# Triple-product correlations in semileptonic decays

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The use of triple-product correlations to signal the presence of $CP$ violations in decays of charged pseudoscalar mesons is studied. It is pointed out that observation of such correlations in the semileptonic modes is likely to be evidence for new physics. The effect of unitarity phases is discussed and a detailed analysis of the Weinberg Higgs-boson model of $CP$ violation is carried out for charged $B$ mesons and kaons.

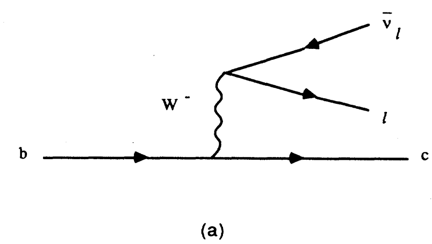
## I. $B$ SEMILEPTONIC DECAY IN THE HIGGS-BOSON MODEL

$CP$ violation has thus far been observed only in the $K^0$-$\bar{K}^0$ system.[1] Two kinds of effects have been found: a nonzero $K_L \to \pi\pi$ branching ratio and an asymmetry in $K_L \to \pi l \bar{\nu}$ semileptonic decay. Other manifestations of $CP$ violations have been searched for, most notably the electric dipole moment of the neutron. In this paper we shall consider the occurrence of $CP$-violating triple-product correlations in the semileptonic decay modes of charged $B$ mesons and kaons.

Experimental studies of $CP$ violation in the system of hadrons containing a $b$ quark will be of particular importance.[2] The reason is that the $b$-quark mass provides a large energy scale relative to that of the light quarks. This allows in principle a probe of the energy dependence of $CP$-violating interactions. If the Kobayashi-Maskawa (KM) model[3] is indeed the source of these effects, then no important energy dependence is expected. However, $CP$ violations are presumably associated with electroweak symmetry breaking which, via Higgs-type mechanisms, could lead to nontrivial energy-dependent structure.

The recent determination[4] of $\epsilon'$ has eliminated the superweak models as viable candidates for $CP$ violations. Although kaon data is in accord with the KM description, other models pass the test as well. For definiteness, we shall consider one of these, due to Weinberg[5] and called the Higgs-boson model hereafter. In this approach, $CP$ violations are generated from interactions of charged Higgs bosons. The occurrence of three Higgs doublets in this model could arise from underlying supersymmetric or technicolor dynamics. In the Higgs-boson model, the dominant $CP$-violating amplitude for kaons is a penguin-type transition: $s$ quark$\to d$ quark$+$gluon. The related penguin amplitude $b$ quark$\to s$ quark$+$gluon has been studied in Ref. 6 by using the observed strength of kaon $CP$ violations as input normalization.[7] The phenomenology of Higgs-boson-induced $b$-quark nonleptonic decays is developed in Ref. 6, with perhaps the most striking signature being a "charm deficit."[8] Interestingly, the Higgs-boson model predicts a neutron electric dipole moment near to the present level of experimental sensitivity.[9] Although there are indications in both the Grenoble and Leningrad experiments that a signal is being seen, more analysis is required before firm conclusions are forthcoming. At any rate, we shall use the Higgs-boson model in developing the phenomenology of triple-product correlations in semileptonic decay.[10]

We begin by carrying out an analysis at the quark level. The semileptonic quark transition $b(\mathbf{k},s_b) \to c(\mathbf{q},s_c)+l(\mathbf{p},s_l)+\bar{\nu}_l(\mathbf{p}',s_\nu)$ can proceed via the exchange of the gauge boson $W^-$ and Higgs particles $H_i^-$ $(i=1,2)$ as depicted in Figs. 1(a) and 1(b), respectively. Neither transition is of a penguin type. The charged-Higgs-boson vertices[11] favor $\tau$-lepton emission, the total invariant amplitude of which is



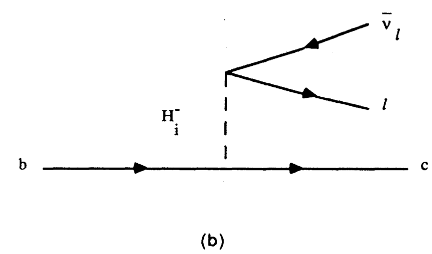




FIG. 1. Transition $b \to c l \bar{\nu}_l$ as mediated by (a) $W^-$ gauge boson, (b) $H_i^-$ Higgs bosons.

$$M=M_W+M_H=\frac{G_F}{\sqrt{2}}V_{cb}[\bar{u}(\mathbf{q},s_c)\gamma^\mu(1+\gamma_5)u(\mathbf{k},s_b)\bar{u}(\mathbf{p},s_l)\gamma_\mu(1+\gamma_5)v(\mathbf{p}',s_\nu)$$
$$+m_b m_\tau \bar{u}(\mathbf{q},s_c)(C+D\gamma_5)u(\mathbf{k},s_b)\bar{u}(\mathbf{p},s_l)(1+\gamma_5)v(\mathbf{p}',s_\nu)]\ , \tag{1}$$

where

$$C=\sum_i \frac{\alpha_i\bar{\alpha}_i^*}{m_{H_i}^2}+\frac{m_c}{m_b}\sum_i\frac{\beta_i\bar{\alpha}_i^*}{m_{H_i}^2}\ ,$$
$$D=-\sum_i \frac{\alpha_i\bar{\alpha}_i^*}{m_{H_i}^2}+\frac{m_c}{m_b}\sum_i\frac{\beta_i\bar{\alpha}_i^*}{m_{H_i}^2}\ . \tag{2}$$

