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Meaning of the BRS Lagrangian theory

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A simplified treatment of the Becchi-Rouet-Stora (BRS) Lagrangian theory is presented. With this treatment we show that the BRS Lagrangian theory in general, and the Feynman-gauge field theory in particular, are effective theories, not the physical theory, and the Feynman gauge is not, strictly speaking, a gauge. The relationship between the quantum states in the BRS Lagrangian theory and those in the physical theory is explicitly given. We also show that one may obtain matrix elements of gauge-invariant operators in the physical theory by calculating corresponding ones in the BRS Lagrangian theory. The formulas which equate such matrix elements are called correspondence formulas. The correspondence formula for the S matrix enables us to equate the scattering amplitudes in the physical theory with those in the BRS Lagrangian theory, thus a proof of the unitary of the Feynman-gauge (as well as other covariant gauges) Feynman rules is rendered unnecessary. This treatment can be applied to various gauge field theories and the examples of the pure Yang-Mills theory and a gauge field theory with a Higgs field is explicitly worked out.

I. INTRODUCTION

The Feynman gauge[1] is perhaps the most convenient gauge to use in a perturbative calculation of scattering amplitudes in gauge field theories. The physical meaning of this gauge is also among the most obscure. In quantum field theories, it is notable that Fermi's formulation of QED in the Coulomb gauge[2] preceded the formulation of the Feynman-gauge QED (Refs. 1 and 3) by 16 years. History essentially repeated itself in the development of non-Abelian gauge field theories. In 1962, Schwinger already successfully formulated non-Abelian gauge field theories in the Coulomb gauge.[4–6] In 1963, Feynman[7] correctly guessed the Feynman-gauge Feynman rules, which contain the contribution of ghosts. Two years later Faddeev and Popov[8] deduced the general existence and the interactions of ghosts, using the approach of path integration and group-theoretic arguments. However, the Faddeev-Popov formalism remains to this day a heuristic one. The heuristic nature of the formalism is illustrated by the incorrect answer it yields when it is applied to the quantization in the Coulomb gauge.[9] It is also demonstrated by the necessity in this formalism to verify unitarity, which is especially cumbersome for non-Abelian gauge field theories with a spontaneously broken vacuum symmetry. Indeed, the original diagrammatic proof by 't Hooft of the unitarity of the Feynman-gauge (as well as other covariant gauges) Feynman rules in this theory was fairly elaborate.[10,11] The task of proving unitarity in the Feynman gauge was simplified by Kugo and Ojima,[12] who treated canonically the Becchi-Rouet-Stora (BRS) Lagrangian theory.[13]

The work of Kugo and Ojima depends, however, on the validity of two postulates. Furthermore, their work still leaves many important questions unanswered. For example, the BRS Lagrangian involves unphysical ghost fields, while the gauge-invariant Lagrangian does not. Indeed, the equations of motion in these Lagrangian theories are different. The work of Kugo and Ojima does not say if the scattering amplitudes in these two theories are the same. One may also ask (i) can one obtain any other physical quantity in one theory by calculating a corresponding one in the other? (ii) In particular, can one obtain the energy spectrum of one theory by calculating the energy spectrum of the other theory? (iii) Are these two quantum theories equivalent to each other and does there exist a unitary operator which connects the quantum states and operators in these two theories?

In a previous Letter[14] we have found that there exist formulas which enable us to obtain the matrix element of gauge-invariant operators in a guage theory by calculating a corresponding quantity in the BRS Lagrangian theory. In this paper we shall give a complete yet simpler presentation of our arguments. The present formulation enables us to conclude that the relation between these two theories is quite different from that between the gauge theory in the temporal gauge and the gauge theory in the Lorentz gauge, for example. We recall that, to quantize in the temporal gauge, we set $A_0=0$ in the gauge-invariant Lagrangian and impose the Gauss law on the quantum states as a supplementary condition; and to quantize in the Lorentz gauge we add a term $-\frac{1}{2}(\partial_\mu A^\mu)^2$ to the gauge-invariant Lagrangian and impose the supplementary condition $\partial_\mu A^\mu=0$ on the quantum states. The Hamiltonians and the quantum states in these two quantization schemes are related by a unitary transformation together with a separation of variables.[6] Thus these two theories are equivalent and we shall call them the physical theory. In contrast, the Hamiltonian and the quantum states in the BRS Lagrangian theory and those in the physical theory are not so related. In short, the BRS Lagrangian theory in general and the Feynman-gauge field theory in particular are *effective* theories, not the physical theory. Indeed, the Feynman gauge is not, strictly speaking, a gauge. For, unlike other gauges, the Feynman gauge does not impose a condition on the components of

A_μ and does not even exist in the classical theory. We have found that, given a physical wave function ψ_w in the temporal gauge, it is useful to construct a wave function ψ_{eff} in the BRS Lagrangian theory by setting

$$\psi_{\text{eff}} \equiv e^{\theta}\psi_w \ , \tag{1.1}$$

where θ is given by (2.16) below. This is because the matrix element of $\mathcal{H}_{\text{eff}}$ (the BRS Hamiltonian) between two states of the form of (1.1) is equal to that of $\mathcal{H}_w$ (the temporal gauge Hamiltonian) between the two corresponding states in the temporal gauge. This does not, however, mean that e^{θ} is the operator which transforms quantum states in the physical theory into the quantum states of the BRS Lagrangian theory. The reason is that

$$\mathcal{H}_{\text{eff}}\psi_{\text{eff}} \neq e^{\theta}\mathcal{H}_w\psi_w \ , \tag{1.2}$$

a result which will be proved in Sec. II.

One of the implications of (1.2) is that, if ψ_w is an eigenstate of $\mathcal{H}_w$, ψ_{eff} is not necessarily an eigenstate of $\mathcal{H}_{\text{eff}}$. Indeed, $H_{\text{eff}}\psi_{\text{eff}}$ is not even in the form of (1.1). Rather, we have, as will be shown in the next section,

$$\mathcal{H}_{\text{eff}}\psi_{\text{eff}} = e^{\theta}\mathcal{H}_w\psi_w + Q\phi \ , \tag{1.3}$$

where Q is the BRS charge given by (2.12) below and ϕ is some wave function the precise form of which can be easily calculated but is unimportant. Mathematically, (1.3) implies that $\mathcal{H}_{\text{eff}}$ cannot be defined if we restrict ourselves to the Hilbert space of wave functions of the form (1.1). Physically, it implies that a quantum state initially in this Hilbert space does not remain in this space as time evolves. Thus we need to expand this Hilbert space into the space of wave functions of the form of

$$e^{\theta}\psi_w + Q\phi \ . \tag{1.4}$$

This expanded space is closed under time evolution, but is *not* isomorphic to the Hilbert space of the physical theory. Indeed, this expanded space is not even a Hilbert space, as the inner product is semipositive definite, not positive definite.

