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Neutral Kaon Decays into Muon Pairs and the Violation of *CP* Invariance*

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Some implications of nonconservation of *CP* invariance for $K_L \to \mu^+\mu^-$ are discussed with reference to a recent experimental result on this decay.

There is a new experimental result on the K_L decay into muon pairs,[1]

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \text{all})} \leq 1.8\times 10^{-9}. \tag{1}$$

The result is of particular interest because it contradicts a theoretical lower bound to this branching ratio. The lower bound is[2,3]

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \text{all})} \geq \alpha^2\beta\theta^2 \frac{\Gamma(K_L \to \gamma\gamma)}{\Gamma(K_L \to \text{all})}, \tag{2}$$

where

$$\alpha \simeq 1/137,\quad \beta = (1-4m^2/M^2)^{1/2},$$

and

$$\theta = \frac{1}{\sqrt{2}}\,\frac{m}{M}\,\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta} .$$

Using the known experimental rate[4]

$$\frac{\Gamma(K_L \to \gamma\gamma)}{\Gamma(K_L \to \text{all})} = 5.2\times 10^{-4}, \tag{3}$$

one obtains

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \text{all})} \geq 6\times 10^{-9}, \tag{4}$$

in contradiction with Eq. (1).

Equation (2) is derived from the following assumptions[3]:

(A) *CPT* invariance and the validity of quantum electrodynamics,

(B) *CP* invariance,

(C) the absorptive part of the $K_L \to \mu^+\mu^-$ amplitude, Abs $A(K_L \to \mu^+\mu^-)$, is given entirely by the contribution of the 2γ intermediate state,

(D) the absorptive part of the $K_L \to 2\gamma$ amplitude is zero.

Estimates of the contributions to Abs $A(K_L \to \mu^+\mu^-)$ from $2\pi\gamma$ and 3π intermediate states suggest[3,5] that it is difficult to lower this bound by more than ~20%. In this note we will discuss some of the consequences of relaxing assumption (B), above, and, in addition, comment on some recent work on this topic.

We define states

$$|K_S\rangle = p|K^0\rangle - q|\overline{K}^0\rangle,$$

$$|K_L\rangle = p|K^0\rangle + q|\overline{K}^0\rangle,$$

with

$$CP|K^0\rangle = -|\overline{K}^0\rangle \text{ and } \epsilon = (p-q)/(p+q),$$

and decay amplitudes

$$A(K^0 \to \mu^+\mu^-) = i\overline{u}(F_1 + \gamma_5 F_2)v,$$

$$A(\overline{K}^0 \to \mu^+\mu^-) = i\overline{u}(G_1 + \gamma_5 G_2)v.$$

The decay rate for $K_L \to \mu^+\mu^-$ can be written

$$\Gamma(K_L \to \mu^+\mu^-) = \frac{M}{4\pi}\beta[\beta^2|\omega_1|^2 + |\Omega_2|^2 + O(\epsilon\omega_1\Omega_1) + O(\epsilon\omega_2\Omega_2)], \tag{5}$$

where

$$2\omega_1 = F_1 + G_1, \quad 2\Omega_1 = F_1 - G_1,$$
$$2\omega_2 = F_2 - G_2, \quad 2\Omega_2 = F_2 + G_2.$$

At the limit of CP invariance, $\omega_1 = 0$, $\omega_2 = 0$, and $\epsilon = 0$.

We define analogous quantities for $K \to 2\gamma$ as follows:

$$A[K^0 \to \gamma(k)\gamma(k')] = \frac{1}{\sqrt{2}}\frac{H_1}{M}F^{\mu\nu}F'_{\mu\nu} + \frac{1}{\sqrt{2}}\frac{H_2}{M}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F'_{\rho\sigma},$$
$$A[\overline{K}^0 \to \gamma(k)\gamma(k')] = \frac{1}{\sqrt{2}}\frac{B_1}{M}F^{\mu\nu}F'_{\mu\nu} + \frac{1}{\sqrt{2}}\frac{B_2}{M}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F'_{\rho\sigma},$$

