Quasipotential Theory of High-Energy Hadron Scattering

V.R. Garsevanishvili,* V.A. Matveev,† L.A. Slepchenko,* and A.N. Tavkhelidze*
International Atomic Energy Agency, International Centre for Theoretical Physics, Miramare, Trieste, Italy
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A quasipotential approach to the high-energy scattering of hadrons is developed. The consideration is based on the quasipotential equation for the scattering amplitude in quantum field theory, and on the physical assumption of the nonsingular character of hadron interactions at high energies. The results of the theoretical calculation are compared with experimental data on elastic nucleon-nucleon and isobar production processes.

I. INTRODUCTION

In this work we shall give the quasipotential description of high-energy hadron scattering.[1,2] This approach is based on the quasipotential equation for the scattering amplitude in quantum field theory.[3,4]

To begin with, let us consider the simplest case of scattering of two spinless particles with equal masses. In this case the quasipotential equation reads

$$T(E;\vec{p},\vec{k}) = V(E;(\vec{p}-\vec{k})^2) + \int \frac{d\vec{q}}{(m^2+\vec{q}^2)^{1/2}}\,\frac{V(E;(\vec{p}-\vec{q})^2)\,T(E;\vec{q},\vec{k})}{\vec{q}^2+m^2-E^2-i0}, \tag{1.1}$$

where E is the energy, and $\vec{p}$, $\vec{k}$, and $\vec{q}$ are the (center-of-mass system) relative momenta of particles in the initial, final, and intermediate states, respectively. The physical relativistically invariant scattering amplitude $T(s,t)$ is defined by the condition

$$T(s,t) = 32\pi^3 T(E;\vec{p},\vec{k})\Big|_{\substack{s=4E^2=4(\vec{p}^{\,2}+m^2)=4(\vec{k}^{\,2}+m^2),\\ t=-(\vec{p}-\vec{k})^2.}} \tag{1.2}$$

Equation (1.1) presents the generalization of the Lippmann-Schwinger equation for the case of relativistic quantum field theory. In contrast to quantum mechanics, however, the quasipotential in Eq. (1.1) is, in general, a complex function of the energy E. The imaginary part of the quasipotential is due to inelastic processes in two-particle scattering and is a positive definite quantity.

It can be rigorously proved that the condition of positivity of the imaginary part of the quasipotential ensures the validity of the so-called "underunitarity" condition

$$SS^\dagger \leqslant 1 \tag{1.3}$$

for the two-particle scattering matrix S.[5] For a pure real quasipotential, for example, we have the relativistic condition of elastic unitarity.

As was shown in Refs. 3 and 4, for the case of weak coupling a general method exists for constructing the local quasipotential using the perturbation expansion for the scattering amplitude. The quasipotential constructed in this way gives a solution which coincides on the mass shell with the physical scattering amplitude. This method was used by Faustov[6] for investigating the spectra of positronium and hydrogenlike atoms in the framework of quantum electrodynamics.

For strong interactions, there is no general method of constructing the quasipotential. For this reason we develop a model of high-energy hadron scattering which is based on the phenomenological choice of the quasipotential in Eq. (1.1). In fact, we will use essentially the following general principles:

(a) The existence of the local quasipotential $V(s,\vec{r})$ which gives an adequate description of high-energy hadron scattering.

(b) The positivity of the imaginary part of the quasipotential:

$$\operatorname{Im} V(s,\vec{r}) \geqslant 0. \tag{1.4}$$

(c) Smooth and nonsingular behavior of the quasipotential at the origin as a function of the relative coordinate of two particles.

The last principle, we believe, concerns the dynamics of hadron interactions at high energies. Probably it means that in high-energy collisions hadrons behave as extended objects with finite sizes.[7,8]

The important consequence of the assumption (c) is that the high-energy hadron scattering has a so-called semiclassical character when the wavelengths of the colliding particles become very small as compared to the size of an effective range of interaction. As a result we have the eikonal or Glauber representation for the scattering amplitude at small angles,[1,2] and the exponential decrease of the scattering amplitude with energy at large angles.[8]

Next, the assumption (c) allows the solution of Eq. (1.1) to be found, in the high-energy limit, as a convergent series of Born approximations. The interesting point is that under certain conditions the first Born term in this series should suffice at small enough momentum transfers.

On the other hand, as is known from experiments, the elastic hadron scattering at high energies and small momentum transfers has a diffractive character, and can be approximately described by a pure imaginary amplitude of the form

$$T(s,t)\approx is\sigma_{\rm tot}(\infty)e^{a(s)t}. \qquad (1.5)$$

The formula (1.5) ensures the constancy of the total cross section and the exponential decrease of the differential cross section with increasing square of momentum transfer. The quantity $a(s)$ is related to the diffraction-slope parameter and, in general, varies slowly with energy.

It is natural to consider the expression (1.5) as the Born approximation for the scattering amplitude. The corresponding local quasipotential can be found by the Fourier transformation and has the Gaussian form

$$V(s,\vec{r})=isg_0(\pi/a)^{3/2}e^{-r^2/4a}. \qquad (1.6)$$

Here

$$\sigma_{\rm tot}(\infty)=32\pi^3 g_0, \qquad (1.7)$$

and thus g_0 is a constant and positive parameter. One can easily see that the local quasipotential (1.6) has a positive imaginary part, and is a smooth and nonsingular function of r.

We will consider Eq. (1.6) as a simplest form of quasipotential which obeys all the principles listed above and also the requirement of diffraction behavior at small momentum transfers. In what follows, we will show that the quasipotential (1.6) allows the main features of high-energy hadron scattering at small and large angles to be reproduced. We should like to stress that some results do not depend on the specific form of the quasipotential (1.6) and have a general character.

