac{pp'}{M_1M_0}\right]$ $+\frac{g_M}{\sqrt{2\beta}}\frac{m_2}{m_1}\left[q_\rho-\frac{qp}{pp'}p'_\rho\right]\left[\frac{qp'}{2M_0M_1}p_\mu-\frac{qp}{2M_0M_1}p'_\mu\right]\Bigg]\bar{P}(p')P_\rho(p)$ $+(m_1\leftrightarrow m_2,\bar{M}\leftrightarrow M)$ $\rightarrow e\epsilon_1\bar{P}(p')P_\mu(p)$

Coupling parameter

$$\mu_0=\frac{g_M}{M_0}d_0$$

$$\mu=\frac{g_M}{\sqrt{2\beta}}\frac{1}{2}\left[\frac{1}{M_0}+\frac{1}{M_1}\right]\left[-\frac{M_1^2}{pp'}\right]f=\frac{g_M}{\sqrt{2\beta}}\frac{M_1(M_0+M_1)}{M_0(M_0^2+M_1^2)}f$$

TABLE I. (*Continued*).

Coupling parameter

$$\epsilon_1=\sqrt{2\beta}\frac{1}{2}\left[1-\frac{pp'}{M_1M_0}\right]d_1=\sqrt{2\beta}\frac{(M_0+M_1)^2}{4M_0M_1}d_1$$

$$\epsilon_2=\sqrt{2\beta}\frac{1}{2M_0M_1}d_1$$

$$\mu_1=-\frac{g_M}{\sqrt{2\beta}}\frac{1}{2}\left[1-\frac{pp'}{M_1M_0}-\frac{qp}{M_1M_0}\right]\left[-\frac{M_1^2}{pp'}\right]d_2=-\frac{g_M}{\sqrt{2\beta}}\frac{M_1^2(M_0+M_1)}{M_0(M_0^2+M_1^2)}d_2$$

$$\mu_2=\frac{g_M}{\sqrt{2\beta}}\frac{1}{2}\left[1-\frac{pp'}{M_1M_0}+\frac{qp'}{M_1M_0}\right]\left[-\frac{M_1^2}{pp'}\right]d_2=\frac{g_M}{\sqrt{2\beta}}\frac{M_1(M_0+M_1)}{M_0^2+M_1^2}d_2$$

$$d_0=\left[1+\frac{m_2}{m_1}\right]\langle\overline{M}QM\rangle+\left[1+\frac{m_1}{m_2}\right]\langle\overline{M}MQ\rangle,\quad \begin{bmatrix}d_1\\ d_2\end{bmatrix}=\begin{bmatrix}1+\frac{m_2}{m_1}\\ \frac{m_2}{m_1}\end{bmatrix}\langle\overline{M}QM\rangle F+\begin{bmatrix}1+\frac{m_1}{m_2}\\ \frac{m_1}{m_2}\end{bmatrix}\langle\overline{M}MQ\rangle\overline{F}$$

$$f=\frac{m_2}{m_1}\langle\overline{M}QM\rangle F-\frac{m_1}{m_2}\langle\overline{M}MQ\rangle\overline{F}$$

$$I=\left[-\frac{M_0M_1}{pp'}\right]\exp\left[-\frac{1}{4\beta}\left[\frac{m_2}{m_1+m_2}\right]^2\left[q^2-2\frac{(qp)(qp')}{pp'}\right]\right]\to\begin{cases}1 & \text{for } S\text{-state}\to S\text{-state transitions}\\ F=\frac{2M_0M_1}{M_0^2+M_1^2}\exp\left[-\frac{m_2}{m_1}\frac{\Omega}{2(M_0^2+M_1^2)}\right] & \text{for } P\text{-state}\to S\text{-state transitions}\end{cases},\quad \overline{F}=F(m_1\leftrightarrow m_2)$$

M ($\overline{M}$): initial (final) meson nonet matrix

Q: charge matrix

$\to$: extraction of terms contributing to radiative decays

The results for the respective processes $^3S_1(^1S_0)\to{}^1S_0(^3S_1)+\gamma$, $^3P_{2,1}(^1P_1)\to{}^1S_0(^3S_1)+\gamma$, $^3P_{2,1,0}\to{}^3S_1+\gamma$, and $^1P_1\to{}^1S_0+\gamma$ are given, respectively, in Tables II, III, IV, and V. In these tables the coupling coefficients ϵ and μ for each decay channel are given in terms of our fundamental parameters and quantities. The result for decays of the physical axial-vector strange mesons, which are mixtures of the $^1P_1$ and $^3P_1$ states, is given separately in Table VI.

In these tables the mixing angles ϕ_i for the two physical isoscalar states I_{0i} and I'_{0i} of respective $^{2S+1}L_J$ nonets are defined from the ideal quark basis,[14,15] $(n\bar{n})=(u\bar{u}+d\bar{d})/\sqrt{2}$ and $(-s\bar{s})$, as

$$(I_{0i})=(n\bar{n})\cos\phi_i+(-s\bar{s})\sin\phi_i\ ,$$
$$(I'_{0i})=(-s\bar{s})\cos\phi_i-(n\bar{n})\sin\phi_i\ . \qquad (3.7)$$

These angles ϕ_i are related to the singlet-octet mixing angles θ_i as $\phi_i\equiv\theta_i-35.26°$. In Table VI we have defined the mixing angle ψ for the physical strange axial-vector mesons as

$$|K_1(1270)\rangle=\cos\psi|K_{1B}(^1P_1)\rangle-\sin\psi|K_{1A}(^3P_1)\rangle\ ,$$
$$|K_1(1400)\rangle=\cos\psi|K_{1A}(^3P_1)\rangle+\sin\psi|K_{1B}(^1P_1)\rangle\ . \qquad (3.8)$$

Here it may be noted that for the antistrange mesons $|\overline{K}_1(1270)\rangle$ and $|\overline{K}_1(1400)\rangle$ the corresponding formula for Eq. (3.8) should be defined with the change of sign as $|\overline{K}_{1A}(^3P_1)\rangle\to-|\overline{K}_{1A}(^3P_1)\rangle$, reflecting the different charge-conjugation property of the $^3P_1$ and $^1P_1$ states. In Tables IV and VI the complex q_γ dependence of the decay width Γ is due to the fact that both of the convection and the spin currents contribute to the decays of $^3P_{2,1,0}$ states into $^3S_1+\gamma$ states.

IV. RADIATIVE DECAY WIDTH OF S-WAVE MESONS IN COMPARISON WITH EXPERIMENT

As is well known, now the assignment of the ground S-wave $q\bar{q}$ system, the $^1S_0$ pseudoscalar- and the $^3S_1$ vector-meson nonet, is experimentally established and there is rather rich experimental information on the radiative transition processes among them. The detailed analysis on these processes was already made in our previous work[16] based upon the same theoretical framework as in this paper. In this paper we have reanalyzed these processes and compared the results with the renewed experiments in Table VII.

For these processes we have the following five kinds of parameters:

$$g_M,\ x(\equiv m_n/m_s),\ \Omega,\ M_0^2,\ \phi\ , \tag{4.1}$$

which may be dependent on the flavor channels of constituent quarks in the relevant meson systems (throughout this paper we shall assume the isospin symmetry between u and d quarks, denoting them as n quarks).

As for the parameters Ω and M_0^2 we take the values

$$\begin{aligned} &\Omega(\text{universal})=1.15\ \text{GeV}^2\ , \\ &M_0^2(n\bar{n})=0.592\ \text{GeV}^2\ , \\ &M_0^2(n\bar{s}\text{ or }s\bar{n})=0.849\ \text{GeV}^2\ , \\ &M_0^2(s\bar{s})=1.12\ \text{GeV}^2\ , \end{aligned} \tag{4.2}$$

determined from the detailed analysis of the mass spectra in our previous work.[5] As for the fundamental parameters g_M, assuming $g_M(n)=g_M(s)$, and x we have determined from the experimental values[17] of $\Gamma(\rho^{\pm}\to\pi^{\pm}\gamma)=68$ keV and $\Gamma(K^{*\pm}\to K^{\pm}\gamma)=50$ keV, respectively, as

$$g_M=0.850,\quad x=0.797\ . \tag{4.3}$$

(In our previous analysis[16] we took the values $g_M=0.931$ and $x=0.674$ determined from the analysis[18] of baryon magnetic moments.) As for the mixing angles of the pseudoscalar and vector nonets, we take the typical values of

$$\begin{aligned} &\phi_P=-55°\ (\text{Ref. }19)\ , \\ &\phi_V=3.7°\ (\text{Ref. }17)\ , \end{aligned} \tag{4.4}$$

and also use these values in the analysis of the P-wave meson decay widths.

By inspecting Table VII we may conclude that our predicted values are in good agreement with the present experimental ones.

TABLE II. Formulas for radiative decay widths of the transitions ${}^3S_1\to{}^1S_0+\gamma$ and ${}^1S_0\to{}^3S_1+\gamma$.

