On an exact low-energy duality in heavy-quark semileptonic decay

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Shifman and Voloshin have pointed out that in a certain limit there would be an exact and calculable duality between quark- and hadronic-level pictures of meson semileptonic decay. I show here that this duality is local in the Dalitz plot and correct to next-to-leading order in the heavy quarks' mass difference. I then discuss some implications of these results for extracting V_{cb} from data on $B \to De\bar{\nu}_e$ and $D^* e\bar{\nu}_e$.

Shifman and Voloshin[1] (SV) have recently pointed out that in a certain limit there would be an exact and *calculable* duality between quark- and hadronic-level pictures of meson semileptonic decay. At the quark level these transitions would be treated as the free-quark decay $Q \to qe\bar{\nu}_e$. For a sufficiently large mass difference $m_Q - m_q$ one would, of course, expect to find a duality between this rate and one at the hadronic level involving a sum over a dense set of q-containing hadronic final states. The remarkable feature of the SV limit is that in it $Q \to qe\bar{\nu}_e$ is exactly dual to the hadronic processes $P_Q \to X_q e\bar{\nu}_e$ after summing over only the *two* final states $X_q = P_q$ and V_q, where P_α and V_α are the ground-state pseudoscalar and vector mesons made of quark α and a light spectator antiquark (henceforth called $\bar{d}$, but the following is also valid for $\bar{u}$ and $\bar{s}$).

In this paper I would like to examine this duality in some detail in the context of the quark potential model description of meson semileptonic decays presented in Ref. 2. I will pay particular attention to the ramifications of this exact duality for studying the Kobayashi-Maskawa[3] (KM) matrix element V_{cb} responsible for the $b \to c$ transition. My main conclusions will be that (1) the SV limit is violated only in second order in the relevant expansion parameter and so is more robust than it might have been, so that (2) the $b \to c$ transition is now sufficiently close to the limiting case that the probable range of model-dependent error in the extraction of V_{cb} from data is substantially decreased.

I begin with an idiosyncratic description of the SV limit designed to facilitate the ensuing discussion. For the free-quark description of the semileptonic decay of P_Q to hold, the kinetic energy T_q of the produced heavy quark q (and, therefore, its three-momentum) must be large with respect to the typical mass scale of the $q\bar{d}$ system (characterized by $b^{1/2} \sim 0.4$ GeV, where b is the mesonic linear potential or string tension). Since

$$0 \le T_q \le \frac{(m_Q - m_q)^2}{2m_Q} \tag{1}$$

this condition will be met over all but a negligible region of the $Q \to qe\bar{\nu}_e$ Dalitz plot so long as

$$x_m \equiv \frac{m_Q^2 - m_q^2}{2m_Q^2} \gg \left[\frac{b}{m_Q^2}\right]^{1/4} . \tag{2}$$

On the other hand, if x_m is small then the invariant mass m_X of the $q\bar{d}$ system will be in a range Δm_X above its minimum with

$$\Delta m_X \sim b^{1/2} x_m \tag{3}$$

if the spectator $\bar{d}$ quark is relativistic ($m_d \ll b^{1/2}$) or

$$\Delta m_X \sim b^{1/2} x_m^2 \tag{4}$$

if it is nonrelativistic ($m_d \gg b^{1/2}$). In either model, if x_m is small then Δm_X is small with respect to the characteristic excitation scale $b^{1/2}$ of the $q\bar{d}$ system so that the recoiling states X will all be orbital ground states. The crucial observation is that $x_m \ll 1$ is compatible with (2): for any fixed x_m, condition (2) can be met by taking m_Q sufficiently large. The physical content of these conditions is that if q is sufficiently heavy it can recoil with large kinetic energy but small velocity.

The final condition for the applicability of the free-quark decay formulas is that both Q and q be effectively free, i.e., that both m_Q and m_q be much larger than $b^{1/2}$. Given (2) and that x_m is small these conditions are, in some sense, already met, but we shall have more to say on them below.