Now let us briefly discuss the connection between the quasipotential approach and that which is based on the Regge-pole hypothesis.

It was suggested several years ago[9] that the diffraction behavior of high-energy hadron scattering is due to the exchange in the t channel of the Pomeranchuk Regge pole with the amplitude

$$T(s,t)=-\beta(t)s^{\alpha_P(t)}\frac{1+e^{-i\pi\alpha_P(t)}}{\sin\pi\alpha_P(t)}. \qquad (1.8)$$

In fact, if one assumes the linear rising of the trajectory and the smooth behavior of the residue function, i.e.,

$$\alpha_P(t)=1+\alpha' t,\quad \beta(t)=\beta(0)e^{bt}, \qquad (1.9)$$

one gets just the expression (1.5) with

$$\sigma_{\rm tot}(\infty)=\beta(0),\quad a(s)=b+\alpha'\ln s. \qquad (1.10)$$

However, as is well known now, the single Pomeranchuk pole, if it exists, gives a very crude approximation for the high-energy scattering amplitude.

There appears in general an infinite sequence of Regge cuts in the complex angular momentum plane, which should be taken into account. Unfortunately, in spite of considerable efforts in the investigation of these Regge cuts,[10] there is as yet no method of calculating their contributions in an unambiguous way. For this reason, the quasipotential approach has a considerable advantage, because it allows the corrections to the Born term of the type (1.5) to be found in a simple way.

It should be noticed that we consider here a rather simplified model of a "pure elastic" scattering. In the intermediate-energy region, it may become necessary to take into account secondary effects such as, for example, nonleading Regge-pole exchange or direct-channel resonances. To do so in a consistent way, the requirements of analyticity and crossing symmetry in the form of finite-energy sum rules[11] may be used.

II. SMALL-ANGLE ELASTIC SCATTERING

Here we will consider the small-angle elastic scattering at high energies,

$$|t/s|\ll 1 \quad\text{and}\quad as\gg 1, \qquad (2.1)$$

where the parameter a is connected with an effective range of interaction by

$$4a=R^2. \qquad (2.2)$$

We will assume here that the quasipotential is pure imaginary and of the simple Gaussian form (1.6), and put off the discussion of other quasipotentials to the end.

A. Small Momentum Transfers

Let us consider first the case of small momentum transfers,

$$|at|<1 \quad\text{and}\quad as\gg 1. \qquad (2.3)$$

In this case, Eq. (1.1) can be solved by iteration procedure:

$$T(\vec{\Delta}^2;E)=V(\vec{\Delta}^2;E)+\delta T(\vec{\Delta}^2;E)+\cdots, \qquad (2.4)$$

and one can expect that only the first few terms in the series (2.4) should suffice.

The expression for the first correction to the Born approximation is of the form

$$\delta T(\vec{\Delta}^2; E) = \int \frac{d\vec{q}}{(m^2+\vec{q}^2)^{1/2}} \times \frac{V((\vec{p}-\vec{q})^2; E)\, V((\vec{q}-\vec{k})^2; E)}{\vec{q}^2+m^2-E^2-i0} = (isg_0)^2 e^{at/2} A(\vec{\Delta}^2; E), \tag{2.5}$$

where

$$A(\vec{\Delta}^2; E) = \int \frac{d\vec{q}}{(m^2+\vec{q}^2)^{1/2}} \frac{e^{-2a(\vec{q}-\vec{\lambda})^2}}{\vec{q}^2+m^2-E^2-i0}, \quad \vec{\lambda} = \tfrac{1}{2}(\vec{p}+\vec{k}). \tag{2.6}$$

After integration over the angles, we get

$$A(\vec{\Delta}^2; E) = \frac{\pi}{2a\lambda} \int_{-\infty}^{\infty} \frac{q\,dq}{(m^2+q^2)^{1/2}} \frac{e^{-2a(q-\lambda)^2}}{q^2+m^2-E^2-i0}, \tag{2.7}$$

where $\lambda = \frac{1}{2}|\vec{p}+\vec{k}|$ or, on the mass shell,

$$\lambda = (p^2 + \tfrac{1}{4}t)^{1/2} = p\cos\tfrac{1}{2}\theta. \tag{2.8}$$

Let us rewrite (2.7) as follows:

$$A = R + iI, \tag{2.9}$$

where

$$I = \frac{\pi^2}{4a\lambda(m^2+p^2)^{1/2}} \left(e^{-2a(p-\lambda)^2} - e^{-2a(p+\lambda)^2}\right) \tag{2.10}$$

and R is determined by the principal value of the integral (2.7). In the high-energy small-momentum-transfer limit (2.3), one can get

$$I = \frac{\pi^2}{as} + O(1/s^2), \tag{2.11a}$$

$$R = \frac{\pi^2}{as} \frac{1}{(2\pi as)^{1/2}} + O(1/s^2). \tag{2.11b}$$

Thus the first two terms in the series (2.4) give

$$T(\vec{\Delta}^2; E) = isg_0 e^{at} - isg_0(\pi^2 g_0/a)(1+i\alpha)e^{at/2} + \cdots, \tag{2.12}$$

where $\alpha = 1/(2\pi as)^{1/2}$.

The conditions of validity of the Born approximation read

$$\pi^2 g_0/a < 1, \quad |at| < 2\ln(a/\pi^2 g_0). \tag{2.13}$$

Under these conditions the first term in Eq. (2.12) describes the diffraction scattering at small momentum transfers with the diffraction-peak slope $A \approx 2a$.

