Tests for "new physics" from τ spin correlation functions for $Z^0 \to \tau^+\tau^- \to A^+B^-X$

Charles A. Nelson
Department of Physics, State University of New York at Binghamton, Binghamton, New York 13901
(Received 9 February 1989)

The fundamental parameters ξ_A, ξ_B, and α_H can be independently determined by measurement of the energy correlation function $I(E_A,E_B)$ where ξ is the Michel polarization parameter for $\tau^- \to \ell^-\nu\bar{\nu}$, $\xi_A = (|g_L|^2 - |g_R|^2)/(|g_L|^2 + |g_R|^2)$ is the chiral polarization parameter for $\tau^- \to A^-\nu$, and $\alpha_H \simeq -2a_\tau v_\tau/(a_\tau^2 + v_\tau^2)$ where a_τ, v_τ describe $Z^0 \to \tau^+\tau^-$ at the tree level. In contrast, measurement of $I(E_A)$ by the τ-polarization technique only determines $\xi_A\alpha_H$. For 10^7 Z^0 events and for $\sin^2\theta_W = 0.23$, the ideal statistical percentage errors in the determination of the Michel parameters are for ξ, 2.6%; for δ, 3.5%; and for ρ, 1.0%; and of the chiral parameters are for ξ_π, 1.4%; for ξ_ρ, 3.0%; and for ξ_{K^*}, 18%. For α_H, the ideal statistical error is $\sigma(\alpha_H) = 0.005$ (3.3%). Equivalently, for $\sin^2\theta_W$ it is 0.3%. Explicit formulas for the full sequential decay correlation function $I(E_{\bar{\mu}}, E_e, \cos\psi_{\bar{\mu}e})$ are given for arbitrary Z mass and for $I(E_{\bar{\mu}}, E_B)$ for arbitrary Michel parameters. These results can be used for analyzing the decay of a Z' boson and for $q\bar{q}$ modes.

I. INTRODUCTION AND DEFINITION OF τ COUPLING PARAMETERS

The first round of experiments at the SLAC Linear Collider (SLC) and the CERN e^+e^- collider LEP will use unpolarized beams in e^+e^- collisions at the peak of the Z^0. Our primary objectives in this paper[1] are to provide (i) explicit formulas and plots of the tree-level energy correlation functions $I(E_A, E_B)$ for the Z^0 sequential decay

$$\begin{array}{l} Z^0 \to \tau^+\tau^- \\ \quad \tau^- \to B^-X \\ \quad \tau^+ \to A^+X \end{array} \quad , \tag{1.1}$$

for arbitrary Z^0 mass and arbitrary chiral structure in the τ couplings, and (ii) associated "ideal statistical errors" (for definition of this term, see Sec. VII) for determination of the τ coupling parameter α_H for describing $Z^0 \to \tau^+\tau^-$, the Michel parameters (ξ,δ,ρ) for $\tau^- \to \ell^-\nu\bar{\nu}$, and the chiral polarization parameters

$$\xi_A = \frac{|g_L|^2 - |g_R|^2}{|g_L|^2 + |g_R|^2} \tag{1.2}$$

for $\tau^+ \to A^+\nu$ where, respectively, $A = \pi, \rho, K^*$. We hope that in the context of τ-pair Monte Carlo simulations[4] this will enable readers to evaluate the merit of using this technique for the measurement of these τ coupling parameters by some of the A^+B^- sequential decays of Eq. (1.1). To do this, this information must be supplemented with that about (a) the measurement limitations, systematic errors, and background distributions for specific modes for detectors[6] at SLC/LEP and about (b) the important theoretical contributions,[7–13] omitted here, from electroweak and QED radiative corrections, QCD corrections, and finite-width Z^0, A^+, and/or B^- resonance corrections.

Before giving the sectional contents of this paper, we define the necessary quantities to describe the τ's electroweak coupling. For describing the $Z^0 \to \tau^+\tau^-$ coupling, we introduce the usual a_τ and v_τ coupling constants so

$$\begin{aligned} \alpha_H &\equiv \frac{|t_{-+}|^2 - |t_{+-}|^2}{|t_{-+}|^2 + |t_{+-}|^2} \\ &\simeq \frac{-2a_\tau v_\tau[1-(2m_\tau/M)^2]^{1/2}}{v_\tau^2 + a_\tau^2[1-(2m_\tau/M)^2]} \\ &\simeq \frac{-2a_\tau v_\tau}{v_\tau^2 + a_\tau^2} \simeq -0.1591 \ , \end{aligned} \tag{1.3}$$

where $t_{\lambda_1\lambda_2}$ are the usual helicity amplitudes describing the couplings of the Z^0 to τ^+ and τ^-. The numerical value in Eq. (1.3) assumes lepton universality so $r = a_\tau/v_\tau = (1-4\sin^2\theta_W)^{-1}$. We use $\sin^2\theta_W = 0.23$ throughout this paper. At the tree level a definition of a distribution parameter in terms of helicity amplitudes is equivalent to a definition based on effective coupling parameters in the covariant coupling approach. The Michel parameters are defined as in polarized-muon decay. For τ decays $\tau^- \to A^-\nu_\tau$, suppressing the A subscripts we introduce the covariant amplitudes

$$\mathcal{M}(\tau^- \to \pi^-\nu) = k_\pi^\alpha \bar{u}_\nu \gamma_\alpha (v - a\gamma_5) u_\tau \tag{1.4}$$

for $A = \pi, K$ and similarly, for $A = \rho, K^*, a_1$,

$$\mathcal{M}(\tau^- \to \rho^-\nu) = \rho^\alpha \bar{u}_\nu \gamma_\alpha (v - a\gamma_5) u_\tau \tag{1.5}$$

and define $g_L = v + a$, $g_R = v - a$. Equivalent to Eq. (1.2),

$$\xi_A = \frac{2\,\mathrm{Re}(v_A a_A^*)}{|v_A|^2 + |a_A|^2} \ , \tag{1.6}$$

where $\xi_A = 1$ for a pure $V-A$ effective coupling.[14]

By measurement of the energy correlation function $I(E_A, E_B)$ for a definite A^+B^- sequential decay mode, Eq. (1.1), the basic parameters ξ_A, ξ_B, and α_H can be independently determined. [This is evident for the μ^+e^- mode from Eqs. (3.17) and (3.18) below and can be used to test $\mu \leftrightarrow e$ universality of the Michel parameters ξ in τ decay. Similarly, for the lepton-hadron $I(E_{\bar{\mu}}, E_h)$ from Eq. (5.10) below; and for the hadron-hadron $I(E_A, E_B)$ from Eq. (6.5).]

In contrast, the elegant τ-polarization technique[15,16,11] determines the combination $\xi_A \alpha_H$ since only the single-particle energy distribution $I(E_A)$ is measured. If precision experiments find $\xi_\pi \alpha_H$ from $I(E_\pi)$ and, for example, $\xi_\rho \alpha_H$ from $I(E_\rho)$ to be equal within errors, then presumably one would assume ξ_π and ξ_ρ are equal; however, the further assumption that $\xi_\pi = \xi_\rho$ equals 1 would be required before a value for α_H, i.e., $\sin^2\theta_W$, would be available for testing the standard model and alternatives for "new physics." On the other hand, given the significance of a precision measurement of $\sin^2\theta_W$, it is very important to use the energy correlation function $I(E_A, E_B)$ to test these assumptions about the value of the ξ's.

In Sec. II, we briefly review the relation of τ-coupling-parameter values to expectations for "new physics." In Sec. III, the muon-energy–electron-energy correlation function $I(E_{\bar{\mu}}, E_e)$ for massless final leptons μ^+ and e^- is obtained for arbitrary Z^0 mass in terms of ξ, δ, ρ. In Appendix A, we give explicit formulas for the full sequential decay correlation function $I(E_{\bar{\mu}}, E_e, \cos\psi_{\bar{\mu}e})$ where $\psi_{\bar{\mu}e}$ is the angle between the $\bar{\mu}$ and e momentum. In Sec. IV formulas for the harder lepton's energy spectrum $I(x_H)$ are obtained. In Sec. V, the lepton energy-hadron energy correlation function $I(E_l, E_h)$ is obtained, and in Sec. VI the purely hadronic energy correlation function $I(E_A, E_B)$.

In Appendix B, formulas for energy-energy correlations $I(E_{\bar{\mu}}, E_e)$ and $I(E_\ell, E_h)$ in the nonrelativistic regime are listed which would be needed if there were a second neutral boson Z' and charged spin-$\frac{1}{2}$ fermions from a fourth family.

In Sec. VII for a 10^7 event Z^0 sample we list in Table II the "ideal statistical errors" for α_H and for the Michel parameters ξ, δ, and ρ for the various A^+B^- modes of Eq. (1.1).

The next three sections compare the $I(E_A, E_B)$ technique with the τ^- polarization technique. They are compared for determination of α_H in Sec. VIII, for determination of the Michel parameters in Sec. IX, and for determination of the chiral polarization parameters ξ_π, ξ_ρ, and ξ_{K^*} in Sec. X.

The principal conclusions are listed in Sec. XI.

II. BRIEF REVIEW OF SIGNATURES FOR NEW PHYSICS

In Table I are tabulated the shifts in α_H expected from some of the schemes of "new physics." We refer the reader to the Refs. 17–21 and to recent investigations[22] of how results from high-precision measurements might be combined to identify the origin of any small discrepancy versus the predictions of the standard model.

The significance of the Michel parameters is well known. It is manifested that ξ and ξ_π, ξ_ρ, and ξ_{K^*} of Eqs. (1.4) and (1.5) are all sensitive to right-handed currents. As with a potential discovery of a reliable small discrepancy in α_H, there are more than a few origins:

TABLE I. Summary of sensitivities of the $Z^0 \to \tau^+\tau^-$ coupling parameter α_H to "new physics." These estimates follow (Ref. 1) using the approximate relationship $\alpha_H \simeq -A_{LR}^{\mu^+\mu^-}$ where $A_{LR}^{\mu^+\mu^-}$ is the initial-state longitudinal-polarization asymmetry in muon pair production by a longitudinally polarized e^- beam in e^+e^- annihilation.

New physics	$-\delta\alpha_H$	$\delta A_{FB}^{\mu^+\mu^-}$	Reference
$m_H = 10^{2\pm 1}$ GeV	± 0.009		17
$m_t = 110 \pm 20$ GeV	± 0.005		17
Additional Higgs bosons			
$M_1^0 = M_2^0 = M_Z$, $M^{\mathrm{ch}} > 300$ GeV	> 0.02	> 0.005	18
Extra Z^0			
$m_{Z'} < \sim 700$ GeV	0.01–0.04		19
Fourth generation and/or superparticles			
Heavy-quark (or -squark) pair (large splitting)	0.02	0.01	20
Heavy-lepton (or -slepton) pair (large splitting, $m_\nu = 0$)	0.012	0.006	20
W-inos ($M_{3/2} \ll 100$ GeV)	0.005	0.0025	20
Technicolor			
SU(8)×SU(8)	0.04	−0.018	20
O(16)	0.07	−0.032	20
Ideal statistical error			From
$\mu^+(\pi,K)^-$ mode	0.010		Table II
Sum of modes	0.005		Table II

There may exist nonconserved currents,[23] or other than combinations of V and A couplings. Both left-right-symmetric models[24] and massive-neutrino models[25] (motivated in part by the missing-light, i.e., dark-matter, puzzle and the solar-neutrino puzzle) frequently contain right-handed currents. In particular, by modification of a left-right-symmetric model of Zee,[26] recently Fukagita and Yanagida,[27] Babu and Mathur,[28] and later Mohapatra[29] have considered models with a singlet charged Higgs field and used the muon's ξ measurements to constrain the couplings. Unfortunately, in spite of the large mass splittings between the families and current notions about nonzero neutrino masses, without decisive empirical data about the (lack of?) couplings of right-handed currents to the particles in the third family, it continues to be very difficult to assess the importance of competing ideas about the chiral structure of nature.

The ξ parameter in polarized-muon decay is known to a few tenth's of a percent level.[30] We find ideal statistical errors for the ξ's describing τ decays to be at the few percent level.

The results listed in Table I can be compared with those expected[31] from instrumentation of longitudinally polarized beams. In particular, the 0.3% ideal-statistical-error level for $\sin^2\theta_W$ from $I(E_A,E_B)$ measurement can be compared with the current world average 3% error and also with the 0.13% precision level expected from A_{LR} measurement at LEP I after instrumentation of polarized beams.

III. MUON-ENERGY–ELECTRON-ENERGY CORRELATION FUNCTION $I(E_{\bar{\mu}},E_e)$

We consider the sequential decay

$$\begin{array}{l} Z^0 \to \tau_1^+ \tau_2^- \\ \quad \tau_2^- \to e_2^- \bar{\nu}_2 \nu_2 \\ \quad \tau_1^+ \to \mu_1^+ \nu_1 \bar{\nu}_1 \end{array} \tag{3.1}$$

Assuming that the $Z^0 \to \tau^+\tau^-$ decay helicity amplitude $t_{\lambda_1\lambda_2}$, defined by

$$\langle \Theta,\Phi,\lambda_1,\lambda_2 | JM \rangle = D^J_{M\lambda}(\Phi,\Theta,-\Phi) t_{\lambda_1\lambda_2} , \tag{3.2}$$

where $\lambda=\lambda_1-\lambda_2$ and $J=1$ are invariant under the CP operation, we obtain

$$t_{\lambda_1\lambda_2} = \gamma_{CP} t_{-\lambda_2,-\lambda_1} , \tag{3.3}$$

where γ_{CP} is the CP quantum number of the Z^0 system. By Lorentz invariance, the density matrices describing $\tau^\pm \to \ell^\pm \nu\bar{\nu}$ are, respectively, of the form

$$\rho_{1/2\,1/2}(\theta_i^\tau,E_i^\tau) = R(x_i^\tau) \pm \xi_i \cos\theta_i^\tau S(x_i^\tau), \quad i=1,2 , \tag{3.4}$$

for the decay of a fully polarized τ^+ (τ^-) at rest with $x^\tau = E^\tau/\mathcal{E}$ where $\mathcal{E}=(m_\tau^2+\mu^2)/(2m_\tau)$ is the maximum ℓ^+ (ℓ^-) energy in the τ rest frame with μ the charged-lepton mass. In Eq. (3.4), ξ is the Michel polarization parameter. The argument x_i in R and S here (and in f and g below) is to be understood to label the quantities for $\tau^+ \to \mu^+\nu\bar{\nu}$ versus those for $\tau^- \to e^-\bar{\nu}\nu$. It then follows by Lorentz invariance that the "standard decay correlation function" for this sequential decay, Eq. (3.1), is

$$I(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau;\phi) = C(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau) + A_0(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau)\cos\phi , \tag{3.5}$$

where

$$A_0 = -\sigma\gamma_{CP}\xi_1 S(E_1^\tau)\xi_2 S(E_2^\tau)\sin\theta_1^\tau\sin\theta_2^\tau \tag{3.6}$$

and

$$C = \sigma S(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau) + \tau T(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau) + \upsilon U(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau) \tag{3.7}$$

with

$$\begin{aligned} S(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau) &= R(E_1^\tau)R(E_2^\tau) - \xi_1 S(E_1^\tau)\xi_2 S(E_2^\tau) \times \cos\theta_1^\tau\cos\theta_2^\tau , \\ T(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau) &= R(E_1^\tau)R(E_2^\tau) + \xi_1 S(E_1^\tau)\xi_2 S(E_2^\tau) \times \cos\theta_1^\tau\cos\theta_2^\tau , \\ U(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau) &= \xi_1 S(E_1^\tau)R(E_2^\tau)\cos\theta_1^\tau + R(E_2^\tau)\xi_2 S(E_2^\tau)\cos\theta_2^\tau \end{aligned} \tag{3.8}$$

and the σ,τ,υ coefficients are given below.

In the τ_1^+ rest frame θ_1^τ and ϕ_1^τ are the polar and azimuthal angles of the μ^+ in the usual helicity coordinate system and E_1^τ is the μ^+ energy. Correspondingly θ_2^τ, ϕ_2^τ, and E_2^τ specify the e^- in the τ_2^- rest frame. The important azimuthal angle ϕ between the $\tau^+\mu^+$ momenta plane and τ^-e^- momenta plane in the Z^0 rest frame is defined by

$$\phi = \phi_1 + \phi_2 . \tag{3.9}$$

While the dependence of R and S of Eqs. (3.6) and (3.8) on the other Michel parameters is of course implicit, we are emphasizing explicitly the dependence of the polarization parameter ξ. Assuming (i) lepton universality so $\xi_1=\xi_2=\xi$, and (ii) almost a $V-A$ charged-current coupling so ξ almost is $+1$, we see that both the T and the U term of the "standard decay correlation function" are sensitive to ξ. The U term and T term, as we discuss below, dominate in the limit of small μ, e, and τ masses versus the Z mass because of approximate helicity conservation in the $Z^0 \to \tau^+\tau^-$ amplitudes.

Because of the kinematics of the sequential decay, the A_0 term in Eq. (3.5) which is proportional to $\cos\phi$ *will not contribute to the muon-energy–electron-energy correlation function* $I(E_{\bar{\mu}},E_e)$. [This fact is easily seen to follow from the ϕ independence of Eqs. (A1) and (A2) of Ref. 2.]