The semileptonic decay rate for $Q \to qe\bar{\nu}_e$ is

$$\frac{d^2\Gamma^q}{dx\,dy} = |V_{qQ}|^2 \frac{G_F^2 m_Q^5}{8\pi^3}[4x(x_m - x) + 2y(2x - x_m) - y^2] \tag{5}$$

while for $P_Q \to P_q e\bar{\nu}_e$ and $P_Q \to V_q e\bar{\nu}_e$ it is

$$\frac{d^2\Gamma^P}{dx\,dy} = |V_{qQ}|^2 \frac{G_F^2 m_{P_Q}^5}{16\pi^3}\{\beta_{++}^P[4x(x_m - x) - y(1 - 2x)]\} \tag{6}$$

and

$$\frac{d^2\Gamma^V}{dx\,dy}=|V_{qQ}|^2\frac{G_F^2 m_{P_Q}^5}{32\pi^3}\times\left[\frac{\alpha^V y}{m_{P_Q}^2}+2\beta_{++}^V[4x(x_m-x)-y(1-2x)]-2\gamma^V y(x_m-2x+\tfrac{1}{2}y)\right], \tag{7}$$

respectively. In these formulas x is the ratio of the electron energy to the rest mass m of the decaying particle, $y\equiv m_{e\nu}^2/m^2$, where $m_{e\nu}$ is the mass of the electron-neutrino system (i.e., $m_{e\nu}^2=t$, the hadronic four-momentum transfer), and β_{++}^P, α^V, β_{++}^V, and γ^V are functions of form factors appearing in the $P_Q\to P_q$ and $P_Q\to V_q$ weak-current matrix elements. With $V_\mu\equiv\bar{q}\gamma_\mu Q$ and $A_\mu\equiv\bar{q}\gamma_\mu\gamma_5 Q$ the relevant form factors are f_+, g, f, and a_+, where

$$\langle P_q(p')|V_\mu|P_Q(p)\rangle\equiv f_+(p+p')_\mu+f_-(p-p')_\mu\ , \tag{8}$$

$$\langle V_q(p'\epsilon)|V_\mu|P_Q(p)\rangle\equiv ig\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p+p')^\rho(p-p')^\sigma\ , \tag{9}$$

and

$$\langle V_q(p'\epsilon)|A_\mu|P_Q(p)\rangle\equiv f\epsilon_\mu^*+a_+(\epsilon^*\cdot p)(p+p')_\mu+a_-(\epsilon^*\cdot p)(p-p')_\mu \tag{10}$$

in terms of which

$$\beta_{++}^P=|f_+|^2\ , \tag{11}$$

$$\alpha^V=f^2+4m_{P_Q}^2 g^2\mathbf{p}'^2\ , \tag{12}$$

$$\beta_{++}^V=\frac{f^2}{4m_{V_q}^2}-m_{P_Q}^2 g^2 y+\frac{1}{2}\left[\frac{m_{P_Q}^2}{m_{V_q}^2}(1-y)-1\right]fa_+ +\frac{m_{P_Q}^2 a_+^2}{m_{V_q}^2}\mathbf{p}'^2\ , \tag{13}$$

and

$$\gamma^V=2gf\ , \tag{14}$$

where

$$\mathbf{p}'^2=\frac{[m_{P_Q}^2(1-y)+m_X^2]^2}{4m_{P_Q}^2}-m_X^2 \tag{15}$$

is the square of the recoil three-momentum of the system X. The form factors are, of course, in general functions of $\mathbf{p}'^2$ or, equivalently, y. The Dalitz-plot variables x and y are restricted to the ranges

$$0\le x\le x_m\equiv\frac{m^2-m_X^2}{2m^2} \tag{16}$$

and

$$0\le y\le\frac{4x(x_m-x)}{1-2x}\ . \tag{17}$$

In Ref. 2 the hadronic form factors are calculated in the quark potential model with the results