Near the point

$$t \approx -\frac{2}{a}\ln(a/\pi^2 g_0), \tag{2.14}$$

where the correction becomes comparable to the first Born approximation, a minimum can be observed in the differential cross section.

Further, from Eq. (2.12), at $t=0$ we get for the total cross section

$$\sigma_{tot} = 32\pi^3 g_0(1-\pi^2 g_0/a). \tag{2.15}$$

If one assumes that the parameter a increases with energy, say, as a logarithm of s, it can be seen that the total cross section will tend to its asymptotic value

$$\sigma_{tot}(\infty) = 32\pi^3 g_0 \tag{2.16}$$

from below.[12] We notice that this behavior of the total cross section is preserved when the whole series of Born approximations is taken into account.

Next, in the approximation (2.12) the ratio of the real and the imaginary parts of the scattering amplitude is determined by

$$\rho(s,t) = \frac{\mathrm{Re}\,T(s,t)}{\mathrm{Im}\,T(s,t)} = \frac{\pi^2 g_0}{a(2\pi as)^{1/2}} e^{-at/2} \tag{2.17}$$

and is a small, positive quantity.

Apparently, both phenomena, the growth of the total cross section to its asymptotic value and the positivity of the real part of the scattering amplitude at very high energies, are connected by general principles such as analyticity, crossing symmetry, etc.

It should be noticed that the observable behavior of the total cross sections and the real parts of the scattering amplitudes at accessible energies do not contradict, generally speaking, the behavior discussed above, and can be explained by secondary effects such as the nonleading Regge-pole or the direct-channel resonance contributions. In any case, the smallness of the real part of the scattering amplitude at high energies and small transfer momenta is in agreement with experimental data. For this reason, in what follows we shall neglect the real part of the scattering amplitude at high energies and small angles.

B. Large Momentum Transfers

Let us consider now the scattering at large momentum transfers outside the diffraction region, when

$$|t/s| \ll 1 \quad \text{and} \quad |at| > 1. \tag{2.18}$$

In this case the scattering amplitude can be found as a convergent series of the Born approximations:

$$T(\vec{\Delta}^2; E) = isg_0 \sum_{n=1}^{\infty} \frac{e^{at/n}}{n n!} \left(-\frac{4\pi^2 g_0}{a}\right)^{n-1}. \tag{2.19}$$

It is easy to see that at large momentum transfer the main contributions to the sum (2.19) are given by the terms with large numbers n.

Now we study the asymptotic behavior of the series (2.19) in the limit $|at| \gg 1$.

Using the Sommerfeld-Watson transform, we rewrite Eq. (2.19) as follows:

$$T(\vec{\Delta}^2; E) = \frac{as}{8\pi^2} \int_C \frac{dz\, e^{-\sigma z - \omega/z}}{2z\Gamma(z+1)\sin\pi z}, \qquad (2.20)$$

where

$$\sigma = \ln(a/4\pi^2 g_0), \quad \omega = |at|. \qquad (2.21)$$

The integration contour C surrounds the positive real semiaxis in a clockwise direction and contains the integers $z = 1, 2, \ldots$.

It can be shown that in the region (2.18) the main contributions to the integral (2.20) come from the first terms of the series expansions of the function $1/\sin\pi z$,

$$\frac{1}{\sin\pi z} = \mp 2i \sum_{n=0}^{\infty} e^{\pm 2\pi i z(n+1/2)}, \qquad (2.22)$$

on the upper and lower branches of the contour C, respectively.

Using the saddle-point method, we get

$$T(\vec{\Delta}^2; E) \approx -\frac{isa}{4\pi^2} \operatorname{Re}\left(\frac{e^{-2(2a|t|)^{1/2}\sinh\gamma + \gamma/2}}{(a|t|\cosh\gamma)^{1/2}}\right). \qquad (2.23)$$

Here the parameter γ is related to the position of a saddle point z_0 by

$$z_0 = (2a|t|)^{1/2} e^{-\gamma} \qquad (2.24)$$

and is determined by the following condition:

$$e^{2\gamma} + 2\gamma = 2[\sigma - i\pi + \ln(2a|t|)^{1/2}]. \qquad (2.25)$$

Equation (2.23) can be written as

$$T(\vec{\Delta}^2; E) \approx \frac{is\alpha}{(a|t|)^{1/2}} e^{-\beta(a|t|)^{1/2}} \cos\psi(s,t), \qquad (2.26)$$

where the parameters α and β are slowly varying functions of s and t.

Thus one can see that in the region (2.18) the scattering amplitude is an exponentially decreasing function of the momentum transfer $|t|^{1/2}$, with possible oscillations[13] near the points where

$$\psi(s,t) = \pi(\kappa + \tfrac{1}{2}), \quad \kappa \text{ an integer}. \qquad (2.27)$$

However, as is seen from Eq. (2.25), at sufficiently large momentum transfers the oscillations, if they exist, should disappear, and the scattering amplitude should behave as follows:

$$T(\vec{\Delta}^2; E) \approx -\frac{isa}{2\pi^2} (\ln 2\omega)^{1/4} \frac{e^{-(2\omega \ln 2\omega)^{1/2}}}{(2\omega \ln 2\omega)^{1/2}}. \qquad (2.28)$$

C. Eikonal Description of Small-Angle Scattering

The eikonal approximation of the scattering amplitude at high energies and small angles has been known for a long time and is very popular nowadays.[14] It is based on the semiclassical picture of high-energy scattering when the wavelengths of the colliding particles are very small as compared to the size of an effective range of interaction. Further, we shall see that the semiclassical character of high-energy small-angle hadron scattering is essentially due to the assumption of nonsingular or smooth behavior of the quasipotential as a function of the relative coordinate of two particles.