This is not to say that the BRS Lagrangian theory is not useful, especially if one is content to restrict oneself to the consideration of certain *matrix elements.* Quite the opposite, the BRS Lagrangian theory is perhaps more useful than it has been generally given credit for. It can be used not only for the calculation of the S matrix, which is the transition amplitude of infinite time duration, but also for the calculation of transition amplitude of any finite time duration. As a matter of fact, one can obtain the matrix elements of a gauge-invariant operator in the physical theory by calculating the corresponding ones in the BRS Lagrangian theory, with the wave functions in these two theories related by (1.1). We call such formulas equating matrix elements in these two theories correspondence formulas. A correspondence formula is exact, valid for all values of the coupling constant. Indeed, no reference to perturbation is necessary. If and when the coupling constant is small so that perturbation can be used, the correspondence formula implies that the Feynman-gauge Feynman rules, for one, are consistent with the Coulomb-gauge Feynman rules. The validity of the correspondence formula dwells on no assumption, nor is it plagued by the difficulty of the Gribov ambiguity.[15,16]

The formulation in this paper is readily applicable to any specific gauge field theory. As an explicit example, we shall treat, in Sec. III, the quantum theory of the BRS Lagrangian with spontaneous broken vacuum symmetry. We shall show that there exists a correspondence between such a theory and the BRS Lagrangian theory. Since the former is unitary in the physical sector, so is the latter. Thus the proof of unitarity of the Feynman-gauge Feynman rules in theories of spontaneous broken symmetry is rendered unnecessary.

II. THE CANONICAL FORMALISM

Consider a general non-Abelian gauge field theory with the Lagrangian

$$L = -\tfrac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu,a} + (D_\mu\phi)^{\dagger}(D^{\mu}\phi) - V(\phi\phi^{\dagger}) \ . \tag{2.1}$$

In (2.1),

$$F^{a}_{\mu\nu} = \partial_\mu A^{a}_{\nu} - \partial_\nu A^{a}_{\mu} - gf^{abc}A^{b}_{\mu}A^{c}_{\nu} \ , \tag{2.2a}$$

with A^{a}_{μ} the gauge field of group index a and polarization μ, g the coupling constant, f^{abc} the structure constant of the gauge group, and ϕ a scalar field which may belong to any of the group representations, with

$$D_\mu\phi \equiv (\partial_\mu + igA^{a}_{\mu}T^{a})\phi \ , \tag{2.2b}$$

where T^a is an infinitesimal group generator in the representation of ϕ. Also included in (2.1) is a possible potential term V. We may also add to (2.1) a Lagrangian term for fermion fields. This extension is straightforward and for the sake of simplicity of presentation will not be elaborated on.

The Lagrangian (2.1) is gauge invariant and, as it is independent of $\dot{A}^{a}_{0}$, the dynamical variable conjugate to A^{a}_{0} cannot be defined. A way to get around this is to choose the temporal gauge $A^{a}_{0}=0$. The usual procedure of canonical quantization can then be applied. In particular the Hamiltonian density, denoted by H_w, is

$$H_w = \frac{\boldsymbol{\pi}^{a}\cdot\boldsymbol{\pi}^{a} + \mathbf{B}^{a}\cdot\mathbf{B}^{a}}{2} + \pi_{\phi^{\dagger}}\pi_{\phi} + (\mathbf{D}\phi)^{\dagger}\cdot(\mathbf{D}\phi) + V(\phi\phi^{\dagger}) \ . \tag{2.3}$$

The Gauss law is missing, and one imposes it as a supplementary condition on the quantum state:

$$G^{a}|\psi_w\rangle = 0 \ , \tag{2.4}$$

where

$$G^{a} = (\mathbf{D}\cdot\boldsymbol{\pi})^{a} + \rho^{a} \ , \tag{2.5}$$

with

$$\rho^{a} = -ig\pi^{T}_{\phi}T^{a}\phi + \text{c.c.}$$

As H_w commutes with G^a, (2.4) holds at all times if it holds at the initial time. The Schrödinger equation is

$$i\frac{\partial}{\partial t}|\psi_w\rangle = \mathcal{H}_w|\psi_w\rangle \ , \tag{2.6}$$

where

$$\mathcal{H}_w \equiv \int H_w d^3x \ . \tag{2.7}$$

We shall call the theory defined by (2.3)–(2.7) the physical theory and states satisfying (2.4) the physical states.

Another way to quantize this non-Abelian gauge field theory is to add to the Lagrangian (2.1) a term $-\frac{1}{2}(\bar{l}^{\,a})^2$, where

$$\bar{l}^{\,a} = \dot{A}^{\,a}_0 + h^a(\mathbf{A},\phi,\phi^\dagger) \ , \tag{2.8}$$

with h^a any real-valued functional of $\mathbf{A}$, ϕ, and $\phi^\dagger$. With this addition, π^a_0 can be defined. While the Lagrangian $L-\frac{1}{2}(\bar{l}^{\,a})^2$ is not gauge invariant, it is possible to prove, after imposing the supplementary condition $\bar{l}^{\,a}=0$ on the quantum states, that this Lagrangian theory is exactly the same as the physical theory. This is is done by eliminating the extra degree of freedom A^a_0 in the former theory by separation of variables.[6]

In this paper we shall pursue the theory obtained by adding to the Lagrangian $L-\frac{1}{2}(\bar{l}^{\,a})^2$ a term to make it invariant under the BRS transformation of

$$\delta A^{\mu a} = (D^\mu\xi)^a \equiv \partial^\mu\xi^a - gf^{abc}A^{\mu b}\xi^c \ , \tag{2.9a}$$

and

$$\delta\phi = -ig\xi^a T^a\phi \ ,$$

where ξ^a is a Hermitian Grassmann field[12] with group index a. The variation of ξ^a under the BRS transformation is chosen to be, as usual,

$$\delta\xi^a = \tfrac{1}{2}gf^{abc}\xi^b\xi^c \ . \tag{2.9b}$$

As is well known,

$$\delta^2 A^\mu = \delta^2\phi = 0 \ ;$$

i.e., the BRS variation of a BRS variation vanishes. One then introduces another Hermitian Grassmann field[12] η^a, the BRS transformation of which is

$$\delta\eta^a = -i\bar{l}^{\,a} \ . \tag{2.9c}$$

Then the Lagrangian

$$L - \tfrac{1}{2}(\bar{l}^{\,a})^2 + i\eta^a\delta\bar{l}^{\,a} \tag{2.10}$$

is invariant under the BRS transformation.