$$f_+=F_3\left[1+\frac{m_Q}{2\mu_-}-\frac{m_Q m_q}{4\mu_+\mu_-}\frac{m_d}{\tilde{m}_q}\frac{\beta_Q^2}{\beta_{Qq}^2}\right], \tag{18}$$

$$f=2\tilde{m}_Q F_3\ , \tag{19}$$

$$g=F_3\left[\frac{1}{2m_q}-\frac{1}{4\mu_-}\frac{m_d}{\tilde{m}_q}\frac{\beta_Q^2}{\beta_{Qq}^2}\right], \tag{20}$$

and

$$a_+=-\frac{F_3}{2\tilde{m}_q}\left[1+\frac{m_d}{m_Q}\left[\frac{\beta_Q^2-\beta_q^2}{\beta_Q^2+\beta_q^2}\right]-\frac{m_d^2}{4\mu_-\tilde{m}_Q}\frac{\beta_q^4}{\beta_{Qq}^4}\right], \tag{21}$$

where m_i is the constituent mass of quark i, $\tilde{m}_\alpha=m_\alpha+m_d$, $\mu_\pm^{-1}=m_q^{-1}\pm m_Q^{-1}$, β_α is a mass which characterizes the size of the $\alpha\bar{d}$ system, $\beta_{Qq}^2=\frac{1}{2}(\beta_Q^2+\beta_q^2)$ and

$$F_3=\left[\frac{\tilde{m}_q}{\tilde{m}_Q}\right]^{1/2}\left[\frac{\beta_Q\beta_q}{\beta_{Qq}^2}\right]^{3/2}\exp\left[-\left[\frac{m_d^2}{4\tilde{m}_Q\tilde{m}_q}\right]\frac{t_m-t}{\kappa^2\beta_{Qq}^2}\right], \tag{22}$$

where $t_m=(m-m_X)^2$, and κ is an empirical factor.

Let us now take the SV limit: for $m_Q\to\infty$ and $x_m\to 0$, the range of the Dalitz-plot variables becomes $0\le x\le x_m\simeq(m_Q-m_q)/m_Q$, $0\le y\le 4x(x_m-x)$ so that

$$\frac{d^2\Gamma^q}{dx\,dy}\to|V_{qQ}|^2\frac{G_F^2 m_Q^5}{2\pi^3}[x(x_m-x)]\ , \tag{23}$$

$$\frac{d^2\Gamma^P}{dx\,dy}\to|V_{qQ}|^2\frac{G_F^2 m_Q^5}{4\pi^3}\{\beta_{++}^P[(x(x_m-x)-\tfrac{1}{4}y]\}\ , \tag{24}$$

$$\frac{d^2\Gamma^V}{dx\,dy}\to|V_{qQ}|^2\frac{G_F^2 m_Q^5}{32\pi^3}\left[\frac{\alpha^V}{m_Q^2}y+2\beta_{++}^V[4x(x_m-x)-y]\right] \tag{25}$$

while, since in this limit $\beta_Q=\beta_q\sim b^{1/2}\sim m_d$,

$$f_+\to 1\ , \tag{26}$$

$$f\to 2m_Q\ , \tag{27}$$

$$g\to\frac{1}{2m_Q}\ , \tag{28}$$

$$a_+\to-\frac{1}{2m_Q} \tag{29}$$

giving

$$\beta_{++}^P\to 1\ , \tag{30}$$

$$\alpha^V\to 4m_Q^2\ , \tag{31}$$

$$\beta_{++}^V\to 1\ , \tag{32}$$

$$\gamma^V\to 2 \tag{33}$$

so that

$$\frac{d^2\Gamma^P}{dx\,dy}\to|V_{qQ}|^2\frac{G_F^2 m_Q^5}{4\pi^3}[x(x_m-x)-\tfrac{1}{4}y]\ , \tag{34}$$

$$\frac{d^2\Gamma^V}{dx\,dy} \to |V_{qQ}|^2 \frac{G_F^2 m_Q^5}{4\pi^3}[x(x_m-x)+\tfrac{1}{4}y] \ . \tag{35}$$

