Polarization in $\bar{B}\to D^*e^-\bar{\nu}_e$ and $D\to\bar{K}^*e^+\nu_e$

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We examine the predictions of the quark potential model for the ratio of longitudinal to transverse vector-meson production in semileptonic decays such as $\bar{B}\to D^*e^-\bar{\nu}_e$ and $D\to\bar{K}^*e^+\nu_e$. We also make some general comments on strategies for extracting reliable values of weak mixing angles from such decays.

I. INTRODUCTION

Extracting the Kobayashi-Maskawa (KM) matrix elements[1] from measurements of semileptonic decay rates necessarily involves theory since the rates depend on hadronic dynamics. Given our present rudimentary ability to calculate hadronic current matrix elements,[2-8] the determination of the KM angles will also involve an active interplay between experiment and theory which will test existing theoretical methods and help to define which measurements can be most useful for this purpose.

It is now widely accepted that semileptonic $\bar{B}$ decay[9,10] is dominated by the $\bar{B}\to De^-\bar{\nu}_e$ and $\bar{B}\to D^*e^-\bar{\nu}_e$ transitions, that semileptonic D decay[11-13] is dominated by $D\to\bar{K}e^+\nu_e$ and $D\to\bar{K}^*e^+\nu_e$, and that the vector final states contribute a large fraction of the total semileptonic rates. While it has been argued[14] that calculations for $\bar{B}\to D$ and $D\to\bar{K}$ can be made reliably and should be emphasized in the extraction of V_{cb} and V_{sc}, it is clearly important to have an independent check on the values deduced. The alternative of using inclusive rates for such a check is compromised by large uncertainties arising from the unknown effective quark masses and doubts about whether the rate for a process dominated by two exclusive final states can be calculated reliably in the parton model.[3,4] Thus the studies of $\bar{B}\to D^*e^-\bar{\nu}_e$ and $D\to\bar{K}^*e^+\nu_e$ have a special role to play in the effort to convincingly determine these fundamental parameters of the standard model.

Let P_α and V_α denote the ground-state pseudoscalar and vector meson with (dominant) quark composition $\alpha\bar{d}$ (or $\alpha\bar{u}$, or $\alpha\bar{s}$). Following the methods of Refs. 3 and 4, the semileptonic decay rates for $P_Q\to P_qe^-\bar{\nu}_e$ and $P_Q\to V_qe^-\bar{\nu}_e$ are

$$\frac{d^2\Gamma^P}{dx\,dy}=|V_{qQ}|^2\frac{G_F^2m_{P_Q}^5}{16\pi^3}\{\beta_{++}^P[4x(x_m-x)-y(1-2x)]\} \quad (1)$$

and

$$\frac{d^2\Gamma^V}{dx\,dy}=|V_{qQ}|^2\frac{G_F^2m_{P_Q}^5}{32\pi^3}\left[\frac{\alpha^Vy}{m_{P_Q}^2}+2\beta_{++}^V[4x(x_m-x)-y(1-2x)]-2\gamma^Vy(x_m-2x+\tfrac{1}{2}y)\right], \quad (2)$$

respectively. [For $P_Q\to V_qe^+\nu_e$ the sign of the γ^V term (see below) is reversed.] In these formulas x is the ratio of the electron energy to the rest mass m of the decaying particle, $x_m\equiv(m^2-m_X^2)/2m^2$ is the maximum value of x, $y\equiv m_{e\nu}^2/m^2$ where $m_{e\nu}$ is the mass of the electron-neutrino system (i.e., $m_{e\nu}^2=t$, the hadronic four-momentum transfer to the recoiling system X), and β_{++}^P, α^V, β_{++}^V, and γ^V are functions of form factors appearing in the $P_Q\to P_q$ and $P_Q\to V_q$ weak-current matrix elements. With $V_\mu\equiv\bar{q}\gamma_\mu Q$ and $A_\mu\equiv\bar{q}\gamma_\mu\gamma_5Q$, the relevant form factors are f_+, g, f, and a_+ where

$$\langle P_q(p')|V_\mu|P_Q(p)\rangle\equiv f_+(p+p')_\mu+f_-(p-p')_\mu\ , \quad (3)$$