Process	$\Gamma=\frac{\alpha}{2J_I+1}\mu_0^2q_\gamma^3$ μ_0
$\rho\to\pi\gamma$	$\frac{2g_M}{3M_{0N}}$
$\to\eta\gamma$	$-2\frac{g_M}{M_{0N}}\sin\phi_P$
$K^{*\pm}\to K^{\pm}\gamma$	$\frac{2-x}{3}\left[1+\frac{1}{x}\right]\frac{g_M}{M_{0K}}$
$K^{*0}\to K^0\gamma$	$-\frac{1+x}{3}\left[1+\frac{1}{x}\right]\frac{g_M}{M_{0K}}$
$\omega\to\pi^0\gamma$	$2\frac{g_M}{M_{0N}}\cos\phi_V$
$\to\eta\gamma$	$-\frac{2}{3}\frac{g_M}{M_{0N}}\cos\phi_V\sin\phi_P-\frac{4}{3}\frac{g_M}{M_{0S}}\sin\phi_V\cos\phi_P$
$\phi\to\pi^0\gamma$	$-2\frac{g_M}{M_{0N}}\sin\phi_V$
$\to\eta\gamma$	$\frac{2}{3}\frac{g_M}{M_{0N}}\sin\phi_V\sin\phi_P-\frac{4}{3}\frac{g_M}{M_{0S}}\cos\phi_V\cos\phi_P$
$\to\eta'\gamma$	$-\frac{2}{3}\frac{g_M}{M_{0N}}\sin\phi_V\cos\phi_P-\frac{4}{3}\frac{g_M}{M_{0S}}\cos\phi_V\sin\phi_P$
$\eta'\to\rho^0\gamma$	$2\frac{g_M}{M_{0N}}\cos\phi_P$
$\to\omega\gamma$	$\frac{2}{3}\frac{g_M}{M_{0N}}\cos\phi_P\cos\phi_V-\frac{4}{3}\frac{g_M}{M_{0S}}\sin\phi_P\sin\phi_V$

$\alpha=\frac{e^2}{4\pi}$ $\quad J_I$: spin of initial mesons $\quad x=\frac{m_n}{m_s}$

V. RADIATIVE DECAY WIDTH OF P-WAVE MESONS IN COMPARISON WITH EXPERIMENT

A. Experimental candidates for P-wave meson nonets

As is well known, the assignment[20] of the excited P-wave $q\bar{q}$ systems is still controversial and not fixed except for the ${}^3P_2$ tensor-meson nonet. A main reason for this situation is that around the relevant mass region there may exist other kinds of particles such as a hybrid-quark-gluon system, a glueball, and a four-quark ($q^2\bar{q}^2$) system. Accordingly in this paper we have picked up all established and/or typical possible candidates and analyzed their radiative decay width. In Table VIII we have shown all the experimental candidates to be examined, on which are made some remarks in the following.

${}^3P_2$ nonet. All the members belonging to this nonet are established. Concerning the two isoscalar members, the $f_2(1270)$ and the $f_2'(1525)$ are supposed to have mainly $n\bar{n}$ and $s\bar{s}$ contents, respectively. The value of the mixing angle ϕ_T determined from two-photon decay widths of the $a_2^0(1320)$, $f_2(1270)$, and $f_2'(1525)$ by current algebra and SU(3) is about $-8.7°$ (Ref. 21), while the quadratic mass formula gives its value of about $-3.6°$. We shall take the value $\phi_T=-8°$ to calculate the radiative decay widths.

${}^3P_1$ nonet. All the members belonging to this nonet except for the isodoublet one $K_{1A}(1340)$ are still controversial. The mass value of K_{1A}, 1340 MeV, is obtained from the mean of mass squared of the two physical isodoublets $K_1(1400)$ and $K_1(1270)$. Concerning the isotriplet member a_1 there is a long history of controversy. Recently the standard choice seems to take the $a_1(1260)$ with the mass of 1260±30 MeV and the width of 300–600 MeV (Ref. 17). Concerning the isoscalar member with the mainly $n\bar{n}$ content, the $f_1(1285)$ seems to be almost a unique candidate. Concerning the other isoscalar member with the, mainly, $s\bar{s}$ content the

$f_1(1420)$ has been usually supposed to be the candidate. Recently another promising candidate, $f_1(1530)$ with $M=1527\pm5$ MeV and $\Gamma=106\pm14$ MeV (Ref. 17), has been observed[22,23] in the reaction $K^-p\to(K\bar{K}\pi)\Lambda$. In the case of the combination of two isoscalar members, $f_1(1285)$ and $f_1(1530)$, the quadratic mass formula gives the mixing angle $\phi_A\approx21°$. In the case of the other combination, $f_1(1285)$ and $f_1(1420)$, the quadratic mass formula gives $\phi_A\approx6.7°$, while the ratio of the decay width $\Gamma(f_1(1420)\to\gamma\gamma^*)$ to $\Gamma(f_1(1285)\to\gamma\gamma^*)$ gives[24] $\phi_A\approx10°$. We shall take the value $\phi_A=21°$ (10°) in the former (latter) combination. We have also examined the case of assignment recently proposed by Iizuka, Masuda, and Miura[25] [$a_1(1075)$, K_{1A}, and f_1 ($\sim n\bar{n}$) (unobserved), f_1' ($\sim s\bar{s}$)$=f_1(1285)$].

$^3P_0$ *nonet.* The experimental situation on the members of this nonet is quite confused. The $K_0^*(1430)$ and the $f_0(1400)$ seem to be favorable members of the isodoublet and the isosinglet with the mainly $n\bar{n}$ content, respectively. The $a_0(980)$ and the $f_0(975)$ are usually assigned to the isotriplet member and the isosinglet one with mainly $s\bar{s}$ content, respectively. Against these assignments, it has been suggested[26] that both the $a_0(980)$ and the $f_0(975)$ may well be weakly bound $K\bar{K}$ systems. Recently there has been the following report.[27] An amplitude analysis of the $K_S^0K_S^0$ system produced in the reaction $K^-p\to(K_S^0K_S^0)\Lambda$ at 11 GeV/c has provided evidence for the resonance $f_0(1525)$ to be assigned as an isoscalar member with the mainly $s\bar{s}$ content instead of the $f_0(975)$. If this assignment is taken, it is remarkable that there exists an approximate mass degeneracy among the isoscalar members belonging to the different ^3P_J nonets [viz., among the $f_0(1400)$, the $f_1(1285)$, and the $f_2(1270)$ and among the $f_0(1525)$, the $f_1(1530)$, and the $f_2'(1525)$]. A similar degeneracy may be seen in the isodoublet sector [viz., among the $K_0^*(1430)$, the $K_{1A}(1340)$, and the $K_2^*(1430)$]. If this degeneracy is valid in the isovector sector, the mass of the $^3P_0$ state in this sector is expected to be ~1300 MeV. In fact an amplitude analysis of the isovector $K\bar{K}$ system produced in the reaction $\pi^-p\to(K^-K_S^0)p$ at 10 GeV/c has provided[28] evidence for an S-wave enhancement peaking at ~1.3 GeV. In this paper we investigate both the cases of [$a_0(980)$, $f_0(975)$] and [$a_0(1300)$, $f_0(1525)$] with the ideal mixing $\phi_S=0°$.

$^1P_1$ *nonet.* All the members of this nonet seem to be established except for h_1', the isoscalar one with the mainly $s\bar{s}$ content. The $b_1(1235)$ and the K_{1B} are unambiguous candidates for the isotriplet and the isodoublet members, respectively. The mass of K_{1B} is calculated to be 1340 MeV from the mean of mass squared of the $K_1(1400)$ and the $K_1(1270)$. The $h_1(1170)$ is a good candidate for the

TABLE III. Formulas of radiative decay widths of the transitions $^3P_{2,1}\to{}^1S_0+\gamma$ and $^1P_1\to{}^3S_1+\gamma$.

$$\Gamma=\begin{Bmatrix}\frac{1}{2}\\1\end{Bmatrix}\frac{\alpha}{2J_I+1}\mu^2q_\gamma^5\quad\begin{matrix}\text{for } {}^3P_{2,1}\\ \text{for } {}^1P_1\end{matrix}$$

Process	μ
$a_2^\pm, a_1^\pm\to\pi^\pm\gamma$; $b_1^\pm\to\rho^\pm\gamma$	$\pm2\dfrac{g_M}{\sqrt{\Omega_N}}A_NF_N$
$K_2^{*\pm}, K_{1A}^\pm\to K^\pm\gamma$; $K_{1B}^\pm\to K^{*\pm}\gamma$	$\pm\left[x+\dfrac{1}{x}+2\right]^{1/2}\dfrac{g_M}{\sqrt{\Omega_K}}A_K\left[\dfrac{2}{3x}F_K+\dfrac{x}{3}\bar{F}_K\right]$
$K_2^{*0}, K_{1A}^0\to K^0\gamma$; $K_{1B}^0\to K^{*0}\gamma$	$\pm\left[x+\dfrac{1}{x}+2\right]^{1/2}\dfrac{g_M}{\sqrt{\Omega_K}}A_K\left[-\dfrac{1}{3x}F_k+\dfrac{x}{3}\bar{F}_K\right]$

where $\begin{bmatrix}+\\-\end{bmatrix}$ for $\begin{bmatrix}K^0\\ \bar{K}^0\end{bmatrix}$

$$A_i=\frac{M_{1i}(M_{0i}+M_{1i})}{M_{0i}(M_{0i}^2+M_{1i}^2)}$$

$$F_j=\frac{2M_{0j}M_{1j}}{M_{0j}^2+M_{1j}^2}\exp\left[-\frac{\Omega_j}{2(M_{0j}^2+M_{1j}^2)}\right]\quad(j=N\text{ or }S)$$

$$F_K=\frac{2M_{0K}M_{1K}}{M_{0K}^2+M_{1K}^2}\exp\left[-\frac{\Omega_K}{2x(M_{0K}^2+M_{1K}^2)}\right]$$

$$\bar{F}_K=F_K\left[x\to\frac{1}{x}\right]$$

TABLE IV. Formulas for radiative decay widths of the transitions $^3P_{2,1,0} \to {}^3S_1 + \gamma$.