where $F_{\mu\nu} = \epsilon_\mu k_\nu - \epsilon_\nu k_\mu$, $F'_{\mu\nu} = \epsilon'_\mu k'_\nu - \epsilon'_\nu k'_\mu$, and ϵ, ϵ' are the photon-polarization vectors. The decay rate for $K_L \to \gamma\gamma$ can be written

$$\Gamma(K_L \to \gamma\gamma) = \frac{M}{4\pi}[\tfrac{1}{4}|\delta_1|^2 + |\Delta_2|^2 + O(\epsilon\delta_1\Delta_1) + O(\epsilon\delta_2\Delta_2)], \tag{6}$$

where

$$2\delta_1 = H_1 + B_1, \quad 2\Delta_1 = H_1 - B_1,$$
$$2\delta_2 = H_2 - B_2, \quad 2\Delta_2 = H_2 + B_2.$$

At the limit of CP invariance, $\delta_1 = 0$, $\delta_2 = 0$, and $\epsilon = 0$. The branching ratio is therefore

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\beta\Gamma(K_L \to \gamma\gamma)} = \frac{|\Omega_2|^2 + \beta^2|\omega_1|^2 + O(\epsilon\omega_1\Omega_1) + O(\epsilon\omega_2\Omega_2)}{|\Delta_2|^2 + \frac{1}{4}|\delta_1|^2 + O(\epsilon\delta_1\Delta_1) + O(\epsilon\delta_2\Delta_2)} \tag{7}$$
$$\geqslant \frac{(\mathrm{Im}\,\Omega_2)^2 + \beta^2(\mathrm{Re}\,\omega_1)^2 + O(\epsilon\omega_1\Omega_1) + O(\epsilon\omega_2\Omega_2)}{|\Delta_2|^2 + \frac{1}{4}|\delta_1|^2 + O(\epsilon\delta_1\Delta_1) + O(\epsilon\delta_2\Delta_2)}. \tag{8}$$

Assumptions (A) and (C) lead, via the unitarity condition, to the relations

$$\mathrm{Im}\,\Omega_2 = \alpha\theta\,\mathrm{Re}\,\Delta_2 \quad \text{and} \quad \mathrm{Re}\,\omega_1 = \alpha\tfrac{1}{2}\theta\,\mathrm{Im}\,\delta_1. \tag{9}$$

From these equations, and neglecting terms proportional to $\epsilon \sim 10^{-3}$, we have the bound

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} \geqslant \alpha^2\beta\theta^2\frac{(\mathrm{Re}\,\Delta_2)^2 + \frac{1}{4}\beta^2(\mathrm{Im}\,\delta_1)^2}{|\Delta_2|^2 + \frac{1}{4}|\delta_1|^2}, \tag{10}$$

and, using (D), this can be written

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} \geqslant \alpha^2\beta\theta^2\left[1 - \frac{4m^2}{M^2}\frac{\frac{1}{4}(\mathrm{Im}\,\delta_1)^2}{(\mathrm{Re}\,\Delta_2)^2 + \frac{1}{4}(\mathrm{Im}\,\delta_1)^2}\right]. \tag{11}$$

The minimum of the right-hand side occurs when the $K_L \to \gamma\gamma$ transition proceeds entirely via a CP-nonconserving mechanism, i.e., when $\Delta_2 = 0$. Then

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \text{all})} \geqslant \alpha^2\beta\theta^2\frac{\Gamma(K_L \to \gamma\gamma)}{\Gamma(K_L \to \text{all})}\left(1 - \frac{4m^2}{M^2}\right). \tag{12}$$

By comparing this result with Eq. (2), we conclude that *even in the case where the assumption of CP invariance is dropped* [but still maintaining assumptions (A), (C), and (D), and the neglect of terms proportional to $\epsilon \sim 10^{-3}$] the unitarity bound, Eq. (4), cannot be lowered by more than 18%.

