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Canonical formalism and the Leibbrandt-Mandelstam prescription for noncovariant gauges

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A very simple and elegant approach to the Leibbrandt-Mandelstam regularization is given within the canonical formalism. For any value of n^2 with n_0 *and* $\mathbf{n}$ different from zero, it consists of introducing the hyperbolic operator $n^*\cdot\partial n\cdot\partial=(n_0\partial_0)^2-(\mathbf{n}\cdot\boldsymbol{\partial})^2$ inside the field equations. The Cauchy problem is solved in the free field theory leading to a propagator regularized by means of the Leibbrandt-Mandelstam prescription. In the non-Abelian theory, these gauges are now on the same footing as relativistic gauges but the contribution of ghost loops vanishes.

I. INTRODUCTION

Since the advent of non-Abelian gauge theories, some special attention has been paid to axial gauges[1] because of the possible ghost decoupling. In addition, the light-cone gauge plays a special role in string theory. In the case of non-Abelian gauge theories, the ghost decoupling is obtained at the cost of the introduction of an unphysical pole at $n\cdot k=0$. This pole was first assumed to be regularized by the principal-value prescription. This regularization can be justified[2] in the case of $n=(1,0,0,0)$ but the violation of the Gauss law does not guarantee that such a gauge really describes the non-Abelian gauge theory. In contrast with this, Wilson-loop calculations[3] actually showed the principal-value prescription to be incorrect. During the last years, a large amount of work[4-11] has been devoted to this question of pole regularization in the temporal gauge.

In the light-cone gauge the principal-value prescription is also incorrect. Leibbrandt[12] and Mandelstam[13] independently suggested the following regularization for the pole:

$$\frac{1}{k\cdot n}\to\frac{k\cdot n^*}{k\cdot nk\cdot n^*+i\epsilon}\ , \tag{1}$$

where $n=(1,0,0,1)$ and $n^*=(1,0,0,-1)$. All the calculations using this prescription were successful and it has been claimed that such a regularization could result from the canonical formalism.[14] However, in this approach, the vector n^* is not present in the starting Lagrangian and only appears, in an artificial way, in the course of the momentum-space expansion of fields.

In the spacelike axial gauge the principal-value prescription does not seem to lead to any difficulty, but the canonical theory is far from being consistent.[15] There are only physical degrees of freedom but the metric is not positive definite. Attempts[16] to remedy this situation have only been partially successful.

A year ago, Leibbrandt[17] proposed to use the regularization (1) for all values of n^2, the dual vector n^* depending on the value of n^2. He computed the gluon self-energy using this prescription and got encouraging results. Moreover, the regularization (1) also implies time factorization in the Wilson loop.[18] Similar proposals have also been made by other authors.[19,20]

The aim of this paper is to present, within a strictly canonical framework, a formulation of the axial gauges automatically giving rise to the Leibbrandt-Mandelstam prescription for any value of n^2. This is done by rewriting the field equations in terms of the operator $n^*\cdot\partial n\cdot\partial$ in such a way that the latter takes the form of a hyperbolic second-order differential operator. This condition is imposed in order to solve unambiguously the Cauchy problem for the commutation relations. The causal propagator is then uniquely defined. The choice $n=(n_0,\mathbf{n}),n^*=(n_0,\to\mathbf{n})$ with n_0 *and* $\mathbf{n}\neq 0$ is particularly suited for time evolution and leads to $n^*\cdot\partial n\cdot\partial=(n_0\partial_0)^2-(\mathbf{n}\cdot\boldsymbol{\partial})^2$. This operator is not very different from the d'Alembert operator and part of our discussion actually consists of translating properties of the solutions of $\Box S=G$ in terms of $n^*\cdot\partial n\cdot\partial S=G$.

In Sec. II the relevant aspects of free field theory are developed. In Sec. III the non-Abelian theory is discussed. Since the theory actually consists of replacing the operator ∂_ν in the covariant gauge-fixing term by $n^*\cdot\partial n_\nu$, a ghost term is present in the Lagrangian. The ghost propagator is well defined and it is easy to show that any ghost loop vanishes. Section IV involves some remarks and conclusions. The Appendix summarizes the properties of the various special functions which are introduced in Sec. II.

II. THE FREE FIELD THEORY

A. Canonical quantization

Let us start with the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+Sn^*\cdot\partial n\cdot A-\tfrac{1}{2}aS^2\ , \tag{2}$$

where

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ , \tag{3}$$

S is a Lagrange multiplier, the so-called Nakanishi-Lautrup field[21] and a a parameter which, in the usual axial gauges, will be set equal to zero. The vectors n and n^* are, respectively,

$$n=(n_0,\mathbf{n}),\quad n^*=(n_0,-\mathbf{n})$$

and it is crucial for our discussion that n_0 *and* $\mathbf{n}$ be different from zero. There is no need to distinguish between the different values of n^2. It is clear that the Lagrangian (2) involves four independent fields, Sn_0^2 being the variable canonically conjugate to A_0.

The field equations are

$$\partial^\mu F_{\mu\nu}-n^*\cdot\partial n_\nu S=0\ . \tag{4}$$

$$n^*\cdot\partial n\cdot A=aS\ . \tag{5}$$

After some elementary algebra, they may be shown to lead to

$$n^*\cdot\partial n\cdot\partial S=0\ , \tag{6}$$

$$n^*\cdot\partial n\cdot\partial\partial\cdot A=[a\Box-n^2(n^*\cdot\partial)^2]^S\ , \tag{7}$$

$$\Box A_\nu=\partial_\nu\cdot A+n^*\cdot\partial n_\nu S\ . \tag{8}$$

In the following, the operator $a\Box-n^2(n^*\cdot\partial)^2$ will often be denoted by K.