Let us first discuss the properties of the solution (2.19). There is a useful formula:

$$\frac{e^{at/n}}{n} = \frac{1}{4\pi a} \int d^2\rho\, e^{i\vec{\rho}\cdot\vec{\Delta}_\perp} e^{-n\vec{\rho}^2/4a}, \qquad (2.29)$$

where the vector $\vec{\Delta}_\perp = (\vec{p} - \vec{k})$ varies in a plane which is perpendicular to the vector $\frac{1}{2}(\vec{p} + \vec{k})$.

Substituting the formula (2.29) into Eq. (2.19), we get

$$T(\vec{\Delta}^2; E) = \frac{s}{(2\pi)^3} \int d^2\rho\, e^{i\vec{\rho}\cdot\vec{\Delta}} \frac{e^{2i\chi(\rho)} - 1}{2i}, \qquad (2.30)$$

where

$$2i\chi(\rho) = -(4\pi^2 g_0/a) e^{-\vec{\rho}^2/4a}. \qquad (2.31)$$

Equation (2.30) is just the eikonal representation for the scattering amplitude in the region (2.1), and the phase function is related to the quasipotential (1.6) by

$$\chi(\rho) = \frac{1}{s} \int_{-\infty}^{\infty} V(s, (\rho^2 + z^2)^{1/2})\, dz. \qquad (2.32)$$

Obviously, this representation would hold for a large class of smooth quasipotentials.

To show this, it is convenient to consider the quasipotential equation for the wave function of two particles in configuration space:

$$(E^2 - m^2 + \vec{\nabla}^2)\psi(\vec{r}) = -\frac{1}{\omega} V(s, \vec{r})\psi(\vec{r}), \qquad (2.33)$$

where

$$\omega = (m^2 - \vec{\nabla}^2)^{1/2}. \qquad (2.34)$$

Owing to the presence of the operator (2.34), Eq. (2.33) is nonlocal. However, under the condition of nonsingular or smooth behavior of the quasipotential $V(s, \vec{r})$, Eq. (2.33) takes an effectively local form in the high-energy limit. Actually, let us look for a solution of Eq. (2.33) in the form

$$\psi(\vec{r}) = e^{ipz} \varphi(\vec{r}), \qquad (2.35)$$

where $\varphi(\vec{r})$ is expected to be a slowly varying function and $E = (\vec{p}^2 + m^2)^{1/2}$.

It can easily be shown that on a space of slowly varying functions, in the high-energy limit,

$$e^{-ipz} \omega e^{ipz} = p - i\partial/\partial z + O(1/p), \qquad (2.36a)$$

or

$$e^{-ipz} (1/\omega) e^{ipz} = 1/p + O(1/p^2). \qquad (2.36b)$$

Thus the function $\varphi(\vec{r})$ obeys the equation

$$-2ip\frac{\partial\varphi(\vec{r})}{\partial z}=\frac{1}{p}V(s,\vec{r})\varphi(\vec{r}), \tag{2.37}$$

which coincides with the one which follows from the local Klein-Gordon equation with effective potential $(1/p)V(s,\vec{r})$. As a result we have the eikonal representation (2.30) with the phase function $\chi(\rho)$ determined by (2.32). So, the eikonal or, as we now usually call it, the Glauber representation should not be considered as a primary dynamical principle (see, for example, Arnold[15]), but it is a consequence of the assumption of the nonsingular character of hadron interactions at high energies.

Recently, many attempts have been made to formulate the eikonal representation of the scattering amplitude in a way which is not restricted to small angles, and which is based only on general principles such as relativistic invariance, unitarity, and analyticity.[16,17] One may suspect, however, that in all these attempts some additional assumptions are introduced in a nonevident way.

D. The Unitarity-Equation Approach

There is a very attractive approach to the description of high-energy hadron scattering, based on the unitarity equation in quantum field theory.[18-20] Here we discuss it briefly and compare it with the quasipotential approach.

The unitarity equation for the scattering amplitude of two spinless particles of equal mass has the form

$$\mathrm{Im}T(s,t)=\int d\omega\, T(s,t')T^\dagger(s,t'')+F(s,t), \tag{2.38}$$

where in the center-of-mass system

$$t=-(\vec{p}-\vec{k})^2,\quad t'=-(\vec{p}-\vec{q})^2,\quad t''=-(\vec{q}-\vec{k})^2,$$
$$s=4(m^2+\vec{p}^2)=4(m^2+\vec{q}^2)=4(m^2+\vec{k}^2), \tag{2.39}$$

and

$$d\omega=\frac{1}{8\pi^2}\frac{d\vec{q}d\vec{q}'}{2q_0 2q_0'}\delta(p+p'-q-q') \tag{2.40}$$

is related to the two-particle phase volume. The quantity $F(s,t)$, which is known as the overlap function of Van Hove, represents the contributions of the inelastic states in Eq. (2.38).