$$\langle V_q(p'\epsilon)|V_\mu|P_Q(p)\rangle\equiv ig\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p+p')^\rho(p-p')^\sigma\ , \quad (4)$$

and

$$\langle V_q(p'\epsilon)|A_\mu|P_Q(p)\rangle\equiv f\epsilon_\mu^*+a_+(\epsilon^*\cdot p)(p+p')_\mu+a_-(\epsilon^*\cdot p)(p-p')_\mu \quad (5)$$

in terms of which

$$\beta_{++}^P=|f_+|^2\ , \quad (6)$$

$$\alpha^V=f^2+4m_{P_Q}^2g^2\mathbf{p}'^2\ , \quad (7)$$

$$\beta_{++}^V=\frac{f^2}{4m_{V_q}^2}-m_{P_Q}^2g^2y+\frac{1}{2}\left[\frac{m_{P_Q}^2}{m_{V_q}^2}(1-y)-1\right]fa_++\frac{m_{P_Q}^2a_+^2}{m_{V_q}^2}\mathbf{p}'^2\ , \quad (8)$$

and

$$\gamma^V=2gf\ , \quad (9)$$

where

$$\mathbf{p}'^2=\frac{[m_{P_Q}^2(1-y)+m_X^2]^2}{4m_{P_Q}^2}-m_X^2 \quad (10)$$

is the square of the recoil three-momentum of the system X. The form factors can of course be functions of $\mathbf{p}'^2$ or, equivalently, y. The Dalitz-plot variables x and y are restricted to the ranges

$$0 \le x \le x_m \equiv \frac{m^2 - m_X^2}{2m^2} \tag{11}$$

and

$$0 \le y \le \frac{4x(x_m - x)}{1-2x} \ . \tag{12}$$

We thus see that $P_Q \to V_q$ decays are considerably more complex than $P_Q \to P_q$ decays: their Dalitz-plot distributions depend on three form factors (and interferences between them) instead of just one. It consequently seems unlikely that theory can predict the $P_Q \to V_q$ rate as reliably as $P_Q \to P_q$.

While this is true, it is not necessarily relevant for extracting KM angles: the experimental study of $P_Q \to V_q$ can, in principle, tell us α^V, β^V_{++}, and γ^V (or f, g, and a_+) at each point in the Dalitz plot by fitting to Eq. (2). Thus, as in $P_Q \to P_q$, we ultimately only require that theory be able to predict reliably one form factor (or a linear combination of form factors). With this long-term goal in mind we will discuss some possible strategies for obtaining such reliable predictions and, in particular, the use of measurements of the V_q polarization (or, more precisely, alignment[15]) as a test of the quark potential model[3,4] for $P_Q \to V_q$ form factors. This study was prompted in part by the recent observation[13] of an unexpectedly large longitudinal fraction in the decays $D \to \bar{K}^* e^+ \nu_e$.

II. $P \to V$ FORM FACTORS IN THE QUARK POTENTIAL MODEL

It was originally thought[3,6,7] that the a_+ form factor of Eq. (5) could not be predicted within the framework of the quark potential model since its calculation appeared to require accuracy to order $(v/c)^2$. It was later shown[4,16] that this is not the case. Since understanding the nature of the form factors f, g, and a_+ is essential to our discussion, we will recall the argument for their calculability.