$$\Gamma = \frac{\alpha}{2J_I+1}\frac{q_\gamma}{2m_I^2} \times \begin{cases} \left[\frac{10}{3}\epsilon_1^2 + \left[(\epsilon_2+\mu_2)^2\frac{m_I^2}{m_F^2} + \frac{4}{3}\mu_1^2\right]q_\gamma^4 + \left[\frac{\epsilon_1^2}{m_F^2} - \epsilon_1(\epsilon_2+\mu_2)\left[1+\frac{m_I^2}{m_F^2}\right] + \frac{4}{3}\epsilon_1\mu_1\right]q_\gamma^2\right] & \text{for } {}^3P_2 \\ 2\epsilon_1^2 + (\epsilon_2-\mu_2)^2\frac{m_I^2}{m_F^2}q_\gamma^4 + \left[\frac{\epsilon_1^2}{m_F^2} - \epsilon_1(\epsilon_2-\mu_2)\left[1+\frac{m_I^2}{m_F^2}\right]\right]q_\gamma^2 & \text{for } {}^3P_1 \\ \frac{2}{3}\epsilon_1^2 + \frac{2}{3}\mu_1^2 q_\gamma^4 - \frac{4}{3}\epsilon_1\mu_1 q_\gamma^2 & \text{for } {}^3P_0 \end{cases}$$

Process	ϵ_1 or ϵ_2	μ_1 or μ_2
$a_2, a_1, a_0 \to \rho\gamma$	$\frac{1}{3}\sqrt{\Omega_N}B_N F_N$	$\frac{2}{3}\frac{g_M}{\sqrt{\Omega_N}}C_N F_N$
$\to \omega\gamma$	$\sqrt{\Omega_N}B_N F_N\cos\phi_V$	$2\frac{g_M}{\sqrt{\Omega_N}}C_N F_N\cos\phi_V$
$\to \phi\gamma$	$-\sqrt{\Omega_N}B_N F_N\sin\phi_V$	$-2\frac{g_M}{\sqrt{\Omega_N}}C_N F_N\sin\phi_V$
$K_2^{*\pm}, K_{1A}^{\pm}, K_0^{*\pm} \to K^{*\pm}\gamma$	$\left[\frac{\Omega_K}{x+\frac{1}{x}+2}\right]^{1/2} B_K\left[\frac{2}{3}F_K - \frac{x}{3}\bar{F}_K\right]\left[1+\frac{1}{x}\right]$	$\left[x+\frac{1}{x}+2\right]^{1/2}\frac{g_M}{\sqrt{\Omega_K}}C_K\left[\frac{2}{3x}F_k - \frac{x}{3}\bar{F}_K\right]$
$K_2^{*0}, K_{1A}^{0}, K_0^{*0} \to K^{*0}\gamma$	$\left[\frac{\Omega_K}{x+\frac{1}{x}+2}\right]^{1/2} B_K\left[-\frac{1}{3}F_K - \frac{x}{3}\bar{F}_K\right]\left[1+\frac{1}{x}\right]$	$\left[x+\frac{1}{x}+2\right]^{1/2}\frac{g_M}{\sqrt{\Omega_K}}C_K\left[-\frac{1}{3x}F_K - \frac{x}{3}\bar{F}_K\right]$
$f_2, f_1, f_0 \to \rho^0\gamma$	$\sqrt{\Omega_N}B_N F_N\cos\phi_i$	$2\frac{g_M}{\sqrt{\Omega_N}}C_N F_N\cos\phi_i$
$\to \omega\gamma$	$\frac{1}{3}\sqrt{\Omega_N}B_N F_N\cos\phi_i\cos\phi_V - \frac{2}{3}\sqrt{\Omega_S}B_S F_S\sin\phi_i\sin\phi_V$	$\frac{2}{3}\frac{g_M}{\sqrt{\Omega_N}}C_N F_N\cos\phi_i\cos\phi_V - \frac{4}{3}\frac{g_M}{\sqrt{\Omega_S}}C_S F_S\sin\phi_i\sin\phi_V$
$\to \phi\gamma$	$-\frac{1}{3}\sqrt{\Omega_N}B_N F_N\cos\phi_i\sin\phi_V - \frac{2}{3}\sqrt{\Omega_S}B_S F_S\sin\phi_i\cos\phi_V$	$-\frac{2}{3}\frac{g_M}{\sqrt{\Omega_N}}C_N F_N\cos\phi_i\sin\phi_V - \frac{4}{3}\frac{g_M}{\sqrt{\Omega_S}}C_S F_S\sin\phi_i\cos\phi_V$
$f_2', f_1', f_0' \to \rho^0\gamma$	$-\sqrt{\Omega_N}B_N F_N\sin\phi_i$	$-2\frac{g_M}{\sqrt{\Omega_N}}C_N F_N\sin\phi_i$
$\to \omega\gamma$	$-\frac{1}{3}\sqrt{\Omega_N}B_N F_N\sin\phi_i\cos\phi_V - \frac{2}{3}\sqrt{\Omega_S}B_S F_S\cos\phi_i\sin\phi_V$	$-\frac{2}{3}\frac{g_M}{\sqrt{\Omega_N}}C_N F_N\sin\phi_i\cos\phi_V - \frac{4}{3}\frac{g_M}{\sqrt{\Omega_S}}C_S F_S\cos\phi_i\sin\phi_V$
$\to \phi\gamma$	$\frac{1}{3}\sqrt{\Omega_N}B_N F_N\sin\phi_i\sin\phi_V - \frac{2}{3}\sqrt{\Omega_S}B_S F_S\cos\phi_i\cos\phi_V$	$\frac{2}{3}\frac{g_M}{\sqrt{\Omega_N}}C_N F_N\sin\phi_i\sin\phi_V - \frac{4}{3}\frac{g_M}{\sqrt{\Omega_S}}C_S F_S\cos\phi_i\cos\phi_V$

$$B_j = \begin{cases} \frac{(M_{0j}+M_{1j})^2}{4M_{0j}M_{1j}} & \text{for } \epsilon_1 \\ \frac{1}{2M_{0j}M_{1j}} & \text{for } \epsilon_2 \end{cases} \qquad C_j = \begin{cases} -\frac{M_{1j}^2(M_{0j}+M_{1j})}{M_{0j}(M_{0j}^2+M_{1j}^2)} & \text{for } \mu_1 \\ \frac{M_{1j}(M_{0j}+M_{1j})}{M_{0j}^2+M_{1j}^2} & \text{for } \mu_2 \end{cases}$$

m_I (m_F): initial (final) meson mass $\qquad \phi_i = \phi_T, \phi_A, \phi_S$

isoscalar member with mainly $n\bar{n}$ content. Recently another isoscalar ($J^{PC}=1^{+-}$) meson with $M=1380\pm20$ MeV and $\Gamma=80\pm30$ MeV, to be classified as h'_1, has been observed[22] in the reaction $K^-p\to(K\bar{K}\pi)\Lambda$ where has also been observed the $f_1(1530)$ mentioned above. The quadratic mass formula for these two isoscalar members $h_1(1170)$ and $h_1(1380)$ gives the mixing angle $\phi_B=-22°$. We apply this value for our analysis.

B. Predicted decay width in comparison with experiment

Most of the parameters appearing in our formulas for the decay width of the P-wave mesons (Tables III–VI) have already been determined through the analysis of the S-wave mesons in Sec. IV [see Eqs. (4.2) and (4.3)]. Another new parameter M_1, the symmetrical mass of P-wave mesons in the HO scheme, is determined through the relation Eq. (3.5) from the values of M_0 and Ω given in Eq. (4.2). Using these values of the parameters and taking the values of the mixing angles given in Sec. V A, we can calculate the numerical decay widths for relevant respective processes. At present, experimental information[29–36] on the P-wave meson decays is limited to a few processes. We have given our predicted values for these processes, separately from the predictions for general processes to be given in the next subsection, and compared them with experiment in Table IX.