Thus we find

$$\frac{d^2\Gamma^q}{dx\,dy} = \frac{d^2\Gamma^P}{dx\,dy} + \frac{d^2\Gamma^V}{dx\,dy} \tag{36}$$

which is the local (in the Dalitz-plot) version of the SV result

$$\Gamma^q = \Gamma^P + \Gamma^V \tag{37}$$

with

$$\Gamma^P = \tfrac{1}{3}\Gamma^V = |V_{qQ}|^2 \frac{G_F^2 m_Q^5 x_m^5}{60\pi^3} \ . \tag{38}$$

It is interesting to note that the "local" duality relation (36) is satisfied by an exact cancellation between the y dependences of $P_Q \to P_q e\bar{\nu}_e$ and $P_Q \to V_q e\bar{\nu}_e$ across the Dalitz plot, and that the 1:3 ratio of $P_q:V_q$ is satisfied locally in x.

I would now like to show that (36) is valid even when one allows corrections to the limit $x_m \to 0$ of order

$$x_m = \frac{m_Q^2 - m_q^2}{2m_Q^2} \simeq \frac{m_Q - m_q}{m_Q} \ .$$

(There is no analogous insensitivity to the limit $m_Q \to \infty$: corrections to the SV limit occur at order $\eta \equiv b^{1/2}/m_Q$.) We begin by noting that $\beta_q/\beta_Q = 1 + O(x_m\eta)$ since as either $x_m \to 0$ or $m_Q \to \infty$, $\beta_q \to \beta_Q$. It follows that, to order x_m^2,

$$f_+ = 1 \ , \tag{39}$$

$$f = 2m_Q(1 - \tfrac{1}{2}x_m) \ , \tag{40}$$

$$g = -a_+ = (2m_Q)^{-1}(1 + \tfrac{1}{2}x_m) \tag{41}$$

so that to this same order

$$\beta_{++}^P = 1 \ , \tag{42}$$

$$\alpha^V = 4m_Q^2(1 - x_m) \ , \tag{43}$$

$$\beta_{++}^V = 1 \ , \tag{44}$$

$$\gamma^V = 2 \ . \tag{45}$$

The quadratic "shielding" of f_+ from $m_Q - m_q$ differences was already noted in Ref. 2. With these results it follows that

$$\frac{d^2\Gamma^q}{dx\,dy} \to |V_{qQ}|^2 \frac{G_F^2 m_Q^5}{2\pi^3}[x(x_m-x)+\tfrac{1}{2}y(2x-x_m)] \ , \tag{46}$$

$$\frac{d^2\Gamma^P}{dx\,dy} \to |V_{qQ}|^2 \frac{G_F^2 m_Q^5}{4\pi^3}[x(x_m-x)-\tfrac{1}{4}y(1-2x)] \ , \tag{47}$$

$$\frac{d^2\Gamma^V}{dx\,dy} \to |V_{qQ}|^2 \frac{G_F^2 m_Q^5}{4\pi^3}[x(x_m-x)+\tfrac{1}{4}y(6x+1-4x_m)] \tag{48}$$

and (36) continues to hold. To this order in x_m the Dalitz-plot boundaries are $0 \le x \le x_m$, $0 \le y \le 4x(1+2x)(x_m-x)$ so that now

$$\Gamma^q = |V_{qQ}|^2 \frac{G_F^2 m_Q^5 x_m^5}{15\pi^3}(1+x_m) \ , \tag{49}$$

$$\Gamma^P = |V_{qQ}|^2 \frac{G_F^2 m_Q^5 x_m^5}{60\pi^3}(1+x_m) \ , \tag{50}$$

$$\Gamma^V = |V_{qQ}|^2 \frac{G_F^2 m_Q^5 x_m^5}{20\pi^3}(1+x_m) \ . \tag{51}$$

Note that all ratios remain unmodified. At the same time, Eqs. (46)–(48) show that the local duality between $d^2\Gamma^q/dx\,dy$ and $d^2\Gamma^P/dx\,dy + d^2\Gamma^V/dx\,dy$ runs deep: its validity extends to include the y (but not the y^2) dependence in Eq. (5).