The quantities $m_{H_i}$ $(i=1,2)$ are the charged-Higgs-boson masses and the following combinations of Higgs-boson mass-matrix mixing angles appear:

$$\alpha_1=s_1^Hc_3^H/c_1^H,\quad \alpha_2=s_1^Hs_3^H/c_1^H\ ,$$
$$\bar{\alpha}_1=(c_1^Hs_2^Hc_3^H-c_2^Hs_3^He^{i\delta_H})/s_1^Hs_2^H\ ,$$
$$\bar{\alpha}_2=(c_1^Hs_2^Hs_3^H+c_2^Hc_3^He^{i\delta_H})/s_1^Hs_2^H\ , \tag{3}$$
$$\beta_1=(c_1^Hc_2^Hc_3^H+s_2^Hs_3^He^{i\delta_H})/s_1^Hc_2^H\ ,$$
$$\beta_2=(c_1^Hc_2^Hs_3^H-s_2^Hc_3^He^{i\delta_H})/s_1^Hc_2^H\ .$$

We refer the reader to Refs. 6, 7, and 11 for a review of the Higgs-boson model, noting briefly that the analysis of Ref. 6 mitigates against the heavy-mass limit $m_{H_i}\geq 100$ GeV. Taking $m_c=1.5$ GeV, $m_b=5.0$ GeV, we obtain for the spin-averaged decay rate

$$\Gamma(b\to c\tau\bar{\nu}_\tau)=\frac{G_F^2m_b^5|V_{cb}|^2}{192\pi^3}\times\left[0.13+0.15\,\mathrm{Re}\sum_i\alpha_i\bar{\alpha}_i^*\left[\frac{m_\tau}{m_{H_i}}\right]^2+0.03\left|\sum_i\alpha_i\bar{\alpha}_i^*\left[\frac{m_c}{m_{H_i}}\right]^2\right|^2\right]\ . \tag{4}$$

The effect of the Higgs-boson-exchange diagrams appears mainly in the interference term. Given the current mass bound for charged scalars[12] $m_{H_i}>19$ GeV, we estimate with large uncertainties this term to contribute at the level of a percent over all but a limited range of the parameter space of Higgs-boson angles. Because of the lepton-mass dependence, electron and muon emissions are greatly suppressed, so universality is violated. Of course, a lack of universality need not imply $CP$ violation. Therefore, we turn next to the subject of $CP$-violating signals.

## II. *CP*-VIOLATING SIGNALS

We can divide possible signals into three types. The first two types correspond to asymmetries in the angular distribution of the decay products. One of them appears from the scalar product of a momentum and polarization, provided that both the momentum and polarization of the $c$ quark and the lepton can be observed. Another appears as a triple-product correlation. For inclusive semileptonic decay, this involves some combination of the momentum and spin vectors, say, both momenta and one of the polarizations, or both polarizations and one momentum. The third type corresponds to a partial-rate asymmetry of the form

$$\frac{\Gamma(b\to c+l+\bar{\nu})-\Gamma(\bar{b}\to\bar{c}+\bar{l}+\nu)}{\Gamma(b\to c+l+\bar{\nu})+\Gamma(\bar{b}\to\bar{c}+\bar{l}+\nu)}\ . \tag{5}$$

It is well known[1] that the $CPT$ theorem requires the presence of a "unitarity" phase (arising from final-state interactions or from real intermediate states) in addition to a $CP$-violating phase in order to generate either the partial-rate asymmetry of Eq. (5) or an angular correlation of the first type. Triple-product signals, on the other hand, arise from the interference of two amplitudes with a relative $CP$-violating phase, and do not require any additional unitarity phases. The issue of unitarity phase is an important, but nettlesome, aspect of observing $CP$-violating correlations. For any given process, if unitarity phases are much smaller than the $CP$-violating phase associated with two interfering amplitudes, triple-product correlations are the dominant signals of $CP$ violation and indeed are the only ones likely to be detectable. Even if unitarity phases and intrinsic $CP$-violating phases are small but of the same order (say, $\eta$), triple-product correlations will be $O(\eta)$ whereas the partial-rate asymmetries will be $O(\eta^2)$ and again much smaller. Unfortunately there is no guarantee in general that unitarity phases are negligible. In some instances, as in semileptonic kaon decay, they have been estimated. We shall return to this point in Sec. VI. In other cases, as in $b$ decay theoretical estimates are less meaningful due, e.g., to the presence of many intermediate states. At any rate, there is no reason to assume that they are suppressed. Fortunately there are at least two ways to identify the truly $CP$-violating signals for the case under discussion of $b$ decay. The first one, already suggested by Weinberg,[5] consists of comparing the asymmetry for decay into $\tau^-$ and decay into $e^-$. If one is seeing a $CP$ violation, the latter is much smaller. On the contrary, if one is seeing unitarity phases the two should be of comparable size. The second standard method consists of comparing the asymmetry in $b\to\bar{c}+\tau^++\nu_\tau$ to the corresponding asymmetry in the $CP$-conjugate process $\bar{b}\to\bar{c}+\tau^++\nu_\tau$. The difference in asymmetries vanishes if they are being faked by unitarity phases.