The remaining coefficients σ, τ, and υ in Eqs. (3.7) and (3.8) are quadratic functions of four $t_{\lambda_1\lambda_2}$ amplitudes for $Z^0 \to \tau^+\tau^-$ and are given by

$$\begin{aligned} \sigma &= \tfrac{1}{2}(|t_{++}|^2+|t_{--}|^2) = |t_{++}|^2 , \\ \tau &= \tfrac{1}{2}(|t_{-+}|^2+|t_{+-}|^2) , \\ \upsilon &= \tfrac{1}{2}(|t_{-+}|^2-|t_{+-}|^2) . \end{aligned} \tag{3.10}$$

A. Tree-level evaluation of Eq. (3.5)

At the tree level, these quadratic functions describing $Z^0 \to \tau^+\tau^-$ can be expressed in the standard model in terms of the ratio of the axial-vector to the vector coupling coefficients

$$r = \frac{a_\tau}{v_\tau} = (1 - 4\sin^2\theta_W)^{-1} , \tag{3.11}$$

so

$$\sigma \simeq m_\tau^2 ,$$
$$\tau \simeq \tfrac{1}{2}[(1+r^2)M^2 - 4r^2 m_\tau^2] , \tag{3.12}$$
$$v \simeq -rM^2\sqrt{1-(2m_\tau/M)^2} ,$$

where M is the Z mass and m_τ is the τ mass. Since $(m_\tau/M)^2 \simeq 3.8\times10^{-4}$, the σ-dependent terms in Eq. (3.5) can be neglected and

$$I(\theta_1^\tau, E_1^\tau; \theta_2^\tau, E_2^\tau; \phi) \simeq \tau T(E_1^\tau, \theta_1^\tau; E_2^\tau, \theta_2^\tau) \times \left[1 + \alpha_H \frac{U(E_1^\tau, \theta_1^\tau; E_2^\tau, \theta_2^\tau)}{T(E_1^\tau, \theta_1^\tau; E_2^\tau, \theta_2^\tau)} \right] \tag{3.13}$$

with the important τ coupling parameter for $\sin^2\theta_W = 0.23$,

$$\alpha_H \equiv \frac{|t_{-+}|^2 - |t_{+-}|^2}{|t_{-+}|^2 + |t_{+-}|^2} \simeq \frac{-2a_\tau v_\tau}{a_\tau^2 + v_\tau^2} \simeq -0.1591 \tag{3.14}$$

and there is no dependence of the right-hand side of Eq. (3.13) on the azimuthal angle ϕ. However, there is sensitivity to α_H and to ξ_1 and ξ_2.

B. Muon-energy–electron-energy correlation function

Because of the four missing neutrinos, the $\tau^\pm$ rest frames are not directly accessible, but it is straightforward and simple to analytically obtain the $E_{\bar\mu}$-E_e energy-energy correlation function

$$I(E_{\bar\mu}, E_e) = T(E_{\bar\mu}, E_e)[1 + A(E_{\bar\mu}, E_e)] \tag{3.15}$$

in the Z rest frame for massless final μ and e leptons where $E_{\bar\mu} \equiv E_1$, and $E_e \equiv E_2$ are their respective energies in the Z rest frame. It is convenient to introduce the scaled lepton energies

$$x \equiv E_{\bar\mu}/E_{\max}, \quad y \equiv E_e/E_{\max} . \tag{3.16}$$

With γ and β the relativistic boost variables connecting a τ rest frame to the Z rest frame ($\gamma = M/(2m_\tau)$, $E_{\max} = m_\tau/[2\gamma(1-\beta)]$) is the maximum available charged-lepton energy in the Z rest frame.

For $E_{\bar\mu}$ and E_e both greater than $E_I = m_\tau/[2\gamma(1+\beta)] \simeq 0.0173$ GeV, equivalently for both x and $y > \gamma^{-2}(1+\beta)^{-2}$, we obtain from Eqs. (3.8) using the general lepton-number-conserving, four-fermion couplings for $\tau^\pm \to \ell^\pm \nu\bar\nu$ that[32]

$$T(E_{\bar\mu}, E_e) = \frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[f(x)f(y) + \xi_1\xi_2 g(x)g(y)]$$
$$\to \frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[f(x)f(y) + g(x)g(y)] \tag{3.17}$$

and

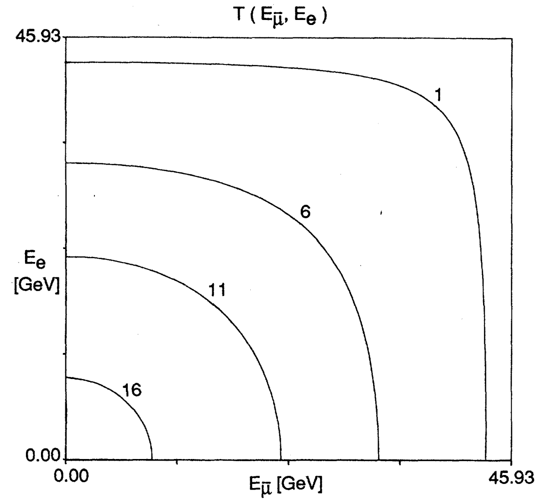

FIG. 1. The contour plot of the $T(E_{\bar\mu}, E_e)$ factor in the muon-energy–electron-energy correlation function $I(E_{\bar\mu}, E_e) = T(E_{\bar\mu}, E_e)[1 + A(E_{\bar\mu}, E_e)]$ in the Z^0 rest frame, for $Z^0 \to \tau^+\tau^- \to \mu^+ e^- X$ for a Z^0 mass $M = 91.9$ GeV. For the precision level of current interest at the SLC and LEP the final μ^+ and e^- mass corrections can be neglected so I is symmetric in $E_{\bar\mu} \leftrightarrow E_e$.

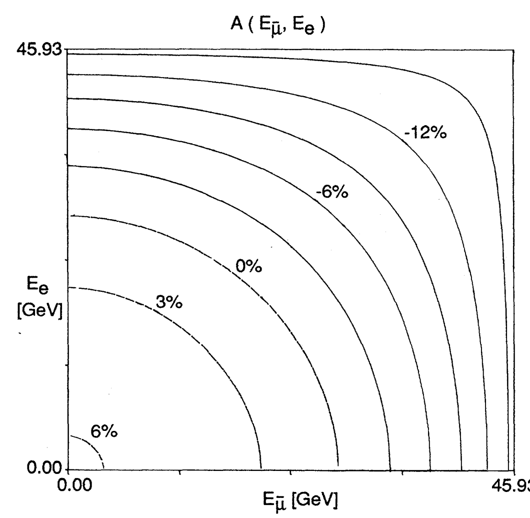

FIG. 2. The contour plot of the $A(E_{\bar\mu}, E_e)$ term in the $E_{\bar\mu}$ and E_e correlation function for $\sin^2\theta_W = 0.23$, $\xi = +1$. At the tree level $A(E_{\bar\mu}, E_e)$ is proportional to the τ couplings' parameter $\xi\alpha_H$ where ξ is the Michel polarization parameter for $\tau^- \to \ell^- \nu\bar\nu$, and $\alpha_H \sim -2a_\tau v_\tau/(a_\tau^2 + v_\tau^2)$ where a_τ, v_τ describe $Z^0 \to \tau^+\tau^-$ at the tree level.

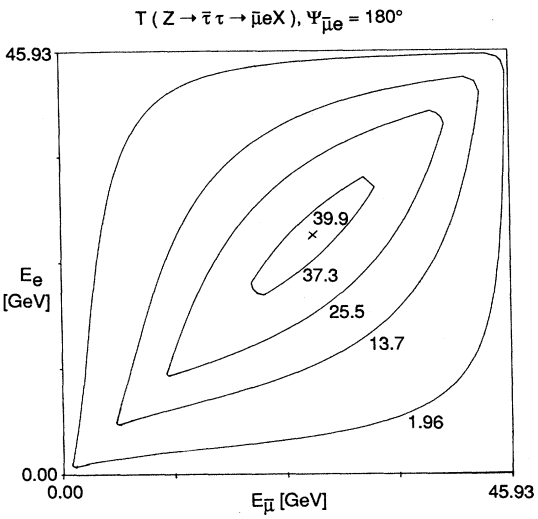

FIG. 3. For μ^+ and e^- back to back, the contour plot of the T term in the "standard decay correlation function" $I(E_{\bar\mu},E_e,\cos\psi_{\bar\mu e})=T(E_{\bar\mu},E_e,\cos\psi_{\bar\mu e})[1+A(E_{\bar\mu},E_e,\cos\psi_{\bar\mu e})]$ in the Z^0 rest frame where $\psi_{\bar\mu e}$ is the opening angle between the μ^+ and e^- momenta.

$$U(E_{\bar\mu},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[\xi_1 g(x)f(y)+\xi_2 f(x)g(y)]$$
$$\to\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6\xi[g(x)f(y)+f(x)g(y)] \tag{3.18}$$

so

$$A(E_{\bar\mu},E_e)=\alpha_H\frac{U(E_{\bar\mu},E_e)}{T(E_{\bar\mu},E_e)}$$
$$\to\xi\alpha_H\left[\frac{g(x)f(y)+f(x)g(y)}{f(x)f(y)+g(x)g(y)}\right]. \tag{3.19}$$

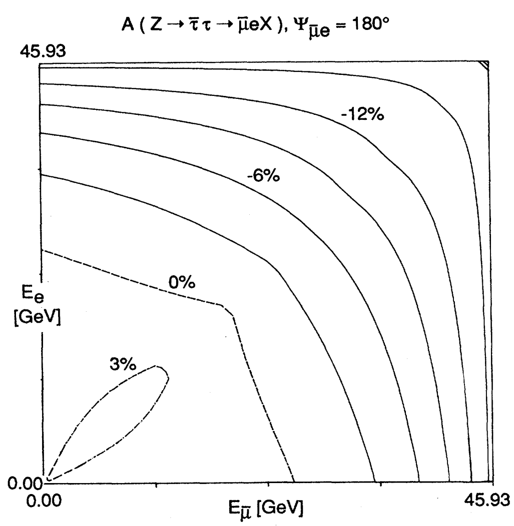

FIG. 4. For μ^+ and e^- back to back, the contour plot of the A term in the "standard decay correlation function" in the Z^0 rest frame. Note A is positive for $E_{\bar\mu}$ and $E_e<(\sim M/4)$ where M is the Z^0 mass

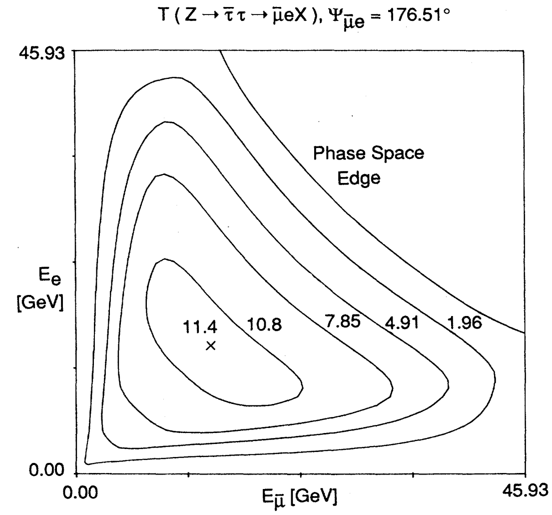

FIG. 5. The T term for $\psi_{\bar\mu e}=176.51°$ ($Z=\cos\psi_{\bar\mu e}=-0.998\,142$).

In these expressions, with $f(x)=a(x)+\rho b(x)$,

$$f(x)=1-3x^2+2x^3+\tfrac{2}{9}\rho(-1+9x^2-8x^3)$$
$$\to\tfrac{5}{6}-\tfrac{3}{2}x^2+\tfrac{2}{3}x^3 \tag{3.20}$$

and with $g(x)=c(x)+\delta\, d(x)$,

$$g(x)=\frac{1}{\beta}\{-\tfrac{1}{3}+2x-3x^2+\tfrac{4}{3}x^3-(1-\beta)x(1-x)^2$$
$$+\tfrac{2}{9}\delta[1-12x+27x^2-16x^3$$
$$+(1-\beta)6x(1-x)(1-2x)]\}$$
$$\to\frac{1}{\beta}[-\tfrac{1}{6}+\tfrac{3}{2}x^2-\tfrac{4}{3}x^3+(1-\beta)2x^2(x-1)]\,, \tag{3.21}$$

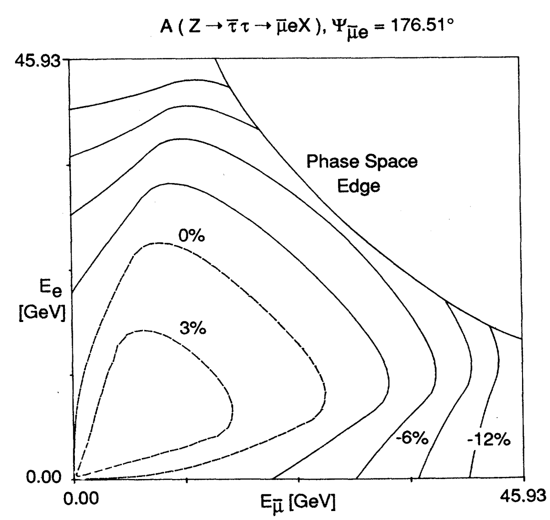

FIG. 6. The A term for $\psi_{\bar\mu e}=176.5°$. Note that as the opening angle $\psi_{\bar\mu e}$ decreases the A term remains positive in the region approximately bounded by $E_{\bar\mu}=E_e\simeq M/4$.

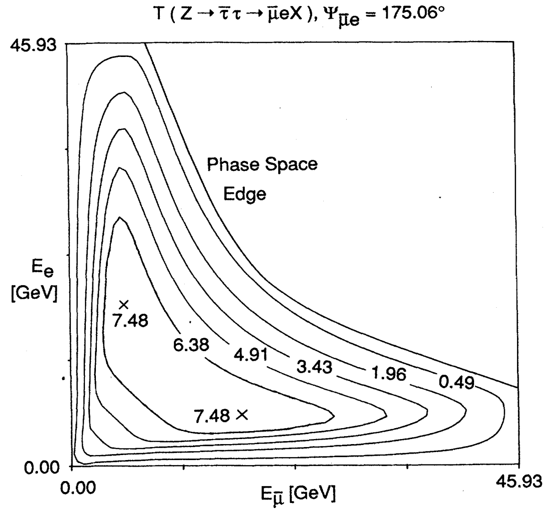

FIG. 7. The T term for $\psi_{\bar{\mu}3}=175.06°$ ($z=\cos\psi_{\bar{\mu}e}=-0.996\,283$).

where ρ and δ are the other Michel[33] parameters ($\rho=\delta=\frac{3}{4}$ for a $V\mp A$ charged-current coupling). The $(1-\beta)\simeq 0.000\,75$ terms in $g(x)$ could be dropped.

The second (arrowed) lines follow in Eqs. (3.17)–(3.21) assuming lepton universality and a $V\mp A$ charged-current coupling. The two functions, $T(E_{\bar{\mu}},E_3)$ and the "analyzing power" $A(E_{\bar{\mu}},E_e)$, are displayed in Figs. 1 and 2 for a $V-A$ coupling ($\xi=+1$) [for a $V+A$ coupling the sign of $A(E_{\bar{\mu}},E_e)$ is opposite]. Note that as the μ^+ or e^- approaches its maximum energy, $A(E_{\bar{\mu}},E_e)\rightarrow\alpha_H$.

In the Z^0 rest frame, the opening angle $\psi_{\bar{\mu}e}$ between the μ^+ and e^- can be measured so we have also analytically obtained the full distribution $I(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})$ for the sequential decay of Eq. (3.1), for massless final leptons μ^+ and e^-. The explicit formulas are listed in Appendix

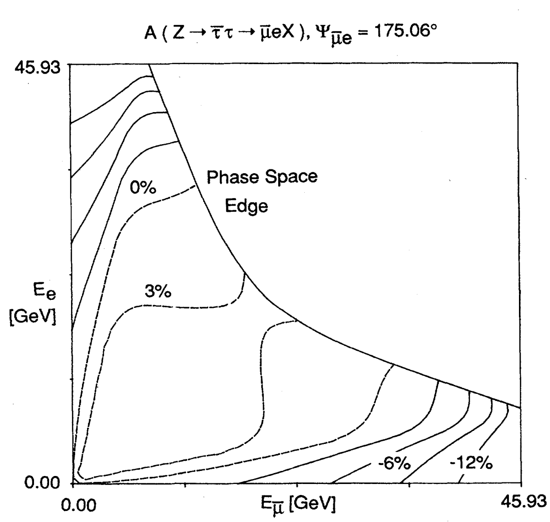

FIG. 8. The A term for $\psi_{\bar{\mu}e}=175.06°$.

A. For μ^+ and e^- back to back, Fig. 3 shows the contour plot of the T term in the standard decay correlation function

$$I(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})=T(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})[1+A(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})] \quad (3.22)$$

and Fig. 4 shows the A term which is proportional to the τ couplings' parameter $\xi\alpha_H$. Figures 5 and 6 show these distributions for $\psi_{\bar{\mu}e}=176.51°$, and Figs. 7 and 8 show them for $\psi_{\bar{\mu}e}=175.06°$. These figures show that as $\psi_{\bar{\mu}e}$ decreases, A remains positive in the region approximately bounded by $E_{\bar{\mu}}=E_e\simeq M/4$ so usage of the simpler $E_{\bar{\mu}}$-E_e energy-energy correlation function $I(E_{\bar{\mu}},E_e)$ of Eq. (3.15) instead of the full distribution $I(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})$ of Eq. (3.22), has insignificantly reduced the $\xi\alpha_H$ signature.