To summarize, we have shown that SV duality is local in the Dalitz plot, and that corrections to this local duality are of second order in $(m_Q^2 - m_q^2)/2m_Q^2$. The existence of such a correspondence significantly enhances the confidence we may have in the "sum over exclusive channels" method of Ref. 2 for calculating semileptonic decay rates and guarantees its accuracy for decays which approximate the SV limit.

We now consider the practical implications of the above discussion for the extraction of V_{cb} from semileptonic decay data. The first point to be made is that in $b \to ce\bar{\nu}_e$ the quarks are not "free" with great accuracy [see the comments just above Eq. (5)]: with $\eta_i \equiv b^{1/2}/m_i$ we have $\eta_b \simeq 0.08$ and $\eta_c \simeq 0.23$. Thus the kinematic parameter x_m in, e.g., Eq. (38) is not very well defined: since m_b and m_c are undetermined to within $\pm b^{1/2}$, Γ^q in Eq. (38) has an uncertainty of $\pm 50\%$. Even if we make the dubious assumption that they are undetermined by the *same* additive constant, there is an uncertainty of about 20%. This means that if the limiting formulas are to be useful in $b \to c$, they must be used at the level of squared matrix elements. We thus propose assessing the model dependence of our understanding of the $B \to De\bar{\nu}_e$ and $B \to D^*e\bar{\nu}_e$ transitions by comparing the rate formulas (6) and (7) with f_+, f, g, and a_+ given by (18)–(21) (the model of Ref. 2) and by (39)–(41).

We begin with $B \to De\bar{\nu}_e$. From Eq. (18) we have

$$f_+(t) = 1.13 \exp\left[-0.34\frac{t_m - t}{t_m}\right] \tag{52}$$

while Eq. (39) gives

$$f_+(t) = 1 \tag{53}$$

independent of t. The result (53) gives, when inserted into Eq. (6),

$$\Gamma^D = |V_{cb}|^2 \frac{G_F^2 m_B^5}{60\pi^3}\left(\tfrac{5}{64} - \tfrac{5}{8}u + \tfrac{5}{8}u^3 - \tfrac{5}{64}u^4 - \tfrac{15}{16}u^2 \ln u\right) \ , \tag{54}$$

where $u \equiv 1 - 2x_m = M_D^2/M_B^2$, which gives $\Gamma^D = 0.14 \times 10^{14}|V_{cb}|^2$ sec^{-2}, while (52) gives $0.11 \times 10^{14}|V_{cb}|^2$

$\sec^{-1}$. We might, therefore, make a first estimate of the theoretical uncertainty in deducing V_{cb} from accurate data of approximately 10%. This estimate should be scrutinized carefully, however, since the differences between (52) and (53) are canceling in the rate calculation: $f_+(t_m)$ is 13% larger in (52) than in (53), but the decrease of f_+ as $t\to 0$ means that the average value of (52) across the Dalitz plot is actually less than unity. We have two comments: (1) all models of which we are aware predict a comparable decrease of $f_+(t)$ as $t\to 0$, e.g., the t dependence in (52) is consistent with vector-meson dominance, which would give

$$f_+(t)=f_+(t_m)\left[1-0.32\left[\frac{t_m-t}{t_m}\right]\right]$$

for small t_m-t; (2) the uncertainty arising from this t dependence can in any event be eliminated by extrapolating the data to $t=t_m$. Thus the real uncertainty is in the deviation of $f_+(t_m)$ in (52) from unity, which once again appears to produce an uncertainty of order 10% in V_{cb}. This is a conservative estimate: I believe that $f_+(t_m)-1$ is itself a reasonably reliable theoretical quantity [note that the SV estimate[1] that $f_+(t_m)=0.88$ is flawed since f_- is of order x_m] so that the actual uncertainty in V_{cb} deduced by this method is considerably smaller, perhaps even at the few percent level. In any event this kind of accuracy is probably sufficient for present needs.