Perhaps, the most experimentally straightforward kind of triple-product correlation in a decay process is one involving just momentum three-vectors. Such a correlation must necessarily involve at least *four* final-state particles.

This can be understood by considering the rest frame of the decaying particle and invoking momentum conservation. The number of independent three-momenta is one less than the number of final-state particles, so a triple product composed entirely of momenta requires four particles in the final state. We shall encounter an example of this later in Sec. V. Unfortunately, a four-particle final state can be difficult to make theoretical predictions about. If experimentally feasible, a reasonable compromise between theory and experiment might instead be to consider two spatial momenta and one polarization vector, or perhaps one momentum and two polarizations. For this class of triple-product correlations, the inclusive semileptonic mode, where the charged lepton and a quark jet are detected, is a natural place to look

### III. TRIPLE-PRODUCT CORRELATIONS: INCLUSIVE *B* SEMILEPTONIC DECAY

In the standard model, the lowest-order amplitude contributing to the decay $b \to c + l + \bar{v}_l$ is the *W*-exchange graph of Fig. 1(a). Before analyzing the Higgs model, we first investigate whether radiative corrections *within the standard model* can generate the interference necessary for triple-product correlations. One class of higher-order amplitudes, as in Figs. 2(a) and 2(b), has the same *CP* phase as the *W*-exchange amplitude. Such processes do not give rise to a *CP*-odd observable. However, Fig. 3 displays a radiative correction which does lead to *CP*-odd observables, but the effect appears not to be large. For the semileptonic decay $P \to P' + l + \bar{v}_l$, where $P\,(P')$ is the initial (final) pseudoscalar meson, the *CP*-violating (*CPV*) amplitude of Fig. 3 is estimated to be

$$M(CPV) = \langle P' l \bar{v}_l | H_W | P'' \rangle (m_{P''}^2 - m_{P'}^2)^{-1} \mathrm{Im} \langle P'' | H_W | P \rangle \ , \tag{6}$$

where $P''$ is an intermediate-state pseudoscalar meson. The relative *CP* violation is

$$M(CPV)/M(CP) = R\,(m_{P''}^2 - m_{P'}^2)^{-1} \mathrm{Im} \langle P'' | H_W | P \rangle \ , \tag{7}$$

where $R = \langle P' l \bar{v}_l | H_W | P'' \rangle / \langle P' l \bar{v}_l | H_W | P \rangle$ typically depends on KM mixing angles. The Fermi constant establishes the scale of this effect, and, for example, we show in Sec. IV that for kaon decay it is extremely small.

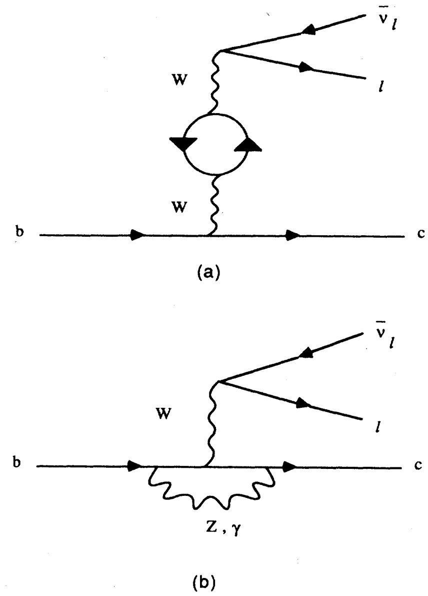



FIG. 2. Standard-model radiative corrections which have the same *CP* phase as the tree diagram.

The Higgs-model amplitude of Fig. 1(b) also interferes with that of Fig. 1(a) to give rise to triple-product correlations signaling *CP* violation. This possibility was actually pointed out originally by Weinberg.[5] It turns out that because such correlations are proportional to the lepton mass, they are relatively suppressed in the kaon sector. In the *B* system we have the possibility of a decay into a massive $\tau$ lepton and the signals are expected to be larger. In the following, consider inclusive semileptonic decay with the appearance of a quark jet. Then still restricting ourselves to the quark level, we first consider the case of two momenta and a spin. Suppose the $\tau$ lepton has a definite spin $s_\tau$, $b(\mathbf{k}) \to c(\mathbf{q}) + \tau(\mathbf{p}, s_\tau) + \bar{v}_\tau$. We define the experimental observable

$$A = \frac{N_{\text{events}}(\mathbf{q}\cdot\mathbf{p}\times\mathbf{s}_\tau > 0) - N_{\text{events}}(\mathbf{q}\cdot\mathbf{p}\times\mathbf{s}_\tau < 0)}{N_{\text{events}}(\mathbf{q}\cdot\mathbf{p}\times\mathbf{s}_\tau > 0) + N_{\text{events}}(\mathbf{q}\cdot\mathbf{p}\times\mathbf{s}_\tau < 0)} \ . \tag{8}$$