The discussion is most transparent for the $V_q \to P_Q$ matrix elements $\langle P_Q(\mathbf{p})|V_\mu(0)|V_q(\mathbf{0},\epsilon)\rangle$ and $\langle P_Q(\mathbf{p})|A_\mu(0)|V_q(\mathbf{0},\epsilon)\rangle$ related to (4) and (5) by complex conjugation and Lorentz invariance. Consider first the matrix element of V_μ. Its time component acting on $|V_q(\mathbf{0},\epsilon)\rangle$ produces a state with $J^P=1^-$; this is forbidden for P_Q (which can have only $J^P=0^-,1^+,2^-,\ldots$). Its space components can produce $J^P=0^+$, 1^+, or 2^+. Angular momentum conservation then requires P_Q to recoil in an $L=1$ state and we see that we can uniquely associate the form factor g of Eq. (4) with this $L=1$ amplitude. Note that $\langle P_Q(\mathbf{p})|\mathbf{V}(0)|V_q(\mathbf{0},\epsilon)\rangle \sim p \sim v/c$ as it must. Next consider the matrix element of A_μ. Its time component acting on $|V_q(\mathbf{0},\epsilon)\rangle$ produces a $J^P=1^+$ state which will once again lead to an $L=1$ partial-wave amplitude, while the space components $\mathbf{A}$ lead to an $L=0$ and an $L=2$ amplitude. These three partial-wave amplitudes can be uniquely associated with linear combinations of the three Lorentz form factors f, a_+, and a_-, which are necessarily proportional to v/c, 1, and $(v/c)^2$, respectively. This shows that their calculation does not require a model that can correctly predict relativistic corrections of order $(v/c)^2$: *this* $(v/c)^2$ is associated with the purely nonrelativistic behavior of a D-wave amplitude.

The calculations of Ref. 4 give for the four form factors which enter (6)–(9) the results

$$f_+ = F_3\left[1 + \frac{m_Q}{2\mu_-} - \frac{m_Q m_q}{4\mu_+\mu_-}\frac{m_d}{\tilde{m}_q}\frac{\beta_Q^2}{\beta_{Qq}^2}\right], \tag{13}$$

$$f = 2\tilde{m}_Q F_3 \ , \tag{14}$$

$$g = F_3\left[\frac{1}{2m_q} - \frac{1}{4\mu_-}\frac{m_d}{\tilde{m}_q}\frac{\beta_Q^2}{\beta_{Qq}^2}\right], \tag{15}$$

and

$$a_+ = -\frac{F_3}{2\tilde{m}_q}\left[1 + \frac{m_d}{m_Q}\left[\frac{\beta_Q^2-\beta_q^2}{\beta_Q^2+\beta_q^2}\right] - \frac{m_d^2}{4\mu_-\tilde{m}_Q}\frac{\beta_q^4}{\beta_{Qq}^4}\right], \tag{16}$$

where m_i is the constituent mass of quark i, $\tilde{m}_\alpha = m_\alpha + m_d$, $\mu_\pm^{-1} = m_q^{-1} \pm m_Q^{-1}$, β_α is a mass which characterizes the size of the $\alpha\bar{d}$ system, $\beta_{Qq}^2 = \frac{1}{2}(\beta_Q^2 + \beta_q^2)$ and

$$F_3 = \left[\frac{\tilde{m}_q}{\tilde{m}_Q}\right]^{1/2}\left[\frac{\beta_Q\beta_q}{\beta_{Qq}^2}\right]^{3/2}\exp\left[-\left[\frac{m_d^2}{4\tilde{m}_Q\tilde{m}_q}\right]\frac{t_m - t}{\kappa^2\beta_{Qq}^2}\right], \tag{17}$$

where $t_m = (m - m_X)^2$, and $\kappa \simeq 0.7$ is an empirical factor which corrects for relativistic effects. The f_+ form factor of the $P_Q \to P_q$ transition is "nearly" a simple overlap factor. In the limit $m_q \to m_Q$ it is guaranteed to be unity since then the vector current will be conserved; moreover, departures of f_+ from unity are of second order in the mass difference $m_Q - m_q$. The f axial-vector form factor corresponds to an allowed Gamow-Teller transition and is analogous to G_A in ordinary β decay, while the vector-current form factor g is analogous to the weak magnetism term. The direct analogue of a_+ is forbidden in β decay by G-parity invariance. However, it may be considered to be related generically to ordinary radiative transitions from excited states which leave the ground state recoiling in a D wave. A simple example of this type is $N^*\frac{5}{2}^-(1675) \to N\gamma$ in which the nucleon recoils with $L=2$.