By inspecting Table IX we may generally conclude that our predicted values for the established candidates [$a_2^+(1320)$, $K_2^{*+}(1430)$, and $K_2^{*0}(1430)$] are in good agreement with the present experimental ones except for the one [$b_2^+(1235)$] which is a factor of about 3–4 below the experimental one, and our values for the usually supposed candidates [$a_1^+(1260)$ and $f_1(1285)$] are consistent with the experiments.

In the following we shall add some comments on the comparison of respective processes.

$^3P_2\to{}^1S_0+\gamma(a_2^+\to\pi^+\gamma,\ K_2^{*+}\to K^+\gamma,\ K_2^{*0}\to K^0\gamma)$. Our predictions for these processes are in good agreement with the experiments[30–32] and seems to confirm the usual assignment of the resonances $a_2(1320)$ and $K_2^*(1430)$ from our radiative-decay analysis. The process $K_2^{*0}\to K^0\gamma$ is forbidden in the SU(3) limit, and our predicted value, which is smaller than the experimental upper bound,[32] depends strongly on the value of x.

$^3P_1\to{}^1S_0+\gamma[a_1^+(1260)\to\pi^+\gamma]$. Our predicted width for the standard mass value of the a_1 resonance is consistent with the present experiment.[33] Here it may be worthwhile to note the controversies concerning the properties of the a_1 resonance. From the analysis of τ decays ($\tau^\pm\to\pi^\pm\pi^+\pi^-\nu_\tau$) it has been recently reported[37,38] that the mass and the total width of the a_1 resonance are, for example, $M=1056\pm20\pm15$ MeV and $\Gamma=476^{+132}_{-120}\pm54$ MeV (Ref. 37) different from the average values[17] of $M=1275\pm28$ MeV and $\Gamma=316\pm45$ MeV using the hadronically produced data alone, while the subsequent reexamination[39] of the same τ decays reports that they are consistent with those average values. Experimentally the values of radiative decay width of a_1 have been reported[33,34] as $\Gamma(a_1^+\to\pi^+\gamma)=640\pm246$ keV and 240 ± 90 keV for the cases of $M=1280$ MeV and $\Gamma=300$ MeV, and $M=1056$ MeV and $\Gamma=476$ MeV, respectively, through the measurement of Primakoff production of the a_1 resonance. Our predicted width, which is $\Gamma(a_1\to\pi\gamma)=125$ keV, for the case of $M=1056$ MeV also seems to be con-

TABLE V. Formulas for radiative decay widths of the transition $^1P_1\to{}^1S_0+\gamma$.

Process	$\Gamma=\frac{\alpha}{3m_I^2}\epsilon_1^2 q_\gamma$ ϵ_1
$b_1\to\pi\gamma$	$\frac{1}{3}\sqrt{\Omega_N}D_N F_N$
$\to\eta\gamma$	$-\sqrt{\Omega_N}D_N F_N\sin\phi_P$
$\to\eta'\gamma$	$\sqrt{\Omega_N}D_N F_N\cos\phi_P$
$K_{1B}^\pm\to K^\pm\gamma$	$\left[\frac{\Omega_K}{x+\frac{1}{x}+2}\right]^{1/2} D_K\left[\frac{2}{3}F_K-\frac{x}{3}\bar{F}_K\right]\left[1+\frac{1}{x}\right]$
$K_{1B}^0\to K^0\gamma$	$\left[\frac{\Omega_K}{x+\frac{1}{x}+2}\right]^{1/2} D_K\left[-\frac{1}{3}F_K-\frac{x}{3}\bar{F}_K\right]\left[1+\frac{1}{x}\right]$
$h_1\to\pi^0\gamma$	$\sqrt{\Omega_N}D_N F_N\cos\phi_B$
$\to\eta\gamma$	$-\frac{1}{3}\sqrt{\Omega_N}D_N F_N\cos\phi_B\sin\phi_P-\frac{2}{3}\sqrt{\Omega_S}D_S F_S\sin\phi_B\cos\phi_P$
$\to\eta'\gamma$	$\frac{1}{3}\sqrt{\Omega_N}D_N F_N\cos\phi_B\cos\phi_P-\frac{4}{3}\sqrt{\Omega_S}D_S F_S\sin\phi_B\sin\phi_P$
$h'_1\to\pi^0\gamma$	$-\sqrt{\Omega_N}D_N F_N\sin\phi_B$
$\to\eta\gamma$	$\frac{1}{3}\sqrt{\Omega_N}D_N F_N\sin\phi_B\sin\phi_P-\frac{2}{3}\sqrt{\Omega_S}D_S F_S\cos\phi_B\cos\phi_P$
$\to\eta'\gamma$	$-\frac{1}{3}\sqrt{\Omega_N}D_N F_N\sin\phi_B\cos\phi_P-\frac{2}{3}\sqrt{\Omega_S}D_S F_S\cos\phi_B\sin\phi_P$
	$D_j=\frac{(M_{0j}+M_{1j})^2}{4M_{0j}M_{1j}}$

TABLE VI. Formulas for radiative decay widths of the $K_1(1270)$ and the $K_1(1400)$.

$\langle K_1 \to K\gamma \rangle$

$$J_\mu = \epsilon_1 \bar{P}(p')K_{1B\mu}(p) + \frac{\mu}{\sqrt{2}}\frac{m_I^2 - m_F^2}{2} q_\gamma \bar{P}(p')K_{1A\mu}(p)$$

$$\Gamma = \frac{\alpha}{3m_1^2} q_\gamma \left[\epsilon_1 \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix} + \eta \frac{\mu}{\sqrt{2}} \frac{m_I^2 - m_F^2}{2} q_\gamma \begin{bmatrix} -\sin\psi \\ \cos\psi \end{bmatrix} \right]^2 \quad \begin{matrix} \text{for } K_1(1270) \\ \text{for } K_1(1400) \end{matrix}$$

	ϵ_1	μ
$K_1^\pm \to K^\pm\gamma$	$\left[\frac{\Omega_K}{x+\frac{1}{x}+2}\right]^{1/2} D_K \left[\frac{2}{3}F_K - \frac{x}{3}\bar{F}_K\right]\left[1+\frac{1}{x}\right]$	$\pm\left[x+\frac{1}{x}+2\right]^{1/2} \frac{g_M}{\sqrt{\Omega_K}} A_K \left[\frac{2}{3x}F_K + \frac{x}{3}\bar{F}_K\right]$
$K_1^0 \to K^0\gamma$, $\bar{K}_1^0 \to \bar{K}^0\gamma$	$\left[\frac{\Omega_K}{x+\frac{1}{x}+2}\right]^{1/2} D_K \left[-\frac{1}{3}F_K - \frac{x}{3}\bar{F}_K\right]\left[1+\frac{1}{x}\right]$	$\pm\left[x+\frac{1}{x}+2\right]^{1/2} \frac{g_M}{\sqrt{\Omega_K}} A_K \left[-\frac{1}{3x}F_K + \frac{x}{3}\bar{F}_K\right]$
	D_K: see Table V	A_K: see Table III

$\langle K_1 \to K^*\gamma \rangle$

$$J_\mu = ie\epsilon_{\mu\nu\kappa\lambda}\left[\frac{\epsilon_1}{\sqrt{2}\,m_I}p_\nu\delta_{\kappa\rho} + \frac{\epsilon_2}{\sqrt{2}\,m_I}p_\nu p_\rho p'_\kappa - \frac{\mu_2}{\sqrt{2}m_I}p_\nu q_\rho p'_\kappa\right]\bar{V}_\rho(p')K_{1A\lambda}(p) - ie\mu\epsilon_{\mu\nu\kappa\lambda}q_\nu p'_\kappa p'_\rho \bar{V}_\lambda(p')K_{1B\rho}(p)$$

$$\Gamma = \frac{\alpha}{6m_I^2}q_\gamma \left[\left\{2\epsilon_1^2 + (\epsilon_2-\mu_2)^2\frac{m_I^2}{m_F^2}q_\gamma^4 + \left[\frac{\epsilon_1^2}{m_F^2} - \epsilon_1(\epsilon_2-\mu_2)\left[1+\frac{m_I^2}{m_F^2}\right]\right]q_\gamma^2\right\}\begin{bmatrix}\sin^2\psi \\ \cos^2\psi\end{bmatrix} + 2m_1^2\mu^2\begin{bmatrix}\cos^2\psi \\ \sin^2\psi\end{bmatrix}q_\gamma^4\right.$$

$$\left. \pm\eta\left[\sqrt{2}\,\epsilon_1\mu m_I q_\gamma^2 + 2\sqrt{2}(\epsilon_2-\mu_2)\mu\frac{m_I^3}{m_I^2+m_F^2}q_\gamma^4\right]\cos\psi\sin\psi\right] \quad \begin{matrix} \text{for } K_1(1270) \\ \text{for } K_1(1400) \end{matrix}$$

	ϵ_1 or ϵ_2	μ_2
$K_1^\pm \to K^{*\pm}\gamma$	$\left[\frac{\Omega_K}{x+\frac{1}{x}+2}\right]^{1/2} B_K \left[\frac{2}{3}F_K - \frac{x}{3}\bar{F}_K\right]\left[1+\frac{1}{x}\right]$	$\left[x+\frac{1}{x}+2\right]^{1/2} \frac{g_M}{\sqrt{\Omega_K}} C_K \left[\frac{2}{3x}F_K - \frac{x}{3}\bar{F}_K\right]$
$K_1^0 \to K^{*0}\gamma$, $\bar{K}_1^0 \to \bar{K}^{*0}\gamma$	$\left[\frac{\Omega_K}{x+\frac{1}{x}+2}\right]^{1/2} B_K \left[-\frac{1}{3}F_K - \frac{x}{3}\bar{F}_K\right]\left[1+\frac{1}{x}\right]$	$\left[x+\frac{1}{x}+2\right]^{1/2} \frac{g_M}{\sqrt{\Omega_K}} C_K \left[-\frac{1}{3x}F_K - \frac{x}{3}F_K\right]$
	B_K: see Table IV	C_K: see Table IV

$$K_{\mu\nu}(^3P_1) = \frac{i}{\sqrt{2}\,m_{K1}}\epsilon_{\mu\nu\kappa\lambda}p_\kappa K_{1A\lambda}(p) \qquad \eta = \begin{cases} +1 & \text{for } K_1^+ \text{ and } K_1^0 \\ -1 & \text{for } K_1^- \text{ and } \bar{K}_1^0 \end{cases}$$

TABLE VII. Radiative decay widths of the S-wave mesons in comparison with experiment. The input values are underlined.