The total amplitude for *W* and *H* exchange is given in Eq. (1). If we square this, average over the *b* spin, and sum over the *c* and $\bar{v}_\tau$ spins, the interference term has among others a piece that looks like

$$\tfrac{1}{2} \sum_{s_b s_c s_{\bar{v}}} |M_W + M_H|^2 = \cdots + \frac{G_F^2}{2} |V_{cb}|^2 2 m_b m_\tau \mathrm{Im} \left[ \sum_i \frac{\alpha_i \bar{\alpha}_i^*}{M_{H_i}^2} \right] \epsilon^{\mu\nu\alpha\beta} k_\mu q_\nu p_\alpha s_\beta \ , \tag{9}$$

where we neglect the term $O(m_c/m_b)$. Observe that in the *b*-quark rest frame this term corresponds to the triple product $\mathbf{q}\times\mathbf{p}\cdot\mathbf{s}$ of the type we are after. To calculate the asymmetry of Eq. (9) we divide the phase space into the regions where the triple product has a definite sign, and integrate. For the ratio $m_c/m_b = 0.3$ we obtain

$$A = \frac{0.013 m_b m_\tau \dfrac{c_2^H s_3^H c_3^H s_\delta^H}{c_1^H s_2^H} \displaystyle\sum_i \frac{(-)^{i+1}}{m_{H_i}^2}}{0.13 + 0.15 m_\tau^2 \mathrm{Re} \displaystyle\sum \frac{\alpha_i \bar{\alpha}_i^*}{m_{H_i}^2} + 0.03 \left| \sum_i \alpha_i \bar{\alpha}_i^* \left( \frac{m_c}{m_{H_i}} \right)^2 \right|^2} \ . \tag{10}$$

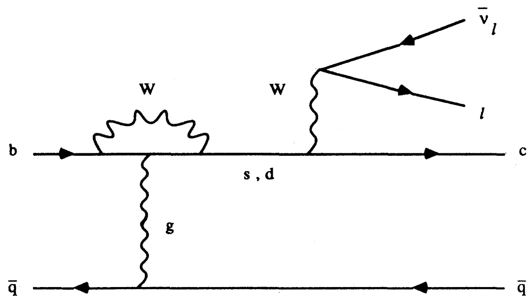



FIG. 3. Standard-model radiative correction which has a different $CP$ phase than the tree diagram.

Observe that this asymmetry is independent of the KM phase $\delta$. This is particularly useful in models with explicit $CP$ violation in the Higgs sector where in addition to the phase $\delta_H$ one can also have $\delta_{\rm KM}\neq 0$. Also, it does not seem to be very sensitive to the quark mass because details of phase space cancel out in the ratio of Eq. (10). The $CP$-violating signal is seen to vanish in any of the limits $\delta^H=0$, $\theta_3^H=0$, $\theta_2^H=\pi/2$, or $m_{H_1}=m_{H_2}$. To estimate $A$, we can rewrite the finding of Ref. 7 for kaon $CP$ violation in the Higgs model as

$$\frac{c_2^H s_3^H c_3^H s_\delta^H}{c_1^H s_2^H}\sum_i(-)^{i+1}\frac{m_b m_\tau}{m_{H_i}^2}\left[\ln\frac{m_{H_i}^2}{m_c^2}-\frac{3}{2}\right]\simeq -0.1\cot^2\theta_2^H \tag{11}$$

provided, as seems likely, the Higgs-penguin vertex is $c$-quark dominated. Incidentally, this analysis incorporates corrections to the Higgs model of Ref. 7. Previous studies of semileptonic transitions were based on earlier and incorrect studies of $K\to 2\pi$ within the Higgs model,[13,14] as well as unrealistically low estimates of the Higgs-boson mass. A simple expression for the magnitude of the asymmetry which is valid over most of parameter space is obtained by replacing the slowly varying logarithmic factor in Eq. (11) by an estimated value of, say, 5 and noting that for $\theta_2^H$ bounded away from 0 and $\pi$, the first term in the denominator of Eq. (10) will dominate the others. We then find $|A|\approx 0.002\cot^2\theta_2^H$, which is mainly dependent on the angle $\theta_2^H$. The significance of the quantity $\cot^2\theta_2^H$ can be understood as follows. In the Higgs-boson model, there are three vacuum expectation values $v_{1,2,3}$ each associated with one of the input Higgs doublets, whose squared sum is constrained to equal $(2\sqrt{2}G_F)^{-1}$. It turns out that $\cot^2\theta_2^H=(v_2/v_3)^2$, so the asymmetry $A$ gives information on the ratio of certain vacuum expectation values in the Higgs-boson model. As $\theta_2^H$ nears $0,\pi$, the exact expression of Eq. (10) should be employed.