One way to assess the reliability of the predictions (13)–(16) is thus by comparing their analogues to data. In the case of f_+ we can compare with the measured $K \to \pi$ and $D \to \bar{K}$ form factors. The predictions are $f_+^{K\to\pi}(t_m) = 1.02$ and $f_+^{D\to\bar{K}}(t_m) = 1.16$, while the experimental values[17] are 1.12 ± 0.05 and 1.35 ± 0.28, respectively. In $K \to \pi$ the f_- form factor, which in the language of the preceding discussion corresponds to an $L=1$

recoil, is also known. Measurements of $\xi \equiv f_-/f_+$ give $\xi \simeq -0.35\pm0.15$ in $K^{\pm}$ decay and -0.11 ± 0.09 in K_L^0 decay, corresponding reasonably well with the theoretical prediction -0.27. These checks, combined with the theoretical constraints already mentioned, lead to the expectation that f_+ is predicted by (13) to an accuracy of about 10%. The formulas for the $P_Q \to V_q$ form factors f, g, and a_+, on the other hand, can only be expected to be accurate at the $\pm20\%$ level typical of the nonrelativistic quark potential model for analogous amplitudes. For example, G_A (analogous to f) is predicted to be $\frac{5}{3}$ (versus 1.259 ± 0.004), the $\omega\to\pi\gamma$ transition magnetic moment (a "typical" magnetic amplitude analogous to g) is predicted to be equal to one nuclear magneton μ_N [versus $(1.31\pm0.08)\mu_N$], and $A_{3/2}^n$ for the $N^*(1675)\to N\gamma$ transition (analogous to a_+) is predicted to be -0.053 $\text{GeV}^{-1/2}$ (versus -0.069 ± 0.019 $\text{GeV}^{-1/2}$).

Another way to assess the reliability of the quark-model predictions for f_+, f, g, and a_+ is by comparing to the Shifman-Voloshin (SV) limit.[18] In the SV limit $[(\Lambda_{\rm QCD}/m_Q)^{1/2} \ll x_m \ll 1$, corresponding to a transition with $m_Q - m_q$ small but m_Q large] the free-quark transition $Q\to qe^-\bar{\nu}_e$ is exactly dual to the hadronic-level transitions $P_Q \to P_q e^- \bar{\nu}_e$ plus $P_Q \to V_q e^- \bar{\nu}_e$. It has been shown,[14] moreover, that this duality is local in the Dalitz plot and that violations of SV duality are of second order in x_m. Indeed, ignoring terms of order $\Lambda_{\rm QCD}/m_Q$, one has, in the SV limit of Eqs. (13)–(16),

$$f_+ = 1 , \tag{18}$$

$$f = 2m_Q(1-\tfrac{1}{2}x_m) , \tag{19}$$

$$g = -a_+ = (2m_Q)^{-1}(1+\tfrac{1}{2}x_m) , \tag{20}$$

which give, to this same order,

$$\beta^P_{++} = 1 , \tag{21}$$

$$\alpha^V = 4m_Q^2(1-x_m) , \tag{22}$$

$$\beta^V_{++} = 1 , \tag{23}$$

$$\gamma^V = 2 . \tag{24}$$

These results, through their duality with free-quark decay to this order, are model independent and therefore provide us with an indirect but nevertheless potentially powerful theoretical assessment of the reliability of the formulas (13)–(16). They also indicate that, while each of f, g, and a_+ receives corrections of order x_m, β^V_{++}, and γ^V are, like β^P_{++}, shielded from corrections to the SV limit up to terms of order x_m^2.

For the $\bar{B}\to D$ and $\bar{B}\to D^*$ transitions, the theoretical constraints of the SV limit are very potent. As discussed in Ref. 14, actual model dependence of the $\bar{B}\to D$ and $\bar{B}\to D^*$ formulas [corresponding to differences between (18)–(24) and the "exact" results from (13)–(16)] is at the 10% level. The SV values of the form factors at zero recoil are compared with the full quark model values in Table I.

For the $D\to\bar{K}$ and $D\to\bar{K}^*$ transitions it is not apparent that the SV limit is appropriate, since with $\eta_i \equiv b^{1/2}/m_i$, where b is the mesonic string tension, we have $\eta_c \simeq 0.23$, $\eta_s \simeq 0.77$. However, $x_m^{D\to\bar{K}}$ is essentially the same as $x_m^{\bar{B}\to D}$ so the SV constraints are not irrelevant. This is shown explicitly in Table I, where one sees better than expected agreement between the quark-model formulas and the SV limit for these $c\to s$ transitions.