The canonical equal-time commutation relations are

$$[A_\mu(x),A_\nu(y)]_{x_0=y_0}=[\pi^\mu(x),\pi^\nu(y)]_{x_0=y_0}=0\ , \tag{9a}$$

$$[A_\mu(x),\pi^\nu(y)]_{x_0=y_0}=i\delta_\mu^\nu\delta^{(3)}(\mathbf{x}-\mathbf{y})\ , \tag{9b}$$

where

$$\pi^0=Sn_0n_0^*=Sn_0^2\ , \tag{10}$$

$$\pi^k=F^{k0}+Sn_0n^k\ . \tag{11}$$

The reader who is not interested by the details of the formalism may directly switch to Sec. II E.

B. Solution of the Cauchy problem

The first step in a free field theory consists in solving the Cauchy problem associated with Eqs. (6)–(8). The operator $n^*\cdot\partial n\cdot\partial$ is not very different from the d'Alembert operator $\Box$. In momentum space, one indeed deals with $n_0^2k_0^2-(\mathbf{n}\cdot\mathbf{k})^2$ instead of $k_0^2-|\mathbf{k}|^2$. The solutions corresponding to $n^*\cdot\partial n\cdot\partial$ may, therefore, be obtained by inspection from those corresponding to the d'Alembert operator. Details are given in the Appendix where the formally covariant odd special function

$$\frac{D_n(x)}{n_0^2}$$

defined by

$$D_n(x)=\frac{-in_0^2}{(2\pi)^3}\int d^4k\ \theta(k_0)\delta(n^*\cdot kn\cdot k)\times(e^{-ik\cdot x}-e^{ik\cdot x})\ , \tag{12}$$

is introduced. It corresponds for the operator $n^*\cdot\partial n\cdot\partial$ to the usual D function for the d'Alembert operator and satisfies

$$n^*\cdot\partial n\cdot\partial D_n=0 \tag{13}$$

as well as

$$D_n(0,\mathbf{x})=0,\quad (\partial_0D_n)(0,\mathbf{x})=-\delta^{(3)}(\mathbf{x})\ . \tag{14}$$

The causal Green's function

$$D_{F,n}=\frac{-1}{(2\pi)^4}\int\frac{d^4k\ e^{-ik\cdot x}}{n^*\cdot kn\cdot k+i\epsilon} \tag{15}$$

satisfying

$$n^*\cdot\partial n\cdot\partial D_{F,n}(x)=\delta^{(4)}(\mathbf{x}) \tag{16}$$

is also needed.

With the help of these functions as well as of those, the familiar D and D_F, corresponding to the d'Alembert operator, it is easy to write the solution of the Cauchy problem associated with Eqs. (6)–(8). One gets

$$S(x)=\int d^3y[\partial_0^yD_n(x-y)S(y)-D_n(x-y)\partial_0^yS(y)]\ , \tag{17}$$

$$\begin{aligned}(\partial\cdot A)(x)=&\int d^3y[\partial_0^yD_n(x-y)(\partial\cdot A)(y)-D_n(x-y)(\partial_0\partial\cdot A)(y)]\\&+\int d^4z\ D_{F,n}(x-z)(KS)(z)-\int d^3y\ d^4z[\partial_0^yD_n(x-y)D_{F,n}(y-z)-D_n(x-y)\partial_0^yD_{F,n}(y-z)](KS)(z)\ ,\end{aligned} \tag{18}$$

$$\begin{aligned}A_\nu(x)=&\int d^3y[\partial_0^yD(x-y)A_\nu(y)-D(x-y)\partial_0A_\nu(y)]+\int d^4zD_F(x-z)V_\nu(z)\\&-\int d^3y\ d^4z[\partial_0^yD(x-y)D_F(y-z)-D(x-y)D_F(y-z)]V_\nu(z)\ ,\end{aligned} \tag{19}$$

where

$$V_\nu=\partial_\nu\partial\cdot A+n_\nu n^*\cdot\partial S \tag{20}$$

and y_0 is completely arbitrary.

C. Commutation relations for any time

Using Eq. (17) as well as equal-time commutation relations between A_μ and S and between A_μ and ∂_0S, it is easy to get

$$[A_\mu(x),S(x')]=\frac{-i}{n_0^2}\partial_\mu D_n(x-x')\ . \tag{21}$$

In the same way, to get $[A_\mu(x),\partial\cdot A(x')]$, one first computes the equal-time commutators

$$[A_0(x),\partial\cdot A(y)]_{x_0=y_0}=\frac{i}{n_0^2}\left(\frac{a}{n_0^2}+|\mathbf{n}|^2\right)\delta^{(3)}(\mathbf{x}-\mathbf{y})\ , \tag{22}$$

$$[A_k(x),\partial\cdot A(y)]_{x_0=y_0}=\frac{in_k}{n_0}\delta^{(3)}(\mathbf{x}-\mathbf{y})\ , \tag{23}$$

$$[A_0(x),\partial_0\partial\cdot A(y)]_{x_0=y_0} =\frac{i\mathbf{n}\cdot\boldsymbol{\partial}}{n_0}\delta^{(3)}(\mathbf{x}-\mathbf{y})-\frac{2i|\mathbf{n}|^2}{n_0^3}\mathbf{n}\cdot\boldsymbol{\partial}\delta^{(3)}(\mathbf{x}-\mathbf{y})\ , \tag{24}$$

$$[A_k(x),\partial_0\partial\cdot A(y)]_{x_0=y_0}=\frac{-i}{n_0^2}\left[\frac{a}{n_0^2}+|\mathbf{n}|^2\right]\partial_k\delta^{(3)}(\mathbf{x}-\mathbf{y}) +i\partial_k\delta^{(3)}(\mathbf{x}-\mathbf{y}) -\frac{in_k}{n_0^2}\mathbf{n}\cdot\boldsymbol{\partial}\delta^{(3)}(\mathbf{x}-\mathbf{y})\ . \tag{25}$$