The deviation of the magnitude of ξ from 1 is a signature for other than a pure $V-A$ lepton-number-conserving, four-fermion coupling for describing $\tau^+\rightarrow\ell^{\pm}\nu\bar{\nu}$, i.e., for the presence of right-handed currents. This is discussed in Sec. II; $|\xi|\neq 1$ is predicted for an arbitrary mixture of V and A components, instead of either $V\mp A$. To determine ξ from $I(E_{\bar{\mu}},E_e)$, we write, from Eqs. (3.17) and (3.18),

$$T(E_{\bar{\mu}},E_e)=T_0(E_{\bar{\mu}},E_e)+\xi^2T_2(E_{\bar{\mu}},E_e) \quad (3.23)$$

so

$$I(E_{\bar{\mu}},E_e)=T_0(E_{\bar{\mu}},E_e)[1+\xi B(E_{\bar{\mu}},E_e)+\xi^2C(E_{\bar{\mu}},E_e)] \quad (3.24)$$

with

$$T_0(E_{\bar{\mu}},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6 f(x)f(y)\,, \quad (3.25)$$

$$\xi B(E_{\bar{\mu}},E_e)=\alpha_H\frac{U(E_{\bar{\mu}},E_e)}{T_0(E_{\bar{\mu}},E_e)}=\xi\alpha_H\frac{g(x)f(y)+f(x)g(y)}{f(x)f(y)}\,, \quad (3.26)$$

$$C(E_{\bar{\mu}},E_e)=\frac{T_2(E_{\bar{\mu}},E_e)}{T_0(E_{\bar{\mu}},E_e)}=\frac{g(x)g(y)}{f(x)f(y)}\,. \quad (3.27)$$

These three functions $T_0(E_{\bar{\mu}},E_e)$, $B(E_{\bar{\mu}},E_e)$, and $C(E_{\bar{\mu}},E_e)$ are discussed further and plotted in Sec. VIII.

IV. THE HARDER LEPTON'S ENERGY SPECTRUM $I(x_H)$

Since $I(E_{\bar{\mu}},E_e)$ is symmetric in $E_{\bar{\mu}}\leftrightarrow E_e$, events from SLC/LEP can be folded about the diagonal $E_{\bar{\mu}}=E_e$. Then to obtain a simple single variable distribution, the energy of the softer lepton can be integrated out. The resulting harder lepton's energy spectrum can be compared with the theoretical prediction obtained[1] from Eqs. (3.15), (3.17), and (3.18) that

$$I(x_H)=T(x_H)[1+A(x_H)],\quad x_H=E_H/E_{\max}\,, \quad (4.1)$$

where, for arbitrary Michel parameters ρ,δ,ξ,

$$T(x)=\frac{\pi}{2}E_{\max}^{7}\gamma^{-8}x(1-x)\{[x^3-2x^2+2+\tfrac{4}{9}\rho(-2x^3+3x^2-1)][-2x^2+x+1+\tfrac{2}{9}\rho(8x^2-x-1)]$$
$$+\tfrac{2}{9}\xi^2(1-x)^2[x-1+\tfrac{2}{3}\delta(-4x+1)][-4x^2+5x-1+\tfrac{2}{3}\delta(16x^2-11x+1)]\} \tag{4.2}$$

and

$$A(x_H)=\alpha_H U(x_H)/T(x_H) \tag{4.3}$$

with

$$U(x)=\xi\frac{\pi}{6}E_{\max}^{7}\gamma^{-8}x(1-x)\{2(1-x)^2[x-1+\tfrac{2}{3}\delta(-4x+1)][-2x^2+x+1+\tfrac{2}{9}\rho(8x^2-x-1)]$$
$$+[x^3-2x^2+2+\tfrac{4}{9}\rho(-2x^3+3x^2-1)][-4x^2+5x-1+\tfrac{2}{3}\delta(16x^2-11x+1)]\} \ . \tag{4.4}$$

Corrections of order $\gamma^{-2}\simeq 0.0015$ relative to the leading term have not been kept. These expressions are for $E_H \gg E_I \simeq 0.0173$ GeV; in the soft limit $E_H \to E_I$ the neglected γ^{-2} terms are important. These and the following expressions have factors of $(1-x)$ explicitly displayed to enable easy assessment of the hard limit $E_H \to E_{\max}$.

For $\rho=\delta=\frac{3}{4}$, as for an arbitrary V and A charged-current coupling, the remaining ξ-dependent distribution $I(x_H)$ is in terms of

$$T(x)|_{\rho=\delta=3/4}=\frac{\pi}{36}E_{\max}^{7}\gamma^{-8}x(1-x)[25+25x-35x^2-10x^3+17x^4-4x^5+\xi^2(1-x)^2(1+3x-6x^2-16x^3)] \tag{4.5}$$

and

$$U(x)|_{\rho=\delta=3/4}=\xi\frac{\pi}{36}E_{\max}^{7}\gamma^{-8}x(1-x)$$
$$\times(-10-10x+62x^2+7x^3-47x^4+16x^5) \ . \tag{4.6}$$

Plots of $T(x_H)$ and $A(x_H)$ for $\xi=1$ are given in Ref. 1. In Eq. (4.5) the ξ^2 term's coefficient vanishes at $x\simeq 0.42$ and at 1, and $U(x)$ of Eq. (4.6) vanishes at $x\simeq 0.52$, 0, and 1. The relative importance of the terms in $I(x)$ for determining their respective coefficients is indicated by the average of their absolute value: for α_H, $|U/T|_{\text{ave}}=0.0546$; with $T(x)\equiv T_0(x)+\xi^2 T_2(x)$, for ξ, $|U/T_0|_{\text{ave}}=0.0548$ and $|T_1/T_0|=0.0168$.

To display the δ dependence, we set $\rho=\frac{3}{4}$ and $\xi=+1$ in Eqs. (4.2) and (4.4) which gives

$$I(x_H)=\frac{\pi}{36}E_{\max}^{7}\gamma^{-8}(D+E\delta+F\delta^2) \ , \tag{4.7}$$

where

$$D=x(1-x)[29-7x+53x^2-122x^3+85x^4-20x^5$$
$$+2\alpha_H(1-x)(-10+25x+12x^2$$
$$-26x^3+8x^4)] \ ,$$
$$E=\tfrac{4}{3}x(1-x)[2(1-x)^3(-2+19x-32x^2)$$
$$+\alpha_H(10-80x+88x^2+83x^3$$
$$-115x^4+32x^5)] \ , \tag{4.8}$$
$$F=\tfrac{16}{9}x(1-x)^3(1-15x+60x^2-64x^3) \ .$$

The average of their absolute value is $\frac{3}{4}|E/D|_{\text{ave}}=0.0506$ and $\frac{9}{16}|F/D|_{\text{ave}}=0.005\,27$.

For determination of both ξ and δ their linear, not their quadratic term, is more important.

Finally, for completeness we set $\delta=\frac{3}{4}$ and $\xi=+1$ to display the ρ dependence:

$$I(x_H)=\frac{\pi}{36}E_{\max}^{7}\gamma^{-8}(J+L\rho+M\rho^2) \ , \tag{4.9}$$

where

$$J=x(1-x)[(1-x)(37+74x-45x^2-64x^3+52x^4)$$
$$+3\alpha_H(-4-4x+28x^2+3x^3$$
$$-33x^4+16x^5)] \ ,$$
$$L=\tfrac{4}{3}x(1-x)[-3(4+4x-28x^2-3x^3-33x^4-16x^5)$$
$$+2\alpha_H(1-x)^2(1+3x-6x^2-16x^3)] \ , \tag{4.10}$$
$$M=\tfrac{16}{9}x(1-x)^3(1+3x-6x^2-16x^3) \ .$$

Since as $x_H\to 1$, $J\to 18\alpha_H(1-x)$ with $L\to 24(1-x)$ and $M=0$, the L term dominates over the J term. Therefore, $I(x_H)$ is quite sensitive to ρ. The average of the absolute values $\frac{3}{4}|L/J|_{\text{ave}}=3.8$ and $\frac{9}{16}|M/J|_{\text{ave}}=0.026$.

V. LEPTON-ENERGY–HADRON-ENERGY CORRELATION FUNCTION $I(E_\ell E_A)$

In this and the following section we incorporate measured two-body τ decays $\tau^+\to h_A^+\bar{\nu}$, $\tau^-\to h_B^-\nu$, into our energy-energy correlation analysis of $Z^0\to\tau^+\tau^-$ sequential decays. In the Z^0 rest frame, the angle between such a charged hadron's momentum and its associated τ's momentum is determined by measurement of the charged hadron's energy E_A. Consequently,[3] E_A and $\cos\theta_A^\tau$ are equivalent sequential decay variables where θ_A^τ, $0\le\theta_A^\tau\le\pi$, is the helicity polar angle of the charged hadron in its τ's rest frame. This linear relationship is

$$\cos\theta_A^\tau = \frac{4E_A m_\tau^2 - M(m_\tau^2-\mu_A^2)}{(M_\tau^2-\mu_A^2)(M^2-4m_\tau^2)^{1/2}}, \quad 0 \le \theta_A^\tau \le \pi$$
$$= [2(E_A/E_A^{\max})-1]\left[1+O\left[\left[\frac{m_\tau}{M}\right]^2\right]\right]. \quad (5.1)$$

We find it simpler to use $\cos\theta_A^\tau$ instead of E_A in our analysis and in the accompanying figures.

For the sequential decay

$$\begin{array}{l} Z^0 \to \tau_1^+ \tau_2^- , \\ \quad\quad\quad \hookrightarrow h_B^- \nu_2 , \\ \quad\quad \hookrightarrow \mu_1^+ \nu_1 \bar{\nu}_1 \end{array} \quad (5.2)$$

the standard decay correlation function is

$$I(\theta_1^\tau, E_1^\tau; \theta_B^\tau; \phi) = C(\theta_1^\tau, E_1^\tau; \theta_B^\tau) + A_0(\theta_1^\tau, E_1^\tau; \theta_B^\tau)\cos\phi , \quad (5.3)$$

where θ_1^τ, E_1^τ are as in Eq. (3.5) and ϕ is the azimuthal angle between the τ^+ μ^+ momenta plane and the $\tau^- h_B^-$ momenta plane in the Z^0 rest frame. In Eq. (5.3),

$$A_0 = \sigma\gamma_{CP}\xi_1 S(E_1^\tau)\xi_B \mathcal{S}_B \sin\theta_1^\tau \sin\theta_B^\tau \quad (5.4)$$

where ξ_B was defined in the Introduction in Eqs. (1.2) and (1.6), and

$$C = \sigma S(E_1^\tau, \theta_1^\tau; \theta_B^\tau) + \tau T(E_1^\tau, \theta_1^\tau; \theta_B^\tau) + \upsilon U(E_1^\tau, \theta_1^\tau; \theta_B^\tau) \quad (5.5)$$

with

$$S(E_1^\tau, \theta_1^\tau; \theta_B^\tau) = R(E_1^\tau) + \xi_1 S(E_1^\tau)\xi_B \mathcal{S}_B \cos\theta_1^\tau \cos\theta_B^\tau ,$$
$$T(E_1^\tau, \theta_1^\tau; \theta_B^\tau) = R(E_1^\tau) - \xi_1 S(E_1^\tau)\xi_B \mathcal{S}_B \cos\theta_1^\tau \cos\theta_B^\tau , \quad (5.6)$$
$$U(E_1^\tau, \theta_1^\tau; \theta_B^\tau) = \xi_1 S(E_1^\tau)\cos\theta_1^\tau - R(E_1^\tau)\xi_B \mathcal{S}_B \cos\theta_B^\tau ,$$

where, as in Sec. III, R and S describe $t^+ \to \mu^+ \nu\bar{\nu}$ and the σ, τ, υ coefficients are quadratic functions of the four helicity amplitudes for $Z^0 \to \tau^+\tau^-$. The normalized parameter $\mathcal{S}_B$ partially characterizes the decay density matrix for $\tau^- \to h_B^- \nu$ with[34]

$$\mathcal{S}_B = \begin{cases} 1 \ \text{for } B=\pi, K, \\ \dfrac{m_\tau^2 - 2m_B^2}{m_\tau^2 + 2m_B^2} \ \text{for } B = \rho, K^*, a_1. \end{cases} \quad (5.7)$$

The A_0 term will not contribute to the muon-energy–hadron-energy correlation function $I(E_{\bar{\mu}}, \cos\theta_B^\tau)$.

Proceeding as in Sec. III, at the tree level,

$$I(\theta_1^\tau, E_1^\tau; \theta_B^\tau; \phi) \simeq \tau T(E_1^\tau, \theta_1^\tau; \theta_B^\tau) \times \left[1 + \alpha_H \frac{U(E_1^\tau, \theta_1^\tau; \theta_B^\tau)}{T(E_1^\tau, \theta_1^\tau; \theta_B^\tau)}\right] \quad (5.8)$$

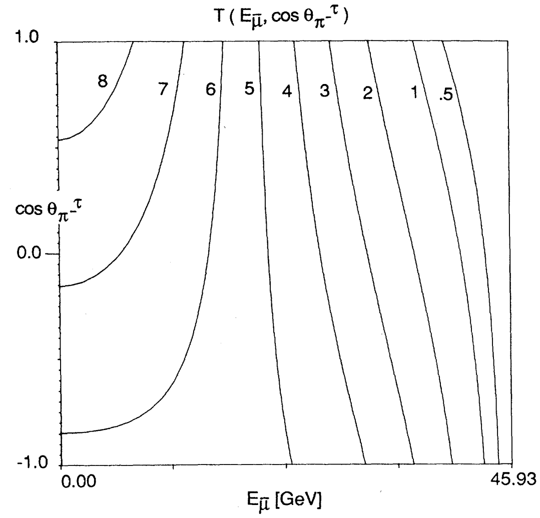

FIG. 9. The contour plot of the $T(E_{\bar{\mu}}, \cos\theta_\pi^\tau)$ factor in the muon-energy–pion-energy correlation function $I(E_{\bar{\mu}}, \cos\theta_\pi^\tau) = T(E_{\bar{\mu}}, \cos\theta_\pi^\tau)[1 + A(E_{\bar{\mu}}, \cos\theta_\pi^\tau)]$ in the Z^0 rest frame for $Z^0 \to \tau^+\tau^-$ with the $\tau^+ \to \mu^+ \nu_\mu \bar{\nu}_\tau$, $\tau^- \to \pi^- \nu_\tau$. The pion energy E_π in the Z^0 rest frame by Eq. (5.1) is linearly related to $\cos\theta_\pi^\tau$ where θ_π^τ is the polar angle of the π^- momentum in the π^- rest frame in the usual helicity coordinate system. In general, E_A and $\cos\theta_A^\tau$ are equivalent sequential decay variables for two-body τ decays. Note that $\cos\theta_A^\tau = 1$ (-1) correspond, respectively, to E_A maximum (minimum).

and again there is no dependence of the right-hand side on the azimuthal angle ϕ. The associated $E_{\bar{\mu}}$-E_B correlation function is

$$I(E_{\bar{\mu}}, \cos\theta_B^\tau) = T(E_{\bar{\mu}}, \cos\theta_B^\tau)[1 + A(E_{\bar{\mu}}, \cos\theta_B^\tau)] , \quad (5.9)$$

where, for E_μ greater than $E_I = m_\tau/[2\gamma(1+\beta)] \simeq 0.0173$ GeV,

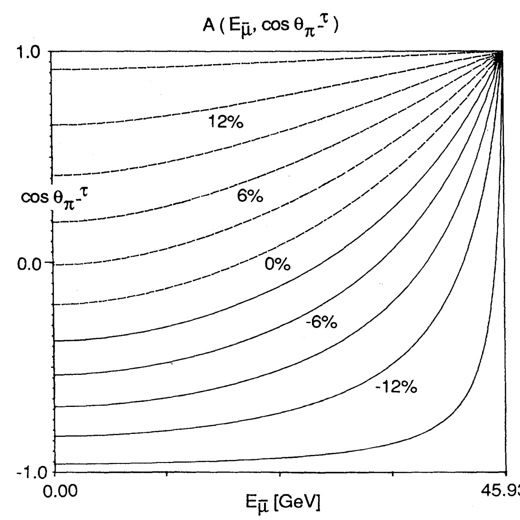

FIG. 10. The contour plot of the $A(E_{\bar{\mu}}, \cos\theta_\pi^\tau)$ term in the $E_{\bar{\mu}}$ and E_π correlation function. In the following figures for the A term at the tree level the analyzing power A is proportional to the α_H parameter describing $Z^0 \to \tau^+\tau^-$.

