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T Invariance and the Unitarity Limit in $K_L^0 \to \mu^+\mu^-$

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The role of T invariance in the unitarity (lower) limit of the decay $K_L^0 \to \mu^+\mu^-$ is examined. We note that the assumption of T invariance is necessary only insofar as ϵ, the CP-violating parameter in the neutral K system, is nonzero. Because of the smallness of this parameter, a chance cancellation appears unlikely. Therefore, the unitarity (lower) limit for $K_L^0 \to \mu^+\mu^-$ can be decreased only (with the above exception) by including in the unitarity sum intermediate states other than the two-photon state (e.g., three-pion states).

The decay

$$K_L^0 \to \mu^+\mu^- \qquad (1)$$

has been of considerable recent interest. Theoretically, this interest arises from possible effects of

(i) higher-order weak interactions,[1]

(ii) the existence of neutral leptonic weak currents,[2] and

(iii) lowest-order weak interactions plus order-α^2 electromagnetic interactions.[3,4]

An experimental study of this reaction can also provide interesting tests for CP and CPT invariance in presently unexplored dynamical situations.[5]

It has been shown that in the absence of observable effects from (i) and (ii) above, nevertheless (iii) provides a lower limit for the rate at which reaction (1) must proceed.[3,4] This lower limit has been derived assuming

(a) unitarity and CPT invariance,

(b) time-reversal invariance, and

(c) dominance of the unitarity sum by the two-photon intermediate state.

Taking the observed branching ratio[6] for

$$K_L^0 \to \gamma\gamma, \qquad (2)$$

i.e., $(5.0\pm0.5)\times10^{-4}$, the lower limit for the branching ratio of reaction (1) is calculated,[7] on this basis, to be[3,4]

$$R_L^{\rm cal}(\mu^+\mu^-) \geq (6.0\pm0.6)\times10^{-9}. \qquad (3)$$

The contribution of states other than the two-photon state has been estimated.[4] From simple dimensional arguments, it has been shown that the contribution of such states (in particular, the three-pion states) is on the order of 10% of the two-photon state.[4]

Recently, the results of experiments in search of reaction (1) have been reported. Thus far, there is no clear evidence that reaction (1) proceeds. In particular, an upper limit has been established. It is[8]

$$R_L^{\rm exp}(\mu^+\mu^-) \leq 1.8\times10^{-9} \quad (90\% \text{ confidence limit}). \qquad (4)$$

This number is significantly below the calculated lower limit of Eq. (3).

In view of this discrepancy, and noting the well-known observation of CP violation in the neutral K system,[9] the assumption of T invariance in the derivation of the theoretical lower limit given by Eq. (3) is suspect. Therefore, the role of T invariance in this derivation deserves further examination.

The matrix elements for $K^0 \to \mu^+\mu^-$ and $\bar{K}^0 \to \mu^+\mu^-$ are taken to be, respectively,

$$\begin{aligned} &\bar{u}(l)(F_1 + i\gamma_5 F_2)v(\bar{l}), \\ &\bar{u}(l)(G_1 + i\gamma_5 G_2)v(\bar{l}). \end{aligned} \qquad (5)$$

Then from CP invariance one gets

$$F_1 = -G_1, \quad F_2 = G_2, \qquad (6)$$

while from CPT invariance and the neglect of final-state interactions, one gets

$$F_1 = G_1^*, \quad F_2 = G_2^*. \qquad (7)$$

The matrix element for reaction (1) is given by

$$\begin{aligned} N^{-1}\bar{u}(l)[(pF_1+qG_1)+i\gamma_5(pF_2+qG_2)]v(\bar{l}) \\ = \left(\frac{p+q}{2N}\right)\bar{u}(l)\{[(F_1+G_1)+\epsilon(F_1-G_1)] \\ +i\gamma_5[(F_2+G_2)+\epsilon(F_2-G_2)]\}v(\bar{l}), \end{aligned} \qquad (8)$$

with $N=(|p|^2+|q|^2)^{1/2}$; p and q are the standard parameters which give the admixture of K^0 and $\bar{K}^0$ states in K_L^0,[10] and $\epsilon \equiv (p-q)/(p+q)$. With CP invariance, $\epsilon = 0$ and reaction (1) proceeds only with the lepton pairs in the $^1S_0$ state, i.e., $F_1 + G_1 = 0$, $F_2 + G_2 \neq 0$.

With CPT invariance (not T invariance)[11] and the use of the unitarity equation, one can calculate the imaginary (absorptive) parts of $F_1 + G_1$ and $F_2 + G_2$, and the real (still absorptive) parts of $F_1 - G_1$ and $F_2 - G_2$.