The process just considered involves measuring lepton polarization. Alternatively we can sum over the lepton spin and allow the $c$ quark to have a definite polarization. This appears to have two features noteworthy of comment. First one can no longer look at a jet originating from the $c$ quark, but one must instead consider the exclusive channel where the $c$ quark hadronizes into a $D^*$. We shall consider this case in Sec. V. Apart from the hadronization issue, a quark-level calculation reveals (denoting the asymmetry by $\bar{A}$) a source of suppression arising from mass factors,

$$\begin{aligned}\bar{A}&=a\left[\frac{m_c m_\tau}{m_b^2}\right]\mathrm{Im}\sum_i\frac{\beta_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b m_\tau\\&=\left[\frac{m_c m_\tau}{m_b^2}\right]\frac{\cot^2\theta_1}{\cos^2\theta_2}A\ .\end{aligned} \tag{12}$$

If the phases in $\beta_i$ are similar to those in $\alpha_i$ this is smaller than the result in Eq. (10) by a factor of about 10 due to the modified mass ratio. Note, however, that if the angle $\theta_1^H$ is small, $\bar{A}$ could be larger than $A$.

Finally we can consider the situation where *both* the $c$ quark and the lepton are polarized. In this case we encounter two types of triple-product correlations. The first one involves two momenta and one of the spins occurring in the combination $(\mathbf{s}_i\cdot\mathbf{p}_j)\mathbf{s}'\cdot\mathbf{p}_i\times\mathbf{p}_k$. However, this is just a more complicated form of the asymmetries already discussed. Therefore, we concentrate on the second kind, one that involves both spins. The relevant terms in the square of the matrix element appear as

$$\begin{aligned}&\frac{m_c}{m_b}(E_c-m_b)\\&\quad\times\mathrm{Im}\left[\sum_i\frac{\alpha_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b^2+\frac{E_c}{m_b}\sum_i\frac{\beta_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b^2\right]\mathbf{p}\cdot(\mathbf{s}_c\times\mathbf{s}_\tau)\\&\quad+\frac{m_c}{m_b}\mathrm{Im}\left[-E_\tau\sum_i\frac{\alpha_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b^2+\frac{p\cdot p'}{m_b}\sum_i\frac{\beta_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b^2\right]\\&\quad\times\mathbf{q}\cdot(\mathbf{s}_c\times\mathbf{s}_\tau)\ .\end{aligned} \tag{13}$$

These give rise to two asymmetries $B$ and $\bar{B}$ we define as

$$B=\frac{N_{\rm events}(\mathbf{p}\cdot\mathbf{s}_c\times\mathbf{s}_\tau>0)-N_{\rm events}(\mathbf{p}\cdot\mathbf{s}_c\times\mathbf{s}_\tau<0)}{N_{\rm total}} \tag{14}$$

and $\bar{B}$ the same with $\mathbf{p}$ replaced by $\mathbf{q}$. Upon integration over phase space we obtain

$$\begin{aligned}B&=\frac{m_\tau m_c}{m_b^2}\left[a_1\mathrm{Im}\sum_i\frac{\alpha_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b^2+a_2\mathrm{Im}\sum_i\frac{\beta_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b^2\right],\\\bar{B}&=\frac{m_\tau m_c}{m_b^2}\left[\bar{a}_1\mathrm{Im}\sum_i\frac{\alpha_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b^2+\bar{a}_2\mathrm{Im}\sum_i\frac{\beta_i\bar{\alpha}_i^*}{m_{H_i}^2}m_b^2\right],\end{aligned} \tag{15}$$

where $a_1\approx 0.29$, $a_2\approx 0.1$, $\bar{a}_1\approx 0.2$, $\bar{a}_2\approx 0.07$. We then see that $B$, $\bar{B}$ are smaller than the asymmetry $A$ of Eq. (10) by a factor $m_c/m_b$. In this case we note that in order to eliminate signals faked by unitarity phases one would add the asymmetries of $b$ and $\bar{b}$ decays. Since these two-spin asymmetries are presumably more difficult to measure, it appears that the quantities $A$, $\bar{A}$ represent the better hope for observation.

## IV. TRIPLE-PRODUCT CORRELATIONS: KAON SEMILEPTONIC DECAY

For definiteness, we shall consider throughout this section the transition $K^+(\mathbf{k})\to\pi^0(\mathbf{q})+\mu^+(\mathbf{p},s_\mu)+\nu_\mu(\mathbf{p}',s_\nu)$. Because there is a good deal of experimental information regarding kaon decays, we can extend the analysis done for $B$ decays in several respects.