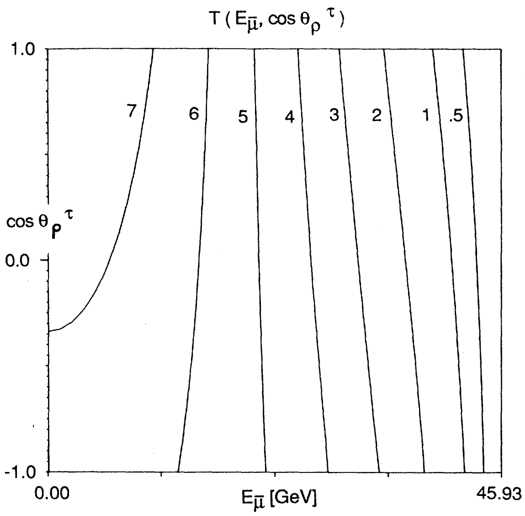

FIG. 11. The $T(E_{\bar{\mu}}, \cos\theta_\rho^\tau)$ term in the $E_{\bar{\mu}}$ and E_ρ correlation function.

$$T(E_{\bar{\mu}}, \cos\theta_B^\tau) = \frac{\pi}{8}\gamma^{-1}\beta^{-1}m_\tau^3[f(x) - \xi g(x)\xi_B \mathcal{S}_B \cos\theta_B^\tau] \ , \tag{5.10}$$

$$U(E_{\bar{\mu}}, \cos\theta_B^\tau) = \frac{\pi}{8}\gamma^{-1}\beta^{-1}E_\tau^3[\xi g(x) - f(x)\xi_B \mathcal{S}_B \cos\theta_B^\tau] \ ,$$

with

$$A(E_{\bar{\mu}}, \cos\theta_B^\tau) = \alpha_H \frac{U(E_{\bar{\mu}}, \cos_\beta^\tau)}{T(E_{\bar{\mu}} \cos_\beta^\tau)} \ ,$$

where $f(x)$ and $g(x)$ are given in Eqs. (3.20) and (3.21).

For the $\mu^+\pi^-$ mode the two functions, $T(E_{\bar{\mu}}, \cos\theta_\pi^\tau)$ and the "analyzing power" $A(E_{\bar{\mu}}, \cos\theta_\pi^\tau)$, are displayed in Figs. 9 and 10 for a $V-A$ coupling ($\xi = +1$) for $\tau^+ \to \mu^+ \nu\bar{\nu}$. Note that as the μ^+ approaches its maximum energy or as the π^- approaches its minimum energy (so $\cos\theta_\pi^\tau \to -1$), $A(E_{\bar{\mu}}, \cos\theta_\pi^\tau) \to \alpha_H$, and that as the π^- approaches its maximum energy, $A(E_{\bar{\mu}}, \cos\theta_\pi^\tau) \to -\alpha_H$.

Figures 11 and 12 show these distributions for the $\mu^+\rho^-$ mode for a $V-A$ coupling for $\tau^+ \to \mu^+\nu\bar{\nu}$. As the μ^+ approaches its maximum energy, $A(E_{\bar{\mu}} \cos\theta_\rho^\tau) \to \alpha_H$. Since the K^* and ρ masses are almost degenerate, the distributions for the μ^+K^{*-} mode are similar to those for the $\mu^+\rho^-$. For the $\mu^+a_1^-$ mode, $\mathcal{S}_{a_1} \simeq 0.0$ so $A(E_{\bar{\mu}}, \cos\theta_{a1}^\tau) \simeq \xi\alpha_H g(x)/f(x)$ is approximately independent of the a_1^- energy.

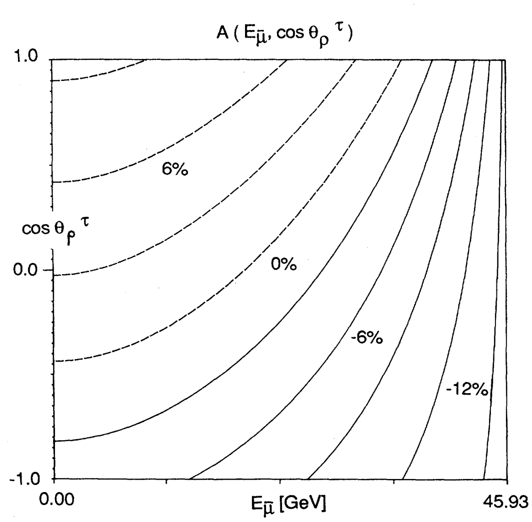

FIG. 12. The analyzing power $A(E_{\bar{\mu}}, \cos\theta_\rho^\tau)$ in the $E_{\bar{\mu}}$ and E_ρ correlation function.

VI. HADRONIC-ENERGY CORRELATION FUNCTION $I(E_A, E_B)$

For the decay sequence

$$Z^0 \to \tau_1^+ \tau_2^- \ , \quad \tau_2^- \to h_B^- \nu_2 \ , \quad \tau_1^+ \to h_A^+ \bar{\nu}_1 \tag{6.1}$$

the standard decay correlation function assuming CP invariance is

$$I(\theta_1^\tau; \theta_2^\tau; \phi) = C(\theta_1^\tau; \theta_2^\tau) + A_0(\theta_1^\tau; \theta_2^\tau)\cos\phi \ , \tag{6.2}$$

where in the Z^0 rest frame ϕ is the azimuthal angle between the $\tau^+h_A^+$ momenta plane and the $\tau_B^- h_B^-$ momenta plane.

The hadronic helicity polar angles θ_1^τ and θ_2^τ in the τ rest frames are, respectively, related to the hadronic energies E_A and E_A as in Eq. (5.1). In Eq. (6.2),

$$A_0 = -\sigma\gamma_{CP}\xi_A \mathcal{S}_A \xi_B \mathcal{S}_B \sin\theta_1^\tau \sin\theta_2^\tau \tag{6.3}$$

and

$$C = \sigma S(\theta_1^\tau; \theta_2^\tau) + \tau T(\theta_1^\tau; \theta_2^\tau) + v U(\theta_1^\tau; \theta_2^\tau) \ , \tag{6.4}$$

where

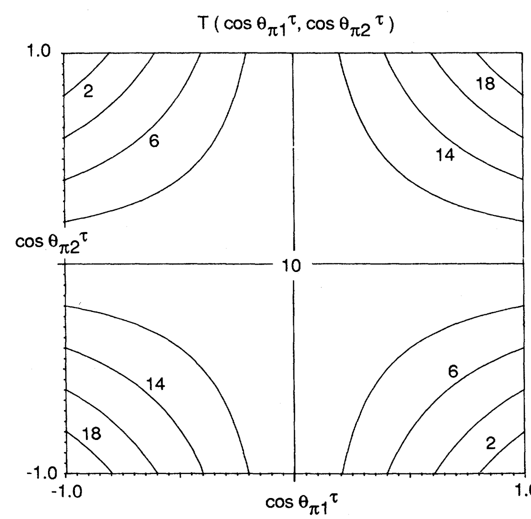

FIG. 13. The $T(\cos\theta_{\pi 1}^\tau, \cos\theta_{\pi 2}^\tau)$ term in the $E_{\pi 1}$ and $E_{\pi 2}$ correlation function. By CP invariance I is symmetric in $E_{\pi 1} \leftrightarrow E_{\pi 2}$.

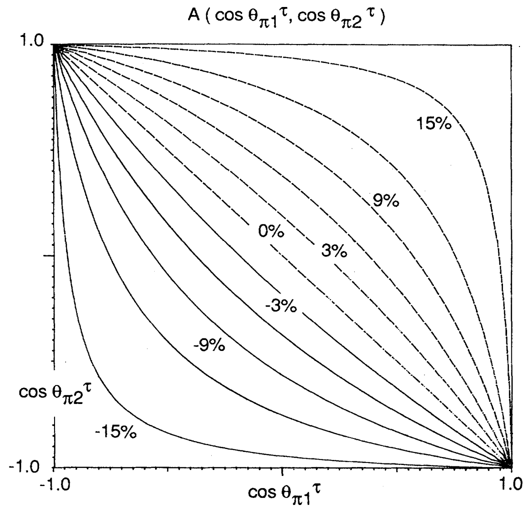

FIG. 14. The analyzing power $A(\cos\theta^\tau_{\pi 1},\cos\theta^\tau_{\pi 2})$ in the $E_{\pi 1}$ and $E_{\pi 2}$ correlation function.

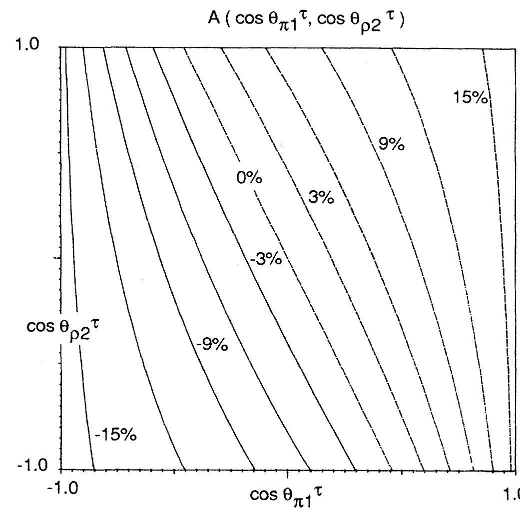

FIG. 16. The analyzing power A in the E_π and E_ρ correlation function.

$$S(\theta^\tau_1;\theta^\tau_2)=1-\xi_A\mathcal{S}_A\xi_B\mathcal{S}_B\cos\theta^\tau_1\cos\theta^\tau_2\ ,$$

$$T(\theta^\tau_1;\theta^\tau_2)=1+\xi_A\mathcal{S}_A\xi_B\mathcal{S}_B\cos\theta^\tau_1\cos\theta^\tau_2\ , \qquad (6.5)$$

$$U(\theta^\tau_1;\theta^\tau_2)=-\xi_A\mathcal{S}_A\cos\theta^\tau_1-\xi_B\mathcal{S}_B\cos\theta^\tau_2$$

in the same notation as previously. As before, at the tree level neglecting the σ terms,

$$I(\theta^\tau_1;\theta^\tau_2;\phi)\simeq\tau T(\theta^\tau_1;\theta^\tau_2)[1+A(\theta^\tau_1;\theta^\tau_2)] \qquad (6.6)$$

with

$$A(\theta^\tau_1;\theta^\tau_2)=\alpha_H\frac{U(\theta^\tau_1;\theta^\tau_2)}{T(\theta^\tau_1;\theta^\tau_2)} \qquad (6.7)$$

and Eq. (6.6) is the desired hadronic-energy correlation function $I(E_AE_B)$. Note that A of Eq. (6.3) depends only on $\sin\theta^\tau_{1,2}$, whereas C of Eq. (6.4) and the following defined functions depend only on $\cos\theta^\tau_{1,2}$. In discussing specific hadronic sequential decay modes we often will explicitly write $T(\cos\theta^\tau_1,\cos\theta^\tau_2)$, etc.

For the $\pi^+\pi^-$ mode, Figs. 13 and 14 show $T(\cos\theta^\tau_{\pi 1},\cos\theta^\tau_{\pi 2})$ and the analyzing power $A(\cos\theta^\tau_{\pi 1},\cos\theta^\tau_{\pi 2})$. As either pion approaches its maximum (minimum) energy in the Z^0 rest frame, $A(\cos\theta^\tau_{\pi 1},\cos\theta^\tau_{\pi 2})\to\mp\alpha_H$. Figures 15 and 16 show the distributions for the $\pi^+\rho^-$ mode. When the pion energy is maximum (minimum), $A(\cos\theta^\tau_\pi,\cos\theta^\tau_\rho)\to\mp\alpha_H$. Figures 17 and 18 for the $\rho^+\rho^-$ mode show the effects of the $\mathcal{S}_\rho$ factors in T and A. The distributions involving K^*'s are similar to those for the ρ's. For h^+_A, the a^+_1 meson, since $\mathcal{S}_{a1}\simeq 0.0$,

$$I(\theta^\tau_1;\theta^\tau_2)\simeq\tau(1-\alpha_H\xi_B\mathcal{S}_B\cos\theta^\tau_2)\ . \qquad (6.8)$$

Obviously, these hadronic-energy correlation functions are extremely simple.

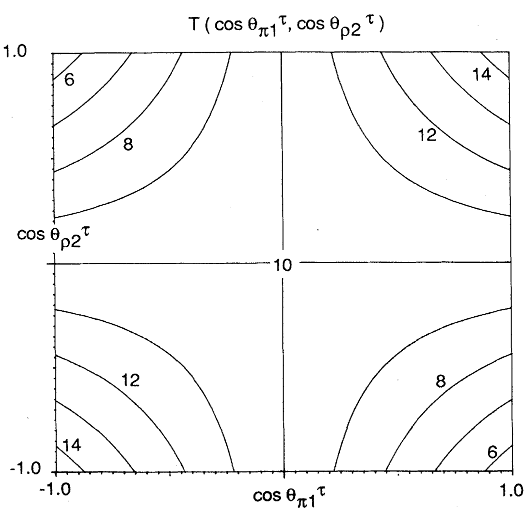

FIG. 15. The T term in the E_π and E_ρ correlation function.

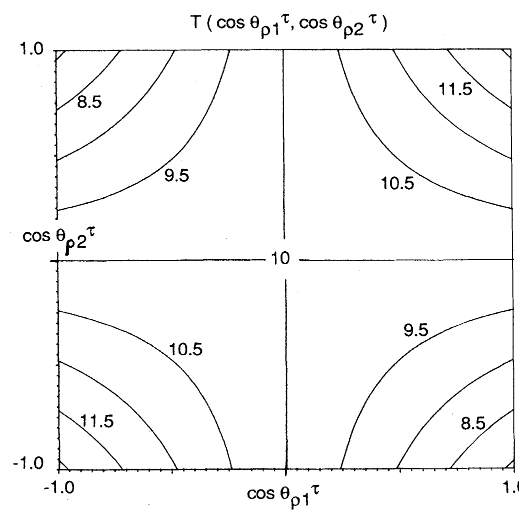

FIG. 17. The T term in the $E_{\rho 1}$ and $E_{\rho 2}$ correlation function. By CP invariance, I is symmetric in $E_{\rho 1}\leftrightarrow E_{\rho 2}$.

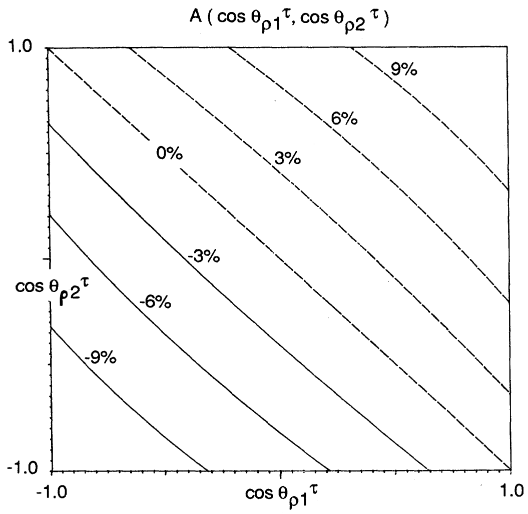

FIG. 18. The analyzing power A in the $E_{\rho 1}$ and $E_{\rho 2}$ correlation function.

VII. IDEAL STATISTICAL ERRORS FOR A 10^7-Z^0-EVENT SAMPLE

We consider a 10^7-Z^0-event sample and assure a $Z^0 \to \tau^+\tau^-$ branching ratio of 3.31%. For other choices any of the ideal statistical errors listed here can be rescaled by the relation

$$\sigma_{\text{other}} = \left[\frac{3.31 \times 10^5}{NB(Z^0 \to \tau^+\tau^-)} \right]^{1/2} \sigma_{\text{here}} . \tag{7.1}$$

We take all τ into one-charged particle branching ratios from the tabulation of Hayes and Perl,[35] except for the branching ratio for $\tau^- \to a_1^{\text{ch}-}\nu$ with $a_1^{\text{ch}-} \to \pi^-\pi^+\pi^-$ which is the "formal average" listed by Gan and Perl in Ref. 36. For other choices of branching ratios, a simple rescaling of the σ's in the tables can be easily performed.

For clarity, we explained how the σ's given in the tables were determined: Given N_{AB} events, we want to know the "ideal statistical error" σ_a for a least-squares-fit measurement of the parameter a from the energy-energy correlation function for the sequential decay

$$\begin{array}{l} Z^0 \to \tau_1^+ \tau_2^- \\ \quad\;\; | \quad \hookrightarrow B^- X_2 \\ \quad\;\; \hookrightarrow A^+ X_1 \end{array} \quad . \tag{7.2}$$

We distribute the N_{AB} events for this mode ideally over a two-dimensional ij grid according to the corresponding theoretical tree-level formula from earlier in this paper, for example, using

$$I(x,y) = Z_0(x,y) + aZ_1(x,y) \tag{7.3}$$

with the standard model's value for the parameter a. Then, the "ideal error" in bin ij is

$$_{ij} = \sqrt{I(x_i,y_j)}$$

and the "ideal statistical error" in the measurement of a is σ_a where[37]

$$\frac{1}{\sigma_a} = \left\{ \sum_{ij} \frac{1}{\sigma_{ij}^2} [Z_1(x_i,y_j)]^2 \right\}^{1/2} . \tag{7.4}$$

In the case of a parameter which appears quadratically, so

$$I(x,y) = Z_0(x,y) + aZ_1(x,y) + a^2 Z_2(x,y) , \tag{7.5}$$

the ideal statistical error is instead given by

$$\frac{1}{\sigma_a} = \left\{ \sum_{ij} \frac{1}{\sigma_{ij}^2} [Z_1(x_i,y_j)^2 + 4aZ_1(x_i,y_j)Z_2(x_i,y_j) + 4a^2 Z_2(x_i,y_j)^2] \right\}^{1/2} . \tag{7.6}$$

For the case of the single variable distribution $I(x_H)$ the summation in Eqs. (7.4) and (7.6) is only over i. Since the distribution giving σ_{ij} is assumed ideal, we take an arbitrarily large number of bins in the summation in Eq. (7.4) and (7.6).