It is convenient to choose the following directions of particle momenta:

$$\vec{p}+\vec{k}=(2p_z,0,0),\qquad \vec{k}-\vec{p}=(0,\vec{\Delta}_\perp), \tag{2.41a}$$

or

$$\vec{p}=(p_z,\tfrac{1}{2}\vec{\Delta}_\perp),\quad \vec{k}=(p_z,-\tfrac{1}{2}\vec{\Delta}_\perp). \tag{2.41b}$$

From Eqs. (2.41) it follows that

$$s=4(m^2+p_z^2)+\vec{\Delta}_\perp^2,$$
$$t=-\vec{\Delta}_\perp^2. \tag{2.42}$$

At high energies and small scattering angles, when $|t/s|\ll 1$, we have

$$s\approx 4p_z^2 \text{ and } t=-\vec{\Delta}_\perp^2=-(\vec{k}_\perp-\vec{p}_\perp)^2. \tag{2.43}$$

If one assumes that the main contributions to the integral in Eq. (2.38) are given by a region of small scattering angles, one can put in Eq. (2.38)

$$t'=-(\vec{p}_\perp-\vec{q}_\perp)^2,\quad t''=-(\vec{q}_\perp-\vec{k}_\perp)^2. \tag{2.44}$$

For the same reason we have

$$\int d\omega\approx\frac{1}{8\pi^2 s}\int_{4\vec{q}_\perp^2\le s}d^2q_\perp. \tag{2.45}$$

Thus, in this approximation the unitarity equation (2.38) takes the following form:

$$\mathrm{Im}T(s,-(\vec{p}_\perp-\vec{k}_\perp)^2=F(s,-(\vec{p}_\perp-\vec{k}_\perp)^2)+\frac{1}{8\pi^2 s}\int_{4\vec{q}_\perp^2\le s}d^2q_\perp T(s,-(\vec{p}_\perp-\vec{q}_\perp)^2)T^\dagger(s,-(\vec{q}_\perp-\vec{k}_\perp)^2). \tag{2.46}$$

If we assume that the integration region in Eq. (2.46) can be expanded out of the circle $4\vec{q}^2\le s$ on the whole plane of transverse momenta, we get the well-known convolution formula

$$\mathrm{Im}T=\frac{1}{8\pi^2 s}\,T*T^\dagger+F. \tag{2.47}$$

We would like to note, however, that such an extension is, generally speaking, a nontrivial thing and needs continuation off the mass shell. Anyhow, it is connected with the assumption about the nonsingular character of high-energy hadron scattering.

Let us consider now the approximation of the pure imaginary amplitude

$$T(s,t)=isA(s,t) \tag{2.48}$$

and assume, as was suggested in Ref. 18, that the overlap function is of the Gaussian form

$$F(s,t)=s\sigma_{\mathrm{inel}}e^{at}. \tag{2.49}$$

Equation (2.47) can easily be solved using the Fourier transformation

$$A(s,t)=\int d^2\rho\, e^{i\vec{\rho}\cdot\vec{\Delta}_\perp}a(s,\vec{\rho}), \tag{2.50}$$

where $t=-\vec{\Delta}_\perp^2$. In doing so we obtain the equation

$$a(s,\vec{\rho})=\tfrac{1}{2}a^2(s,\vec{\rho})+\frac{\sigma_{\mathrm{inel}}}{4\pi a}e^{-\rho^2/4a} \tag{2.51}$$

with the following solution[21]:

$$a(s, \vec{\rho}) = 1 - [1 - \eta(s, \rho)]^{1/2}, \qquad (2.52)$$

where

$$\eta(s, \rho) = \frac{\sigma_{\text{inel}}}{2\pi a}\, e^{-\rho^2/4a}. \qquad (2.53)$$

Assuming that

$$\eta(s, \rho) < 1, \qquad (2.54)$$

we can expand the solution (2.52) in a power series of $\eta(s, \rho)$,

$$a(s, \rho) = \sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} \frac{e^{-n\rho^2/4a}}{2n-1} \left(\frac{\sigma_{\text{inel}}}{8\pi a}\right)^n. \qquad (2.55)$$

Substituting the expression (2.55) in the formula (2.50), we get the solution for the scattering amplitude:

$$T(s, t) = is\sigma_{\text{inel}} \sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} \frac{e^{-at/n}}{2n(2n-1)} \left(\frac{\sigma_{\text{inel}}}{8\pi a}\right)^{n-1}. \qquad (2.56)$$

Thus we obtain for the scattering amplitude at high energies and small angles a series of the type (2.19), but with coefficients which do not alternate in sign and which decrease much more slowly with increasing n.

The constancy of signs of the terms in such a series has already been mentioned in some works as a difficulty of this approach, because it prevents dips and oscillations of the differential cross section from being explained.

Another difficulty consists in the following. It can be shown that the series (2.56) decreases with increasing momentum transfers as an exponent of $-|t|^{1/2}$, whereas the overlap function (2.49) decreases as an exponent of $-|t|$. It is known, however, that the behavior of elastic processes and that of inelastic processes do not differ considerably at large momentum transfer.

Contrary to this, the quasipotential equation (1.1) with the quasipotential (1.6) corresponds to the unitarity equation with an overlap function of a rather complicated form,

$$F(s, t) = s\beta e^{at} + s\beta \sum_{n=2}^{\infty} \frac{e^{at/n}}{nn!} (2^n - 3)\left(-\frac{4\pi^2 g_0}{a}\right)^{n-1}, \qquad (2.57)$$

where $\beta = 32\pi^3 g_0$. The expression (2.57) can be obtained by straightforward calculations using Eq. (2.19) and the unitarity condition.

One can see that the series (2.57) has the same properties as the elastic scattering amplitude.

E. Application to Elastic *pp* Scattering

Here we would like to give a comparison of the results obtained previously on the basis of the quasipotential Eq. (1.1) with the experimental data on high-energy elastic *pp* scattering at small angles.