This combination of empirical evidence and theoretical constraints indicates to us that the probable errors in the predictions of Refs. 3 and 4 for the zero recoil values of form factors in both $\bar{B}\to D^*$ and $D\to\bar{K}^*$ are of the order of 20%, and that it is possible that certain combinations of form factors (e.g., β^V_{++} and γ^V) are, like f_+, considerably more reliable. With these expectations in mind, we now turn to the prediction of the polarizations in $P_Q\to V_q$ decays.

III. POLARIZATION OF V_q IN $P_Q \to V_q e^- \bar{\nu}_e$

The measurement of the alignment[15] of the vector meson in the $P_Q\to V_q$ transition is another method of testing predictions for the form factors f, g, and a_+. Generalizing Eq. (2), one obtains for the differential decay rate for $P_Q\to V_q^T e^- \bar{\nu}_e$, where V_q^T is transversly polarized,

$$\frac{d^2\Gamma^{V_T}}{dx\,dy} = |V_{qQ}|^2 \frac{G_F^2 m_{P_Q}^5}{32\pi^3} \left[\frac{\alpha^{V_T} y}{m_{P_Q}^2} + 2\beta^{V_T}_{++}[4x(x_m-x)-y(1-2x)] - 2\gamma^{V_T} y(x_m-2x+\tfrac{1}{2}y) - \frac{\delta^{V_T}}{2m_{P_Q}^2} y \left[\frac{4x(x_m-x)-y(1-2x)}{(x_m+\frac{1}{2}y)^2-y} \right] \right] \tag{25}$$

TABLE I. The zero recoil values of form factors and hadronic tensor coefficients of the quark model compared to those of the SV limit.

		f_+	$f/2m_{P_Q}$	g (GeV^{-1})	a_+ (GeV^{-1})	β^P_{++}	$\alpha^V/4m_{P_Q}^2$	β^V_{++}	γ^V
$B\to D$, D^*	SV limit	1	0.76	0.12	-0.12	1	0.53	1	2
	quark model	1.13	0.65	0.16	-0.15	1.28	0.42	0.99	2.23
$D\to K$, K^*	SV limit	1	0.75	0.34	-0.34	1	0.52	1	2
	quark model	1.16	0.72	0.49	-0.36	1.34	0.52	1.00	2.64

in the rest frame of P_Q, where

$$\alpha^{V_T}=\alpha^{V}=f^2+4m_{P_Q}^2 g^2 \mathbf{p}'^2 , \quad (26)$$

$$\beta_{++}^{V_T}=-m_{P_Q}^2 y g^2 , \quad (27)$$

$$\gamma^{V_T}=\gamma^{V}=2gf , \quad (28)$$

and

$$\delta^{V_T}=|f|^2 , \quad (29)$$

involve only f and g since the $(\epsilon^*\cdot p)$ multiplying a_+ in Eq. (5) ensures that it does not contribute to transverse decay.

Since the same form factors appear here as in the unpolarized rate formula, our comments on empirical measures of their accuracy are as relevant to Eq. (25) as they were to Eq. (2). To complete the analogous discussion of the accuracy of Eq. (25), however, we must also discuss the Shifman-Voloshin limit, in which it becomes

$$\left.\frac{d^2\Gamma^{V_T}}{dx\,dy}\right|_{\rm SV}=|V_{qQ}|^2\frac{G_F^2 m_Q^5}{8\pi^3}y[(1-x_m)F(x,y)+(2x-x_m)] . \quad (30)$$

Here,

$$F(x,y)=\frac{(x_m+\frac{1}{2}y)^2-\frac{1}{2}y-2x(x_m-x+\frac{1}{2}y)}{(x_m+\frac{1}{2}y)^2-y} \quad (31)$$

is a factor that arises from the angles that the leptons' momenta make with the V_q direction of recoil. In the free-quark model the corresponding SV limit for the polarized decay rate is less obvious than for spin-summed rates since it involves not only the spin of q, but also that of the spectator antiquark. However, in the decaying P_Q there are equal probabilities for the spin of Q to be up or down along the q recoil direction. Moreover, in the SV limit where the recoil momentum vectors of q and V_q become equal, the residual "spectator" (which in the simplest view could be a constituent quark but in general corresponds to the full state vector of P_Q with a "hole" left in it) has a spin along the q recoil direction which can be added to the q helicity to obtain the total hadronic helicity. Thus the rate we seek is the average spin-flip rate (for $Q\downarrow\to q\uparrow$ and $Q\uparrow\to q\downarrow$):