Process		Radiative decay width Γ (keV) Theory[a]	Experiment[b]
$\rho \to \pi\gamma$	$^3S_1 \to {}^1S_0$	<u>68</u>	68±7
$\to \eta\gamma$		54	50±13[c]
$K^{*\pm} \to K^\pm\gamma$		<u>50</u>	50±5
$K^{*0} \to K^0\gamma$		109	117±10
$\omega \to \pi^0\gamma$		646	680±80
$\to \eta\gamma$		6.0	$3.0^{+2.5}_{-1.8}$[c]
$\phi \to \pi^0\gamma$		6.1	5.78±0.58
$\to \eta\gamma$		49	56.7±2.8
$\to \eta'\gamma$		0.36	<1.8[d]
$\eta' \to \rho^0\gamma$	$^1S_0 \to {}^3S_1$	57	62±6
$\to \omega\gamma$		6.8	6.2±0.9

[a]$\eta = -(n\bar{n})\sin(-55°) + (-s\bar{s})\cos(-55°)$, $\eta' = (-s\bar{s})\sin(-55°) + (n\bar{n})\cos(-55°)$, $\omega = (n\bar{n})\cos3.7° + (-s\bar{s})\sin3.7°$, $\phi = (-s\bar{s})\cos3.7° - (n\bar{n})\sin3.7°$.
[b]From Ref. 17, unless otherwise noted.
[c]Reference 51.
[d]Reference 52.

sistent with the corresponding experimental one.

$^3P_1 \to {}^3S_1 + \gamma$ [$f_1(1285) \to \phi\gamma$]. Our radiative widths of $f_1(1285) \to \phi\gamma$ in the cases of the mixing angle $\phi_A = 21°$ and 10° in combination with $f_1(1530)$ and $f_1(1420)$ are, respectively, 17 and 4.1 keV. The former case seems to be consistent, while the latter case is not consistent with the present experiment.[35] Conversely the experimental width (23±11 keV) gives the mixing angle between $\phi_A = 16°–29°$. Here we note that about 90% of this theoretical width is due to contributions from the convection current [the first term in Eq. (2.6b)] which reflects essentially the current conservation in our scheme.

$^1P_1 \to {}^1S_0 + \gamma$ [$b_1^+(1235) \to \pi^+\gamma$]. Our predicted width (66 keV) for this process is much smaller than the experimental one (230±60 keV).[36] We consider that this discrepancy is quite serious for our scheme, since to this process contributes only the convection current, whose form is almost uniquely determined from the conservation of EM current. Remembering that our prediction for the process $f_1(1285) \to \phi\gamma$, where the contribution of the same convection current is dominant, reproduces well the experiment, we might suspect the experiment. There

TABLE VIII. Experimental candidates, established and/or typically proposed, to be examined are listed. Some comments based on the results of our analysis of the radiative decays are also given.

$^{2S+1}L_J$	J^{PC}	Candidates Isospin 1	$\frac{1}{2}$	O_N	O_S	Our comments
$^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2(1270)$	$f_2'(1525)$	Consistent with the assignments of $a_2(1320)$ and $K_2^*(1430)$
$^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}(1340)$	$f_1(1285)$	$f_1(1530)$[a]	Consistent with the assignment of $a_1(1260)$
					$f_1(1420)$	$f_1(1285)$: mainly the $n\bar{n}$ member with mixing angle $\phi_A \approx 16°-29°$; its $\rho^0\gamma$
		$a_1(1075)$[b]		Not seen	$f_1(1285)$[b]	branching fraction large enough to be observed $f_1(1530)$: favored over $f_1(1420)$ as mainly the $s\bar{s}$ member; experimental research of their decays into $\phi\gamma$ expected $K_1^{\pm}(1400) \to K^{\pm}\gamma$: This width may be most feasibly obtained in the Primakoff production
$^3P_0$	0^{++}	$a_0(980)$	$K_0^*(1430)$	$f_0(1400)$	$f_0(975)$	
		$a_0(1300)$[c]			$f_0(1525)$[c]	
$^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}(1340)$	$h_1(1170)$	$h_1(1380)$[d]	$b_1(1235) \to \pi\gamma$: $\Gamma^{\rm expt}/\Gamma^{\rm th} \sim 3-4$; reexamination of $\Gamma^{\rm expt}$ required
					$h_1(1420)$[b]	

[a]References 22 and 23.
[b]Reference 25.
[c]Reference 27.
[d]Reference 22.

seems to be the possibility[40] that the resonance "$b_1(1235)$" observed through the Primakoff process might be contaminated with any other state than the relevant $q\bar{q}$ state which is located in the similar mass region. Here we note that a theory in Ref. 41 gives good agreement with the experiment for this process. However, we think that its theoretical scheme is rather special as will be discussed in Sec. V D. It is also to be noted that the same theory gives a much larger value than the experimental one for the width $\Gamma(a_1 \to \pi\gamma)$.

TABLE IX. Radiative decay widths of the P-wave mesons in comparison with experiment. Theoretical values without the form-factor effects ($F=\bar{F}=1$) are also, for reference, given in parentheses.

Process		Radiative decay width Γ (keV) Theory (without F)	Experiment
$a_2^+(1320) \to \pi^+\gamma$	$^3P_2 \to {}^1S_0$	235(508)	295±60[a]
$K_2^{*+}(1430) \to K^+\gamma$	$^3P_2 \to {}^1S_0$	183(347)	240±45[b]
$K_2^{*0}(1430) \to K^0\gamma$	$^3P_2 \to {}^1S_0$	2.3(6.6)	<84[c]
$a_1^+(1260) \to \pi^+\gamma$	$^3P_1 \to {}^1S_0$	311(672)	640±246[d]
$f_1(1285) \to \phi\gamma$	$^3P_1 \to {}^3S_1$	20(32)[e]	23±11[g]
		5.6(9.3)[f]	
$b_1^+(1235) \to \pi^+\gamma$	$^1P_1 \to {}^1S_0$	66(144)	230±60[h]

[a]Reference 30.
[b]Reference 31.
[c]Reference 32.
[d]Reference 33.
[e]$f_1(1285) = (n\bar{n})\cos\phi_A + (-s\bar{s})\sin\phi_A$: $\phi_A = 21°$.
[f]Same as the above equation: $\phi_A = 10°$.
[g]Reference 35.
[h]Reference 36.

The above comments on the results of our analysis are summarized in Table VIII.

C. Prediction of decay widths for all possible radiative processes

Our predicted values of the P-wave meson decay width and their branching fractions ($=\Gamma_\gamma/\Gamma_{tot}$; the total width taken from the experimental value[17]) for all the possible radiative channels are given systematically in Tables X–XIV; Table X for the decay of the $^3P_2$ nonet, Table XI for the decay of the $^3P_1$ nonet, Table XII for the decay of the $^3P_0$ nonet [including the process $\phi\to f_0(975)\gamma$] (see Ref. 53), Table XIII for the decay of the $^1P_1$ nonet, and Table XIV for the decay of the $K_1(1270)$ and the $K_1(1400)$.

Here we note that decays of nonstrange neutral mesons through the processes $^3P_{2,1,0}\to {}^1S_0+\gamma$ and $^1P_1\to {}^3S_1+\gamma$ are forbidden due to the change-conjugation symmetry, and are omitted from these tables. In Table XII the processes $f_0(975)$ and $f_0(1525)\to\rho^0\gamma$ vanish because of our choice of the mixing angle for the scalar nonet $\phi_S=0°$. In Table XIV we have typically given the results for the following three values of the mixing angle[42,43] between the states $^3P_1$ and $^1P_1$ defined in Eq. (3.8): (i) $\psi=56°$ determined[43] from the decay rates of the processes $K_1(1400)$ and $K_1(1270)\to K^*\pi$ or $K\rho$; (ii) $\psi=45°$, where the $K_1(1270)$ decouples completely from $K^*\pi$ and the $K_1(1400)$ from $K\rho$; (iii) $\psi=0°$, where no mixing occurs and $K_1(1270)=K_{1B}$ and $K_1(1400)=K_{1A}$.