Instead of the Lagrangian (2.10) we shall define

$$L_{\text{eff}} = L - \tfrac{1}{2}(\bar{l}^{\,a})^2 - i\dot{\eta}^a(D^0\xi)^a + i\eta^a\delta h^a \ , \tag{2.11}$$

which differs from the Lagrangian of (2.10) by a total time derivative.

The Lagrangian L_{eff} is the BRS Lagrangian with the conserved quantity

$$Q = \int d^3x\,(\xi^a G^a + i\pi^a_\eta\pi^a_0 + \tfrac{1}{2}gf^{abc}\pi^a_\xi\xi^b\xi^c) \ , \tag{2.12}$$

where

$$\pi^a_0 = -\bar{l}^{\,a}, \quad \pi^a_\eta = -i(D^0\xi)^a, \quad \pi^a_\xi = i\dot{\eta}^a \ ,$$

with

$$[\pi^a_\eta(\mathbf{x}),\eta^b(\mathbf{y})]_+ = [\pi^a_\xi(\mathbf{x}),\xi^b(\mathbf{y})]_+ = -i\delta^{ab}\delta^{(3)}(\mathbf{x}-\mathbf{y}) \ . \tag{2.13}$$

[Note that (2.13) implies that π_η and π_ξ are anti-Hermitian operators.] It is well known that Q is a Grassmann operator satisfying

$$Q^2 = 0 \ . \tag{2.14}$$

Note that Q is actually independent of the choice of h^a. The variation of a field under a BRS transformation can now be alternatively expressed as the commutator (for a field with bose statistics) or anticommutator (for a Grassmann field) of this field with Q. For example,

$$\delta A^{\mu,a} = i[Q, A^{\mu,a}]$$

and

$$\delta\xi^a = i[Q,\xi^a]_+ \ .$$

With this definition, δ^2 of any operator vanishes. For instance, let R be a bosonic operator; we have

$$\delta^2 R = -[Q,[Q,R]]_+ = 0 \ ,$$

as can be proved by writing out all the terms in the expression above and making use of (2.14). The Hamiltonian corresponding to L_{eff} is given by

$$H_{\text{eff}} = H_w + i[Q,\Delta]_+ \ , \tag{2.15}$$

where H_w is given by (2.3) and

$$\Delta \equiv A^a_0\pi^a_\xi + ih^a\eta^a + \tfrac{1}{2}i\pi^a_0\eta^a \ .$$

The derivation of (2.15) is presented in Appendix A. The particular form for Δ appears immaterial. The important point in (2.15) is that H_{eff} differs from H_w by a BRS variation.

Let $|\psi_w\rangle$ be a physical quantum state in the physical theory, hence satisfying (2.4), and let us denote

$$\psi_w(\mathbf{A},\phi,\phi^*) \equiv \langle\mathbf{A},\phi,\phi^*|\psi_w\rangle \ ,$$

where $|\mathbf{A},\phi,\phi^*\rangle$ is an eigenstate of the field operators (the group indices have been omitted for brevity). We shall construct from this physical wave function a wave function ψ_{eff} in the BRS Lagrangian theory:

$$\psi_{\text{eff}}(A_\mu,\phi,\phi^*,\xi,\eta) \equiv e^\theta\psi_w(\mathbf{A},\phi,\phi^*) \ , \tag{2.16a}$$

where

$$\theta = \theta_1 + \theta_2 \ , \tag{2.16b}$$

with

$$\theta_1 \equiv i\int d^3x\ \eta^a\delta I^a(\mathbf{A},\phi,\phi^*) \tag{2.16c}$$

and

$$\theta_2 \equiv \tfrac{1}{2}\int d^3x\,(A^a_0 - iI^a)\sqrt{-\nabla^2}(A^a_0 - iI^a) \ , \tag{2.16d}$$

where I^a may be any real-valued functional of $\mathbf{A}$, ϕ, and

ϕ^*. Note that, unlike ψ_w which is a functional of $\mathbf{A}$, ϕ, and ϕ^* but not of ξ, η, and A_0, ψ_{eff} is a functional of all of these fields. This wave function satisfies

$$Q\psi_{\text{eff}}=0 , \tag{2.17}$$

which will be proved in Appendix B. Equations (2.15) and (2.17) give

$$H_{\text{eff}}\psi_{\text{eff}}=H_w\psi_{\text{eff}}+iQ\Delta\psi_{\text{eff}} . \tag{2.18}$$

Thus, as operators on ψ_{eff}, H_{eff} and H_w are not equal. Indeed,

$$(H_{\text{eff}})^n\psi_{\text{eff}}=(H_w)^n\psi_{\text{eff}}+Q\phi_n, \quad n=1,2,\dots, \tag{2.19}$$

where ϕ_n is some wave function the precise form of which does not matter. [Equation (2.19) is easily proved by using (2.14) and the fact that H_w is gauge invariant and hence commutes with Q.] As a consequence of (2.19) we have

$$e^{-i\mathcal{H}_{\text{eff}}t}\psi_{\text{eff}}=e^{-i\mathcal{H}_w t}\psi_{\text{eff}}+Q\phi . \tag{2.20}$$

Indeed

$$f(H_{\text{eff}})\psi_{\text{eff}}=f(H_w)\psi_{\text{eff}}+Q\phi ,$$

where f is any analytic function.