$$\frac{d^2\Gamma^{qT}}{dx\,dy}=|V_{qQ}|^2\frac{G_F^2 m_Q^5}{8\pi^3}y[(1-x_m-\tfrac{1}{2}y)F(x,y)+(2x-x_m-\tfrac{1}{2}y)] \quad (32)$$

which agrees with Eq. (30) to next-to-leading order in the SV limit, where y can be neglected with respect to x and x_m.

From this analysis we can conclude that the transverse decay rate formula (25) is also protected by the SV limit. We would also like to point out that although $\Gamma^V:\Gamma^P=3:1$ in next-to-leading order in the SV limit[14] as expected from "spin counting," this naive "explanation" of $\Gamma^V:\Gamma^P$ is false. Integration of Eq. (30) gives

$$\Gamma^{V_T}=|V_{qQ}|^2\frac{G_F^2 m_Q^5}{45\pi^3}x_m^5(1+x_m) \quad (33)$$

as compared to[14]

$$\Gamma^{V}=|V_{qQ}|^2\frac{G_F^2 m_Q^5}{20\pi^3}x_m^5(1+x_m) \quad (34)$$

so that

$$\frac{\Gamma_L}{\Gamma_T}=\frac{5}{4} \quad (35)$$

up to at least next-to-leading order in the SV limit.

IV. ESTIMATES OF UNCERTAINTIES IN RATES AND POLARIZATIONS

As mentioned in Sec. I, this study was prompted in part by the recent observation[13] that $\Gamma_L/\Gamma_T=2.4^{+1.7}_{-0.9}\pm0.2$ in $D\to\bar{K}^*e^+\nu_e$, compared to the Ref. 4 prediction of 1.09. This same experiment also finds $\Gamma(D\to\bar{K}^*e^+\nu_e)=(0.41\pm0.07\pm0.05)\times10^{11}$ sec^{-1}, about a factor of 2 smaller than the Ref. 4 prediction of 0.94×10^{11} sec^{-1}. In contrast with this mismatch between the model and experiment for the $c\to s$ transitions is the agreement observed in $b\to c$ transitions. $\Gamma(\bar{B}\to D^*e^-\bar{\nu}_e)$ is predicted to be 60% of the total semileptonic rate, while experimentally[10] it is found[19] to be (60±21)%. Moreover, a recent measurement[10] giving $\Gamma_L/\Gamma_T=0.85\pm0.45$ in $\bar{B}\to D^*e^-\bar{\nu}_e$ is consistent with the prediction 0.97 of the model (and with the SV limit).

In this section we explore the implications for such measurements of the preceding discussion of the theoretical uncertainties in the prediction of current matrix elements. One of the main conclusions of this study is that while the observed $D\to\bar{K}^*e^+\nu_e$ rate can be marginally accommodated within the expected inaccuracies of the model, the observed value of Γ_L/Γ_T for this decay cannot be. The situation is summarized in Table II where the sensitivities of the transverse and total rates to 20% variations in f, g, and a_+ are given. The results show

TABLE II. The sensitivity of the transverse and longitudinal decay rates in $\bar{B}\to D^*e^-\bar{\nu}_e$ and $D\to\bar{K}^*e^+\bar{\nu}_e$ to variations in the normalizations of f, g, and a_+ from their calculated values [Eqs. (14)–(16)].