Let us now consider $B\to D^*e\bar{\nu}_e$. This process is intrinsically more complicated since all three form factors f, g, and a_+ contribute, but it is worth considering despite our conclusions on $B\to De\bar{\nu}_e$ both as a check and because it is easier to measure. In this case we have from Eqs. (19)–(21) that

$$\frac{f(t)}{2m_B}=0.65F(t)\ , \tag{55}$$

$$g(t)=0.16F(t)\ \mathrm{GeV}^{-1}\ , \tag{56}$$

$$a_+(t)=-0.15F(t)\ \mathrm{GeV}^{-1}\ , \tag{57}$$

where

$$F(t)=\exp\left[-0.31\frac{t_m-t}{t_m}\right]\ ,$$

while Eqs. (40) and (41) give

$$\frac{f(t)}{2m_B}=0.76\ , \tag{58}$$

$$g(t)=-a_+(t)=0.12\ \mathrm{GeV}^{-1} \tag{59}$$

independent of t. The relation between (55)–(57) and (58) and (59) is more readily appreciated by noticing that, as Eqs. (43)–(45) indicate, β^V_{++} and γ^V are free of corrections to order x_m:

$$\frac{\alpha^{D^*}}{4m_B^2}=0.53\simeq 1-x_m\ , \tag{60}$$

$$\beta^{D^*}_{++}=1\ , \tag{61}$$

$$\gamma^{D^*}=2 \tag{62}$$

while (55)–(57) give

$$\frac{\alpha^{D^*}}{4m_B^2}=0.42\left[1.00+0.26\frac{t_m-t}{t_m}+0.06\left[\frac{t_m-t}{t_m}\right]^2\right]|F(t)|^2\ , \tag{63}$$

$$\beta^{D^*}_{++}=0.99\left[1-0.44\left[\frac{t_m-t}{t_m}\right]+0.15\left[\frac{t_m-t}{t_m}\right]^2\right]|F(t)|^2\ , \tag{64}$$

$$\gamma^{D^*}=2.23|F(t)|^2\ . \tag{65}$$

Thus the zero-recoil ($t=t_m$) values of $\beta^{D^*}_{++}$ and γ^{D^*} are quite close to (61) and (62), and the agreement of α^{D^*} with (60) is comparable to that of β^D_{++} in $B\to De\bar{\nu}_e$. However, a closer examination reveals some reasons for caution. The rate $d\Gamma^{D^*}/dx$ which follows from the order-x_m limit formula (48) has contributions from the α^{D^*}, $\beta^{D^*}_{++}$, and γ^{D^*} terms (43)–(45) in the ratio $\frac{2}{3}:\frac{1}{3}:0$. Thus although γ^{D^*} in (65) is close to (62) is important for γ^{D^*}-induced structure in the Dalitz plot, it is irrelevant to a determination of V_{cb} from the total rate in $B\to D^*e\bar{\nu}_e$, which is determined by α^{D^*} and $\beta^{D^*}_{++}$. The main concern with these two quantities is that they show significant t dependence over and above that contained in $F(t)$, in contrast with the limiting formulas (60) and (61). A direct calculation of the contributions of each of α^{D^*}, $\beta^{D^*}_{++}$, and γ^{D^*} to the rate gives, using the order-x_m results of Eqs. (60)–(62) in Eq. (7),

$$\Gamma^{D^*}_\alpha=|V_{cb}|^2\frac{G_F^2m_B^5}{60\pi^3}(1+u^*)\left[\tfrac{5}{32}+\tfrac{45}{32}u^*-\tfrac{45}{32}u^{*3}-\tfrac{5}{32}u^{*4}+\tfrac{15}{16}u^*(1+u^*)\ln u^*\right]\ , \tag{66}$$

$$\Gamma^{D^*}_{\beta++}=|V_{cb}|^2\frac{G_F^2m_B^5}{60\pi^3}\left(\tfrac{5}{64}-\tfrac{5}{8}u^*+\tfrac{5}{8}u^{*3}-\tfrac{5}{64}u^{*4}-\tfrac{15}{16}u^{*2}\ln u^*\right)\ , \tag{67}$$