There have recently appeared two related papers on this topic. Farrar and Treiman[6] have shown that since assumption (D) is unnecessary for a CP-nonconserving transition into $\gamma\gamma$ with 3π intermediate states, then, using (A) and (C) and neglecting terms proportional to ϵ, there is a relation between the CP-nonconserving rates $\tilde{\Gamma}(K_L \to \mu^+\mu^-)$ and $\tilde{\Gamma}(K_L \to \gamma\gamma)$, i.e.,

$$\tilde{\Gamma}(K_L \to \gamma\gamma) \leqslant \frac{\tilde{\Gamma}(K_L \to \mu^+\mu^-)}{\alpha^2\beta^3\theta^2} = 1.85 \times 10^{-4}. \tag{13}$$

This implies that the rate for the CP-nonconserving transition into two photons is less than 37% of the total decay rate. On the other hand, using assumptions (A), (C), and (D) for the CP-conserving transition into two photons, one finds

$$\Gamma(K_L \to 2\gamma) \leqslant \frac{\Gamma(K_L \to \mu^+\mu^-)}{\alpha^2\beta\theta^2} = 1.5 \times 10^{-4}, \tag{14}$$

and the two rates do not add to 5.2×10^{-4}. One possible way out of this contradiction is that the 3π contribution to the absorptive part of the CP-conserving transition $K_L \to \gamma\gamma$ must be large. However, using the Schwarz inequality in the $K_L \to \gamma\gamma$ CP-conserving transition, we find [Eq. (2.19) of Ref. 3] a bound on the total cross section for $\gamma\gamma \to 3\pi$ in states of total angular momentum zero and negative parity, namely,

$$\sigma(\gamma\gamma \to 3\pi) \geq \frac{8(\mathrm{Im}H_2)^2}{M\Gamma(K_L \to 3\pi)}. \quad (15)$$

If $\mathrm{Im}H_2$ is large, then the right-hand side of Eq. (10) could decrease significantly, provided that the 3π contribution to $\mathrm{Abs}(K_L \to \mu^+\mu^-)$ does not interfere constructively with the 2γ contribution. As an illustration let us take the case where the real and imaginary parts of $H_2 = \Delta_2$ are equal; then

$$\sigma(\gamma\gamma \to 3\pi) \geq \frac{16\pi\,\Gamma(K_L \to 2\gamma)}{M^2\Gamma(K_L \to 3\pi)} = 1\times 10^{-28}\ \mathrm{cm}^2. \quad (16)$$

Considering the fact that the usual cross section for $\gamma\gamma \to 2\pi$ with minimal couplings for the pions [Eq. (3.14) of Ref. 3] is 3.5×10^{-31} cm^2, the large value above could only be explained by a 3π resonance at the mass of the kaon. All the perturbation-theoretical models[7] we have considered for $\sigma(\gamma\gamma \to 3\pi)$ lead to values around 10^{-34} cm^2. If we take Eq. (16) and use it in the Weizsäcker-Williams cross section for $\sigma(e^+e^- \to e^+e^-3\pi)$,[8] assuming a linearly rising cross section for $\sigma(\gamma\gamma \to 3\pi)$ between threshold and the c.m. energy equal to the mass of the kaon, then $\sigma(e^+e^- \to e^+e^-3\pi)$ at the mass of the kaon has a value larger than 10^{-33} cm^2. Although this does not violate the present experimental limit set by colliding beams, clearly future work can either rule out or substantiate this large cross section.[9]

Throughout this discussion, by neglecting terms proportional to ϵ in Eqs. (5) and (6), we have implicitly assumed that the CP-nonconserving amplitudes are at most of the order of the CP-conserving ones. In a recent paper,[10] however, Christ and Lee have discussed the implications of keeping terms proportional to ϵ. In particular, using the same assumptions (A), (C), and (D) and the experimental result of Eq. (3), they show that the $K_L \to \mu^+\mu^-$ result, Eq. (1), can be explained as due to a nonconservation of CP invariance if the rate for $K_S \to \mu^+\mu^-$ is larger than the rate for $K_L \to \mu^+\mu^-$ by at least a factor $O(10^6)$. This implies a destructive interference among the terms in the numerator of Eq. (7). Clearly, further experimental work on $K_{S,L} \to \mu^+\mu^-$ and also on $K_S \to \gamma\gamma$ and $e^+e^- \to e^+e^-3\pi$ is needed.

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