The above commutator will be expressed in terms of the function

$$E_n(x-x')=\int d^4z\, D_{F,n}(x'-z)D_n(x-z) =\frac{-in_0^2}{(2\pi)^3}\int d^4k\, \epsilon(k_0)\delta'(n\cdot kn^*\cdot k)\times e^{-ik\cdot(x-x')} \tag{26}$$

which satisfies

$$\partial_0^p E_n(x)|_{x_0=0}=0 \quad \text{for } 0\le p\le 2\ , \tag{27}$$

$$K\partial_0 E_n(x)|_{x_0=0}=\left[\frac{-a}{n_0^2}+n^2\delta^{(3)}(\mathbf{x})\right]\ , \tag{28}$$

$$K\partial_0^2 E_n(x)|_{x_0=0}=\frac{2n^2}{n_0}\mathbf{n}\cdot\boldsymbol{\partial}^{(3)}(\mathbf{x})\ . \tag{29}$$

Using Eq. (18) as well as Eqs. (21)–(29), one gets, after some algebra,

$$[A_\mu(x),\partial\cdot A(x')]=-\frac{iK}{n_0^2}\partial_\mu E_n(x-x') -\frac{in_\mu}{n_0^2}n^*\cdot\partial D_n(x-x')\ . \tag{30}$$

One is now in position to compute $[A_\mu(x),A_\nu(x')]$ for any time. One uses

$$[A_\mu(x),V_\nu(z)]=\frac{iK\partial_\mu\partial_\nu}{n_0^2}E_n(x-z) +i(n_\mu\partial_\nu+n_\nu\partial_\mu)\frac{n^*\cdot\partial}{n_0^2}D_n(x-z) \tag{31}$$

as well as the new functions

$$F_D(x-x')=\int d^4z\, D_F(x'-z)D_n(x-z) =\frac{in_0^2}{(2\pi)^3}\int\frac{d^4k\,\epsilon(k_0)\delta(n\cdot kn^*\cdot k)}{k^2+i\epsilon}\times e^{-ik\cdot(x-x')}, \tag{32}$$

$$F_E(x-x')=\int d^4z\, D_F(x'-z)E_n(x-z) =\frac{in_0^2}{(2\pi)^3}\int\frac{d^4k\,\epsilon(k_0)\delta'(n\cdot kn^*\cdot k)}{k^2+i\epsilon}\times e^{-ik\cdot(x-x')}\ , \tag{33}$$

where δ' is the derivative of the δ function. They satisfy

$$\Box F_D=D_n,\quad \Box F_E=E_n,\quad n^*\cdot\partial n\cdot\partial F_D=0\ , \quad n^*\cdot\partial n\cdot\partial F_E=F_D \tag{34}$$

as well as

$$F_D(0,\mathbf{x})=0,\quad F_E(0,\mathbf{x})=0\ . \tag{35}$$

However,

$$\partial_0 F_D(0,\mathbf{x})=\frac{n_0^2}{(2\pi)^3}\int\frac{d^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{n}\cdot\mathbf{k})^2-|\mathbf{k}|^2n_0^2+i\epsilon} =(\Delta_\perp^{-1}\delta^{(3)})(\mathbf{x})\ , \tag{36}$$

where the operator $\Delta_\perp^{-1}$ is defined, in momentum space, by

$$\tilde{\Delta}_\perp^{-1}(k)=\frac{n_0^2}{(\mathbf{n}\cdot\mathbf{k})^2-n_0^2|\mathbf{k}|^2+i\epsilon}\ . \tag{37}$$

In the same way,

$$\partial_0 F_E(0,\mathbf{x})=\frac{1}{n_0^2}(\Delta_\perp^{-2}\delta^{(3)})(\mathbf{x})\ . \tag{38}$$

Straightforward manipulations introducing Eqs. (31)–(38) into Eq. (19) lead to

$$[A_\mu(x),A_\nu(x')]=-ig_{\mu\nu}D(x-x') +\frac{i(n_\mu\partial_\nu+\partial_\mu n_\nu)}{n_0^2}n^*\cdot\partial F_D'(x-x') +ia\frac{\partial_\mu\partial_\nu}{n_0^2}E_n(x-x') -i\frac{n^2}{n_0^2}(n^*\cdot\partial)^2\partial_\mu\partial_\nu F_E'(x-x')\ , \tag{39}$$

where the new functions F_D' and F_E' are, respectively,

$$F_D'(x)=F_D(x)+(\Delta_\perp^{-1}D)(x)\ , \tag{40}$$

$$F_E'(x)=F_E(x)+\frac{1}{n_0^2}(\Delta_\perp^{-2}D)(x)\ . \tag{41}$$

Their properties are given in the Appendix.

It should be noted here that the regularization $1/(k^2+i\epsilon)$ of the pole at $k^2=0$ in the definition of F_D and F_E is obtained as a result of using the causal Green's function in writing the particular solution of $\Box F=G$. Had one chosen another solution a different regularization would have been obtained. This is, therefore, just a matter of convention and has no deep meaning here. An expression for $F_D'(x)$ which is independent of this choice is given by Eq. (A27) of the Appendix.