$$\Gamma^{D^*}_\gamma=0\ , \tag{68}$$

respectively, where $u^*=m_{D^*}^2/m_B^2$, giving $\Gamma^{D^*}_\alpha=0.29\times 10^{14}\,|V_{cb}|^2\ \sec^{-1}$, $\Gamma^{D^*}_{\beta++}=0.12\times 10^{14}\,|V_{cb}|^2\ \sec^{-1}$, and $\Gamma^{D^*}_\gamma=0$ (for a total $\Gamma^{D^*}=0.41\times 10^{14}|V_{cb}|^2\ \sec^{-1}$) to be compared with the "exact" results from Eqs. (63)–(65) of $\Gamma^{D^*}_\alpha=0.19\times 10^{14}\ |V_{cb}|^2\ \sec^{-1}$, $\Gamma^{D^*}_{\beta++}=0.06\times 10^{14}|V_{cb}|^2\ \sec^{-1}$, and $\Gamma^{D^*}_\gamma=0$ (for a total $\Gamma^{D^*}=0.25\times 10^{14}|V_{cb}|^2\ \sec^{-1}$). Although these rates superficially indicate a rather large theoretical uncertainty in deducing V_{cb} from $B\to D^*e\bar{\nu}_e$, one should note that the t dependence of Eqs. (63)–(65) is such that the values in Eqs. (60)–(62) are obtained by the coefficients of $|F(t)|^2$ within the Dalitz plot: $\alpha^{D^*}/|F(t)|^2$ goes from 21% below to 5% above its limit value (60) as t goes from t_m to 0, while $\beta^{D^*}_{++}/|F(t)|^2$ goes from 1% below to 30% below its limit value (61) in the same range. That the "exact" rates are less than those given by (67) and (68) is, as for Γ^D, therefore mainly a more or less reliable form-factor effect. Thus, as with Γ^D, we conclude that the main uncertainty is in the deviations of (63)–(65) from (60)–(62) at $t=t_m$. Given that the rate is dominated by $\Gamma_\alpha^{D^*}$, one could assess the resulting uncertainty in V_{cb} at about 10% again. In this case, given the way they depend on dynamics, I am less sanguine about the ability of theory to predict the $B \rightarrow D^*$ form factors and, although I expect the errors are less than the above estimate indicates, I would look to Γ^D, and not Γ^{D^*}, for a determination to better than 10% of this crucial KM matrix element.

Further progress in both extracting the KM amplitudes and understanding hadronic current matrix elements will involve a continuous interplay between theory[4] and experiment. As experiment provides more detailed information on the semileptonic decays [measurement of $F(t)$ in $B \rightarrow D$, separation of $B \rightarrow D^*$ into α, β_{++}, and γ contributions, measurement of the D^* polarization in $B \rightarrow D^*$, determination of the $D+D^*$ fraction of the total $b \rightarrow c$ rate, etc.] we can expect a steady refinement in theory, including a better understanding of which quantities can be most reliably used to extract V_{cb}. [For example, a fit to the Dalitz plot could be used to extract $\Gamma_\gamma^{D^*}$ which, based on (62) and (65), might turn out to be as reliably predicted as Γ^D.] Similar progress can be expected in the study of $b \rightarrow u$, $c \rightarrow s$, $c \rightarrow d$, and even $s \rightarrow u$ and $d \rightarrow u$ transitions. In this paper I have tried to take a step in this direction by showing that the SV limit is an exact and local duality valid in next-to-leading order in the expansion parameter x_m. On this basis I have argued that while real $b \rightarrow c$ transitions are rather far from the limiting case, the violations appear to be due mainly to phase-space and form-factor effects so that V_{cb} can already be deduced from $B \rightarrow De\bar{\nu}_e$ and perhaps $B \rightarrow D^* e\bar{\nu}_e$ with an uncertainty of less than 10%.

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