If the unitarity sum is dominated by the two-photon intermediate state, a straightforward calculation gives the following set of equations[12]:

$$\mathrm{Im}(F_2 + G_2) = 2A\,\mathrm{Re}(f_2 + g_2)\,, \tag{9a}$$

$$\mathrm{Im}(F_1 + G_1) = A\,\mathrm{Re}(f_1 + g_1)\,, \tag{9b}$$

$$\mathrm{Re}(F_1 - G_1) = -A\,\mathrm{Im}(f_1 - g_1)\,, \tag{9c}$$

$$\mathrm{Re}(F_2 - G_2) = -2A\,\mathrm{Im}(f_2 - g_2)\,, \tag{9d}$$

where

$$A = \tfrac{1}{2}\alpha \frac{m_l}{\beta_l M_K} \ln\left(\frac{1+\beta_l}{1-\beta_l}\right) = 2.57\times 10^{-3} \quad (\text{for } m_l = m_\mu)\,,$$

$$\beta_l = \left(1 - \frac{4m_l^2}{M_K^2}\right)^{1/2}\,,$$

and α is the fine-structure constant. The amplitudes for $K^0 \to \gamma\gamma$ and $\bar{K}^0 \to \gamma\gamma$ have been taken to be, respectively,

$$\begin{aligned} &\frac{i}{M_K}\left(f_1 F_{\mu\nu}F'_{\mu\nu} + i f_2 F_{\mu\nu}\tilde{F}'_{\mu\nu}\right)\,, \\ &\frac{i}{M_K}\left(g_1 F_{\mu\nu}F'_{\mu\nu} + i g_2 F_{\mu\nu}\tilde{F}'_{\mu\nu}\right)\,, \end{aligned} \tag{10}$$

where $F_{\mu\nu} = \epsilon_\mu k_\nu - \epsilon_\nu k_\mu$, $\tilde{F}_{\mu\nu} = e_{\mu\nu\lambda\rho} F_{\lambda\rho}$. With the above amplitudes for $K^0 \to \gamma\gamma$ and $\bar{K}^0 \to \gamma\gamma$, a set of equations identical in form to Eqs. (6) and (7) hold from CP and CPT invariance, respectively. [We note that only the dispersive parts enter on the right-hand side of Eqs. (9) when all appropriate intermediate states are included.]

With CP invariance, only Eqs. (9a) and (9c) are relevant. Equation (9a) applies to K_L^0 decay and gives rise to the unitarity (lower) limit for reaction (1),[3,4] and the two-photon state is expected to dominate the unitarity sum by roughly an order of magnitude.[4] Equation (9c) applies to K_S^0 decay, and the two-photon-dominance assumption is not valid. Other intermediate states, such as 2π and $2\pi\gamma$, contribute significantly.[4]

With possible CP violation in neutral K decays (other than $\epsilon \neq 0$), Eqs. (9b) and (9d) are generated by the unitarity equation. If appreciable CP violation occurs in reaction (2) (i.e., to generate a significant portion of the observed $K_L^0 \to \gamma\gamma$ rate), then the assumed two-photon dominance is valid for Eq. (9b). Other contributions to this unitarity sum come from the 2π state generated by a possible $\epsilon' \neq 0$ (the on-mass-shell CP-violating amplitude in $K_L^0 \to 2\pi$),[10] and the corresponding $2\pi\gamma$ inner bremsstrahlung state, and would be negligible. Much less is known about the validity of Eq. (9d) from experiments, since the reactions

$$K_S^0 \to \gamma\gamma\,, \tag{11}$$

$$K_S^0 \to 3\pi\,, \tag{12}$$

have yet to be observed (irrespective of the presence of CP violation).

By using Eqs. (9a) and (9b), it is easy to see that one can set a rigorous (to the extent of two-photon dominance) lower bound on the rate of reaction (1), insofar as ϵ can be neglected. Because ϵ is complex and nonzero, a cancellation can occur, in principle, to destroy the above lower bound. This cancellation can be generated by dispersive parts in $F_1 - G_1$ or $F_2 - G_2$ [see Eq. (8)]. This cancellation appears unlikely because the factor ϵ, which has magnitude $\sim 2\times 10^{-3}$, is a coefficient of $F_1 - G_1$ and $F_2 - G_2$. These amplitudes would have to be large in order for the cancellation to be effective. Such large amplitudes can be observed in the decay

$$K_S^0 \to \mu^+\mu^-\,. \tag{13}$$

In particular, a branching ratio of order 10^6 larger than the corresponding unitarity estimate would be necessary, i.e., the branching ratio $(K_S^0 \to \mu^+\mu^-)/(K_S^0 \to \text{all})$ would be $\sim 10^{-6}$. This seems unlikely, although the possibility is not strictly eliminated by experiment.[13]

To the extent, then, that ϵ can be neglected, and that the experimental limit of Eq. (4) holds, the contribution, in the standard way, of states other than the two-photon state in the unitarity sum on the right-hand side of Eq. (9a) must be important. [This solution would appear to be the most palatable one at present. Other alternatives are possible as, for example, the introduction of a new interaction for muons to generate the cancellation in Eq. (9a), etc. All dispersive contributions to reaction (1) must also nearly cancel.] In particular, the 3π states, though estimated to be too small by an order of magnitude,[4] may

(i) generate sufficient absorptive parts to reaction (2), and

(ii) provide the necessary cancellation in Eq. (9a) to reconcile the present disagreement between Eqs. (3) and (4).

Note. After the completion of the present work, we were informed of similar work performed by Farrar and Treiman,[14] and Christ and Lee.[15] Farrar and Treiman used Eq. (9b) in reverse, i.e., to set upper bounds on CP violation in reaction (2). Christ and Lee considered in detail the implication of cancellations arising from the dispersive parts of $F_1 - G_1$ and/or $F_2 - G_2$. In particular, they placed upper and lower bounds on reaction (13) and compared these with present experimental results. We thank Professor S. B. Treiman and Professor N. H. Christ for bringing the above to our attention.

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