In describing the real physical processes it is necessary, generally speaking, to take into account the spin structure of the scattering amplitude. In the case of elastic proton-proton scattering, for instance, there are five independent invariant amplitudes, which can be chosen in the helicity basis as follows:

$$\begin{aligned} T_1 &= \langle \tfrac{1}{2}, \tfrac{1}{2} | T | \tfrac{1}{2}, \tfrac{1}{2} \rangle, \\ T_2 &= \langle \tfrac{1}{2}, \tfrac{1}{2} | T | -\tfrac{1}{2}, -\tfrac{1}{2} \rangle, \\ T_3 &= \langle \tfrac{1}{2}, -\tfrac{1}{2} | T | \tfrac{1}{2}, -\tfrac{1}{2} \rangle, \\ T_4 &= \langle \tfrac{1}{2}, -\tfrac{1}{2} | T | -\tfrac{1}{2}, \tfrac{1}{2} \rangle, \\ T_5 &= \langle \tfrac{1}{2}, \tfrac{1}{2} | T | \tfrac{1}{2}, -\tfrac{1}{2} \rangle. \end{aligned} \qquad (2.58)$$

However, only two of them, T_1 and T_3, which correspond to the spin-nonflip processes, give nonvanishing contributions to the forward scattering.

The relative magnitudes of the spin-flip amplitudes T_2, T_4, and T_5 at nonzero scattering angles can be determined from the knowledge of the polarization parameter, which does not exceed 10% at high energies and decreases with increasing energy.[22] The remaining spin-nonflip amplitudes T_1 and T_3 are approximately equal to each other.

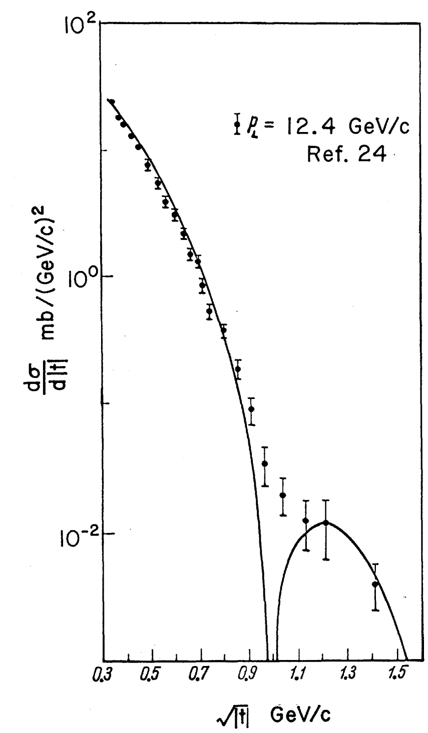

FIG. 1. *pp* scattering at $p_L = 12.4$ GeV/c (data from Ref. 24). The solid curve is the result of our theoretical calculation.

This is a consequence of a "pure elastic" character of high-energy hadron scattering, which is due to the exchange of zero quantum numbers in crossed channels.

Thus, in the description of an unpolarized proton-proton scattering at high energies, one can confine oneself to consider one amplitude $T \approx T_1 \approx T_3$ in the framework of the quasipotential equation (1.1) for spinless particles.[23]

The total and the differential cross sections of unpolarized proton-proton scattering are related to the scattering amplitude $T(E, \vec{\Delta}^2)$, which satisfies the quasipotential equation (1.1), by

$$\sigma_{\rm tot} = \frac{8\pi^3}{pE} T(E; \vec{\Delta}^2 = 0), \qquad (2.59a)$$

$$\frac{d\sigma}{dt} = -4\pi \left| \frac{\pi^2 T(E; \vec{\Delta}^2)}{pE} \right|^2, \quad t = -\vec{\Delta}^2. \qquad (2.59b)$$

The solution of Eq. (1.1) with the quasipotential (1.6), in the region of small scattering angles (2.19), depends on the two real parameters a and g_0 entering the definition of the quasipotential (1.6).

The numerical values of these parameters can be found from the experimental data at small and vanishing momentum transfers, i.e., from the total cross section $\sigma_{\rm tot}$ and the diffraction-peak slope A, in the following manner:

$$\sigma_{\rm tot} = 8\pi a I(x),$$

$$A \approx \left[\frac{d}{dt}\left(\ln \frac{d\sigma}{dt}\right)\right]_{t=0} = 2a \frac{1}{I(x)} \int_0^x \frac{d\xi}{\xi} I(\xi), \qquad (2.60a)$$

$$I(x) = -\sum_{n=1}^{\infty} \frac{(-x)^n}{n n!} = \int_0^x \frac{d\xi}{\xi}(1 - e^{-\xi}), \qquad (2.60b)$$

$$x = 4\pi^2 g_0 / a. \qquad (2.61)$$

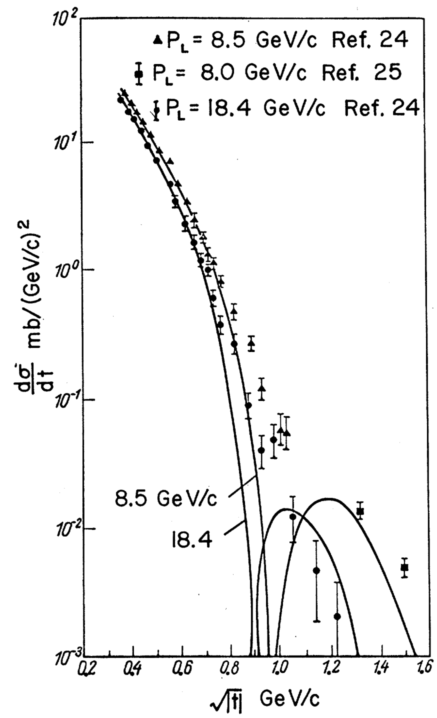

FIG. 2. pp scattering at $p_L = 8.0$ GeV/c (data from Ref. 25), $p_L = 8.5$ GeV/c (data from Ref. 24), and $p_L = 18.4$ GeV/c (data from Ref. 24). The solid curves are the results of our theoretical calculation.