To sum over modes, where σ_A is the error for the Ath mode, we use $\sigma = [\sum_A (1/\sigma_A^2)]^{-1/2}$.

These procedures, though simple and clear, are ideal.[38] In assuming an ideal distribution of the N_{AB} events according to Eq. (7.3) or (7.5) instead of, for example, that generated by a Monte Carlo program, we do not incorporate the statistical error from a presumably Poisson distribution of data in each ij bin. Second, to reduce the errors on the parameter of interest, for instance, α_H, we must sum over several modes. Before doing such a formal average, the Particle Data Group's method[30] is to first combine the systematic and statistical errors in quadrature. Such improvements are indeed desirable to make and, of course, will have to be incorporated once actual data is available.

In Table II are tabulated the ideal statistical errors for measurements of the fundamental parameters α_H, and ξ, δ, and ρ by the energy-energy correlation functions for the decay sequence $Z^0 \to \tau^+\tau^-$ with $\tau^+ \to A^+X$ and $\tau^- \to B^-X$. These were computed using the tree-level formulas of the preceding sections. Because of the sequential factorization property[39,1] of such sequential decay distributions, we expect corrections only of 0.1% in size due to electroweak radiative corrections[1] in $SU(2) \times U(1)$.

These radiative corrections must be included to precisely measure these fundamental parameters. The magnitudes of the ideal statistical errors should only be changed in the third significant figure, and so Table II can be used to assess the importance of measurements of α_H and of ξ, δ, and ρ by the different sequential decay modes, whether separately, or in combination.

In the table, $a_1^{\text{ch}-}$ denotes the expectation for $\tau^- \to \pi^-\pi^+\pi^-\nu$ assuming that it is dominated by the a_1 resonance, as expected. There are significant differences between τ decay experiments which measure this mode and in the branching ratio they obtain. We refer the reader to Refs. 36 and 40. The a_1 is a very broad resonance. The tree-level energy-energy correlation functions in this paper do not include the effects due to a finite resonance width which must be included for both the ρ reso-

TABLE II. Ideal statistical errors for measurements of the fundamental parameters α_H, ξ, δ, and ρ by the energy-energy correlation functions for the sequential decay $Z^0 \to \tau^+\tau^-$ with $\tau^+ \to A^+X$ and $\tau^- \to B^-X$. Double underlined entries denote the smallest ideal statistical error for a single decay mode. Square brackets denote percentage errors.

Sequential decay mode	Number of events	Ideal statistical error $\sigma(\alpha_H)$	$\sigma(\xi)$	$\sigma(\delta)$	$\sigma(\rho)$
μ^+e^-	41 246	0.0155	0.0428	0.0486	$\underline{\underline{0.0116}}$
$\mu^+(\pi,K)^-$	26 874	$\underline{\underline{0.0101}}$	$\underline{\underline{0.0391}}$	$\underline{\underline{0.0374}}$	0.0186
$\mu^+\rho^-$	52 579	0.0128	0.0593	0.0585	0.0141
μ^+K^{*-}	3 272	0.0595	0.2969	0.2937	0.0568
$\mu^+a_1^{\mathrm{ch}-}$	15 657	0.0348	0.2188	0.2182	0.0261
$(\pi,K)^+(\pi,K)^-$	4 377	0.0205			
$(\pi,K)^+\rho^-$	17 129	0.0124			
$(\pi,K)^+K^{*-}$	1 066	0.0511			
$(\pi,K)^+a_1^{\mathrm{ch}-}$	5 101	0.0241			
$\rho^+\rho^-$	16 757	0.0213			
ρ^+K^{*-}	2 085	0.0684			
$\rho^+a_1^{\mathrm{ch}-}$	9 980	0.0379			
$K^{*+}K^{*-}$	65	0.4634			
$K^{*+}a_1^{*-}$	621	0.2083			
Sum of above modes	196 808	0.00532 [3.34%]	0.0257 [2.57%]	0.0262 [3.49%]	0.00764 [1.02%]
Sum of modes without a_1^{ch}	165 450	0.00558	0.0259	0.0263	0.00799

nance and the a_1, as has been done, e.g., in analysis of polarization effects in top quark decays into a real W boson. Since the parameter $\mathcal{S}_{a1} = -0.0105$ almost vanishes, the $t \to Wb$ analysis indicates that the finite-width effect here is at the few percent level.

We emphasize that the present $a_1^{\mathrm{ch}-}$ results using an energy-energy correlation function, assume a two-body τ decay, and therefore are not directly applicable for that part of the $\pi^-\pi^+\pi^-\nu$ channel due to an n-body, $n > 2$, decay. Conversely, as we have emphasized before, sequential decay correlation functions can be powerfully used to test resonance dominance hypotheses (cf. issues raised in Ref. 41), and to eliminate possible background effects.

All sequential decay modes with one-charged-particle τ decays have been tabulated in Table II except for $a_1^{\mathrm{ch}+}a_1^{\mathrm{ch}-}$ which for 1486 events has $\sigma(\alpha_H) = 3.01$. A listing labeled μ^+e^- has, of course, summed over[42] the combinations μ^+e^-, $e^+\mu^-$, $\mu^+\mu^-$, and e^+e^-. The separate contributions for the pseudoscalar modes can be separately obtained by the reader since the branching ratio for $\tau^- \to \pi^-\nu$ is $(10.8 \pm 0.6)\%$ and for $\tau^- \to K^-\nu$ is $(0.7 \pm 0.2)\%$.

In Table III the ideal statistical errors for measurements by the harder lepton's energy spectrum are compared with those for measurements from energy-energy correlation functions for the decay sequence $Z^0 \to \tau^+\tau^-$ with $\tau^+ \to A^+X$ and $\tau^- \to B^-X$. For the determination of α_H and ρ there is, respectively, only a 4% and 3% improvement. However, for both ξ and δ the percentage of improvement is over 30%, in which case it should be worthwhile to either directly analyze the energy-energy correlation function $I(E_{\bar{\mu}}, E_e)$, or else some other one-dimensional distribution which contains the same degree of sensitivity to ξ and δ as does the energy-energy correlation $I(E_{\bar{\mu}}, E_e)$. The necessary starting formulas for

TABLE III. Comparison of standard-model values (Ref. 43) for the $Z^0 \to \tau^+\tau^-$ coupling parameter α_H and for the Michel parameters describing $\tau^+ \to \mu^+\nu\bar{\nu}$ with "ideal statistical errors" for measurements (Ref. 44) by the harder lepton's energy spectrum $I(x_H)$ and by the energy-energy correlation functions for the sequential decay $Z^0 \to \tau^+\tau^-$ with $\tau^+ \to A^+X$ and $\tau^- \to B^-X$.

Quantity	Standard-model value	Ideal statistical error From $I_{\bar{\mu}e}(x)$	From $I(E_{\bar{\mu}}, E_e)$	Percent better	From "Sum of Modes" of Table II	Factor better	Measurement
α_H	-0.1591	0.0161	0.0155	4%	0.0053	3.0	
ξ	1.0	0.0632	0.0428	32%	0.026	2.4	
δ	0.75	0.0734	0.0486	34%	0.026	2.8	
ρ	0.75	0.0119	0.0116	3%	0.0076	1.6	0.73 ± 0.07[a]

[a]DELCO Collaboration, W. Bacin *et al.*, Phys. Rev. Lett. **42**, 749 (1979); CLEO Collaboration, S. Behrends *et al.*, Phys. Rev. D **32**, 2468 (1985); MAC Collaboration, W. T. Ford *et al.*, *ibid.* **36**, 1971 (1987).

studying possible projections and/or projections of moments of $I(E_{\bar{\mu}},E_e)$ and of other energy-energy correlations have been explicitly listed in the preceding sections of this paper.

Along with the ideal statistical error for the case that there is a sum over modes as in Table II, we have listed the "factor of improvement" versus usage of $I(x_H)$. From increased statistics alone, an improvement factor of 2 would be expected so the larger factor indicates that for α_H the "analyzing power" is greater for modes other than μ^+e^-. This more than statistics improvement is also clear by the entries for $\sigma(\alpha_H)$ in Table II for the $\mu^+(\pi K)^-$, $\mu^+\rho^-$, $(\pi,K)^+\rho^-$, and $(\pi,K)^+(\pi K)^-$ modes. [For determination of ξ and δ it is more appropriate to compare the result of using a sum over modes with the ideal statistical error for a measurement by $I(E_{\bar{\mu}},E_e)$ instead of by $I(x_H)$. Then the respective improvement factor is 1.65 for ξ and 1.87 for δ.]

From the entries in Table II, the "relative sequential decay analyzing power" of the various modes for α_H, ξ, δ, and ρ can be easily worked out. These strengths would be useful in consideration of other applications of sequential decay methods, but in the present context since the relative total number of events in the ABth mode cannot be adjusted, the entries listed in Table II for the "ideal statistical errors" is what is relevant.

From Table II we see that the $\mu^+(\pi,K)^-$ mode has the smallest ideal statistical errors for α_H, and ξ and δ. The μ^+e^- mode has the smallest $\sigma(\rho)$.

VIII. DETERMINATION OF α_H: COMPARISON OF $I(E_A,E_B)$ WITH τ-POLARIZATION TECHNIQUE

The τ-polarization technique for determination of α_H uses a single τ decay's energy distribution $I(E_A)$ in the Z^0 rest frame in order to measure $\xi_A\alpha_H$. For instance, for $\tau^+\to\mu^+\nu\bar{\nu}$, using Eq. (5.9),

$$I(E_{\bar{\mu}})=\int_{-1}^{1}d(\cos\theta_B^\tau)I(E_{\bar{\mu}},\cos\theta_B^\tau)\ , \tag{8.1}$$

or Eq. (3.15),

$$I(E_{\bar{\mu}})=\int_{E_I}^{E_F}dE_e I(E_{\bar{\mu}},E_e)\ , \tag{8.2}$$

we obtain

$$I(E_{\bar{\mu}})=f(x)+\xi\alpha_H g(x), x=E_{\bar{\mu}}/E_{\max}\ . \tag{8.3}$$

Similarly, for $\tau^-\to B^-\nu$, from Eq. (5.9),

$$I(E_B)=\int_{E_I}^{E_f}dE_{\bar{\mu}}I(E_{\bar{\mu}},\cos\theta_B^\tau) \tag{8.4}$$

or, from Eq. (6.6),

$$I(E_B)=\int_{-1}^{1}d(\cos\beta_A^\tau)I(\cos\theta_A^\tau,\cos\theta_B^\tau) \tag{8.5}$$

the energy distribution of particle B is

$$I(E_B)=1-\xi_B\alpha_H\mathcal{S}_B\cos\theta_B^\tau \tag{8.6}$$

TABLE IV. Comparison of ideal statistical errors for α_H from measurements by the energy-energy correlation functions $I(E_A,E_B)$ with measurements by the single-particle energy distribution $I(E_i)=I(\cos\theta_i^\tau)$, that is by the τ-polarization method.

Sequential decay mode	Ideal statistical error $\sigma(\alpha_H)$ From $I(E_A,E_B)$	From $I(E_i)=I(\cos\theta_i^\tau)$ i=better, $\sigma(\alpha_H)$		j=worse, $\sigma(\alpha_H)$	
μ^+e^-	0.0155		0.0215		Same
$\mu^+(\pi,K)^-$	0.0101	$(\pi K)^-$	0.0105	μ^+	0.0266
$\mu^+\rho^-$	0.0128	ρ^-	0.0165	μ^+	0.0190
μ^+K^{*-}	0.0595	μ^+	0.0762	K^{*-}	0.0908
$\mu^+a_1^{ch-}$	0.0348	μ^+	0.0348	a_1^{ch-}	1.3122
$(\pi,K)^+(\pi,K)^-$	0.0205		0.0260		Same
$(\pi,K)^+\rho^-$	0.0124	$(\pi,K)^+$	0.0131	ρ^-	0.0289
$(\pi,K)^+K^{*-}$	0.0511	$(\pi,K)^+$	0.0527	K^{*-}	0.1590
$(\pi,K)^+a_1^{ch-}$	0.0241	$(\pi,K)^+$	0.0241	a_1^{ch-}	2.2990
$\rho^+\rho^-$	0.0213		0.0292		Same
ρ^+K^{*-}	0.0684	ρ^+	0.0828	K^{*-}	0.1137
$\rho^+a_1^{ch-}$	0.0379	ρ^+	0.0379	a_1^{ch-}	1.6436
$K^{*-}K^{*-}$	0.4634		0.6446		Same
$K^{*+}a_1^{ch-}$	0.2083	K^{*+}	0.2084	a_1^{ch-}	6.5891
Sum of above modes	0.00532		0.00605 Factor worse =1.4		
Sum of only (πK) modes			0.00736 Factor worse =1.38		

[E_B and $\cos\theta_B^\tau$ are equivalent, see Eq. (5.1)].

Without real data one does not know whether or not it is realistic to assume that the Michel parameter $\xi=1$ for $\tau^\pm \rightarrow \ell^\pm \nu\bar{\nu}$ and/or that the chiral polarization parameters $\xi_\pi, \xi_k, \xi_\rho, \xi_{K^*}$ are all equal to one. However, our objective in this section is to compare the determination of α_H from the energy correlation function $I(E_A, E_B)$ with its determination from the τ-polarization technique which uses $I(E_{\bar{\mu}})$ or $I(E_B)$, so we will assume all the ξ's are one.

In Table IV the ideal statistical errors for α_H from measurements by $I(E_A, E_B)$ and by $I(E_i)$ are compared for the $Z^0 \rightarrow \tau^+\tau^- \rightarrow (A^+X')$ sequential decay modes considered in this paper. This table shows that there is 14% to 38% decrease for the ideal statistical error $\sigma(\alpha_H)$ in not using the energy correlation function $I(E_A, E_B)$. Discussions in the literature of the τ-polarization technique have emphasized the π energy distribution. From comparison with Table I, we conclude that is is very important to analyze how much direct application of the $I(E_A, E_B)$ technique in τ pair Monte Carlo simulations will reduce systematic errors such as due to the $\rho^\pm \rightarrow \pi^\pm\pi^0$ background to $\tau^\pm \rightarrow \pi^\pm \nu$. Can background from Bhabha scattering and $e^+e^- \rightarrow \mu^+\mu^-$ with large initial- and final-state QED radiation be controlled so that the $Z^0 \rightarrow \tau^+\tau^- \rightarrow (e^+\nu\bar{\nu})(e^-\nu\bar{\nu})$ or $(\mu^-\nu\bar{\nu})(\mu^-\nu\bar{\nu})$ modes be used to measure α_H? Some combination of both techniques will probably give the largest statistics and most reliable measurements.[45]

IX. DETERMINATION OF ξ, δ, AND ρ: COMPARISON OF $I(E_{\bar{\mu}}, E_B)$ WITH τ-POLARIZATION TECHNIQUE

The Michel parameters ξ, δ, and ρ can be measured by both techniques from the sequential decay modes $Z^0 \rightarrow (\tau^+\tau^-) \rightarrow (\ell^+\nu\bar{\nu})(B^-\nu)$ and $(A^+X)(\ell^-\nu\bar{\nu})$. This occurs for in Eq. (3.20), $f(x)$ depends on ρ and in Eq. (3.21), $g(x)$ depends on δ. The polarization parameter ξ appears in $I(E_{\bar{\mu}}, E_B)$ in Eqs. (3.17), (3.18), and (5.10), and in $I(E_{\bar{\mu}})$ in Eq. (8.3).

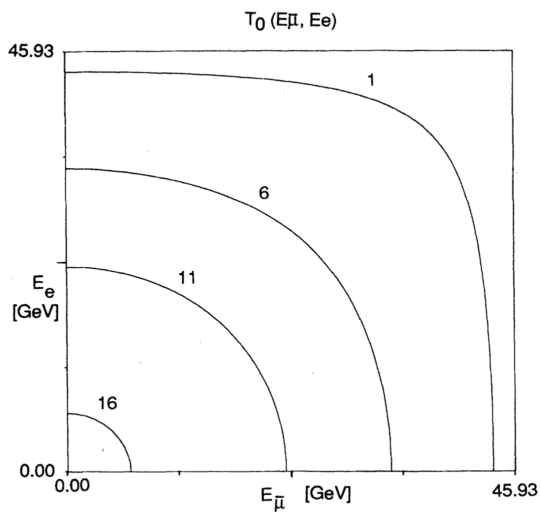

FIG. 19. Contour plot of the $T_0(E_{\bar{\mu}}, E_e)$ factor in the muon-energy–electron-energy correlation function in the Z^0 rest frame, $I(E_{\bar{\mu}}, E_e) = (1+\xi B+\xi^2 C)$, where ξ is the Michel polarization parameter for $\tau^-\beta\ell^-\nu\bar{\nu}$. For a pure $V-A$ coupling, $\xi=1$.