First, the estimate for $CP$-violating signals arising purely from the standard model can be made more precise. In fact, Eq. (7) becomes

$$\frac{M(CPV)}{M(CP)}\approx\frac{V_{ud}}{V_{us}}(m_K^2-m_\pi^2)^{-1}\mathrm{Im}\langle\pi^+|H_W|K^+\rangle$$
$$=\frac{\cos\theta_C}{\sin\theta_C}(m_K^2-m_\pi^2)^{-1}F_\pi\epsilon'\mathrm{Re}A_0\frac{\sqrt{2}}{\omega}\ , \qquad (16)$$

where $A_0$ is the isospin zero $K\pi\pi$ decay amplitude and $\omega\equiv\mathrm{Re}A_2/\mathrm{Re}A_0\simeq\frac{1}{20}$. Inserting numerical values for these quantities, we obtain $M(CPV)/M(CP)\simeq\epsilon\times10^{-7}$.

The above analysis of a standard-model-induced triple-product correlation is an example wherein the $CP$-violating impurity enters via the vector and/or axial-vector currents. For the case where the $K$-to-$\pi$ transition is totally in the form of a vector current $V_\mu$, the decay amplitude becomes

$$M_{\mathrm{vec}}=\frac{G_F}{\sqrt{2}}V_{us}\langle\pi^0(\mathbf{q})|V_\mu|K^+(\mathbf{k})\rangle\bar{u}(\mathbf{p}')\gamma^\mu(1+\gamma_5)v(\mathbf{p})\ . \qquad (17)$$

The hadronic matrix element can be expressed as

$$\langle\pi^0(\mathbf{q})|V_\mu|K^+(\mathbf{k})\rangle=\frac{F_+}{\sqrt{2}}[(k+q)_\mu+\xi(k-q)_\mu]\ , \qquad (18)$$

and $\xi$ is complex valued if $CP$ violation occurs. The contribution of the $CP$-violating interference term to the squared matrix element is

$$|M|^2_{\mathrm{int}}=G_F^2|V_{us}|^2F_+^2\frac{m_\mu m_K}{E_\mu E_\nu}\mathrm{Im}\xi\,\mathbf{s}\cdot\mathbf{p}\times\mathbf{q}\ , \qquad (19)$$

where $\mathbf{s}$ is the muon polarization vector. An integration over phase space yields the decay rate for the asymmetry term:

$$\Gamma_A=\frac{m_K^5G_F^2}{192\pi^3}|V_{us}|^2|F_+|^2\frac{m_\mu}{m_K}0.012\,\mathrm{Im}\xi \qquad (20)$$

or

$$\Gamma_A/\Gamma_{\mathrm{tot}}\simeq\frac{m_\mu}{m_K}\frac{\mathrm{Im}\xi}{3.9+0.8\,\mathrm{Re}\xi+0.1|\xi|^2}\ . \qquad (21)$$

An experimental search[15] on muon polarization implies the limit $\mathrm{Im}\xi\lesssim-0.018$ for the maximal case of $\mathrm{Re}\xi=0$.

Returning to the Higgs model, the effective weak operator is

$$H_{\mathrm{eff}}=\frac{G_F}{\sqrt{2}}V_{us}[\bar{s}\gamma_\mu(1+\gamma_5)u\bar{\mu}\gamma^\mu(1+\gamma_5)\nu_\mu$$
$$+m_sm_\mu\bar{s}(C+D\gamma_5)u\bar{\mu}(1+\gamma_5)\nu_\mu]\ , \qquad (22)$$

with $C,D$ given as in Eq. (2) but with $m_c/m_b$ replaced by $m_u/m_s$. Upon taking the $K^+$-to-$\pi^+$ matrix element and squaring, we obtain, for the interference term,

$$|M|^2_{\mathrm{int}}=G_F^2|V_{us}|^2|F_+|^2\frac{m_Km_\mu}{E_\mu E_\nu}$$
$$\times\mathrm{Im}C(m_K^2-m_\pi^2+\xi t)\mathbf{s}\cdot\mathbf{q}\times\mathbf{p}\ , \qquad (23)$$

where $t=(k-q)^2$. The contribution of this term to the kaon decay rate is

$$\Gamma_A^{(K)}=\frac{m_K^4G_F^2}{32\pi^3}\sin^2\theta_C|F_+|^2m_\mu\sum_i\mathrm{Im}(\alpha_i\bar{\alpha}_i^*)\frac{m_K^2}{m_{H_i}^2}\tilde{I}\ , \qquad (24)$$

where $\tilde{I}=(1.77+0.52\xi)\times10^{-3}$. A more useful form for comparison with $B$ decay is

$$\Gamma_A^{(K)}=\frac{G_F^2m_K^5}{192\pi^3}|V_{us}|^2 0.01\sum_i\mathrm{Im}\alpha_i\bar{\alpha}_i^*\frac{m_Km_\mu}{m_{H_i}^2}\ . \qquad (25)$$