			$\Gamma_\perp/\lvert V_{qQ}\rvert^2$		$\Gamma/\lvert V_{qQ}\rvert^2$		Γ_L/Γ_T	
$f/f^{\rm calc}$	$g/g^{\rm calc}$	$a_+/a_+^{\rm calc}$	$\bar{B}\to D^*$ (10^{13} sec^{-1})	$D\to\bar{K}^*$ (10^{11} sec^{-1})	$\bar{B}\to D^*$ (10^{13} sec^{-1})	$D\to\bar{K}^*$ (10^{11} sec^{-1})	$\bar{B}\to D^*$	$D\to\bar{K}^*$
1	1	1	1.28	0.46	2.52	0.96	0.97	1.09
0.8	1	1	0.87	0.31	1.48	0.58	0.70	0.89
1	0.8	1	1.23	0.44	2.47	0.94	1.01	1.13
1	1	0.8	1.28	0.46	2.78	1.02	1.17	1.23

that the expected errors in Γ_L/Γ_T are also about 20%, while those in the rates (in particular those associated with f) are about twice as large. Figures 1 and 2 show the Dalitz plots for $D\to\bar{K}^*_T$, $D\to\bar{K}^*$, $\bar{B}\to D^*_T$, and $\bar{B}\to D^*$ corresponding to the canonical form factors of the first row of this table. It should be noted that the predicted ratio of Γ_L/Γ_T is usually reasonably close to the value 5/4 of the SV limit given in Eq. (35). Thus the expected errors in the model suggest a value for Γ_L/Γ_T in $D\to\bar{K}^* e^+\nu_e$ more than 1σ below the measured value.

One of the peculiarities of this situation is that $\bar{B}\to D^*$ decays appear to be in much better agreement with theory than $D\to\bar{K}^*$ decays, even though from Table I there is little reason to expect this. However, given that the latter decays are farther from the SV limit (recall $\eta_s\simeq 0.77$), this observation is not paradoxical.

Our conclusion regarding the difficulty of accommodating a large Γ_L/Γ_T is rather different from that drawn in a recent analogous study[20] of the uncertainties in the infinite-momentum frame relativistic quark model of Ref. 5. The reason for this difference is easily uncovered. As we have explained, in the model under discussion here the form factors can be associated with nonrelativistic partial-wave amplitudes and as such are expected to have a typical quark-model uncertainty. In the model of Ref. 5, the analogue of a_+ is a form factor A_2 which is computed as a difference of two matrix elements. The authors of Ref. 20 argue that a 50% change in one of these matrix elements (the matrix element of a "bad" operator) from *its* nominal value could change $A_2(0)$ from its nominal value of 1.15 to -0.10. A similar effect might have occurred in the model under discussion here, since the calculation of a_+ involves (in the vector-particle rest frame introduced earlier) the use of both an $L=2$ amplitude arising from $\mathbf{A}$ and an $L=1$ amplitude arising from A^0: namely,

$$a_+ + a_- = F_3 \frac{m_d}{2m_q m_Q} \frac{\beta_q^2}{\beta_{Qq}^2}\left[1-\frac{m_d}{2\tilde{m}_Q}\frac{\beta_q^2}{\beta_{Qq}^2}\right] \tag{36}$$

and

$$\tilde{m}_Q(a_+ + a_-)+\tilde{m}_q(a_+ - a_-) = -2\tilde{m}_Q F_3\left[\frac{1}{2m_Q}-\frac{m_d}{4\mu_+\tilde{m}_Q}\frac{\beta_q^2}{\beta_{Qq}^2}\right], \tag{37}$$

respectively, so that

$$a_+ = -\frac{\tilde{m}_Q}{\tilde{m}_q}F_3\left[\frac{1}{2m_Q}-\frac{m_d}{4\mu_+\tilde{m}_Q}\frac{\beta_q^2}{\beta_{Qq}^2}\right] - \frac{\tilde{m}_Q-\tilde{m}_q}{2\tilde{m}_q}(a_+ + a_-)\ . \tag{38}$$

However, these two contributions to a_+ are of the same sign; moreover $a_+ + a_-$ is small (0.01 GeV^{-1} for $\bar{B}\to D^*$ and 0.08 GeV^{-1} for $D\to\bar{K}^*$) so that even a 100% error in this $L=2$ amplitude leads to only a small change in a_+ (5% for $\bar{B}\to D^*$ and 17% for $D\to\bar{K}^*$). Thus the calculation of a_+ does not seem particularly delicate in the quark potential model, and the estimates we made above remain intact.