D. Momentum-space operators

Let us consider the Fourier expansion

$$A_\mu(x)=(2\pi)^{-3/2}\int d^4k\ \tilde{A}_\mu(k)e^{-ik\cdot x} \tag{42}$$

or

$$A_\mu(x)=(2\pi)^{-3/2}\int d^4k\ \theta(k_0)\times[a_\mu(k)e^{-ik\cdot x}+b_\mu(k)e^{ik\cdot x}] \tag{43}$$

with

$$a_\mu(k)=\tilde{A}_\mu(k),\quad b_\mu(k)=\tilde{A}_\mu(-k)\ . \tag{44}$$

The operators $a_\mu(k)$ and $b_\mu(k)$ are interpreted, respectively, as linear combinations of creation and annihilation operators. The explicit form of the linear combinations is not obvious to write down. Since it is not needed here, it will be left for another publication. Hermiticity of the field imposes $a_\mu^\dagger=b_\mu$. The commutation relations satisfied by the operators $a_\mu(k)$ and $b_\mu(k)$ are

$$[a_\mu(k),a_\nu(k')]=[b_\mu(k),b_\nu(k')]=0\ , \tag{45}$$

$$\theta(k_0)[a_\mu(k),b_\nu(k')]=\left[-g_{\mu\nu}\delta(k^2)+(n_\mu k_\nu+n_\nu k_\mu)n^*\cdot k\left[\frac{\delta(n\cdot kn^*\cdot k)}{k^2+i\epsilon}+\frac{\delta(k^2)}{n^*\cdot kn\cdot k-i\epsilon}\right]-ak_\mu k_\nu\delta'(n\cdot kn^*\cdot k)\right.$$
$$\left.+n^2(n^*\cdot k)^2k_\mu k_\nu\left[\frac{\delta'(n\cdot kn^*\cdot k)}{k^2+i\epsilon}-\frac{\delta(k^2)}{(n^*\cdot kn\cdot k-i\epsilon)^2}\right]\right]\delta^{(4)}(k-k')\ . \tag{46}$$

E. The propagator

The propagator is defined by

$$D_{\mu\nu}(x)=\langle 0|T[A_\mu(x)A_\nu(0)]|0\rangle\ , \tag{47}$$

where T is the chronological product. It can be obtained from Eqs. (42)–(46) using careful manipulations in the framework of distribution theory. However, here we have

$$\left[\Box g^{\mu\nu}-\partial^\mu\partial^\nu-n^\mu n^\nu\frac{(n^*\cdot\partial)^2}{a}\right]D_{\nu\lambda}(x)=i\delta^\mu_\lambda\delta^{(4)}(x),\quad a\neq 0\ , \tag{48}$$

as can be checked from equal-time commutation relations and field equations. The propagator may be obtained directly by inverting the operator of the left-hand side of Eq. (48) using the causality requirement. It should be noted that the operator which must be inverted is $n^*\cdot\partial n\cdot\partial$ rather than $n\cdot\partial$. There is no ambiguity here in contrast with the pole indeterminacy in the usual axial gauge. In momentum space, the propagator is

$$\tilde{D}_{\mu\nu}(k)=\frac{-i}{k^2+i\epsilon}\left[g_{\mu\nu}-\frac{n_\mu k_\nu+n_\nu k_\mu}{n\cdot kn^*\cdot k+i\epsilon}n^*\cdot k-\frac{ak^2-n^2(n^*\cdot k)^2}{(n\cdot kn^*\cdot k+i\epsilon)^2}k_\mu k_\nu\right]\ , \tag{49}$$

where the $+i\epsilon$ arises, as usual, from causality. For $a=0$, Eq. (49) is the axial gauge propagator with the Leibbrandt-Mandelstam pole prescription.

III. THE NON-ABELIAN THEORY

In non-Abelian theory, the Lagrangian becomes

$$\mathcal{L}=-\tfrac{1}{4}F^\alpha_{\mu\nu}F^{\mu\nu}_\alpha-n^*\cdot\partial S_\alpha n\cdot A^\alpha-\tfrac{1}{2}aS_\alpha S^\alpha+\mathcal{L}_{\text{ghost}}\ , \tag{50}$$

where now

$$F^\alpha_{\mu\nu}=\partial_\mu A^\alpha_\nu-\partial_\nu A^\alpha_\mu+gf^\alpha_{\beta\gamma}A^\beta_\mu A^\gamma_\nu\ . \tag{51}$$

The last part $\mathcal{L}_{\text{ghost}}$ is fixed by the requirement of Becchi-Rouet-Stora (BRS) invariance, i.e., invariance under the transformations

$$\delta A^\alpha_\mu=(D_\mu\eta)^\alpha\delta\lambda,\quad \delta S_\alpha=0\ ,$$
$$\delta\eta^\alpha=gf^\alpha_{\beta\gamma}\eta^\beta\eta^\gamma\delta\lambda,\quad \delta\xi_\alpha=-S_\alpha\delta\lambda\ , \tag{52}$$

where η is the ghost, ξ is the antighost, D_μ the covariant derivative, and $\delta\lambda$ a Grassmann number. It reads

$$\mathcal{L}_{\text{ghost}}=n^*\cdot\partial\xi_\alpha(n\cdot D\eta)^\alpha\ . \tag{53}$$

Ghosts are, therefore, *a priori* present in the formulation with a ghost-ghost-gluon vertex $-in_\mu n^*\cdot\partial f^\alpha_{\beta\gamma}$ and a ghost propagator $1/(n^*\cdot kn\cdot k+i\epsilon)$.