From division by the empirical decay rate[16] $\Gamma_{\mathrm{expt}}=1.69\times10^{-15}$ MeV, and using Eq. (11) as input, we obtain the kaon asymmetry $A(K)$. It is straightforward to relate the kaon and $b$-quark decay asymmetries, and we find $A(K)\simeq A(b)/89$. Most of the relative suppression in the kaon asymmetry arises from the ratio of mass factors $m_Km_\mu/m_bm_\tau\simeq\frac{1}{170}$. Although the experiment of Ref. 15 involves a muon polarization measurement at a specific kinematic point rather than an integrated triple-product correlation measurement, we can use that analysis to place a bound on the asymmetry. We obtain

$$A(K)\le8.5\times10^{-4}\ . \qquad (26)$$

Now input from Eq. (11) implies (for $\theta_2^H\neq0,\pi$) that $A(K)\simeq2.2\times10^{-5}\cot^2\theta_2^H$. Comparison with Eq. (26) implies that even further sensitivity is needed; at present there is only the rather loose bound $\cot^2\theta_2^H\le39$.

## V. TRIPLE-PRODUCT CORRELATIONS: EXCLUSIVE B SEMILEPTONIC DECAY

In this section, we shall indicate how triple-product correlations could be analyzed for an *exclusive* decay into hadrons rather than one where a quark jet is detected. The case $B^+\to D^0\tau^+\nu_\tau$ is analogous to the $K^+$ decay with substitution of the appropriate form factors. Consider the semileptonic decay $B^+(\mathbf{k})\to D^{*0}(\mathbf{q},\lambda)\tau^+(\mathbf{p})+\nu_\tau(\mathbf{p}')$, which is then followed by the strong decay $D^*\to D\pi$. Interestingly, the final configuration is a four-particle state: $D+\pi+\bar{\tau}+\nu_\tau$. Therefore, as explained earlier, it is possible to construct triple-product correlations which involve just spatial momenta. This is, in fact, the kind of correlation which we shall consider in the following.

At the quark level the effective operator which induces the weak transition is

$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}V_{cb}[\bar{b}\gamma_\mu(1+\gamma_5)c\bar{\nu}_\tau\gamma^\mu(1+\gamma_5)\tau +m_b m_\tau \bar{b}(C+D\gamma_5)c\bar{\nu}(1+\gamma_5)\tau] , \quad (27)$$

where $C,D$ are given in Eq. (2). The decay amplitude, specified in terms of the $D^*$ polarization $\lambda$, is

$$\begin{aligned}M=\frac{G_F}{\sqrt{2}}V_{cb}[&\langle D^{*0}(\mathbf{q},\lambda)|\bar{b}\gamma_\mu(1+\gamma_5)c|B^+(\mathbf{k})\rangle\\ &\times\bar{u}(\mathbf{p}')\gamma^\mu(1+\gamma_5)v(\mathbf{p})\\ &+m_b m_\tau\langle D^{*0}(\mathbf{q},\lambda)|\bar{b}(C+D\gamma_5)c|B^+(\mathbf{k})\rangle\\ &\times\bar{u}(\mathbf{p}')(1+\gamma_5)v(\mathbf{p})] . \quad (28)\end{aligned}$$

The hadronic matrix element is decomposed in terms of form factors

$$\begin{aligned}\langle D^{*0}(\mathbf{q},\lambda)|\bar{b}\gamma_\mu(1+\gamma_5)c|B^+(\mathbf{k})\rangle &=ig\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(\lambda)(k+q)^\alpha(k-q)^\beta+f\epsilon^*_\mu(\lambda)\\ &+a_+(\epsilon^*\cdot k)(k+q)_\mu\\ &+a_-(\epsilon^*\cdot k)(k-q)_\mu . \quad (29)\end{aligned}$$

The form factors are not known over the full range of the momentum transfer $t$. They have been estimated at one point $t_0=(m_B-m_{D^*})^2$ and some dynamical model such as pole dominance would be needed to infer their values at other values of $t$ (Refs. 17 and 18).

Having found the decay amplitude for $D^*$ emission, our next step is to perform a density-matrix analysis leading ultimately to a triple product of the form $\mathbf{p}_{D^*}\cdot\mathbf{p}_\tau\times\mathbf{p}_D$. First we construct

$$\rho(\lambda,\lambda')=\frac{1}{N}\sum M[B^+\to D^{*0}(\lambda)\tau^+\nu_\tau] \times M^*[B^+\to D^{*0}(\lambda')\tau^+\nu_\tau] , \quad (30)$$

where

$$N=\sum_\lambda|M[B^+\to D^*(\lambda)\tau^+\nu_\tau]|^2 . \quad (31)$$

It is clear that $\rho(\lambda,\lambda')$ is of the form

$$\rho(\lambda,\lambda')=T^{\mu\nu}\epsilon^*_\mu(\lambda)\epsilon_\nu(\lambda') , \quad (32)$$

where $T^{\mu\nu}$ is given by a lengthy expression involving the form factors of Eq. (29). Next we must consider the $D^*\to D\pi$ decay amplitude,

$$M[D^*(\lambda)\to D\pi]=\epsilon_\mu(\lambda)\Gamma^\mu , \quad (33)$$

where $\Gamma^\mu=h(p_D-p_\pi)^\mu$ and $h$ is the $D^*D\pi$ coupling strength. If we define the quantity