While H_{eff} operating on ψ_{eff} cannot be replaced by H_w, the matrix elements of H_{eff} and H_w are equal, To wit, we have from (2.18) that

$$\langle\psi_{\text{eff}}^{(1)}|H_{\text{eff}}|\psi_{\text{eff}}^{(2)}\rangle=\langle\psi_{\text{eff}}^{(1)}|H_w|\psi_{\text{eff}}^{(2)}\rangle . \tag{2.21}$$

which is a consequence of (2.17) and the fact that Q is Hermitian. Similarly, we have from (2.20) that

$$\langle\psi_{\text{eff}}^{(1)}|e^{-i\mathcal{H}_{\text{eff}}t}|\psi_{\text{eff}}^{(2)}\rangle=\langle\psi_{\text{eff}}^{(1)}|e^{-i\mathcal{H}_w t}|\psi_{\text{eff}}^{(2)}\rangle . \tag{2.22}$$

Therefore, H_{eff} can be replaced by H_w, if matrix elements between effective wave functions are taken.

Equation (2.22) is not yet in the desired form, as $\mathcal{H}_w$ operates on $e^{\theta}\psi_w$ and not directly on ψ_w, as is the case in the physical theory. It will be proved in Appendix C that

$$e^{-i\mathcal{H}_w t}e^{\theta}\psi_w=e^{\theta}e^{-i\mathcal{H}_w t}\psi_w+Q\phi \tag{2.23}$$

and thus, together with (2.20), we have

$$e^{-i\mathcal{H}_{\text{eff}}t}e^{\theta}\psi^w=e^{\theta}e^{-i\mathcal{H}_w t}\psi_w+Q\phi' . \tag{2.24}$$

Equation (2.24) shows that, while a wave function of the form of $e^{\theta}\psi_w$ does not remain in this form as it evolves in time, it differs from this form only by a term of the form of $Q\phi$. Indeed, a wave function of the form of

$$e^{\theta}\psi_w+Q\phi \tag{2.25}$$

remains in this form at all times. We shall call the quantum theory with the Hamiltonian $\mathcal{H}_{\text{eff}}$ in the space of (2.25) the effective theory. The left side of (2.22) is a transition amplitude in the effective theory. Substituting (2.23) into (2.22) we get

$$\langle\psi_{\text{eff}}^{(1)}|e^{-i\mathcal{H}_{\text{eff}}t}|\psi_{\text{eff}}^{(2)}\rangle=\langle\psi_w^{(1)}|e^{\theta^++\theta}e^{-i\mathcal{H}_w t}|\psi_w^{(2)}\rangle . \tag{2.26}$$

Referring to (2.16) we have

$$\theta^\dagger+\theta=2\theta_1+\Omega_3+\Omega_4 ,$$

where

$$\Omega_3\equiv\int d^3x\, A_0^a\sqrt{-\nabla^2}A_0^a \tag{2.27}$$

and

$$\Omega_4\equiv-\int d^3x\, I^a\sqrt{-\nabla^2}I^a . \tag{2.28}$$

The only factor on the right side of (2.26) which is dependent on A_0, η, and ξ is $e^{2\theta_1+\Omega_3}$, as Ω_4, $\mathcal{H}_w$, and ψ_w are independent of these variables. Thus the functional integrations over A_0, η, and ξ which one performs in taking the matrix element of (2.26) can be explicitly carried out. The function Ω_3 is negative definite if A_0 is imaginary. Thus we integrate over the imaginary values of A_0, which is just using the indefinite metric. The integral is a constant, which can be set to unity if the quantum states are properly normalized. The functional integrations over η and ξ can also be carried out, and give a determinant which can be interpreted as the contributions of ghosts. The right side of (2.26) can then be reduced to the transition amplitude of the physical theory, as will be shown in more detail in Appendix C. Thus we have

$$\langle\psi_{\text{eff}}^{(1)}|e^{-i\mathcal{H}_{\text{eff}}t}|\psi_{\text{eff}}^{(2)}\rangle=\langle\psi_w^{(1)}|e^{-i\mathcal{H}_w t}|\psi_w^{(2)}\rangle_w , \tag{2.29}$$

where the subscript outside of the angular brackets indicates that the matrix element on the right side of (2.29) is that in the physical theory; i.e., it involves functional integrations over $\mathbf{A}$, ϕ, and ϕ^* but not those over ξ, η, and A_0, with a weighting factor originated from gauge fixing. The precise form for this matrix element is given by (C5) in Appendix C. Equation (2.29) is the correspondence formula for the transition amplitude. We remind the reader that the functional integral in (C5) is not a path integral.

Similarly, we may derive correspondence formulas for other gauge-invariant operators. More precisely, let M be an operator in the effective theory which is in the form of

$$M=M_w+[Q,0]_+ , \tag{2.30}$$

where M_w is a gauge-invariant operator in the physical theory (i.e., it is gauge invariant and depends only on $\mathbf{A}$, ϕ, ϕ^*, and their conjugates but not on A_0, η, ξ, and their conjugates) then the matrix element of M between effective states $e^{\theta}\psi_{w1}$ and $e^{\theta}\psi_{w2}$ is the same as that of M_w between physical states ψ_{w1} and ψ_{w2}. The proof will be given in Appendix D. Examples of such operators are the elements of the Poincaré group: time translations (already specifically discussed), spatial translations, rotations, and Lorentz boosts. There also exists a correspondence formula for the Wilson loop.[17]

While the matrix elements in the two quantum theories have a precise correspondence, the same cannot be said for the quantum states without modification. As we recall, the wave function in the effective theory is of the form of (2.25). It is straightforward to prove that

$$e^{\theta}\psi_w+Q\phi=0\rightarrow e^{\theta}\psi_w=0 \text{ and } Q\phi=0 . \tag{2.31}$$

This is because the norm of $Q\phi$ is zero, while the norm of $e^{\theta}\psi_w$ is the same as that of ψ_w in the physical space, the latter being zero only if $\psi_w=0$. Because of (2.31), the decomposition of a quantum state in the effective space into a sum in the form of (2.25) is unique. The part $Q\phi$ never contributes to the matrix elements of gauge-invariant operators, and the only physically useful part of the wave function is $e^{\theta}\psi_w$. Consider therefore the mapping which maps all of the wave functions $e^{\theta}\psi_w+Q\phi$ which are of the same $e^{\theta}\psi_w$ and different $Q\phi$ into the same point. The linear space obtained from such a mapping is called a quotient space of the effective space. The operator in the quotient space which performs the same mappings as those of $\mathcal{H}_{\rm eff}$ in the effective space will be called $\mathcal{H}'_{\rm eff}$. Let a point in the quotient space be an eigenstate of $\mathcal{H}'_{\rm eff}$, then

$$\mathcal{H}_{\rm eff}(e^{\theta}\psi_w+Q\phi)=E(e^{\theta}\psi_w+Q\phi') , \tag{2.32}$$

for some ψ_w, ϕ, and ϕ'. By (D1) and the uniqueness of decomposition, ψ_w must be an eigenstate of $\mathcal{H}_w$ with eigenvalue E. We may also prove that the converse is true. Therefore, the quotient space is isomorphic to the physical space, and the energy spectrum of $\mathcal{H}_w$ in the physical space is the same as that of $\mathcal{H}'_{\rm eff}$ in the quotient space.