The ghost contribution to a loop with n-external Yang-Mills legs is proportional to

$$I_n=\int d^4k\prod_{i=1}^{n}\frac{n^*\cdot k_i}{n^*\cdot k_i n\cdot k_i+i\epsilon}\ , \tag{54}$$

where $k_i=k+Q_i$, $Q_1=0$, $Q_i=\sum_{j=1}^{i-1}q_j$, q_j being the four-momentum of the jth Yang-Mills particle inserted in the loop. For the loop with two legs, for instance,

$$I_2=\int d^4k\frac{n^*\cdot k}{n^*\cdot kn\cdot k+i\epsilon}\,\frac{n^*\cdot(k+q)}{n^*\cdot(k+q)n\cdot(k+q)+i\epsilon}\ . \tag{55}$$

If one makes the substitution

$$\frac{n^*\cdot k}{n\cdot kn^*\cdot k+i\epsilon}\frac{n^*\cdot(k+q)}{n^*\cdot(k+q)n\cdot(k+q)+i\epsilon}=\frac{1}{n\cdot q}\left[\frac{n^*\cdot k}{n^*\cdot kn\cdot k+i\epsilon}-\frac{n^*\cdot(k+q)}{n^*\cdot(k+q)n\cdot(k+q)+i\epsilon}\right]$$
$$+i\epsilon\frac{n^*\cdot q}{n\cdot q}\frac{1}{n^*\cdot kn\cdot k+i\epsilon}\frac{1}{n^*\cdot(k+q)n\cdot(k+q)+i\epsilon}, \quad (56)$$

the theory of distributions tells that the last term does not contribute[19] in the limit $\epsilon\to 0$ in contrast with other regularizations. The introduction of Eq. (56) into I_2 and the change of variables from k to $k'=k+q$ in the second integral shows that I_2 vanishes.

Such a proof can easily be extended to n insertions and constitutes a rigorous version of the usual proof of ghost loop vanishing in axial gauges. It must be stressed that the vanishing of I_n is a consequence of the disappearance of the last term of Eq. (56) for $\epsilon\to 0$ and does not hold for an arbitrary regularization. The vanishing of I_n is the main advantage of the Leibbrandt-Mandelstam regularization.

IV. THE PLANAR GAUGE

Calculations with the Leibbrandt-Mandelstam prescription were also carried out in the case of the planar gauge.[22] In this particular case, the application of the formalism developed here is far from obvious and needs a deeper understanding of the structure of the Lagrangian in this gauge.

The main difficulties of the planar gauge come from the fact that the requirement of BRS invariance in its usual formulation leads to second-order derivatives in the Lagrangian[23] which reads

$$\mathcal{L}=-\tfrac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha}-\frac{1}{n^2}n\cdot A_{\alpha}\Box S^{\alpha}$$
$$+\frac{1}{2n^2}S_{\alpha}\Box S_{\alpha}+\frac{1}{n^2}\Box\xi_{\alpha}n\cdot D\eta_{\alpha}. \quad (57)$$

Quantization of a Lagrangian with second-order derivatives is not familiar. It can be carried out by following the Ostrogradski procedure which essentially amounts to introduce an auxiliary field $T=\partial_0 S$. (Internal-symmetry indices will be dropped in the remainder of this section.) Such a choice which is not even formally covariant is far from being elegant. However, it has the advantage of leading to the same result as the first-order Lagrangian differing from (57) by a four-divergence. The n^* vector is not introduced inside the Lagrangian and a strict application of the canonical formalism—i.e., (a) solve the Cauchy problem for commutators with equal-time commutators as initial data, (b) define the vacuum as a state annihilated by annihilation operators, (c) define the propagator as the vacuum expectation value of the time-ordered product of fields—will lead to the principal-value prescription for the pole in the propagator.[2]

Instead of setting $T=\partial_0 S$, let us introduce as many auxiliary fields as needed in order to satisfy BRS invariance and formal covariance as well as to introduce the n^* vector. The procedure amounts to setting

$$\Box S=n^*\cdot\partial S',\quad \Box\xi=n^*\cdot\partial\xi' \quad (58)$$

and inserting these equations into the Lagrangian which finally reads

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{n^2}n\cdot An^*\cdot\partial S'+\frac{1}{2n^2}Sn^*\cdot\partial S'$$
$$+\frac{1}{n^2}n^*\cdot\partial\xi' nD\eta-\partial_{\mu}u\,\partial^{\mu}S-un^*\cdot\partial S'$$
$$-\partial_{\mu}\xi\partial^{\mu}v-n^*\cdot\partial\xi' v. \quad (59)$$

The new auxiliary fields S', ξ', u, and v transform under a BRS transformation as

$$\delta S'=0,\quad \delta\xi'=-S'\delta\lambda,\quad \delta u=-v\delta\lambda,\quad \delta v=0. \quad (60)$$

In the free field case, the field equations in the gluon sector are

$$\partial^{\mu}F_{\mu\nu}-\frac{n_{\nu}}{n^2}n^*\cdot\partial S'=0, \quad (61)$$

$$n^*\cdot\partial(n\cdot A-\tfrac{1}{2}S+un^2)=0, \quad (62)$$

$$\tfrac{1}{2}n^*\cdot\partial S'+\Box u=0, \quad (63)$$

$$\Box S-n^*\cdot\partial S'=0, \quad (64)$$

from which one obtains

$$n^*\cdot\partial n\cdot\partial S'=0, \quad (65)$$

$$n^*\cdot\partial n\cdot\partial\partial\cdot A=0, \quad (66)$$

$$\Box A_{\nu}=\partial_{\nu}\partial\cdot A+n^*\cdot\partial n_{\nu}S', \quad (67)$$

which should be compared with Eqs. (6)–(8). They show that the fields S and u, while having their own dynamics, are really auxiliary fields. Indeed, they do not occur in couplings and they do not contribute to the solution of A_{μ} in the free field case.