$$R_{\lambda\lambda'}=\epsilon_\alpha(\lambda)\epsilon^*_\beta(\lambda')G^{\alpha\beta} , \quad (34)$$

where $G^{\alpha\beta}=\Gamma^\alpha\Gamma^{*\beta}$, then the angular distribution of the final-state $D$ meson is described by

$$\begin{aligned}W&=\sum_{\lambda,\lambda'}\rho_{\lambda\lambda'}R_{\lambda\lambda'}\\ &=T^{\mu\nu}G_{\mu\nu}-\frac{1}{m^2_{D^*}}(q_\mu q_\alpha T^{\mu\nu}G^\alpha_\nu+q_\nu q_\alpha T^{\mu\nu}G^\alpha_\mu) , \quad (35)\end{aligned}$$

where we have used $q_\mu q_\nu T^{\mu\nu}=0$. The part of $W$ which contains the Higgs-boson-induced effect is

$$\begin{aligned}W_{\rm Higgs}=\kappa[&k\cdot(p_D-p_\pi)-k\cdot q(m^2_D-m^2_\pi)/m^2_{D^*}]\\ &\times[\mathbf{q}_{D^*}\cdot(\mathbf{p}_\tau\times\mathbf{p}_D)] , \quad (36)\end{aligned}$$

where

$$\begin{aligned}\kappa=64\,{\rm Im}D|h|^2[&f+a_+(m^2_B-m^2_{D^*})\\ &+a_-(k-q)^2]m_B m_\tau\frac{m_b m_c}{m_b+m_c} . \quad (37)\end{aligned}$$

As anticipated, a triple correlation involving just momenta appears. We have chosen not to carry out the calculation further due to ignorance of the $t$ dependence of the form factors. However, the procedure is clear and the reader is free to employ whatever form-factor model is deemed reasonable.

## VI. CONCLUDING REMARKS

Experiments dealing with $CP$ violations are notoriously difficult. Detection of triple-product correlations is no exception in that asymmetries are predicted to be small. However, the subject is not without interest. Foremost is the possibility that it deals with new physics.

Before an observed triple-product correlation can be associated with new physics, it must be demonstrated that the effect is due neither to unitarity phases nor to the standard model itself. For kaon decays, the impact of unitarity phases has been estimated to occur at the level of $\sim 10^{-6}$ (Ref. 13). This arises from final-state interactions induced by electromagnetism. Also, in Sec. IV, we have shown that interference between the tree-level amplitude and certain radiative corrections can indeed produce a $CP$-odd triple-product correlation, but we estimate its magnitude to be at the $\sim 10^{-7}\epsilon$ level. The $CP$ violation is reflected in the presence of a complex ratio ${\rm Im}\xi$ of form factors, and we reviewed the extent to which this quantity is bounded by existing data. As an example of new physics, the Higgs model contributes at the $\sim 10^{-5}$ level, and would, therefore, be expected to dominate the above mechanisms.

For the case of $B$ decays, we have shown how both inclusive and exclusive decays can give rise to triple-product correlations. Both types of possibilities should be kept in mind by experimentalists, the choice depending on experimental exigencies. For the former, we estimate that the largest triple-product signal involves $\mathbf{s}_\tau\cdot\mathbf{p}_\tau\times\mathbf{p}_c$, where $\mathbf{p}_c$ is the momentum of the $c$-quark jet. As expected in the Higgs-boson model, mass factors cause the effect to be enhanced relative to that occurring in kaon decay. The inclusive decay provides information on ${\rm Im}C$, where $C$ is defined in Eq. (2). Alternatively the exclusive $D^*$ mode contains a triple product proportional to a separate quantity Im$D$ also defined in Eq. (2). Because the strong decay $D^* \to D\pi$ will always occur before detection, the final state (prior to the weak $D$ decay) contains four particles, and, hence, an experimentally advantageous momentum triple product is available. Another relative advantage of the exclusive mode is that at present, the reconstruction of both the spin and momentum of a tau from, say, its muon decay is a formidable task. There is a difficulty, however, due to the unknown momentum transfer dependence of the $B$-to-$D^*$ form factors. Of course, one could also compute correlations which involve spins, as well as those arising from Im$\xi$ factors. The $D^*$ mode contains an interesting theoretical sidelight. One might argue that the $D^*$ decay, which is strong, would obviate $CP$-violating information of the type we are interested in. However, we have explicitly shown that a signal indeed survives. Finally, we have pointed out (in Sec. II) experimental tests which can distinguish between $CP$-violating signals and unitarity phases.

Obviously, $t$-flavored mesons would provide an even better experimental system due to their much larger mass. Quite generally, tests of $CP$ violation at high energies would be most welcome.[19,20] Extension of our analysis to neutral pseudoscalar mesons is immediate, the effect of particle-antiparticle mixing being irrelevant in cases where the decays are self-tagging. Future work includes not only these extensions but also analysis of the unitarity phases expected for the systems of $b$ quarks and $t$ quarks.

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