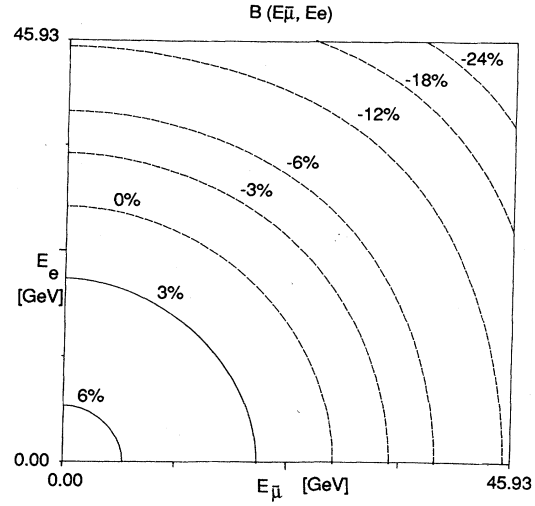

FIG. 20. Contour plot of the $B(E_{\bar{\mu}}, E_e)$ term, the coefficient of ξ, in the $E_{\bar{\mu}}$ and E_e correlation function $I(E_{\bar{\mu}}, E_e)$.

Table V compares their ideal statistical errors. For $\sigma(\xi)$ and $\sigma(\delta)$ the τ^- polarization technique is a factor of about 2.8 worse.

For the μ^+e^- mode, for instance, the origin of this large effect is apparent from Figs. 19–21. The analytic origin is transparent since

$$
\begin{aligned}
I(E_{\bar{\mu}}, E_e) &= T_0(1+\xi B+\xi^2 C) \\
&= f(x)f(y)+\xi[f(x)g(y)+g(x)f(y)] \\
&\quad +\xi^2[g(x)g(y)] ,
\end{aligned}
\tag{9.1}
$$

where, from Eqs. (3.20) and (3.21),

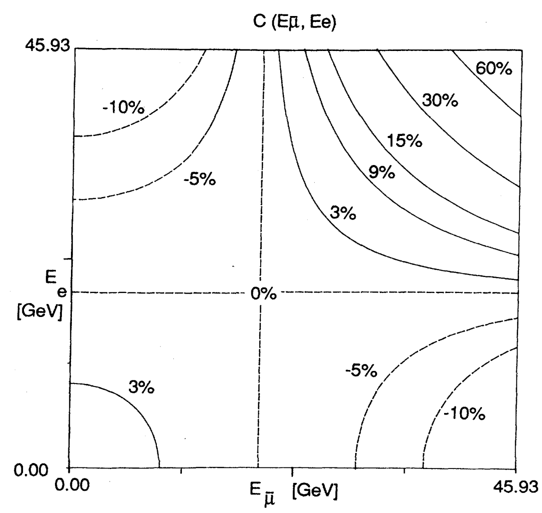

FIG. 21. Contour plot of the $C(E_{\bar{\mu}}, E_e)$ term, the coefficient of ξ^2, in the $E_{\bar{\mu}}$ and E_e correlation function $I(E_{\bar{\mu}}, E_e)$.

TABLE V. Comparison of ideal statistical errors for the Michel parameters ξ, δ, and ρ from measurements by $I(E_{\bar{\mu}},E_B)$ with measurements by $I(E_{\bar{\mu}})$.

Sequential decay mode	$I(E_{\bar{\mu}},E_B)$ $\sigma(\xi)$	$I(E_{\bar{\mu}})$ $\sigma(\xi)$	$I(E_{\bar{\mu}},E_B)$ $\sigma(\delta)$	$I(E_{\bar{\mu}})$ $\sigma(\delta)$	$I(E_{\bar{\mu}},E_B)$ $\sigma(\rho)$	$I(E_{\bar{\mu}})$ $\sigma(\rho)$
μ^+e^-	0.0428	0.1349	0.0486	0.1345	0.011 6	0.016 1
$\mu^+(\pi,K)^-$	0.0391	0.1671	0.0374	0.1667	0.018 6	0.019 9
$\mu^*\rho^-$	0.0593	0.1195	0.0585	0.1192	0.014 1	0.014 3
μ^+K^{*-}	0.2969	0.4790	0.2937	0.4777	0.056 8	0.057 2
$\mu^+a_1^{\mathrm{ch}-}$	0.2188	0.2190	0.2182	0.2184	0.026 1	0.026 1
Sum of above modes	0.0257	0.0733	0.0262	0.0731	0.007 64	0.008 75
		Factor worse =2.86		Factor worse =2.80		Factor worse =1.15

$$f(y)\simeq\tfrac{5}{6}(1-y)(1-0.525y)(1+1.52y)\ ,$$
$$g(y)\simeq-\tfrac{1}{6}(1-y)(1-2.37y)(1+3.37y)\ , \tag{9.2}$$

and the quadratic term ξ^2C dominates the ideal statistical error for $\sigma(\xi)$ in $I(E_{\bar{\mu}},E_e)$. The zeros at $x_0=y_0\simeq(1/2.37)$ in $T_0C=g(x)g(y)$ are responsible for the sign behavior shown in Fig. 18 for $C(x,y)$. After integrating out the E_e energy (i.e., the y dependence), this term does not contribute to $I(E_{\bar{\mu}})$ since $\int dy\, g(y)=0$. [In fact, only the $a(y)$ term in $f(y)=a(y)+\rho b(y)$, $g(y)=d(y)+\delta c(y)$, of Eqs. (3.20) and (3.21) contributes upon integration of E_e from E_I to E_F. The $c(y)$ term upon integration contributes $(1-\beta)$ corrections; $b(y)$ and $d(y)$ both vanish.]

X. DETERMINATION OF CHIRAL POLARIZATION PARAMETERS ξ_π, ξ_ρ, AND ξ_{K^*}: COMPARISON OF $I(E_A,E_B)$ WITH τ-POLARIZATION TECHNIQUE

The τ-polarization technique for the mode $\tau^+\to A^+X$ uses the energy distribution $I(E_A)$ to determine the product $\xi_A\alpha_H$ of τ coupling parameters. To determine ξ_A, the value for α_H must be assumed. For Tables VI–VIII we have used $\sin^2\theta_W=0.23$ so $\alpha_H=-0.1591$. Instead, as discussed in the Introduction $I(E_A,E_B)$ can be used to independently determine $\xi_A\xi_B$, $\xi_A\alpha_H$, and $\xi_B\alpha_H$; therefore, ξ_A, ξ_B, and α_H can be independently determined from $I(E_A,E_B)$. [The $\sigma(\xi_A)$ values for $I(E_A,E_B)$ in these tables, of course, do depend on the values $\sin^2\theta_W=-0.23$.]

In Table VI as throughout this paper, we have combined the $\tau\to\pi\nu$ and $\tau\to K\nu$ modes. Here if only the $\tau\to\pi\nu$ mode is used the numbers for $\sigma(\xi_\pi)$ should be multiplied by 1.03. [For $(\pi,K)^+(\pi,K)^-$ to get $\pi^+\pi^-$, it should be multiplied by 1.06.]

Tables VI and VII show that ξ_π and ξ_ρ can be determined from the energy-energy correlation functions $I(E_A,E_B)$ to the few percent level. Even if α_H were known, $I(E_A,E_B)$ provides a factor of about 3 improvement over the τ-polarization technique.

Table VIII shows that ξ_{K^*} can be determined to about the 20% level.

XI. CONCLUSIONS

(1) The explicit formulas for the energy correlation functions $I(E_A,E_B)$ at the tree level are extremely simple

TABLE VI. Comparison of ideal statistical errors for the chiral polarization parameter ξ_π from measurements by $I(E_\pi,E_B)$ with measurements of $\xi_\pi\alpha_H$ by $I(E_\pi)$. From $I(E_\pi)$, α_H must be given to obtain ξ_π, whereas $I(E_\pi,E_B)$ can be used to independently determine $\xi_\pi\xi_B$, $\xi_\pi\alpha_H$, and $\xi_B\alpha_H$ (see text). For the standard $V-A$ coupling, $\xi_\pi=1$. Square brackets denote the percentage error.

Sequential $(\pi,K)^+B^-$ mode	$I(E_\pi,E_B)$ $\sigma(\xi_\pi)$	$I(E_\pi)$ $\sigma(\xi_\pi)$
B particle		
μ^-	0.0363	0.0660
$(\pi,K)^-$	0.0157	0.1634
ρ^-	0.0410	0.0823
K^{*-}	0.2086	0.3312
$a_1^{\mathrm{ch}-}$	0.1512	0.1515
Sum of above modes	0.0135 [1.35%]	0.0463 Factor worse =3.43

TABLE VII. Comparison of ideal statistical errors for the chiral polarization parameter ξ_ρ from measurements by $I(E_\rho,E_B)$ with measurements of $\xi_\rho\alpha_H$ by $I(E_\rho)$. From $I(E_\rho)$, α_H must be given to obtain ξ_ρ. For the standard $V-A$ coupling, $\xi_\rho=1$.

Sequential ρ^+B^- mode	$I(E_\rho,E_B)$ $\sigma(\xi_\rho)$	$I(E_\rho)$ $\sigma(\xi_\rho)$
B particle		
μ^-	0.0596	0.1037
$(\pi,K)^-$	0.0472	0.1816
ρ^-	0.0517	0.1835
K^{*-}	0.3344	0.5204
$a_1^{\mathrm{ch}-}$	0.2378	0.2382
Sum of above modes	0.0297 [2.97%]	0.0757 Factor worse =2.55

TABLE VIII. Comparison of ideal statistical errors for the chiral polarization parameter ξ_{K^*} from measurements by $I(E_{K^*},B)$ with measurements of $\xi_{K^*}\alpha_H$ by $I(E_{K^*})$. From $I(E_{K^*})$, α_H must be given to obtain ξ_{K^*}. For the standard $V-A$ coupling, $\xi_{K^*}=1$.

Sequential $K^{*+}B^-$ mode	$I(E_{K^*},E_B)$ $\sigma(\xi_{K^*})$	$I(E_K)$ $\sigma(\xi_{K^*})$
B particle:		
μ^-	0.3293	0.5707
$(\pi,K)^-$	0.2646	0.9994
ρ^-	0.3734	0.7146
K^{*-}	1.474	4.052
$a_1^{\text{ch}-}$	1.309	1.310
Sum of above modes	0.178 [17.8%]	0.387 Factor worse =2.18

and easily derived. Since $I(E_A,E_B)$ depends independently on $\xi_A\xi_B$, $\xi_A\alpha_H$, and $\xi_B\alpha_H$, the fundamental parameters ξ_A, ξ_B, and α_H can be independently determined by measurement of $I(E_A,E_B)$. In contrast, measurement of $I(E_A)$ by the τ-polarization technique only determines $\xi_A\alpha_H$. Because of a factorization property, radiative corrections to $I(E_A,E_B)$ are as tractable as for A_{LR}; however, $I(E_A,E_B)$ does not require longitudinal beam polarization.[1]

(2) Using $I(E_A,E_B)$ for 10^7 Z^0 events the ideal statistical percentage errors in the determination of the Michel parameters for $\tau^\pm\to\ell^\pm\nu\bar{\nu}$ are for ξ, 2.57%; for δ, 3.49%; and for ρ, 1.02%. The present world average for ρ only excludes more than 47% of a $V+A$ coupling at the 95% confidence level.[43]

In the determination of the chiral polarization parameters in $\tau^+\to\pi_A^+\nu$, they are for ξ_π, 1.35%; for ξ_ρ, 2.97%; and for ξ_{K^*}, 17.8%. These chiral parameters equal the value "one" in the standard model. Right-handed currents, only approximate conservation of the leptonic axial-vector current, etc., could produce deviations from one.

(3) The $Z^0\to\tau^+\tau^-$ coupling parameter $\alpha_H\simeq -2a_\tau v_\tau/(a_\tau^2+v_\tau^2)$ can also be determined from $I(E_A,E_B)$. From Table II, the ideal statistical error for 10^7 Z^0 events is $\sigma(\alpha_H)=0.0053$, or a percentage error 3.34% for $\sin^2\theta_W=0.23$. Usage of the τ-polarization technique for the (π,K) modes, gives instead $\sigma(\alpha_H)=0.0074$ which is almost 40% worse. It is very important for specific SLC, or LEP, detectors to study how much direct application of the $I(E_A,E_B)$ technique in τ pair Monte Carlo simulations can reduce systematic errors. Some combination of both techniques will probably yield the largest statistics and most reliable measurement of α_H.

(4) Explicit formulas for the full sequential decay correlation function

$$I(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})=T(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})\times[1+A(E_{\bar{\mu}},E_e,\cos\psi_{\bar{\mu}e})]$$

are given for arbitrary Z mass, and for $I(E_{\bar{\mu}},E_h)$ for arbitrary Michel parameters. These results can therefore be used for analyzing the decay of a Z' boson and for $q\bar{q}$ modes[46] such as $t\bar{t}$.

(5) Measurement of the ξ and the other chiral polarization parameters ξ_A will enable precision measurement of $\sin^2\theta_W$ to a 0.3% ideal-statistical-error level from unpolarized e^+e^- collisions at the Z^0. This can be compared with the 0.13% precision level expected from a later measurement of A_{LR} at LEP I after instrumentation of polarized beams.[31] Measurement of charged-particle energy correlation functions $I(E_A,E_B)$ for various one-prong τ modes in $Z^0\to\tau^+\tau^-$ might also be a helpful constraint in solving the τ missing one-prong modes puzzle.

ACKNOWLEDGMENTS

The author thanks Jesse Ernst for assistance with the computer analysis. He thanks Noel K. Yeh for several discussions, and thanks the members and visitors in the theory groups at Brookhaven and Cornell for intellectual stimulation. This work was partially supported by U.S. Department of Energy Grant No. DE-FG02-86ER40291.

APPENDIX A: EXPLICIT EXPRESSIONS FOR $T(E_1,E_2,z)$ AND $U(E_1,E_2,z)$ FOR μ^+e^- MODE

The integral expressions for $I(E_1,E_2,\cos\psi)$ for $Z^0\to\tau^+\tau^-\to(\mu^+\nu\bar{\nu})(e^-\bar{\nu}\nu)$ are the same as those listed in Appendix A in Ref. 2. They enable analytic evaluation in the Z^0 rest frame of the standard decay correlation function $I(E_1,E_2,\cos\psi)$ under the assumption that the final muon and electron masses are set equal to zero. The allowed phase space is divided into the four regions $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, and $\tilde{D}$ shown in Fig. 22. These regions are separated by $E_I=m_\tau/[2\gamma(1+\beta)]$ and the maximum lepton energy is $E_F=m_\tau/[2\gamma(1-\beta)]$ where $\gamma=M/(2m_\tau)$ and β are the relativistic boost variables connecting a τ rest frame to the Z^0 rest frame. For a Z mass $M=91.9$ GeV,

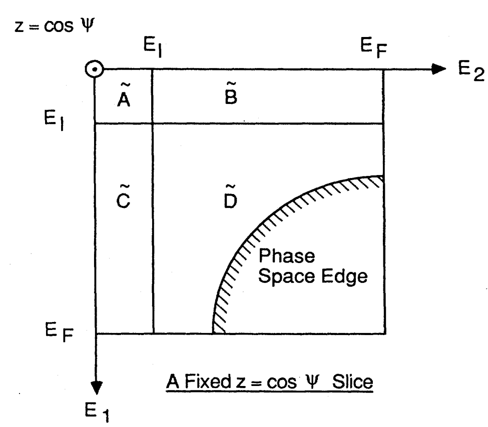

FIG. 22. An illustration showing a fixed $z=\cos\psi_{12}$ slice of the available E_1E_2 phase space. For analytic evaluation of the full sequential decay correlation function $I(E_1,E_2z)$, the phase space is divided into four regions: $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, and $\tilde{D}$.

$E_I = 0.0173$ GeV and $E_F = 45.9$ GeV so the $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ regions are important here for checking the longer and more complicated expressions for the $\tilde{D}$ region. If m_τ/M were significantly larger, these other regions could be of physical interest.

The following explicit expressions are in terms of the Z^0 rest frame observables $E_1 \equiv E_{\bar{\mu}}$, the μ_1^+ energy; $E_2 \equiv E_e$, the e_2^- energy; and $z = \cos\psi$ where $\psi \equiv \psi_{\bar{\mu}e}$ is the opening angle between the μ^+ and e^- momenta. Often, we use in place of these charged-lepton energies the rescaled inverses

$$y_{1,2} \equiv m_\tau / 2\gamma E_{1,2} \ . \tag{A1}$$

For the $\tilde{A}$ region, we find

$$\begin{aligned} T_{\tilde{A}}(E_1,E_2,z) = \frac{\pi}{2} E_1^3 E_2^3 \beta^2 \gamma^8 \Big(& \tfrac{832}{15} - \tfrac{128}{3}(y_1+y_2) + 40 y_1 y_2 + z\left[-\tfrac{128}{3} + \tfrac{64}{3}(y_1-y_2) - \tfrac{40}{3} y_1 y_2\right] + z^2 \tfrac{64}{15} \\ & + \gamma^{-2}\left\{-\tfrac{608}{15} + \tfrac{56}{3}(y_1+y_2) - 4y_1y_2 + z\left[\tfrac{128}{3} - \tfrac{56}{3}(y_1+y_2) + 12 y_1 y_2\right] - z^2 \tfrac{32}{5}\right\} \\ & + \gamma^{-4}\left(\tfrac{16}{15} - \tfrac{16}{3} z + \tfrac{32}{15} z^2\right)\Big) \end{aligned} \tag{A2}$$

and

$$\begin{aligned} U_{\tilde{A}}(E_1,E_2 z) = \frac{\pi}{2} E_1^3 E_2^3 \beta \gamma^8 \Big(& \tfrac{832}{15} - \tfrac{128}{3}(y_1+y_2) + 24 y_1 y_2 + z\left[-\tfrac{128}{3} + \tfrac{64}{3}(y_1+y_2) - 8 y_1 y_2\right] + z^2 \tfrac{64}{15} \\ & + \gamma^{-2}\left\{-\tfrac{1024}{15} + \tfrac{136}{3}(y_1+y_2) - 24 y_1 y_2 + z\left[64 - \tfrac{88}{3}(y_1+y_2) + 8 y_1 y_2\right] - z^2 \tfrac{128}{15}\right\} \\ & + \gamma^{-4}\left\{\tfrac{64}{5} - \tfrac{8}{3}(y_1+y_2) + z\left[-\tfrac{64}{3} + 8(y_1+y_2)\right] + z^2 \tfrac{64}{15}\right\}\Big) \ , \end{aligned} \tag{A3}$$

where the power of β is one less in (A3) vs (A2).