We have made a comparison of the results obtained above with experimental data on elastic pp scattering in the region (2.1) at $p_L = 8.5$, 12.4, and 18.4 GeV/c.[24,25] The theoretical curves on Figs. 1 and 2 correspond to the following values of the parameters g_0 and a:

$$p_L = 8.5\ \text{GeV}/c, \quad g_0 = 0.13\ (\text{GeV}/c)^{-2}, \quad a = 2.6\ (\text{GeV}/c)^{-2},$$

$$p_L = 12.4\ \text{GeV}/c, \quad g_0 = 0.12\ (\text{GeV}/c)^{-2}, \quad a = 2.8\ (\text{GeV}/c)^{-2},$$

$$p_L = 18.4\ \text{GeV}/c, \quad g_0 = 0.14\ (\text{GeV}/c)^{-2}, \quad a = 3.8\ (\text{GeV}/c)^{-2},$$

which were calculated using formulas (2.60) and (2.61) from the experimental values of the total cross section[26] and the diffraction-peak slope[24] at corresponding energies.[27] As is seen from Figs. 1 and 2, the theoretical curves reproduce rather well the behavior of the differential cross section of elastic pp scattering in the region (2.1), as well as the positions of the diffraction minima and their energy dependence.

It is interesting to mention that the numerical value of the parameter a for the energy region considered here corresponds to the effective radius of interaction

$$R \approx 1/2\mu = 0.71\ \text{F} \quad \text{or} \quad a = 3.3\ \text{GeV}^{-2},$$

which is just defined by the position of the nearby singularity in the t channel for the case of elastic scattering. Moreover, the total cross section is very close at these energies to its geometrical value,

$$\sigma_{\rm tot} \approx 2\pi R^2 = \pi/2\mu^2 \approx 32\ \text{mb}.$$

Notice that the qualitative analysis of elastic pp scattering at high energies has been made from various points of view in Refs. 13, 19, and 20, and also by Chiu and Finkelstein,[28] Finkelstein and

Jacob,[29] Frautschi and Margolis,[30] and Frautschi, Kofoed-Hansen, and Margolis.[31] Near the points where the sum (2.19) vanishes, it is necessary to take into account the following terms of the expansion of the scattering amplitude in inverse powers of momentum p. This leads to the so-called "filling of minima." Furthermore, in Fig. 6 below the behavior of the differential cross section of elastic pp scattering at $p_L = 8.5$ GeV/c in the region $0 \leq |t| < 0.6$ (GeV/c)^2 is shown. One can see from Fig. 6 that the existence of a small "shoulder" at $|t| \approx 0.3$ (GeV/c)^2 is in agreement with the results of the theoretical calculations. A similar behavior is observed at other energies as well.

III. LARGE-ANGLE SCATTERING

Let us consider now two-particle scattering at high energies and fixed scattering angles:

$$as \gg 1, \quad |t/s| \approx \sin^2\tfrac{1}{2}\theta \text{ is fixed.} \tag{3.1}$$

In this case the series of the Born approximation for the scattering amplitude has the following form:

$$T(E;\vec{\Delta}^2) = isg_0 \sum_{n=1}^{\infty} \frac{n^{2n}}{(n!)^2}\,\frac{e^{at/n}}{n^{3/2}}\left(\frac{isg_0\pi^{3/2}e^{\varphi(\theta)}}{tpa^{3/2}}\right)^{n-1}, \tag{3.2}$$

where

$$\varphi(\theta) = 1 - \frac{\theta}{2\tan\frac{1}{2}\theta}\,. \tag{3.3}$$

The function $\varphi(\theta)$ is rather small in the region (3.1) and in what follows we will neglect it.

It is easy to see that when $|at| \gg 1$ the main contributions to the sum (3.2) are given by the terms with $n \gg 1$ and we can use the Stirling formula $n! \approx (2\pi n)^{1/2}(n/e)^n$. As a result, we obtain

$$T(E;\vec{\Delta}^2) \approx isg_0\,\frac{e^2}{2\pi}\sum_{n=1}^{\infty}\frac{e^{at/n}}{n^{5/2}}\,(-i\gamma)^{n-1}, \tag{3.4}$$

where

$$\gamma = \frac{g_0 e^2}{p\sin\frac{1}{2}\theta}\,(\pi/a)^{3/2} = \frac{sg_0e^2}{|t|p}\,(\pi/a)^{3/2}. \tag{3.5}$$

Let us use the representation

$$\frac{e^{at/n}}{n^{5/2}} = \frac{1}{(4\pi a)^{5/2}}\int d^5r\, e^{i\vec{r}\cdot\vec{\Delta}}\,(e^{-r^2/4a})^n, \tag{3.6}$$

where $t = -\vec{\Delta}^2$, and $\vec{r}$ is a vector in some subsidiary five-dimensional space.