We may further show that, for operators of the form of (2.30), a complete set of states in the effective space is $e^{\theta}\psi_w^{(n)}$, where $\psi_w^{(n)}$ forms a complete set of physical states. More precisely, we have

$$\langle e^{\theta}\psi_w^{(f)}|M_1M_2|e^{\theta}\psi_w^{(i)}\rangle=\sum_n \langle e^{\theta}\psi_w^{(f)}|M_1|e^{\theta}\psi_w^{(n)}\rangle\langle e^{\theta}\psi_w^{(n)}|M_2|e^{\theta}\psi_w^{(i)}\rangle . \tag{2.33}$$

Equation (2.33) can be proved by using the correspondence formulas for the matrix elements of M_1M_2, M_1, and M_2 and the fact that $\psi_w^{(n)}$ forms a complete set of states in the physical space. That the scattering amplitude in the effective theory is unitary within the physical sector is an easy consequence of (2.33). Of course, in our formalism, such a result hardly needs a proof: the scattering amplitude in the effective theory is equal to that in the physical theory, which is unitary.

III. APPLICATIONS

In this section we apply the formalism developed in the preceding section to specific cases. This is done by making definite choices of h^a and I^a introduced in (2.8) and (2.16c), which, in our formalism, can be any real-valued functions of $\mathbf{A}$, ϕ, and $\phi^\dagger$. (They can even be, for example, nonlinear functions of these fields.) While different choices give the same scattering amplitudes as those in the corresponding physical theory, they represent different gauges, some of which may be more convenient to use than others in a specific situation.

A. The Feynman gauge

The effective theory in the Feynman gauge is obtained by choosing

$$h^a=\sqrt{-\nabla^2}I^a=\nabla\cdot\mathbf{A}^a \tag{3.1}$$

and hence

$$\delta h^a=-(\nabla\cdot\mathbf{D}\xi)^a .$$

The Lagrangian (2.11) is then equal to

$$\begin{aligned}&-\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a}+(D_\mu\phi)^\dagger(D^\mu\phi) ,\\ &-\tfrac{1}{2}(\partial_\mu A^{\mu,a})^2-i(\partial_\mu\eta^a)(D^\mu\xi)^a ,\end{aligned} \tag{3.2}$$

which is indeed the Lagrangian in the Feynman gauge. The wave function is given by

$$\psi_{\rm eff}=e^{\theta}\psi_w(\mathbf{A},\phi,\phi^\dagger) , \tag{3.3a}$$

where

$$\theta=\theta_1+\theta_2 , \tag{3.3b}$$

with

$$\theta_1\equiv -i\int d^3x\, \eta^a\frac{1}{\sqrt{-\nabla^2}}(\nabla\cdot\mathbf{D}\xi)^a , \tag{3.3c}$$

$$\theta_2\equiv\tfrac{1}{2}\int d^3x(A_0^a-iA_L^a)\sqrt{-\nabla^2}(A_0^a-iA_L^a) , \tag{3.3d}$$

and

$$A_L^a\equiv\frac{1}{\sqrt{-\nabla^2}}(\nabla\cdot\mathbf{A}^a) . \tag{3.4}$$

If we want to calculate the on-shell scattering amplitude with the effective Lagrangian (3.2) and the wave function (3.3a), we turn off the coupling g adiabatically in the distant past and the distant future. Then ψ_w is independent of A_L. Also

$$e^{\theta_1}\to\exp\left[i\int d^3x\,\eta^a\sqrt{-\nabla^2}\xi^a\right]$$

is the ground-state wave function for the ghost fields, and e^{θ_2} is related to the ground-state wave function for the longitudinal modes of the gauge field by a unitary transformation

$$e^{\theta_2}=U\exp\left[\tfrac{1}{2}\int d^3x\, A_0^a\sqrt{-\nabla^2}A_0^a -\tfrac{1}{2}\int d^3x\, A_L^a\sqrt{-\nabla^2}A_L^a\right] ,$$

with

$$U=\exp\left[-i\int d^3x\, A_0^a\sqrt{-\nabla^2}A_L^a\right] .$$

After making the unitary transformation of U, we find that the Feynman rules for the scattering amplitude are precisely those in the Feynman gauge. In particular, the gluon propagator is equal to

$$\frac{-ig_{\mu\nu}\delta_{ab}}{k^2+i\epsilon} \ . \tag{3.5}$$

The $i\epsilon$ in (3.5) for the longitudinal modes (as well as for the ghost modes) are obtained as the initial and the final states are in the ground state of these modes.

The treatments in this subsection can be easily extended to cover the case of the α gauge.

B. Gauge field theories with Higgs mesons

In this section, we treat non-Abelian gauge field theories with Higgs bosons.[18]

It is usually thought that the Higgs field $\phi(x)$ has a nonzero vacuum expectation value. This value is assumed to be a constant, defining a preferred direction which breaks the vacuum symmetry. As a consequence, some or all of the gauge mesons acquire masses.

We believe that such a view is erroneous. The quantum states in the physical theory satisfies the Gauss law (2.4). Since G^a in (2.4) is an infinitesimal generator for gauge transformations, all physical quantum states are gauge invariant, i.e.,

$$\psi_w(\mathbf{A},\phi)=\psi_w(\mathbf{A}',\phi') \ , \tag{3.6}$$

as long as $(\mathbf{A},\phi)$ is related to $(\mathbf{A}',\phi')$ by a gauge transformation. As a consequence of (3.6), $\phi(\mathbf{x})$ has no preferred direction and its vacuum expectation value cannot be in any given direction. Furthermore, (3.6) implies that quantum states in the physical theory are not normalizable, and vacuum expectations are not defined until we factor out the group volume by the choice of a gauge. Indeed, the field ϕ is gauge dependent, and the vacuum expectation of ϕ depends on the gauge adopted. The vacuum expectation value of ϕ referred to in the literature should be identified with that in the unitary gauge, and the masses of the gauge mesons are determined by the Hamiltonian in this gauge.