For the $\tilde{B}$ region, for the $T_{\tilde{B}}$ term

$$T_{\tilde{B}}(E_1,E_2,z) = -\frac{\pi}{2} E_1^3 E_2^3 \beta \gamma^8 \bar{S}(1-\beta, y_2) \ , \tag{A4}$$

where the polynomial

$$\bar{S}(q,p) = a(q-p) + \frac{b}{2}(q^2-p^2) + \frac{c}{3}(q^3-p^3) + \frac{d}{4}(q^4-p^4) + \frac{e}{5}(q^5-p^5) \tag{A5}$$

with the coefficients

$$\begin{aligned} a &= 2\beta^{-2}\gamma^{-2} y_2[2 - y_1 + z(4-y_1) + 2z^2 + \gamma^{-2}(-3 + y_1 - 2z + z^2)] \ , \\ b &= 2\beta^{-2}\{-8y_2 + 10 y_1 y_2 + 2 z y_2(-8+5y_1) - 8z^2 y_2 \\ &\quad + \gamma^{-2}[-4 + 2y_1 + 14 y_2 - 10 y_1 y_2 + z(-8 + 2y_1 + 10 y_2 - 8 y_1 y_2) - 4z^2(1+y_2)] \\ &\quad + \gamma^{-4}[6 - 2y_1 - 3y_2 + 2z(2+y_2) - z^2(2-3y_2)]\} \ , \\ c &= 2\beta^{-2}\{8 - 8(y_1+y_2) + 2z(8 - 4y_1 + 8y_2 - 5y_1y_2) + 8z^2(1+3y_2) \\ &\quad + \gamma^{-2}[-16 + 8y_1 + 5y_2 + z(-4 + 4y_1 - 14y_2 + 9y_1y_2) + 3z^2(4-5y_2)] + \gamma^{-4}(2 - 4z - 2z^2)\} \ , \\ d &= 2\beta^{-2}\{8 + 4y_2 + 8z(-2+y_1) - 12z^2(2+y_2) + \gamma^{-2}[-2 - 3y_2 + 6z(2-y_1) + 3z^2(2+3y_2)]\} \ , \\ e &= 4\beta^{-2}[-2 + 6z^2 + \gamma^{-2}(1-3z^2)] \ . \end{aligned} \tag{A6}$$

For the $U_{\tilde{B}}$ term, we obtain

$$U_{\tilde{B}}(E_1,E_2,z) = -\frac{\pi}{2} E_1^3 E_2^3 \beta \gamma^8 \bar{S}(1-\beta, y_2) \ , \tag{A7}$$

where the coefficients of the polynomial $\bar{S}(1-\beta, y_2)$ are

$$\begin{aligned} a &= 2\beta^{-1}\gamma^{-2} y_2[2 - 3y_1 + z(4-3y_1) + 2z^2 + \gamma^{-2}(-1+z^2)] \ , \\ b &= 2\beta^{-1}\{-8y_2 + 6y_1y_2 + 2zy_2(-8+3y_1) - 8z^2y_2 \\ &\quad + \gamma^{-2}[-4 + 6y_1 + 12y_2 - 3y_1y_2 + z(-8 + 6y_1 + 2y_2 + 3y_1y_2) - 2z^2(2+6y_2)] + \gamma^{-4}(2-2z^2)\} \ , \\ c &= 2\beta^{-1}\{8 - 8(y_1+y_2) + 2z(4 - 4y_1 + 8y_2 - 3y_1y_2) + 8z^2(1+3y_2) \\ &\quad + \gamma^{-2}[-12 + 2y_1 - y_2 + 2z(2 - 3y_1 - 3y_2) + z^2(16+3y_2)]\} \ , \\ d &= 4\beta^{-1}[4 + 2y_2 - 4z(2-y_1) - 6z^2(2+y_2) + \gamma^{-2}(1 + 2z - 3z^2)] \ , \\ e &= 8\beta^{-1}(-1+3z^2) \ . \end{aligned} \tag{A8}$$

The $\tilde{D}$ region is divided into four subregions as explained in Appendix A in Ref. 2: For the $\tilde{W}$ subregion,

$$T_{\tilde{W}}(E_1,E_2,z)=-\frac{\pi}{2}E_1^3E_2^3\beta\gamma^8\bar{S}(1-\beta,Y_{\min}(z,y_1)) , \quad \text{(A9)}$$

$$U_{\tilde{W}}(E_1,E_2,z)=-\frac{\pi}{2}E_1^3E_2^3\beta\gamma^8\bar{S}(1-\beta,Y_{\min}(z,y_1)) , \quad \text{(A10)}$$

where the coefficients of the polynomials $\bar{S}$ are, respectively, given in Eqs. (A6) and (A8), and where

$$Y_{\max}(z,y_1)=1+\beta\cos(\psi-\theta_1), \quad Y_{\min}(z,y_1)=1+\beta\cos(\psi+\theta_1) \quad \text{(A11)}$$

with the angles $\theta_{1,2}$ defined by

$$c_{1,2}\equiv\cos\theta_{1,2}=1/\beta(1-y_{1,2}), \quad s_{1,2}\equiv\sin\theta_{1,2}=(1-\cos^2\theta_{1,2})^{1/2} . \quad \text{(A12)}$$

For the $\tilde{X}$ subregion, for the $T_{\tilde{X}}$ term

$$T_{\tilde{X}}(E_1,E_2,z)=E_1^3E_2^3\beta^2\gamma^8\left[\pi\theta(-z-c_1)\bar{I}[-\cos(\psi+\theta_1)]-\cos^{-1}\left[\frac{c_1+zc_2}{s_2\sin\psi}\right]\bar{I}(c_2)+\bar{J}+\bar{K}\right] , \quad \text{(A13)}$$

where the three functions $\bar{I}$, $\bar{J}$, and $\bar{K}$ are given in Appendix B in Ref. 2. They depend on the coefficients

$$\begin{aligned}
f&=12-8y_1-12y_2+10y_1y_2+4z^2(-1+y_2)+\gamma^{-2}[-8+2y_1+6y_2-y_1y_2+4z^2(1-y_2)] ,\\
g&=\beta\{-24+16y_1+12y_2-10y_1y_2+z(16-8y_1-16y_2+10y_1y_2)+4z^2(2-y_2)\\
&\quad+\gamma^{-2}[10-2y_1-3y_2+2z(-2+y_2)+3z^2(-2+y_2)]\} ,\\
h&=8-8y_1+4y_2+z(-32+16y_1+16y_2-10y_1y_2)+4z^2(2-3y_2)\\
&\quad+\gamma^{-2}[-10+8y_1-4y_2+z(32-14y_1-14y_2+9y_1y_2)+6z^2(-1+2y_2)]+\gamma^{-4}(2-4z-2z^2) ,\\
i&=\beta\{8-4y_2+8z(2-y_1)+12z^2(-2+y_2)+\gamma^{-2}[-6+3y_2+6z(-2+y_1)+9z^2(2-y_2)]\} ,\\
j&=2[-2+6z^2+\gamma^{-2}(3-9z^2)+\gamma^{-4}(-1+3z^2)] ,
\end{aligned} \quad \text{(A14)}$$

and

$$\begin{aligned}
k&=2\beta[-6+2y_1+6y_2-3y_1y_2+\gamma^{-2}(2-y_2)] ,\\
l&=24-8y_1-12y_2+6y_1y_2+12z(-1+y_2)+\gamma^{-2}[-26+8y_1+11y_2-6y_1y_2+12z(1-y_1)]+\gamma^{-4}4 ,\\
m&=\beta\{-12+4y_1+12z(2-y_2)+\gamma^{-2}[10-4y_1+9z(-2+y_2)]\} ,\\
n&=-12z+\gamma^{-2}18z-\gamma^{-4}6z .
\end{aligned} \quad \text{(A15)}$$

Similarly, the $U_{\tilde{X}}$ term is given by

$$U_{\tilde{X}}(E_1,E_2,z)=[\text{right-hand side (RHS) of Eq. (A13)}] \quad \text{(A16)}$$

except the coefficients needed for $\bar{I}$, $\bar{J}$, and $\bar{K}$ are

$$\begin{aligned}
f&=\beta^{-1}\{12-8y_1-12y_2+6y_1y_2+4z^2(-1+y_2)\\
&\qquad+\gamma^{-2}[-14+8y_1+13y_2-6y_1y_2+z^2(6-5y_2)]+\gamma^{-4}[2-y_2+z^2(-2+y_2)]\} ,\\
g&=-24+16y_1+12y_2-6y_1y_2+z(16-8y_1-16y_2+6y_1y_2)+4z^2(2-y_2)\\
&\quad+\gamma^{-2}[22-10y_1-10y_2+3y_1y_2+z(-12+6y_1+10y_2-3y_1y_2)+2z^2(-5+2y_2)]+\gamma^{-4}(-2+2z^2) ,\\
h&=\beta\{8-8y_1+4y_2+z(-32+16y_1+16y_2-6y_1y_2)+z^2(8-12y_2)\\
&\quad+\gamma^{-2}[-6+2y_1-y_2+z(16-6y_1-6y_2)+z^2(-2+3y_2)]\} ,\\
i&=8-4y_2+8z(2-y_1)+12z^2(-2+y_2)+\gamma^{-2}[-10+4y_2+4z(-5+2y_1)+6z^2(5-2y_2)]+\gamma^{-4}(2+4z-6z^2) ,\\
j&=\beta[-4+12z^2+\gamma^{-2}(4-12z^2)] ,
\end{aligned} \quad \text{(A17)}$$

and

$$\begin{aligned} k &= -12+4y_1+12y_2-2y_1y_2+\gamma^{-2}(10-4y_1-9y_2+2y_1y_2)\,, \\ l &= \beta\{24-8y_1-12y_2+2y_1y_2+12z(-1+y_2)+\gamma^{-2}[-14+4y_1+6y_2+3z(2-y_2)]\}\,, \\ m &= -12+4y_1+12z(2-y_2)+\gamma^{-2}[16-4y_1+6z(-5+2y_2)]+\gamma^{-4}(-4+6z)\,, \\ n &= \beta(-12z+\gamma^{-2}12z)\,. \end{aligned} \tag{A18}$$

For the Y subregion, the $T_{\tilde{Y}}$ term is

$$T_{\tilde{Y}}(E_1,E_2,z)=E_1^3E_2^3\beta^2\gamma^8\left[\pi\theta(-z-c_1)\bar{I}[-\cos(\psi+\theta_1)]-\pi\theta(z-c_1)\bar{I}(c_2)+\bar{J}'+\bar{K}'-\frac{\pi}{2}\beta^{-1}\bar{S}(y_2,Y_{\max}(z,y_1))\right]\,, \tag{A19}$$

where the new functions $\bar{J}'$ and $\bar{K}'$ are also given in Appendix B in Ref. 2, and where the necessary coefficients are given above in Eqs. (A14) and (A15). For the $U_{\tilde{Y}}$ term,

$$U_{\tilde{Y}}(E_1,E_2,z)=[\text{RHS of Eq. (A19)}] \tag{A20}$$

except the coefficients are those of Eqs. (A17) and (A18). Finally, for the $\tilde{Z}$ subregion of $\tilde{D}$,

$$T_{\tilde{Z}}(E_1,E_2,z)=\frac{\pi}{2}E_1^3E_2^3\beta\gamma^8\bar{S}(y_2,Y_{\min}(z,y_1)) \tag{A21}$$

with the coefficients of Eq. (A6), and

$$U_{\tilde{Z}}(E_1,E_2,z)=[\text{RHS of Eq. (A21)}] \tag{A22}$$

except the coefficients are those of Eq. (A8).

These expressions, which are somewhat lengthy, were evaluated numerically in order to obtain Figs. 3–8. In some of the expressions for the $\tilde{D}$ region, in particular near the "phase-space edge" we found significant cancellations among the terms which required usage of "quad-precision" variables in the numerical computation. While this could be avoided by appropriately rewriting the expressions, in the present form we found that programming errors were relatively easy to ferret out by simple tests. Tests included varying the m_τ/M ratio while checking that $T\geq 0$, that $|U/T|\leq 1$, and for continuity between the $\tilde{A}$, $\tilde{B}$, $\tilde{D}$ regions and between the various $\tilde{D}$ subregions. For some E_1,E_2 points in the $\tilde{D}$ region, $T(E_1,E_2,z)$ and $A(E_1,E_2,z)$ were also numerically integrated and found to agree, respectively, with $T(E_1,E_2)$ and $A(E_1,E_2)$ for Figs. 1 and 2. If requested for a reader's research, a down-loaded copy of the numerical program will be supplied by the author.

From the above expressions, simpler expressions for T and U can be easily worked out in the extreme relativistic limit where $\gamma=M/(2m_\tau)$ is large as was done for $\psi_{12}=180°$ in Ref. 3.

APPENDIX B: FORMULAS FOR ENERGY-ENERGY CORRELATION $I(E_{\bar{\mu}},E_e)$ AND $I(E_\ell,E_k)$ IN NONRELATIVISTIC REGIME

Since the Z mass M is much greater than $2m_\tau$, the $\tilde{A}$ and $\tilde{B}$ regions of Fig. 9 are not relevant to Figs. 1 and 2, for instance, here $E_I=0.0173$ GeV. However, these regions might be of interest, e.g., if there were a second neutral boson Z' and charged spin-$\frac{1}{2}$ fermions from a fourth family.[45] Note that for $E_I\geq\frac{1}{2}E_F$, the γ boost parameter between the M rest frame and a spin-$\frac{1}{2}$ fermion frame satisfies $1\leq\gamma\leq\frac{5}{4}$ so this is a nonrelativistic regime $[\gamma=M/(2m_\tau)]$.

In place of Eqs. (3.17) and (3.18), for the $\tilde{A}$ region,

$$T_{\tilde{A}}(E_{\bar{\mu}},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[f_s(x)f_s(y)+\xi_1\xi_2g_s(x)g_s(y)]\,, \tag{B1}$$

$$U_{\tilde{A}}(E_{\bar{\mu}},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[\xi_1g_s(x)f_s(y)+\xi_2f_s(x)g_s(y)]\,. \tag{B2}$$

Since helicity conservation would not be expected to be a good approximation in the $t_{\lambda_1\lambda_2}$ amplitudes, the σ terms in Eq. (3.7) cannot be dropped so one also needs

$$S_{\tilde{A}}(E_{\bar{\mu}},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[f_s(x)f_s(y)-\xi_1\xi_2g_s(x)g_s(y)] \tag{B3}$$

and in place of Eq. (3.15), the final-lepton energy correlation function is

$$I(E_{\bar{\mu}},E_{e3})=\sigma S(E_{\bar{\mu}},E_e)+\tau T(E_{\bar{\mu}},E_e)+vU(E_{\bar{\mu}},E_e)\,. \tag{B4}$$

In Eqs. (B1)–(B3), letting $\omega_{\bar{\mu}}=(1-\beta)/x_{\bar{\mu}}=m_\tau/(2\gamma E_{\bar{\mu}})$ and similarly for ω_e,

$$\begin{aligned} f_s(x)&=2\beta\left[-8+6\omega+\frac{2}{\gamma^2}+\frac{1}{9}\rho\left[64-36\omega-\frac{16}{\gamma^2}\right]\right]\Big/\omega^3 \\ &\to 2\beta\left[-\frac{8}{3}+3\omega+\frac{2}{3}\frac{1}{\gamma^2}\right]\Big/\omega^3\,, \end{aligned} \tag{B5}$$

$$\begin{aligned} g_s(x)&=4\beta^2[\tfrac{4}{3}-\omega+\tfrac{2}{9}\delta(-16+9\omega)]/\omega^3 \\ &\to 2\beta^2(\omega-\tfrac{8}{3})/\omega^3\,, \end{aligned} \tag{B6}$$

where the arrowed lines follow for a $V\mp A$ coupling. Note that the ω's are rescaled inverses of the charged-lepton energies.