By following the same methods as in the previous sections, one easily gets the propagator

$$D_{\mu\nu}(k)=\frac{-i}{k^2+i\epsilon}\left[g_{\mu\nu}-\frac{k_{\mu}n_{\nu}+n_{\mu}k_{\nu}}{n\cdot kn^*\cdot k+i\epsilon}n^*\cdot k\right]$$

by defining it either as the time-ordered product of free fields or as the causal Green's function. In the latter case, the Leibbrandt-Mandelstam prescription comes out because the fundamental operator in the theory is $n^*\cdot\partial n\cdot\partial$ instead of the $n\cdot\partial$ operator in usual formulations of axial or planar gauges.

The ghost loop vanishing holds also here because the ghost term in (59) has essentially the same structure as in (53).

V. REMARKS AND CONCLUSIONS

Let us first remark that the Lagrangian (50) involves exactly the same number of degrees of freedom as the relativistic gauges. It may, therefore, be studied by the same quantization methods as those used in the relativistic gauges, i.e., BRS quantization. This is in contrast with the usual realization of axial gauges through the gauge-fixing term $Sn\cdot A$, which cannot be treated by BRS methods since it involves only three degrees of freedom. In such a case, one has to proceed to a quantization "in the manner of Dirac," i.e., to restrict physical states by imposing that the Gauss law be verified for such states. In both cases, unphysical states are involved. The structure of the Fock space in the free field theory as well as the discussion of Poincaré covariance in the physical subspace are left for another publication.

Here, physical states are cohomology classes of the BRS operator as in relativistic gauges. The main difference with relativistic gauges is the vanishing of ghost loops, a property which is usually assumed in axial gauges, although not always true.[7,10] It strongly depends on the regularization and, in particular, on the occurrence or not of the last term in decomposition of the type (56). This last term can also give rise to singularities[22] in gluon loop contribution.

The second remark concerns the question "why must n_0 and $\mathbf{n}$ be kept different from zero?" The answer is simply that, if $n_0=0$ in the starting Lagrangian, $\pi^0=0$ is a constraint and the theory reduces to the ill-defined spacelike axial gauge.[15] If $\mathbf{n}=0$, the theory reduces to the usual temporal gauge with the principal-value prescription.[2] Let us note that the limits $n_0\to 0$ or $\mathbf{n}\to 0$ can be taken at the end of the calculations. For $\mathbf{n}\to 0$, what is obtained is not the pure temporal gauge but the static temporal gauge $\partial_0 A_0=0$ which has some advantages in the case of finite-temperature problems.[24] Of course, other values of n^* can be used provided $n^*\cdot\partial n\cdot\partial$ remains a hyperbolic operator. The choice $n^*=(n_0,-\mathbf{n})$ is particularly well suited for equal-time quantization but the final answer is given in a formally covariant way, independently of any choice for n or n^*.

For instance, the choice of Hüffel, Landshoff, and Taylor[18] $n=(1,0,0,0)$, $n^*=(0,1,0,0)$ can be recovered in our formalism by setting

$$x_0=\frac{1}{\sqrt{2}}(x_0'-x_1'),\quad x_1=\frac{1}{\sqrt{2}}(x_0'+x_1')$$

and carrying out the equal-time quantization in the primed coordinate frame. Various other choices are discussed in Ref. 20.

One could raise the objection that in our approach the gauge condition is, for $a=0$, $n^*\cdot\partial n\cdot A=0$ rather than $n\cdot A=0$. However, for $a=0$, the propagator (49) satisfies $n^\mu D_{\mu\nu}=0$ and the solution of $n^*\cdot\partial n\cdot A=0$ which vanishes at infinity is $n\cdot A=0$. Therefore, with this familiar additional constraint, the gauge described here corresponds to the usual axial gauge.

As a last comment, one should note that the Leibbrandt-Mandelstam prescription is limited to the Lagrangian described here [up to a gauge parameter in the planar gauge fixing $(a/2n^2)Sn^*\cdot\partial S'$]. Indeed, if one tries to be more general and replaces the ak^2 term by an arbitrary function $A(k)$ in the propagator (49) and inverts it by imposing locality of the Lagrangian, there are only two solutions $A(k)=ak^2$ and $A(k)=an^2(n^*\cdot k)^2$, which are the cases discussed here.

The conclusion of this paper is very short. The formalism which is built up here sets the Leibbrandt-Mandelstam prescription for axial gauges on exactly the same footing as relativistic gauges but with vanishing ghost loops. It cures all the previous difficulties encountered with axial gauges which are not on the same degree of rigor as relativistic gauges.

APPENDIX: SPECIAL FUNCTIONS IN THE LEIBBRANDT-MANDELSTAM REGULARIZED AXIAL GAUGES

1. Usual special functions

The special functions of interest in free covariant field theory are

$$\Delta(x;m^2)=\frac{-i}{(2\pi)^3}\int d^4k\,\epsilon(k_0)\delta(k^2-m^2)e^{-ik\cdot x}=\Delta^{(+)}(x;m^2)+\Delta^{(-)}(x;m^2)\ ,\qquad\text{(A1)}$$

where

$$\Delta^{(\pm)}(x;m^2)=\mp\frac{i}{(2\pi)^3}\times\int d^4k\,\theta(\pm k_0)\delta(k^2-m^2)e^{-ik\cdot x}\qquad\text{(A2)}$$

are, respectively, positive- and negative-frequency elementary solutions of the Klein-Gordon equation

$$(\Box+m^2)\Delta^{(\pm)}(x;m^2)=0\ .\qquad\text{(A3)}$$

The function $\Delta(x;m^2)$ is odd and satisfies

$$\Delta(0,\mathbf{x};m^2)=0,\quad \partial_0\Delta(0,\mathbf{x};m^2)=-\delta^{(3)}(\mathbf{x})\ .\qquad\text{(A4)}$$