To give an explicit example, consider the Lagrangian gauge field theory in which the gauge group is SU(2) and the Higgs meson is an isovector:

$$\phi\equiv\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix} , \tag{3.7}$$

where the components ϕ_a are Hermitian. The Lagrangian is

$$L=-\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a}+\tfrac{1}{2}(D_\mu\phi)^T(D^\mu\phi)+\tfrac{1}{2}m^2\phi^T\phi -\frac{\lambda}{4}(\phi^T\phi)^2 \ . \tag{3.8}$$

The Hamiltonian in the temporal gauge is

$$H_w=\frac{(\pi^a)^2+(\mathbf{B}^a)^2}{2}+\tfrac{1}{2}\pi_\phi^T\pi_\phi+\tfrac{1}{2}(\mathbf{D}\phi)^T\cdot(\mathbf{D}\phi) -\tfrac{1}{2}m^2\phi^T\phi+\frac{\lambda}{4}(\phi^T\phi)^2 \tag{3.9}$$

and G^a of the Gauss law is

$$G^a\equiv\nabla\cdot\pi^a+g\epsilon^{abc}\mathbf{A}^b\cdot\pi^c+g\epsilon^{abc}\phi_b\pi_{\phi_c} \ . \tag{3.10}$$

Any field configuration $\{\mathbf{A}^a(\mathbf{x}),\phi_a(\mathbf{x})\}$ can be gauge transformed into one with

$$\phi=\begin{bmatrix}0\\ 0\\ \phi_3\end{bmatrix},\quad \phi_3\geq 0 \ . \tag{3.11}$$

We may therefore study the gauge field theory in the unitary gauge in which ϕ is in the direction of the positive I_3 axis. The vacuum expectation of ϕ_3 in this gauge is naturally positive. The classical Hamiltonian in the unitary gauge is equal to H_w with ϕ_1 and ϕ_2 set to zero and with π_{ϕ_1} and π_{ϕ_2} replaced by the roots of $G^1=G^2=0$. The quantum Hamiltonian in the unitary gauge differs from this classical Hamiltonian by operator ordering.[5,6] As it turns out,[6] this means that we simply insert J and J^{-1} into quadratic forms of π, e.g.,

$$\tfrac{1}{2}\pi_{\phi_3}^2\rightarrow\frac{1}{2}\frac{1}{J}\pi_{\phi_3}J\pi_{\phi_3} \ ,$$

where J is the Jacobian in the unitary gauge. Thus we obtain the Hamiltonian in the unitary gauge as

$$H_u=\frac{(\pi^a)^2+(\mathbf{B}^a)^2}{2}+\tfrac{1}{2}(g\phi_3)^{-2}[\nabla\cdot\pi^1+g(\mathbf{A}^2\cdot\pi^3-\mathbf{A}^3\cdot\pi^2)]^2+\tfrac{1}{2}(g\phi_3)^{-2}[\nabla\cdot\pi^2+g(\mathbf{A}^3\cdot\pi^1-\mathbf{A}^1\cdot\pi^3)] + \frac{1}{2J}\pi_{\phi_3}J\pi_{\phi_3}+\tfrac{1}{2}(\nabla\phi_3)^2+\tfrac{1}{2}g^2\phi_3^2(\mathbf{A}^1\cdot\mathbf{A}^1+\mathbf{A}^2\cdot\mathbf{A}^2)-\tfrac{1}{2}m^2\phi_3^2+\frac{\lambda}{4}\phi_3^4 \ .$$

The infinitesimal gauge transformation for ϕ at $\phi_1=\phi_2=0$ is

$$\delta\phi=g\phi_3\begin{bmatrix}-\theta_2\\ \theta_1\\ 0\end{bmatrix} ,$$

where θ_1, θ_2, and θ_3 (not shown) are the group parameters. Thus the Jacobian is

$$J=\det\frac{\partial\phi}{\partial\theta}=(\det g\phi_3)^2 \ .$$

We note that the Gauss law of $a=3$ has not been utilized, hence the wave function in the unitary gauge is required to satisfy

$$G^3|\psi_u\rangle=0 \ ,$$

where

$$G^3=\nabla\cdot\pi^3+g(\mathbf{A}^1\cdot\pi^2-\mathbf{A}^2\cdot\pi^1)\ .$$

As in the literature, we may put

$$\phi_3\equiv v+\chi_3\ ,$$

where $v=m/\sqrt{\lambda}$ is the classical vacuum expectation value of ϕ_3 in the unitary gauge. In the limit $g\to 0$ with $M\equiv gv$ fixed, it is justified to set $g\phi_3$ to M, set g to zero everywhere else, and replace ϕ_3 in J by v (hence J becomes a constant). We then easily find that the two gauge mesons of $a=1$ and 2 both acquire the same mass M, while the gauge meson of $a=3$ remains massless, as is well known.

In the effective theory we put

$$\phi\equiv\begin{bmatrix}0\\0\\v\end{bmatrix}+\chi\ . \tag{3.12}$$

We shall denote

$$\chi=\begin{bmatrix}-\chi^2\\ \chi^1\\ \chi^3\end{bmatrix} \tag{3.13}$$

and choose

$$\begin{aligned}\bar{l}^{\,a}&=\partial^\mu A^a_\mu-M\chi^a,\quad a=1,2\\ &=\partial^\mu A^a_\mu,\quad a=3\ .\end{aligned} \tag{3.14}$$

The reason for such a choice is that the crossed term $MA^a_\mu\partial^\mu\chi^a$ in $-\frac{1}{2}(\bar{l}^{\,a})^2$ cancels out such a term in L, and in the weak-coupling limit, L_{eff} is the free Lagrangian of uncoupled fields. In addition, we choose

$$I^a=A'^a_L\ , \tag{3.15}$$

where

$$\begin{aligned}A'^a_L&\equiv\frac{1}{\sqrt{M^2-\nabla^2}}(-M\chi^a+\sqrt{-\nabla^2}A^a_L),\quad a=1,2\\ &\equiv A^a_L,\quad a=3\ ,\end{aligned} \tag{3.16}$$

(3.17)

with

$$A^a_L\equiv\frac{1}{\sqrt{-\nabla^2}}(\nabla\cdot\mathbf{A}^a)\ .$$

We shall also define

$$\begin{aligned}\chi'^a&\equiv\frac{1}{\sqrt{M^2-\nabla^2}}(\sqrt{-\nabla^2}\chi^a+MA^a_L),\quad a=1,2\\ &\equiv\chi^a,\quad a=3\ .\end{aligned}$$

Note that, if we set $M=0$, the primed and the unprimed fields become the same.