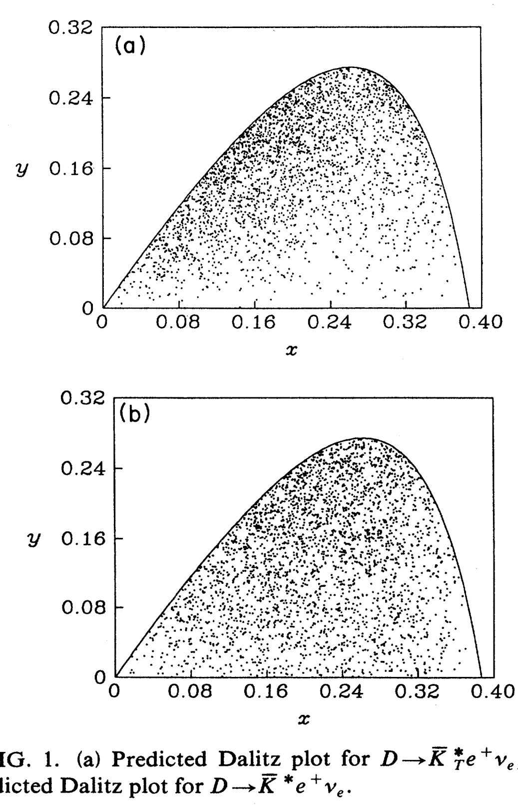

FIG. 1. (a) Predicted Dalitz plot for $D\to\bar{K}^*_T e^+\nu_e$; (b) full predicted Dalitz plot for $D\to\bar{K}^* e^+\nu_e$.

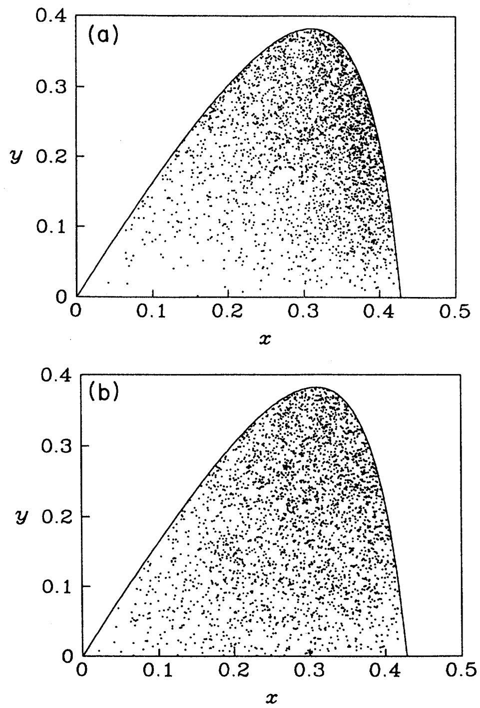

FIG. 2. (a) Predicted Dalitz plot for $\bar{B}\to D^*_T e^-\bar{\nu}_e$; (b) full predicted Dalitz plot for $\bar{B}\to D^* e^-\bar{\nu}_e$.

V. CONCLUSIONS

The ability to deduce with confidence the KM angles from semileptonic decays will depend on an interplay between theory and experiment. This interaction will serve to refine models of hadronic matrix elements and more clearly define those experimental quantities which are most reliably predicted by theory, and therefore most useful for extracting these angles. We believe $\bar{B} \to D^*$ and $D \to \bar{K}^*$ decays have an important role to play in this process.

In this paper we have concentrated on the use of the longitudinal to transverse decay ratios as a probe of models for hadronic matrix elements. While pointing out that there are reasons why the extraction of quantities such as β^P_{++}, β^V_{++}, and γ^V may eventually prove more fruitful in determining V_{cb}, the study of such readily accessible observables as total rates and vector particle alignments is an important step in this process, as well as in the study of other KM angles. In particular we have shown that it appears to be quite difficult for the quark potential model to produce ratios for the vector alignments Γ_L/Γ_T in $\bar{B} \to D^* e^- \bar{\nu}_e$ and $D \to \bar{K}^* e^+ \nu_e$ very different from the value 5/4 of the Shifman-Voloshin limit. The confirmation of the recent report that Γ_L/Γ_T in $D \to \bar{K}^*$ decays is large, as well as a more accurate measurement of this quantity in $\bar{B} \to D^*$ decays, thus seems imperative.

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