The propagator is given by

$$i\Delta_F(x;m^2)=\theta(x_0)\Delta^{(+)}(x;m^2)-\theta(-x_0)\Delta^{(-)}(x;m^2)=\frac{-i}{(2\pi)^4}\int d^4k\frac{e^{-ik\cdot x}}{k^2-m^2+i\epsilon}\ .\qquad\text{(A5)}$$

All these functions have a well-defined limit for $m^2\to 0$, which is usually denoted by the corresponding D symbol.

2. The Δ_n special functions

In some of the equations of this paper, the d'Alembert operator is replaced by $n^*\cdot\partial n\cdot\partial$. It is interesting to keep a parameter m^2, although it cannot be interpreted as a square mass. In the same way as above, one defines

$$\Delta_n(x;m^2)=\frac{-in_0^2}{(2\pi)^3}\int d^4k\,\epsilon(k_0)\times\delta(n\cdot kn^*\cdot k-m^2)e^{-ik\cdot x}\qquad\text{(A6)}$$

as well as the corresponding $\Delta_n^{(\pm)}(x;m^2)$ and $\Delta_{F,n}^{(\pm)}(x;m^2)$ by replacing everywhere k^2 by $n\cdot kn^*\cdot k$.

We still have

$$\Delta_n(0,\mathbf{x};m^2)=0\ ,\quad \partial_0\Delta_n(0,\mathbf{x};m^2)=-\delta^{(3)}(\mathbf{x})\qquad\text{(A7)}$$

as well as

$$(n^*\cdot\partial n\cdot\partial+m^2)\Delta_n^{(\pm)}(x;m^2)=0\ , \quad (A8)$$

$$(n^*\cdot\partial n\cdot\partial+m^2)\Delta_{F,n}(x;m^2)=\delta^{(4)}(\mathbf{x})\ . \quad (A9)$$

3. The E_n special function

When one computes

$$\int d^4z\ \Delta_{F,n}(x'-z)\Delta_n(x-z)\ ,$$

one gets the formal result

$$\frac{in_0^2}{(2\pi)^3}\int d^4k\frac{\epsilon(k_0)e^{-ik\cdot(x-x')}}{k\cdot nk\cdot n^*-m^2+i\epsilon}\delta(n\cdot kn^*\cdot k-m^2+i\epsilon)$$

which is ill defined. The correct definition is

$$E_n(x-x';m^2)=\frac{-in_0^2}{(2\pi)^3}\int d^4k\ \epsilon(k_0)e^{-ik\cdot(x-x')}\times\delta'(n\cdot kn^*\cdot k-m^2+i\epsilon)\ . \quad (A10)$$

It obviously satisfies

$$(n\cdot\partial n^*\cdot\partial+m^2)E_n(x;m^2)=\Delta_n(x;m^2)\ , \quad (A11)$$

$$E_n(x;m^2)=-\frac{\partial}{\partial m^2}\Delta_n(x;m^2)\ . \quad (A12)$$

Since E_n is still odd,

$$E_n(0,\mathbf{x};m^2)=0 \quad (A13)$$

while

$$\partial_0E_n(0,\mathbf{x};m^2)=0 \quad (A14)$$

is a consequence of (A7) and (A12).

The corresponding $E_{F,n}$ satisfies

$$(n\cdot\partial n^*\cdot\partial+m^2)E_{F,n}(x)=D_{F,n}(x) \quad (A15)$$

and is given by

$$E_{F,n}(x)=\frac{in_0^2}{(2\pi)^4}\int d^4k\frac{e^{-ik\cdot x}}{(n\cdot kn^*\cdot k-m^2+i\epsilon)^2}\ . \quad (A16)$$

The corresponding $E(x;m^2)$ is met in relativistic gauges of the free Maxwell theory.

4. The F_Δ and F'_Δ special functions

The F_Δ special function is defined by

$$F_\Delta(x-x';m^2)=\int d^4z\ D_F(x'-z)\Delta_n(x-z;m^2)$$
$$=\frac{in_0^2}{(2\pi)^3}\int d^4k\frac{e^{-ik\cdot(x-x')}}{k^2+i\epsilon}\epsilon(k_0)\times\delta(n\cdot kn^*\cdot k-m^2+i\epsilon)\ . \quad (A17)$$

It obviously satisfies

$$\Box F_\Delta(x;m^2)=\Delta_n(x;m^2)\ , \quad (A18)$$

$$(n^*\cdot\partial n\cdot\partial+m^2)F_\Delta(x;m^2)=0\ . \quad (A19)$$

It is again an odd function, so that

$$F_\Delta(0,\mathbf{x};m^2)=0\ . \quad (A20)$$

One can easily compute, at least formally,

$$\partial_0F_\Delta(0,\mathbf{x};m^2)=\frac{n_0^2}{(2\pi)^3}\int\frac{d^3k\ e^{i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{n}\cdot\mathbf{k})^2+m^2-|\mathbf{k}|^2n_0^2+i\epsilon} \quad (A21)$$

which one formally writes as

$$\partial_0F_\Delta(0,\mathbf{x};m^2)=(\Delta_\perp+m^2)^{-1}\delta^{(3)}(\mathbf{x})\ , \quad (A22)$$

where the operator $(\Delta_\perp+m^2)^{-1}$ is symmetric and is represented in momentum space by

$$(\Delta_\perp+m^2)^{-1}(k)=\frac{n_0^2}{(\mathbf{n}\cdot\mathbf{k})^2+m^2-n_0^2|\mathbf{k}|^2+i\epsilon}. \quad (A23)$$