The effective Lagrangian so chosen is given by (2.11), (3.15), and

$$\begin{aligned}\delta h^a&=-(\nabla\cdot\mathbf{D}\xi)^a+M(g\chi_3+M)\xi^a\\ &\quad+Mg\epsilon^{ab}\chi_b\xi_3,\quad a=1,2\\ &=-(\nabla\cdot\mathbf{D}\xi)^3,\quad a=3\ ,\end{aligned} \tag{3.18}$$

where $\epsilon^{ab},a,b=1$ or 2, is antisymmetric with $\epsilon^{12}=-\epsilon^{21}=1$. It has been so chosen that, in the limit of g and λ going to zero with M fixed, it is the free Lagrangian in which there are uncoupled fields $\mathbf{A}'^a$, χ'^a, η^a, and ξ^a, $a=1,2$, of mass M, uncoupled fields $\mathbf{A}^3$, η^3, and ξ^3 of zero mass, and a free field χ^3 of mass $\sqrt{2}m$. In addition, G^a becomes, in this limit $\sqrt{-\nabla^2}\pi'^a_L$. Thus this case can be treated in exactly the same way as in Sec. III A and Appendix C with A_L replaced by A'_L. In particular, the propagator for the vector meson is

$$\frac{-ig_{\mu\nu}\delta_{ab}}{k^2-M^2(1-\delta_{3a})+i\epsilon}\ .$$

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APPENDIX A

In this appendix we derive the expression (2.15) for H_{eff}.

Our goal is to express the difference of H_{eff} with H_w as BRS variations of operators. First, we list the BRS variations of the π's below. We have

$$\delta\pi^a_0=i[Q,\pi^a_0]=0 \tag{A1}$$

and similarly

$$\delta\pi^a=gf^{abc}\xi^b\pi^c\ , \tag{A2}$$

$$\delta\pi_\phi=ig\xi^aT^a\pi_\phi\ , \tag{A3}$$

$$\delta\pi^a_\xi=G^a+gf^{abc}\pi^b_\xi\xi^c\ , \tag{A4}$$

$$\delta\pi^a_\eta=0\ . \tag{A5}$$

Also, if C_1 is a bosonic operator, we have

$$\delta(C_1C_2)=(\delta C_1)C_2+C_1\delta C_2 \tag{A6}$$

and, if C_1 is a Grassmann operator,

$$\delta(C_1C_2)=(\delta C_1)C_2-C_1\delta C_2\ . \tag{A7}$$

The Hamiltonian corresponding to the Lagrangian in (2.11) is given by

$$\begin{aligned}H_{\text{eff}}=&\dot{A}^a_0\pi^a_0+\mathbf{A}^a\cdot\pi^a+\pi_\phi\dot{\phi}+\pi^\dagger_\phi\dot{\phi}^\dagger+\dot{\xi}^a\pi^a_\xi\\ &+\dot{\eta}^a\pi^a_\eta-L+\tfrac{1}{2}(\pi^a_0)^2+i\dot{\eta}(D^0\xi)^a-i\eta^a\delta h^a\ .\end{aligned} \tag{A8}$$

(A8)

Making use of (A6), (2.9c), and (A1) we get

$$\delta(i\pi^a_0\eta^a)=-(\pi^a_0)^2\ . \tag{A9}$$

Also

$$\begin{aligned}\delta(ih^a\eta^a)&=i(\delta h^a)\eta^a+ih^a\delta\eta^a\\ &=-i\eta^a(\delta h^a)-h^a\pi^a_0\ .\end{aligned} \tag{A10}$$

Adding (A9) to (A10) we get

$$\delta(i\pi_0^a\eta^a+ih^a\eta^a)=-i\eta^a(\delta h^a)+\dot{A}_0^a\pi_0^a \; . \tag{A11}$$

We further observe that the sixth and the ninth terms on the right side of (A8) cancel each other and that

$$\mathbf{A}^a\cdot\boldsymbol{\pi}^a+\pi_\phi\dot{\phi}+\pi_{\phi^\dagger}\dot{\phi}^\dagger-L=H_w+A_0^aG^a \; , \tag{A12}$$

as the left side of (A12) is the Hamiltonian corresponding to the gauge-invariant Lagrangian L. Also, a little calculation gives

$$\delta(A_0^a\pi_\xi^a)=\dot{\xi}^a\pi_\xi^a+A_0^aG^a \; . \tag{A13}$$

Thus (2.15) is proved.

APPENDIX B

In this appendix we shall prove (2.17).

Referring to (2.16c) we have

$$\Theta_1=\Omega_1+\Omega_2 \; , \tag{B1}$$

where

$$\Omega_1\equiv\int d^3x\,[Q,\eta^aI^a]_+ \tag{B2}$$

and

$$\Omega_2\equiv-\int d^3x\;\pi_0^aI^a \; . \tag{B3}$$

Then

$$e^{\Theta}=e^{\Omega_1}e^{\Omega_2}e^{\theta_2} \; . \tag{B4}$$

As is seen from (B2), Ω_1 is a BRS variation, thus it commutes with Q. Therefore, (2.17) is reduced to

$$Qe^{\Omega_2}e^{\Theta_2}\psi_w=0 \; . \tag{B5}$$

The next step involves the use of the formula

$$e^{-g(q)}f(p,q)e^{g(q)}=f(p-ig',q) \; , \tag{B6}$$

where p and q are conjugate to each other with

$$[p,q]=-i \; ,$$

and where f may be any function and g is differentiable. By identifying A_0 and $-\pi_0$ with p and q, respectively, it is easy to prove that

$$e^{\Omega_2}e^{\Theta_2}e^{-\Omega_2}=\exp\left[\tfrac{1}{2}\int d^3x\;A_0^a\sqrt{-\nabla^2}A_0^a\right] \tag{B7}$$

or

$$e^{\Omega_2}e^{\Theta_2}=\exp\left[\tfrac{1}{2}\int d^3x\;A_0^a\sqrt{-\nabla^2}A_0^a\right]e^{\Omega_2} \; . \tag{B8}$$