As may be seen in Table IX, the experimental information is presently very poor and much experimental work is needed to clarify the assignment of the P-wave meson nonets. In this content the present analysis provides a guide to detectable processes. Here we wish to take up particularly the following two examples of interesting processes: $f_1(1285)\to\rho^0\gamma$ and $K_1^{\pm}(1400)\to K^{\pm}\gamma$. The former has a most prominent predicted branching frac-

TABLE X. Predicted radiative decay widths and branching fractions of the $^3P_2$ nonet. The decay modes which have any experimental information are underlined.

Process	[Mass (MeV) Width][a]	Radiative decay width Γ (keV): Width	Fraction (%)
$a_2(1320)$ $\to\underline{\pi^\pm\gamma}$	[1318±5, 110±5]	235	0.21
$\to\rho\gamma$		28	0.03
$\to\omega\gamma$		247	0.22
$\to\phi\gamma$		0.8	7×10^{-4}
$K_2^*(1430)$ $\to\underline{K^\pm\gamma}$	[1426±2, 99±3]	183	0.18
$\to\underline{K^0\gamma}$		2.3	2×10^{-3}
$\to\underline{K^{*\pm}}\gamma$		38	0.04
$\to K^{*0}\gamma$		109	0.11
$f_2(1270)^b$ $\to\rho^0\gamma$	[1274±5, 185±20]	254	0.14
$\to\omega\gamma$		27	0.01
$\to\phi\gamma$		1.3	7×10^{-4}
$f_2'(1525)^b$ $\to\rho^0\gamma$	[1525±5, 76±10]	4.8	6×10^{-3}
$\to\omega\gamma$		~0	~0
$\to\phi\gamma$		104	0.14

[a]Reference 17.
[b]$f_2(1270)=(n\bar n)\cos(-8°)+(-s\bar s)\sin(-8°)$, $f_2'(1525)=(-s\bar s)\cos(-8°)-(n\bar n)\sin(-8°)$.

TABLE XI. Predicted radiative decay widths and branching fractions of the $^3P_1$ nonet. The decay modes which have any experimental information are underlined.

Process	[Mass (MeV) Width][a]	Radiative decay width Γ (keV): Width	Fraction (%)
$a_1(1260)$ $\to\underline{\pi^\pm\gamma}$	[1260±30, 300−600]	311	0.05–0.10
$\to\rho\gamma$		62	0.01–0.02
$\to\omega\gamma$		537	0.09–0.18
$\to\phi\gamma$		0.9	$(2-3)\times10^{-4}$
$\phi_A=21°$			
$f_1(1285)^b$ $\to\rho^0\gamma$	[1283±5, 25±3]	509	2.0
$\to\omega\gamma$		48	0.19
$\to\underline{\phi\gamma}$		20	0.08
$f_1(1530)^b$ $\to\underline{\rho^0\gamma}$	[1527±5, 106±14]	116	0.11
$\to\omega\gamma$		24	0.02
$\to\phi\gamma$		198	0.19
$\phi_A=10°$			
$f_1(1285)^c$ $\to\rho^0\gamma$	[1283±5, 25±3]	565	2.3
$\to\omega\gamma$		57	0.23
$\to\underline{\phi\gamma}$		5.6	0.02
$f_1(1420)^c$ $\to\underline{\rho^0\gamma}$	[1422±10, 55±3]	23	0.04
$\to\omega\gamma$		8.1	0.01
$\to\phi\gamma$		180	0.33

[a]Reference 17.
[b]$f_1(1285)=(n\bar n)\cos21°+(-s\bar s)\sin21°$, $f_1(1530)=(-s\bar s)\cos21°-(n\bar n)\sin21°$.
[c]$f_1(1285)=(n\bar n)\cos10°+(-s\bar s)\sin10°$, $f_1(1420)=(-s\bar s)\cos10°-(n\bar n)\sin10°$.

tion of ~2% out of all possible radiative decay channels of the P-wave mesons and therefore we expect that this process may be observed in the reaction $\pi^- p \to (\pi^+\pi^-\gamma)n$ if the contributions from $f_2(1270)$, $f_0(1400)$, and $\eta(1280)\to\rho^0\gamma$ are properly separated. The latter is one of the processes which are possible but have not yet been investigated through the Primakoff effect. Since it has the largest partial radiative width (~300 keV) among these uninvestigated processes according to our analysis, it may be most feasible to detect this process in the Primakoff production.

TABLE XII. Predicted radiative decay widths and branching fractions of the $^3P_0$ nonet. The prediction for $\phi\to f_0(975)+\gamma$ is also given.

Process [Mass (MeV) Width][a]		Radiative decay width Γ (keV) Width	Fraction (%)
$a_0(980)$ [983±3, 57±11]	$\to\rho\gamma$	40	0.07
	$\to\omega\gamma$	331	0.58
$a_0(1300)$ [~1300, ~250][b]	$\to\rho\gamma$	93	0.04
	$\to\omega\gamma$	800	0.32
	$\to\phi\gamma$	1.3	5×10^{-4}
$K_0^*(1430)$ [1429±7, 287±23]	$\to K^{*\pm}\gamma$	154	0.05
	$\to K^{*0}\gamma$	358	0.12
$f_0(1400)$[c] [~1400, 150–400]	$\to\rho^0\gamma$	1032	.26–0.69
	$\to\omega\gamma$	110	0.03–0.07
	$\to\phi\gamma$	0.2	$(5-13)\times10^{-5}$
$f_0(975)$[c] [976±3, 34±6]	$\to\rho^0\gamma$	0	0
	$\to\omega\gamma$	0.6	2×10^{-3}
$f_0(1525)$[c] [~1525, ~90][e]	$\to\rho^0\gamma$	0	0
	$\to\omega\gamma$	2.4	3×10^{-3}
	$\to\phi\gamma$	302	0.34
$\phi(1020)$ [1019.41±0.01, 4.41±0.05]	$\to f_0(975)\gamma$[c,e]	11	0.25

[a]From Ref. 17, unless otherwise noted.
[b]Reference 28.
[c]$f_0(1400)=n\bar{n}$, $f_0(975)$, and $f_0(1525)=-s\bar{s}$.
[d]Reference 27.
[e]See Ref. 53.

D. Supplementary discussions on our analysis

In this subsection we add some remarks on our theoretical scheme and give some comments on related works.

(i) We have examined the effect of the form factors F_j $(j=N,S,K)$ (appearing in Tables III–V) due to the space-time overlap integral of the internal HO wave functions. In Table IX the theoretical values of decay width without these form factors are also given in the corresponding parentheses. From this we see that the HO wave function satisfying the definite-metric-type subsidiary condition gives desirable moderate damping effects to the radiative decays of P-wave mesons similarly as in the case of the EM form factors[44] of hadrons. As is well known, in most cases of the NR treatment the corresponding form factor gives the strong exponential damping.

(ii) In this paper we have taken the BW scheme for the covariant spin wave function out of two interesting schemes. The reason for this is that in the BW scheme it is easy to introduce covariantly the effects of perturbative QCD, while in the other scheme, the minimally boosted Pauli (MP) scheme,[45] it seems difficult. Actually we have also made our analysis based on the MP scheme. Both the BW and the MP schemes give the same results for the radiative decays of the S-wave mesons, while for the decays of the P-wave mesons the latter gives slightly worse agreement[46] with experiment than the former.

(iii) By using the values of parameters in Eq. (4.2)

TABLE XIII. Predicted radiative decay widths and branching fractions of the $^1P_1$ nonet. The decay mode which has any experimental information is underlined.

Process [Mass (MeV) Width][a]		Radiative decay width Γ (keV) Width	Fraction (%)
$b_1(1235)$ [1233±10, 150±10]	$\to\underline{\pi\gamma}$	66	0.04
	$\to\eta\gamma$	327	0.22
	$\to\eta'\gamma$	79	0.05
	$\to\rho^\pm\gamma$	50	0.03
$h_1(1170)$[b] [1170±40, 335±26]	$\to\pi^0\gamma$	543	0.16
	$\to\eta\gamma$	86	0.03
	$\to\eta'\gamma$	0.5	1×10^{-4}
$h_1(1380)$[b] [1380±20, 80±30][c]	$\to\pi^0\gamma$	75	0.09
	$\to\eta\gamma$	40	0.05
	$\to\eta'\gamma$	115	0.14

[a]From Ref. 17, unless otherwise noted.
[b]$h_1(1170)=(n\bar{n})\cos(-22°)+(-s\bar{s})\sin(-22°)$, $h_1(1380)=(-s\bar{s})\cos(-22°)-(n\bar{n})\sin(-22°)$.
[c]Reference 22.

TABLE XIV. Predicted radiative decay widths and branching fractions of the $K_1(1270)$ and the $K_1(1400)$.