For the $\tilde{B}$ region, since $E_2=E_e>E_I$ whereas $E_1=E_{\bar{\mu}}<E_I$ (or $\omega_e<m_\tau/2\gamma E_I$) and $\omega_{\bar{\mu}}>m_\tau/2\gamma E_I$ the analogous expressions are

$$S_{\tilde{B}}(E_{\bar{\mu}},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[f_s(x)f(y)-\xi_1\xi_2 g_s(x)g(y)] , \tag{B7}$$

$$T_{\tilde{B}}(E_{\bar{\mu}},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[f_s(x)f(y)+\xi_1\xi_2 g_s(x)g(y)] , \tag{B8}$$

$$U_{\tilde{B}}(E_{\bar{\mu}},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[\xi_1 g_s(x)f(y)+\xi_2 f_s(x)g(y)] . \tag{B9}$$

For the $\tilde{D}$ region since Eq. (B4) involves the S term, besides the expressions in the text one needs

$$S(E_{\bar{\mu}},E_e)=\frac{\pi}{64}\gamma^4\beta^{-2}E_{\bar{\mu}}^3E_e^3[f(x)f(y)-\xi_1\xi_2 g(x)g(y)] . \tag{B10}$$

As noted in the text, the A_0 term in Eq. (3.5) does not contribute to the energy correlation function $I(E_{\bar{\mu}},E_e)$.

Likewise for the lepton-energy–hadron-energy correlation function when $E_{\bar{\mu}}<E_I$, in Eqs. (5.10) one must replace $f(x)$ by $f_s(x)$ and $g(x)$ by $g_s(x)$, where f_s and g_s are given in Eq. (B5) and (B6). In place of Eq. (5.9), since the σ terms cannot be dropped

$$I(E_{\bar{\mu}},\cos\theta_B^\tau)=\sigma S(E_{\bar{\mu}},\cos\theta_B^\tau)+\tau T(E_{\bar{\mu}},\cos\theta_B^\tau) +vU(E_{\bar{\mu}},\cos\theta_B^\tau) , \tag{B11}$$

where besides the just obtained analogues of Eqs. (5.10),

$$S(E_{\bar{\mu}},\cos\theta_B^\tau)=\frac{\pi}{8}\gamma^{-1}\beta^{-1}m_\tau^3[f_s(x)+\xi g_s(x)\xi_B\mathcal{S}_B\cos\theta_B^\tau] . \tag{B12}$$

---

[1]There is an earlier, shorter paper: C. A. Nelson, Phys. Rev. Lett. **62**, 1347 (1989); see also Refs. 2 and 3.

[2]J. R. Dell'Aquila and C. A. Nelson, Nucl. Phys. (to be published).

[3]J. R. Dell'Aquila and C. A. Nelson, Nucl. Phys. (to be published); C. A. Nelson, Report No. SUNY BING 10/15/88 (unpublished).

[4]For instance, in D. Cords *et al.*, SLAC Report No. 247, 1982 (unpublished), citation is made to LULEPT (D. Stoker, Mark II) and KORALZ (S. Jadach, Z. Was, and R. Stuart; R. Van Kooten, Mark II; and see Ref. 5).

[5]F. Boillot and Z. Was, Max-Planck-Institut, München Report No. MPI-PAE/Exp. El. 196, 1988 (unpublished).

[6]Some information is available in the literature, particularly from SLAC and CERN publications: H. R. Band, Report No. WISC-EX-88-300, 1988 (unpublished); SLAC Report No. 315, 1987 (unpublished); SLAC Report No. 273, 1984 (unpublished); J. M. Dorfan, in *TASI Lectures in Elementary Particle Physics—1989,* Lectures of the Theoretical Advanced Study Institute, Ann Arbor, Michigan, 1984, edited by D. N. Williams (University of Michigan, Ann Arbor, 1984), and references therein; SLAC Report No. 247, 1982 (unpublished); CERN report (unpublished); J. J. Thresher, in *Proceedings of the XXIV International Conference on High Energy Physics,* Munich, West Germany, 1988, edited by R. Kotthaus and J. Kuhn (Springer, Berlin, 1988); CERN Report No. 88-06, 1988 (unpublished); CERN Report No. 87-08, 1987 (unpublished); Report No. CERN/LEPC/87-6, 1987 (unpublished); in *Physics at LEP,* LEP Jamboree, Geneva, Switzerland, 1985, edited by J. Ellis and R. Peccei (CERN Report No. 86-02, Geneva, 1986); CERN Report No. 79-01, 1979 (unpublished).

[7]See the theoretical contributions in the reports of Ref. 6.

[8]Radiative corrections to some distributions for the process $e^+e^-\to\tau^+\tau^-$ have been investigated by S. Jadach and Z. Was, Acta Phys. Pol. **B15**, 1151 (1984); **B16**, 483 (1985); Z. Was, *ibid.* **B17**, 1099 (1987); CERN Report No. 88-06, 1988 (unpublished); and see Ref. 5.

[9]F. Boillot, Ph.D. thesis, University of Paris, 1987.

[10]M. Secco, thesis, Laurea in Fisica, University of Trieste, 1987.

[11]Some relevant papers in Ref. 6 are J. Chauveau, in *Physics at LEP* (Ref. 6); G. Goggi, CERN Report No. 79-01 (unpublished); J. E. Augustin, *ibid.*

[12]S. L. Jadach *et al.*, Z. Phys. C **38**, 609 (1988); B. A. Kniehl, J. H. Kühn, and R. G. Stuart, Phys. Lett. B **214**, 621 (1988).

[13]D. C. Kennedy and B. W. Lynn, Report No. SLAC-PUB-4039, 1988 (unpublished); earlier calculations include M. Veltman, Nucl. Phys. **B123**, 89 (1977); B. W. Lynn, M. E. Peskin, and R. G. Stuart, in *Physics at LEP* (Ref. 6); R. G. Stuart, thesis, Oxford University, Report No. RAL T 008, 1985; G. Altarelli and G. Martinelli, *ibid.*; G. Burgers, Report No. CERN-TH.5119/88, 1988 (unpublished); B. Grzadkowski *et al.*, Nucl. Phys. **B281**, 18 (1987); G. Burgers and W. Hollik, Report No. CERN-TH.5131/88, 1988 (unpublished).

[14]For A^- with spin 0, e.g., $\tau^-\to\pi^-\nu$, in terms of the helicity amplitude $C(0,\mu_2)$ where

$$\langle\theta_2,\phi_2,0,\mu_2|\tfrac{1}{2},\lambda_2\rangle=D^{1/2}_{\lambda_2,-\mu_2}(\phi_2,\theta_2,-\phi_2)C(0,\mu_2)$$

the chiral polarization parameter

$$\xi_\pi=\frac{|C(0,-\frac{1}{2})|^2-|C(0,\frac{1}{2})|^2}{|C(0,-\frac{1}{2})|^2+|C(0,\frac{1}{2})|^2} .$$

For spin 1, e.g., $\tau^-\to\rho^-\nu$, instead

$$\langle\theta_2,\phi_2,\mu_1,\mu_2|\tfrac{1}{2},\lambda_2\rangle=D^{1/2}_{\lambda_2,\mu}(\phi_2,H_2,-\phi_2)T(\mu_1,\mu_2)$$

here $\mu=\mu_1-\mu_2$ and

$$\xi_\rho\mathcal{S}_\rho=\frac{|T(0,-\frac{1}{2})|^2-|T(-1,-\frac{1}{2})|^2+|T(1,\frac{1}{2})|^2-|T(0,\frac{1}{2})|^2}{|T(0,-\frac{1}{2})|^2+|T(-1,-\frac{1}{2})|^2+|T(1,\frac{1}{2})|^2+|T(0,\frac{1}{2})|^2} ,$$

where $\mathcal{S}_\rho=(m_\tau^2-2m_\rho^2)/(m_\tau^2+2m_\rho^2)$. So for A^- with spin 0, assuming CP invariance, two of the three degrees of freedom can be measured from the energy-energy correlation function. For spin 1, two of seven can be measured by this technique.

[15]Y.-S. Tsai, Phys. Rev. D **4**, 2821 (1971).

[16]See also W. B. Lindquist, Cornell Report No. CLNS 81-485,

1981 (unpublished); J. Dorfan and S. Stone, Cornell Report No. CLNS 81-515, 1981 (unpublished).

[17]From CERN Report No. LEPC/87-6, 1987 (unpublished) based on results of G. Altarelli *et al.*, in *Physics at LEP* (Ref. 6).

[18]W. Hollik, DESY Reports Nos. 88-106, 1988 and 88-003, 1988 (unpublished).

[19]V. G. Angelopoulos, J. Ellis, D. V. Nanopoulos, and N. D. Tracas, Phys. Lett. B **176**, 203 (1986); J. Ellis, K. Enquist, D. V. Nanopoulos, and F. Zwirner, Mod. Phys. Lett. A **1**, 57 (1986); Nucl. Phys. **B276**, 14 (1986).

[20]B. W. Lynn, M. E. Peskin, and R. G. Stuart, in *Physics at LEP* (Ref. 6); B. W. Lynn, CERN Report No. 88-06, 1988 (unpublished); D. C. Kennedy, Report No. SLAC-PUB-4726, 1988 (unpublished).

[21]M. Cvetic and B. W. Lynn, Phys. Rev. D **35**, 51 (1987); G. Gounaris and D. Schildknecht, Report No. CERN-TH. 5078/88, 1988 (unpublished); F. Jegerlehner, Bielefeld Report No. BI-TP 87/16, 1987 (unpublished); J. H. Kühn, Report No. MPI-PAE/PTh 79/87, 1987 (unpublished).

[22]F. Boudjema, F. M. Renard, and C. Verzegnassi, Report No. CERN-TH.5049/88 (unpublished); Phys. Lett. B **214**, 151 (1988); R. Stuart, in *Proceedings of the XXIV International Conference on High Energy Physics* (Ref. 6); T. Riemann and M. Sachwitz, IHE der AdW der DDR-Berlin Report No. PHE-87-12, 1987 (unpublished).

[23]R. Budny, Phys. Rev. D **14**, 2969 (1976). See also P. Langacker and D. London, DESY Report No. 88-082, 1988 (unpublished).

[24]J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); H. Fritzsch and P. Minkowski, Nucl. Phys. **B103**, 61 (1976); R. N. Mohapatra and D. P. Sidhu, Phys. Rev. Lett. **38**, 667 (1977); M. A. Beg, R. V. Budny, R. N. Mohapatra, and A. Sirlin, *ibid.* **38**, 1252 (1977); B. R. Holstein and S. B. Trieman, Phys. Rev. D **16**, 2369 (1977).

[25]See, for example, and references therein, R. E. Shrock, Phys. Rev. D **24**, 1275 (1981); Phys. Lett. **112B**, 382 (1982); L. Wolfenstein, Carnegie Mellon report, 1988 (unpublished); H. Harari, Report No. WIS/88/74-PH, 1988 (unpublished).

[26]A. Zee, Phys. Lett. **93B**, 389 (1980).

[27]M. Fukugita and T. Yanagida, Phys. Rev. Lett. **58**, 1807 (1987).

[28]K. Babu and V. S. Mathur, Phys. Lett. B **196**, 218 (1987).

[29]R. N. Mohapatra, Phys. Lett. B **201**, 517 (1988).

[30]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).

[31]For 40 pb^{-1} (10^6 Z^0 events) it has been estimated that $\delta A_{LR}=\pm 0.003$, or $\delta \sin^2\theta_W=0.0003$, D. Treille, CERN Report No. 88-06 (unpublished); assuming lepton universality this can be compared with $\delta\alpha_H$ values for the τ coupling in Table I; for "sum of modes" $\delta\alpha_H=0.005$, or $\delta \sin^2\theta_W=0.000\,68$.

[32]The analogous expressions for $S(E_{\bar\mu},E_e)$ in comparison with $T(E_{\bar\mu},E_e)$ of Eq. (3.17) follow by replacing $\xi_1\xi_2\rightarrow -\xi_1\xi_2$. Similarly below in Eq. (4.2), for $S(x_H)$ in place of $T(x_H)$ replace $\xi^2\rightarrow -\xi^2$.

[33]The η Michel parameter does not appear for we neglect terms proportional to (m_e/E_e) and $(m_{\bar\mu}/E_{\bar\mu})$ in R and S of Eq. (3.4). A more general, model-independent treatment of observables is reviewed in the Particle Data Group article (Ref. 30); see F. Scheck, Phys. Rep. **44**, 187 (1978), and other references therein; W. Fetscher, Paul Scherrer Institute Report No. PR-88-09, 1988 (unpublished).

[34]This factor suppresses for $J_B\neq 0$ the spin signatures; numerically $\mathcal{S}_\rho=0.457$, $\mathcal{S}_{K^*}=0.333$, and $\mathcal{S}_{a_1}=-0.011$.

[35]K. G. Hayes and M. L. Perl, Phys. Rev. D **38**, 3351 (1988), and see Ref. 30.

[36]K. K. Gan and M. L. Perl, Int. J. Mod. Phys. A **3**, 531 (1988); B. C. Barish and R. Stroynowski, Phys. Rep. **157**, 1 (1988); C. Kiesling, Max-Planck (München) report, 1988 (unpublished); M. Perl, Annu. Rev. Nucl. Sci. **30**, 299 (1980).

[37]If the coefficients of both terms in Eq. (7.3) are considered to be used to minimize χ^2, then for the a parameter σ_a is slightly larger. The analysis, however, is more complicated. Should, for instance, ξ_A, ξ_B, and α_H all be found from $I(E_A,E_B)$ to differ from standard-model values, then the inclusion of the correlation between their errors will definitely be required.

[38]Detector efficiencies and energy calibration uncertainties are not included in these "ideal statistical errors."

[39]T. L. Trueman, Phys. Rev. D **18**, 3423 (1978). See also N. P. Chang and C. A. Nelson, Phys. Rev. Lett. **40**, 1617 (1978), and C. N. Yang, Phys. Rev. **77**, 242 (1950); **77**, 722 (1950). In the case of $I(E_A,E_B)$, it is the C term of Eq. (3.5) that contains the physics signatures. Projections of this term were first considered by Trueman. In later work, C was analyzed as a two-variable distribution, see C. A. Nelson, Phys. Rev. D **30**, 107 (1984).

[40]M. G. Bowler, Phys. Lett. B **182**, 400 (1986).

[41]C. A. Nelson, J. Phys. (Paris) Colloq. **46**, C2-261 (1985); J. R. Dell'Aquila and C. A. Nelson, Phys. Rev. D **33**, 80 (1986).

[42]See discussion in last paragraph of Sec. VIII.

[43]Some theoretical models predict ρ slightly less than 0.75 [Henry Tye cited in R. S. Galik, Cornell Report No. CLNS 87/88, 1987 (unpublished)]. The present world average for ρ only excludes more than 47% of a $V+A$ coupling at 95% confidence level [F. Gilman, Report No. SLAC-PUB-4352 (unpublished)].

[44]See R. Budny, Phys. Lett. **55B**, 227 (1975); J. Ellis and M. K. Gaillard, CERN Report No. 76-18 (unpublished); J. Babson and E. Ma, Phys. Rev. D **26**, 2497 (1982); Z. Phys. C **20**, 5 (1983); H. Kühn and F. Wagner, Nucl. Phys. **B236**, 16 (1984). Recent results from the KEK storage ring TRISTAN are reviewed by A. Maki, KEK Report No. 88-52, 1988 (unpublished); S. K. Kim, KEK Report No. 88-61, 1988 (unpublished); and T. Kamae, in *Proceedings of the XXIV International Conference on High Energy Physics* (Ref. 6). See R. Marshall, Report No. RAL-88-051 (unpublished), contribution to Munich conference. Experiments are in progress by CLEO, ARGUS, and the Crystal Ball Collaborations.

[45]For the τ-polarization technique, the ideal statistical error associated with the energy distribution $I(x)=1-\xi_B\alpha_H\mathcal{S}_B x=1-S_B x$, $S_B=\xi_B\alpha_H\mathcal{S}_B$ is given by

$$\begin{aligned}\sigma(\alpha_H)&=\left\{\frac{N}{2(-S_B)^3}\left[\ln\left[\frac{1-S_B}{1+S_B}\right]+2S_B\right]\right\}^{-1/2}\\ &\simeq\left[\frac{N}{(-S_B)^3}\left[\frac{(-S_B)^3}{3}+\frac{(-S_B)^5}{5}\right.\right.\\ &\qquad\left.\left.+\frac{(-S_B)^7}{7}+\cdots\right]\right]^{-1/2}\\ &\simeq\sqrt{3/N}\,[1-\tfrac{3}{10}(\xi_B\alpha_H\mathcal{S}_B)^2]\simeq\sqrt{3/N}\,[1+O(1\%)]\ ,\end{aligned}$$

where N is the number of events included in $\tau\rightarrow BX$. So this ideal statistical error $\sigma(\alpha_H)$ is not sensitive to the value of $\sin^2\theta_W$. However, the percentage error $[\sigma(\alpha_H)/\alpha_H]$ is very

sensitive to the value of $x_W = \sin^2\theta_W$ since $\partial\alpha_H/\partial x_W \simeq 7.8$.

[46]Polarization effects in possible new Z' physics have been discussed by P. J. Franzini and F. J. Gilman, Phys. Rev. D **35**, 855 (1987); J. L. Rosner, *ibid.* **35**, 2244 (1987); and J. P. Ader, S. Narison, and J. C. Wallet, Montpellier Report No. PM:86-9, 1986 (unpublished). The t-quark electroweak couplings could be measured using $I(E_{\bar{\mu}}, E_e)$ of Sec. III via $Z' \rightarrow t\bar{t}$ if $m_t \lesssim 85$ GeV. However, if $m_t \gtrsim 100$ GeV, the t-quark spin-polarization effects would be very suppressed due to $\mathcal{S}_t$ factors, Eq. (5.7), associated with t and $\bar{t}$ decays into "real" W's, see Ref. 3.