Now, since $\psi_w(\mathbf{A},\phi,\phi^*)$ does not depend on A_0, we have

$$\pi_0\psi_w=-i\frac{\delta}{\delta A_0}\psi_w=0 \; . \tag{B9}$$

It follows from (B9) that

$$e^{\Omega_2}\psi_w=0 \; , \tag{B10}$$

where Ω_2 is given by (B3). With the help of (B8) and (B10), (B5) is reduced to

$$Q\exp\left[\tfrac{1}{2}\int d^3x\;A_0^a\sqrt{-\nabla^2}A_0^a\right]\psi_w=0 \; . \tag{B11}$$

Next we have

$$\pi_\eta\psi_w=\pi_\xi\psi_w=0 \; , \tag{B12}$$

as $\psi_w(\mathbf{A},\phi,\phi^*)$ does not depend on η or ξ. We also have, by (2.4),

$$G\psi_w=0 \tag{B13}$$

By (2.12), Q is a linear superposition of G, π_η, and π_ξ. Since all these three operators commute with A_0, (B11) is easily proved.

APPENDIX C

In this appendix we shall prove that the right side of (2.26) is the transition amplitude in the physical theory.

The physical wave function satisfies the Gauss law (2.4). Since G^a is an infinitesimal generator for time-independent gauge transformations, the physical wave function is gauge invariant and a gauge can be chosen. Let us choose the gauge to be

$$I^a(\mathbf{A},\phi,\phi^\dagger)=f(\mathbf{x}) \; , \tag{C1}$$

where f is any function of $\mathbf{x}$. Under an infinitesimal gauge transform of group parameter χ^a,

$$\begin{aligned}\delta I^a&=\frac{\delta I^a}{\delta A^{j,b}}(-D_j\chi)^b+\frac{\delta I^a}{\delta\phi}(-igT^b\chi^b\phi)\\&\quad+\frac{\delta I^a}{\delta\phi^\dagger}(igT^b\chi^b\phi^\dagger)\\&\equiv M^{ab}\chi^b \; .\end{aligned} \tag{C2}$$

Thus, the phase space with the gauge fixed by the relation (C1) is

$$\mathcal{D}\,\mathbf{A}^a\mathcal{D}\phi\,\mathcal{D}\phi^\dagger\delta(I^a-f^a)\det(M) \tag{C3}$$

and the physical transition amplitude from the state $|\psi_w^{(2)}\rangle$ to the state $|\psi_w^{(1)}\rangle$ after a time duration t is

$$\int\mathcal{D}\,\mathbf{A}^a\mathcal{D}\phi\,\mathcal{D}\phi^\dagger\delta(I^a-f^a)\det M\,\psi_w^{(1)*}e^{-i\mathcal{H}_wt}\psi_w^{(2)} \; . \tag{C4}$$

Since the amplitude in (C4) is independent of f we may multiply (C4) by $\exp(-\int d^3x\,f^a\sqrt{-\nabla^2}f^a)$ and integrate over all f^a. The result is

$$\int\mathcal{D}\,\mathbf{A}^a\mathcal{D}\phi\,\mathcal{D}\phi^*\psi_w^{(1)*}e^{-i\mathcal{H}_wt}\psi_w^{(2)}e^{\Omega_4}\det M \; . \tag{C5}$$

On the other hand, if we carry out the functional integrations over A_0, η, and ξ for the right side of (2.26), we also get the expression in (C5). Thus the result.

Finally, we mention that there is no difficulty with the Gribov ambiguity in the gauge fixing of (C1) as long as we integrate over all $\mathbf{A}$, ϕ, and $\phi^\dagger$ (Ref. 16). The point is that while the number of Gribov copies may vary from one orbit to another, this number is either odd for all orbits or even for all orbits. Furthermore, contributions from Gribov copies cancel pairwise. Thus the integral over all $\mathbf{A}$, ϕ, and ϕ^* is either identically zero (when the numbers of copies are all even), or exactly equal to the contribution of one copy for all orbits (when the numbers of copies are all odd). Since the integral in (C5) is not zero, the latter is the case.

APPENDIX D

In this appendix we shall prove the following theorem. If M is an operator which is in the form of (2.30), then

$$Me^{\Theta}\psi_w = e^{\theta}M\psi_w + Q\psi \ . \tag{D1}$$

Equations (2.23) and (2.24) are special cases of (D1).

First of all, we may ignore the term $[Q,0]_+$ in (2.30). This is because $0Qe^{\Theta}\psi_w$ is equal to zero and $Q0e^{\Theta}\psi_w$ is of the form of $Q\psi$.

Next we note that both

$$M_w(\Omega_1)^n e^{\Omega_2} e^{\Theta_2}\psi_w, \quad n=1,2,\ldots$$

and

$$(\Omega_1)^n M_w e^{\Omega_2} e^{\Theta_2}\psi_w, \quad n=1,2,\ldots,$$

are of the form of $Q\psi$. Thus

$$M_w e^{\Omega_1} e^{\Omega_2} e^{\Theta_2}\psi_w = e^{\Omega_1} M_w e^{\Omega_2} e^{\Theta_2}\psi_w + Q\psi \ . \tag{D2}$$

Next, we have from (B8) and $\pi_0\psi_w=0$ that

$$\begin{aligned} M_w e^{\Omega_2} e^{\Theta_2}\psi_w &= M_w \exp\left[\tfrac{1}{2}\int d^3x\, A_0^a\sqrt{-\nabla^2}A_0^a\right]\psi_w \\ &= \exp\left[\tfrac{1}{2}\int d^3x\, A_0^a\sqrt{-\nabla^2}A_0^a\right] M_w\psi_w \\ &= \exp\left[\tfrac{1}{2}\int d^3x\, A_0^a\sqrt{-\nabla^2}A_0^a\right] e^{\Omega_2} M_w\psi_w \ ; \end{aligned} \tag{D3}$$

the last two steps are obtained as M_w is independent of π_0 and A_0. Equation (D1) follows readily from (D2) and (D3).

Finally, we mention that if M is an operator which commutes or anticommutes with Q and is not dependent on A_0 and π_0, then M also satisfies (D1).

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