Process [Mass (MeV) / Width][a]	Radiative decay width Γ (keV), Mixing angle[b] ψ: 56° Width	56° Fraction (%)	45° Width	45° Fraction (%)	0° Width	0° Fraction (%)
$K_1(1270)$ [1270±10, 90±20] $\to K^{\pm}\gamma$	21	0.02	2.9	3×10^{-3}	91	0.10
$\to K^0\gamma$	60	0.07	106	0.12	252	0.28
$\to K^{*\pm}\gamma$	61	0.07	51	0.06	22	0.02
$\to K^{*0}\gamma$	135	0.15	98	0.11	0.3	3×10^{-4}
$K_1(1400)$ [1401±10, 184±9] $\to K^{\pm}\gamma$	286	0.16	332	0.18	273	0.15
$\to K^0\gamma$	190	0.10	148	0.08	3.4	2×10^{-3}
$\to K^{*\pm}\gamma$	87	0.05	92	0.05	102	0.06
$\to K^{*0}\gamma$	78	0.04	125	0.07	251	0.14

[a]Reference 17.
[b]See Eq. (3.8).

determined in this paper we have recalculated the baryon magnetic moments. The results have become[47] not so much different from the previous ones[18] with the different values of parameters.

(iv) Recently the authors of Ref. 48 have proposed a different viewpoint from the ordinary one concerning the identification of two isoscalar $J^{PC}=1^{++}$ mesons. They assign the $f_1(1285)$ and the $f_1(1530)$ to the $^3P_1$ state with mainly $s\bar{s}$ content and the radially excited $^3P_1$ state with mainly $n\bar{n}$ content, respectively, contrary to the more standard assignment in Table VIII as the $^3P_1$ states with the mainly $n\bar{n}$ and $s\bar{s}$ contents, respectively. If we take their assignment, our model gives $\Gamma[f_1(1285)\to\phi\gamma]=123-135$ keV for 93–100 % $s\bar{s}$, the very large value, being contrary to their prediction ($\approx 16\pm4\pm4$ keV), as compared to the observed one. On the other hand, our model with the standard assignment predicts $\Gamma[f_1(1285)\to\rho^0\gamma]=509$ (565) keV for the case of the mixing angle $\phi_A=21°$ (10°), which is much smaller than their estimate (~ 2 MeV) and not contradictory with the fact that this channel has not been observed yet. These differences reflect the difference of q_γ dependence (q_γ being the photon momentum) of the radiative decay width in the two schemes. They used the NR method in which the radiative width was proportional to q_γ^3. Thus our analysis seems to be out of favor with their proposal.

(v) As was noted in Sec. V B, the authors of Ref. 41, which has been frequently referred to, reproduce quite well the experimental width of $b_1\to\pi\gamma$, being contrary to our result. Using vector-meson dominance (VMD) and assuming some q_π dependence on the relevant strong vertex they derived the formula

$$\Gamma(b_1\to\pi\gamma)=\left[\frac{q_\gamma}{q_\pi}\right]^3\frac{\alpha}{g_\omega^2/4\pi}\Gamma(b_1\to\pi\omega_\perp)\ , \qquad (5.1)$$

where $\omega_\perp$ denotes the transversely polarized ω meson, g_ω is the ω-γ coupling constant, and q_γ [q_π] is the magnitude of photon [pion (supposed $m_\pi=0$)] three-momentum in the rest frame of the b_1 meson. The formula, inputting the experimental values of $g_\omega^2/4\pi=23$ determined from $\Gamma(\omega\to e^+e^-)=0.60$ keV and of $\Gamma(b_1\to\pi\omega_\perp)=131$ MeV from the ratio of the partial-wave amplitudes $D/S=0.26$ and $\Gamma(b_1\to\pi\omega)=150$ MeV for the decay process $b_1\to\pi\omega$, gives $\Gamma(b_1\to\pi\gamma)=219$ keV in good agreement with the experiment. The essential point giving this seemingly good result is the momentum dependence $(q_\gamma/q_\pi)^3$ in Eq. (5.1). However, it seems to us that their argument[49] leading to this dependence is rather special. As a matter of fact we can derive, assuming similarly VMD and using a usual technique[50] of the local field theory in treating the strong decay part $b_1\to\pi\omega$, another formula

$$\Gamma(b_1\to\pi\gamma)=\frac{q_\gamma}{q_\pi}\frac{\alpha}{g_\omega^2/4\pi}\Gamma(b_1\to\pi\omega_\perp)\ . \qquad (5.2)$$

This gives $\Gamma(b_1\to\pi\gamma)=72$ keV which is quite close to our theoretical value given in Table IX.

VI. CONCLUDING REMARKS

In this paper we have applied the COQM for analyzing the radiative decays of the ground S-wave and the first excited P-wave mesons in light-quark $q\bar{q}$ systems. An important feature of our theoretical framework is that all the respective radiative decay processes are, in a unified and systematic way, described by the covariant effective EM currents whose conservation are guaranteed by the internal structure of meson mass spectra in the COQM.

The predicted decay widths of the S-wave mesons seem to be in very good agreement with the present comparatively rich experiments (see Table VII).

Concerning the P-wave meson system experimental information is very poor. Even the assignment of the particles is still not established except for the $^3P_2$ nonet, and the radiative decay width is reported only for a few restricted processes. Our results of analysis are generally in good agreement with the experiments for the established candidates except for the process $b_1^+ \to \pi^+\gamma$ and consistent with the experiments for the usually supposed candidates (see Table IX). Thus, insofar as the present poor experimental information is concerned, regrettably we cannot extract any new definite comment from our analysis on the controversial points of the P-wave meson classification. (However, see some comments in Sec. V.)

We have given our predicted width for all possible radiative decay channels in Tables X–XIV. Correspondingly more experimental efforts are expected.

It is generally believed that around the mass region of the P-wave mesons there may exist many other kinds of particles such as glueballs, hybrid mesons, and four-quark mesons. Recently we have carefully studied the isoscalar $J^{PC}=1^{++}$ mesons thus far observed and argued that the $f_1(1285)$ and the $f_1(1530)$ are more likely isoscalar members of the $^3P_1$ $q\bar{q}$-nonet and the $f_1(1420)$ is presumably a hybrid meson. (The results will be published elsewhere.)

Finally we point out some defects in our treatment. (We do not repeat here the defects concerning the general structure of our theoretical scheme, which were already mentioned in our previous work.[5]) One serious defect is that, in contrast with most other cases of models, our scheme is not directly applicable to heavy-quark systems where we have now fairly rich experimental knowledge, because the symmetric HO description of the system seems to be far from the actual system.

ACKNOWLEDGMENTS

We would like to express our deep gratitude to Dr. N. Honzawa, Dr. K. Yamaguchi, H. Sawazaki, and T. Kurokawa for their cooperation in our related works.

*Present address: Physical Science Laboratory, College of Science and Technology, Nihon University, Funabashi, Chiba 274, Japan.

[1]For further discussions on this point, see S. Ishida *et al.*, Prog. Theor. Phys. **71**, 806 (1984); S. Ishida, K. Yamada, and M. Oda, Phys. Rev. D **28**, 2918 (1983).

[2]D. P. Stanley and D. Robson, Phys. Rev. D **21**, 3180 (1980); J. Carlson, J. Kogut, and V. R. Pandharipande, *ibid.* **27**, 233 (1983); **28**, 2807 (1983); S. Godfrey and N. Isgur, *ibid.* **32**, 189 (1985); C. Habe *et al.*, Prog. Theor. Phys. **77**, 917 (1987).

[3]S. Ishida, Prog. Theor. Phys. **46**, 1570 (1971); **46**, 1905 (1971); R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**, 2706 (1971). For a recent review see S. Ishida, in *Hadron Spectroscopy—1985,* proceedings of the International Conference, University of Maryland, edited by S. Oneda (AIP Conf. Proc. No. 132) (AIP, New York, 1985), p. 277, and the references therein. See also Y. S. Kim and M. E. Noz, *Theory and Applications of the Poincaré Group* (Reidel, Dordrecht, 1986).

[4]S. Ishida *et al.*, Phys. Rev. D **20**, 2906 (1979).

[5]S. Ishida and K. Yamada, Phys. Rev. D **35**, 265 (1987).

[6]J. Babcock and J. L. Rosner, Phys. Rev. D **14**, 1286 (1976); J. L. Rosner, *ibid.* **23**, 1127 (1981); Riazuddin and Fayyazuddin, *ibid.* **32**, 2458 (1985); M. K. Volkov, Yad. Fiz. **40**, 974 (1984) [Sov. J. Nucl. Phys. **40**, 622 (1984)].

[7]Godfrey and Isgur (Ref. 2); A. S. Bagdasaryan, S. V. Esaybegyan, and H. L. Ter-Isaakyan, Yad. Fiz. **42**, 440 (1985) [Sov. J. Nucl. Phys. **42**, 278 (1985)]; I. G. Aznauryan and K. A. Oganesyan, *ibid.* **47**, 1731 (1988) **47**, 1097 (1988)]; N. N. Singh and A. N. Mitra, Phys. Rev. D **38**, 1454 (1988). However, see the following work treating the radiative decays of heavy-quark meson systems from the similar viewpoint to ours but on the different method: F. Daghighian and D. Silverman, Phys. Rev. D **36**, 3401 (1987).