In the commutator, the function F_Δ is replaced by the function

$$F'_\Delta(x;m^2)=F_\Delta(x;m^2)+(\Delta_\perp+m^2)^{-1}D(x)\ . \quad (A24)$$

Its representation by Fourier integral is

$$F'_\Delta(x)=\frac{in_0^2}{(2\pi)^3}\int d^4k\ e^{-ik\cdot x}\epsilon(k_0)\times\left[\frac{\delta(n\cdot kn^*\cdot k-m^2)}{k^2+i\epsilon}+\frac{\delta(k^2)}{n^*\cdot kn\cdot k-m^2-i\epsilon}\right]. \quad (A25)$$

Using

$$\delta(x)=\frac{-1}{2i\pi}\left[\frac{1}{x+i\epsilon}-\frac{1}{x-i\epsilon}\right]\ , \quad (A26)$$

one can write

$$F'_\Delta(x)=\frac{n_0^2}{(2\pi)^4}\int d^4k\ e^{-ik\cdot x}\epsilon(k_0)\times\left[\frac{1}{k^2+i\epsilon}\frac{1}{n^*\cdot kn\cdot k-m^2+i\epsilon}-\frac{1}{k^2-i\epsilon}\frac{1}{n^*\cdot kn\cdot k-m^2-i\epsilon}\right]\ , \quad (A27)$$

where now the $\pm i\epsilon$ are fixed while in (A25) they could be interchanged. It satisfies

$$\Box F'_\Delta=\Delta_n,\quad (n^*\cdot\partial n\cdot\partial+m^2)F'_\Delta=n_0^2\Delta\ ,$$
$$\partial_0F'_\Delta(0,\mathbf{x})=0\ . \quad (A28)$$

The corresponding $F'_{\Delta,F}$ is

$$F'_{\Delta,F}(x)=\frac{in_0^2}{(2\pi)^4}\int d^4k\frac{e^{-ik\cdot x}}{(k^2+i\epsilon)(n^*\cdot kn\cdot k-m^2+i\epsilon)}\ . \quad (A29)$$

It satisfies

$$\Box F'_{\Delta,F}=\Delta_{F,n}, \quad (n^*\cdot\partial n\cdot\partial+m^2)F'_{\Delta,F}=n_0^2\Delta_F \ . \tag{A30}$$

5. The F_E and F'_E special functions

The function F_E is defined by

$$\begin{aligned}F_E(x-x';m^2)&=\int d^4z\, D_F(x'-z)E_n(x-y;m^2)\\&=\frac{in_0^2}{(2\pi)^3}\int d^4k\frac{e^{-ik\cdot(x-x')}}{k^2+i\epsilon}\\&\qquad\times\epsilon(k_0)\delta'(n\cdot kn^*\cdot k-m^2)\ .\end{aligned} \tag{A31}$$

It satisfies

$$\Box F_E(x-x';m^2)=E_n(x-x';m^2)\ , \tag{A32}$$

$$(n^*\cdot\partial n\cdot\partial+m^2)F_E(x-x';m^2)=F_\Delta(x-x';m^2)\ , \tag{A33}$$

$$F_E(x;m^2)=-\frac{\partial}{\partial m^2}F_\Delta(x;m^2)\ . \tag{A34}$$

The properties of $F_E(0,\mathbf{x};m^2)$ and $\partial_0F_E(0,\mathbf{x};m^2)$ are easily obtained from those of $F_\Delta(0,\mathbf{x};m^2)$ using Eq. (A32). One gets

$$\begin{aligned}&F_E(0,\mathbf{x};m^2)=0\ ,\\&\partial_0F_E(0,\mathbf{x};m^2)=\frac{1}{n_0^2}(\Delta_\perp+m^2)^{-2}\delta^{(3)}(\mathbf{x})\ .\end{aligned} \tag{A35}$$

The function

$$F'_E(x;m^2)=F_E(x;m^2)+\frac{1}{n_0^2}(\Delta_\perp+m^2)^{-2}D(x) \tag{A36}$$

satisfies

$$\partial_0F'_E(0,\mathbf{x};m^2)=0\ . \tag{A37}$$

Its properties are

$$F'_E=\frac{in_0^2}{(2\pi)^3}\int d^4k\, e^{-ik\cdot x}\epsilon(k_0)\times\left[\frac{\delta'(n\cdot kn^*\cdot k-m^2)}{k^2+i\epsilon}-\frac{\delta(k^2)}{(n\cdot kn^*\cdot k-m^2-i\epsilon)^2}\right], \tag{A38}$$

$$\Box F'_E=E_n, \quad (n^*\cdot\partial n\cdot\partial+m^2)F'_E=F'_D\ . \tag{A39}$$

The corresponding $F'_{E,F}$ is

$$F'_{E,F}(x)=\frac{-in_0^2}{(2\pi)^4}\int d^4k\frac{e^{-ik\cdot x}}{(k^2+i\epsilon)(n^*\cdot kn\cdot k-m^2+i\epsilon)^2}\ . \tag{A40}$$

It satisfies

$$\Box F'_{E,F}(x)=E_{F,n}(x), \quad (n^*\cdot\partial n\cdot\partial+m^2)F'_{E,F}=F'_{\Delta,F}\ . \tag{A41}$$

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