[8]A. Salam, R. Delbourgo, and J. Strathdee, Proc. R. Soc. London **A284**, 146 (1965); B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965).

[9]S. Ishida and T. Sonoda, Prog. Theor. Phys. **70**, 1323 (1983).

[10]T. Takabayasi, Nuovo Cimento **33**, 668 (1964); S. Ishida and J. Otokozawa, Prog. Theor. Phys. **47**, 2117 (1972); Y. S. Kim and M. E. Noz, Phys. Rev. D **8**, 3521 (1973).

[11]K. Fujimura, T. Kobayashi, and M. Namiki, Prog. Theor. Phys. **43**, 73 (1970).

[12]R. G. Lipes, Phys. Rev. D **5**, 2849 (1972); S. Ishida and J. Otokozawa, Prog. Theor. Phys. **53**, 217 (1975).

[13]Feynman, Kislinger, and Ravndal (Ref. 3).

[14]S. Okubo, Phys. Lett. **5**, 165 (1963); G. Zweig, CERN Report No. 8182/Th. 401, 1964 (unpublished).

[15]S. Ishida, Prog. Theor. Phys. **32**, 922 (1964).

[16]Ishida *et al.* (Ref. 1).

[17]Particle Data Group, G. P. Yost *et al.* Phys. Lett. B **204**, 1 (1988).

[18]Ishida, Yamada, and Oda (Ref. 1).

[19]F. J. Gilman and R. Kauffman, Phys. Rev. D **36**, 2761 (1987).

[20]For a recent review, see, for example, B. Diekmann, Phys. Rep. **159**, 99 (1988). For the problem of scalar mesons, see also N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Usp. Fiz. Nauk **142**, 361 (1984) [Sov. Phys. Usp. **27**, 161 (1984)]; K. L. Au, D. Morgan, and M. R. Pennington, Phys. Lett. **167B**, 229 (1986); Phys. Rev. D **35**, 1633 (1987).

[21]J. Olsson, in *Lepton and Photon Interactions,* proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, West Germany, 1987, edited by W. Bartel and R. Rückl [Nucl. Phys. B (Proc. Suppl.) **3**, 613 (1988)].

[22]D. Aston *et al.*, Phys. Lett. B **201**, 573 (1988).

[23]Ph. Gavillet *et al.*, Z. Phys. C **16**, 119 (1982).

[24]H. Aihara *et al.*, Phys. Rev. D **38**, 1 (1988); see also G. Gidal *et al.*, Phys. Rev. Lett. **59**, 2012 (1987).

[25]J. Iizuka, F. Masuda, and T. Miura, Phys. Rev. D **36**, 1422 (1987); Prog. Theor. Phys. **79**, 141 (1988); J. Iizuka, H. Koibuchi, and F. Masuda, Report No. GK-U. TSUKU/88-01, 1988 (unpublished).

[26]J. Weinstein and N. Isgur, Phys. Rev. Lett. **48**, 659 (1982); Phys. Rev. D **27**, 588 (1983).

[27]D. Aston *et al.*, Nucl. Phys. **B301**, 525 (1988).

[28]A. D. Martin *et al.*, Phys. Lett. **74B**, 417 (1978).

[29]For a recent review, see, for example, L. G. Landsberg, Phys.

Rep. **128**, 301 (1985).

[30]S. Cihangir *et al.*, Phys. Lett. **117B**, 119 (1982).

[31]S. Chihangir *et al.*, Phys. Lett. **117B**, 123 (1982).

[32]D. Carlsmith *et al.*, Phys. Rev. D **36**, 3502 (1987).

[33]M. Zielinski *et al.*, Phys. Rev. Lett. **52**, 1195 (1984).

[34]M. Zielinski, Phys. Rev. Lett. **58**, 2002 (1987).

[35]S. I. Bityukov *et al.*, Phys. Lett. B **203**, 327 (1988).

[36]B. Collick *et al.*, Phys. Rev. Lett. **53**, 2374 (1984).

[37]W. Ruckstuhl *et al.*, Phys. Rev. Lett. **56**, 2132 (1986).

[38]W. B. Schmidke *et al.*, Phys. Rev. Lett. **57**, 527 (1986); H. Albrecht *et al.*, Z. Phys. C **33**, 7 (1986); H. R. Band *et al.*, Phys. Lett. B **198**, 297 (1987).

[39]M. G. Bowler, Phys. Lett. B **182**, 400 (1986); **184**, 419(E) (1987); **209**, 99 (1988); see also N. A. Törnqvist, Z. Phys. C **36**, 695 (1987); **40**, 632(E) (1988).

[40]In fact the mass and total width of the $b_1(1235)$ observed in the Primakoff production process are 1271 ± 11 MeV and 232 ± 29 MeV, respectively. These values are larger than the standard ones of $M=1233\pm10$ MeV and $\Gamma=150\pm10$ MeV: see Ref. 36.

[41]Babcock and Rosner (Ref. 6); Rosner (Ref. 6).

[42]M. G. Bowler *et al.*, Nucl. Phys. **B74**, 493 (1974); **B97**, 227 (1975); M. G. Bowler, J. Phys. G **3**, 775 (1977); R. Barbieri, R. Gatto, and Z. Kunszt, Phys. Lett. **66B**, 349 (1977); R. K. Carnegie *et al.*, *ibid.* **68B**, 287 (1977); M. Mazzucato *et al.*, Nucl. Phys. **B156**, 532 (1979).

[43]C. Daum *et al.*, Nucl. Phys. **B187**, 1 (1981).

[44]Ishida (Ref. 3).

[45]S. Ishida, A. Matsuda, and M. Namiki, Prog. Theor. Phys. **57**, 210 (1977).

[46]In the MP scheme there are two types of spin wave functions, the MP I and MP II, for $q\bar{q}$ systems. For the processes $a_2^{\pm}\to\pi^{\pm}\gamma$, $K_2^{*\pm}\to K^{\pm}\gamma$, $K_2^{*0}\to K^0\gamma$, $a_1^{\pm}\to\pi^{\pm}\gamma$, $f_1(1285)\to\phi\gamma$, $b_1^{\pm}(1235)\to\pi^{\pm}\gamma$, the decay widths in the MP I are 127, 114, 1.4, 169, 20 ($\phi_A=21°$) or 5.7 ($\phi_A=10°$), 58 keV, respectively, and those in the MP II are 375, 269, 3.4, 497, 20 ($\phi_A=21°$) or 5.5 ($\phi_A=10°$), 76 keV.

[47]Our present and previous values of the baryon magnetic moments are as follows. For the baryons p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, $\Lambda\Sigma^0$ the present (previous) results are 2.55 ($\underline{2.79}$), -1.70 (-1.86), -0.618 ($\underline{-0.613}$), 2.13 (2.47), 0.68 (0.76), -0.77 (-0.94), -1.23 (-1.38), -0.51 (-0.49), -1.30 (-1.52) in units of nuclear magnetons, respectively, where input values are underlined. These values should be compared with the corresponding experimental ones: 2.793, -1.913, -0.613 ± 0.004, 2.42 ± 0.05, unmeasured, -1.157 ± 0.025, -1.250 ± 0.014, -0.69 ± 0.04, -1.61 ± 0.08 taken from Ref. 17.

[48]Iizuka, Koibuchi, and Masuda (Ref. 25).

[49]Babcoc and Rosner (Ref. 6); A. J. G. Hey, J. L. Rosner, and J. Weyers, Nucl. Phys. **B61**, 205 (1973); F. J. Gilman, M. Kugler, and S. Meshkov, Phys. Rev. D **9**, 715 (1974); F. J. Gilman and I. Karliner, *ibid.* **10**, 2194 (1974).

[50]The most general form of local interaction Lagrangian for the relevant vertex is given as $H_I^S=g_1\omega_\mu^*(q)B_\mu(p)\pi^*(p')+g_2p'_\mu\omega_\mu^*(q)p'_\nu B_\nu(p)\pi^*(p')$. We also assume the Lagrangian $H_1^{\omega\gamma}=(em_\omega^2/g_\omega)\omega_\mu(q)A_\mu(q)$ for the photon-ω-meson transition part. It is easily shown that only the first g_1-term of H_I^S contributes to the decay process $b_1\to\omega_\perp+\pi$. This scheme leads to the formula (5.2) in the text.

[51]D. E. Andrews *et al.*, Phys. Rev. Lett. **38**, 198 (1977).

[52]V. P. Druzhinin *et al.*, Z. Phys. C **37**, 1 (1987).

[53]Recently it has been reported that the experimental upper limit on the branching fraction for the decay mode $\phi\to f_0(975)\gamma$ is 0.2% (Ref. 52). Our predicted fraction, assuming that the $f_0(975)$ is a pure $s\bar{s}$ state, is 0.25%. To be consistent with this upper limit the $f_0(975)$ should have a fair amount of $n\bar{n}$ content.