Using the formula (3.6), we can rewrite the series (3.4) in the integral form

$$T(E;\vec{\Delta}^2) \approx isg_0\,\frac{e^2}{2\pi(4\pi a)^{5/2}}\int d\vec{r}\; e^{i\vec{r}\cdot\vec{\Delta}}\,\frac{e^{-r^2/4a}}{1 + i\gamma e^{-r^2/4a}}. \tag{3.7}$$

After integrating over the angles in spherical coordinates, we get

$$T(E;\vec{\Delta}^2) \approx isg_0\,\frac{2\pi e^2}{3(4\pi a)^{5/2}}\,\frac{1}{i\Delta}\int_{-\infty}^{\infty} r^3dr\,e^{ir\Delta}\,\frac{e^{-r^2/4a}}{1+i\gamma e^{-r^2/4a}}\,. \tag{3.8}$$

The integral in Eq. (3.8) is convergent and can easily be calculated by means of the residue theorem:

$$T(E;\vec{\Delta}^2) \approx \frac{\Delta p r_0^{\,3}}{12\pi^2}\,e^{i\Delta r_0}, \tag{3.9}$$

where

$$\Delta = |t|^{1/2}.$$

The positions of the poles of the integrand are defined by the equation

$$1 + i\gamma e^{-r_0^2/4a} = 0 \tag{3.10}$$

or

$$r_0^{\,2} = -2\pi i a\left(1 + \frac{2i}{\pi}\ln\gamma\right). \tag{3.11}$$

We find from Eq. (3.11) that the leading pole which lies in the upper half plane is

$$r_0 = i\left[2\pi a\left(1 + \frac{2i}{\pi}\ln\gamma\right)\right]^{1/2}. \tag{3.12}$$

At intermediate energies, when the second term in Eq. (3.11) can be neglected, i.e. $r_0^{\,2} \approx -2\pi i a$, we obtain the following expression for the differential cross section at large angles:

$$\frac{d\sigma}{d\Omega} \approx (\tfrac{1}{3}\pi a)^2\Delta^2 e^{-2\Delta(\pi a)^{1/2}}, \quad \Delta = |t|^{1/2}. \tag{3.13}$$

An interesting feature of the result (3.13) is the fact that at momentum transfers corresponding to large angles, the $d\sigma/d\Omega$ depends weakly on energy. The only energy dependence of $d\sigma/d\Omega$ enters through the parameter a which is related to the forward diffraction-peak width.

At sufficiently high energies, when the second term in Eq. (3.11) gives the main contribution, we get for the differential cross section the following asymptotic behavior:

$$\frac{d\sigma}{d\Omega} \approx s(\tfrac{1}{3}a\sin\tfrac{1}{2}\theta\ln s)^2 e^{-c(\theta)(2as\ln s)^{1/2}}, \tag{3.14}$$

where $c(\theta) = 2\sin\frac{1}{2}\theta$.

The interesting point is that for the logarithmic growth of the parameter a with increasing energy, we obtain just the Cerulus-Martin lower bound for the differential cross section at high energies and fixed scattering angles.[32]

Let us briefly discuss the application of these results to the description of large-angle elastic pp scattering. As we have seen previously, in the energy region $p_L = 10{-}20$ GeV/c the parameters a

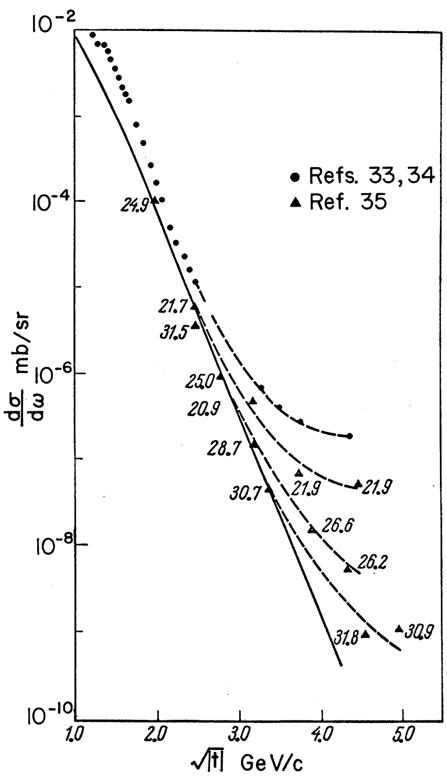

FIG. 3. pp scattering at large angles (data from Refs. 33–35). The solid curve is the result of our theoretical calculations. The dashed lines are hand-drawn to guide the eye. The numbers on the figure indicate the momentum of the incident particle in the laboratory frame.

and g_0 are given approximately by

$$a \approx 3.0\ (\mathrm{GeV}/c)^{-2}, \quad g_0 \approx 0.13\ (\mathrm{GeV}/c)^{-2}. \qquad (3.15)$$

The condition of convergence of the series (3.4), $|\gamma|<1$, for these numerical values of the parameter a and g_0 reads

$$|t| > s^{1/2}(0.3\ \mathrm{GeV}). \qquad (3.16)$$

The theoretical curve on Fig. 3, which is calculated using (3.13) and (3.15), reproduces the absolute value and character of the decrease of the differential cross section[33–35] in the region of large scattering angles, restricted by the condition (3.16). We stress that, in accordance with the remark made earlier,[23] these results cannot be applied, generally speaking, to scattering angles near $\theta = 90°$.

IV. BACKWARD SCATTERING

One can see from the foregoing considerations that the scattering amplitude at large angles [Eq. (3.9)] exponentially decreases with increasing energy. Thus, the solution of Eq. (1.1) with quasipotential (1.6) leads to an exponentially small cross section for backward scattering at high energies, which in a number of cases contradicts the experimental data. As was pointed out in Ref. 2, this fact is due to the neglect of exchange forces in the two-particle system.

In what follows we shall show how the exchange forces can be included in the quasipotential equation, and shall use the results obtained for the analysis of experimental data on np backward scattering.

In the presence of exchange forces, the scattering amplitude $T(\vec{p}, \vec{k}; E)$ can be represented as a sum of two quantities,

$$T(\vec{p}, \vec{k}; E) = G(\vec{p}, \vec{k}; E) + H(\vec{p}, \vec{k}; E), \qquad (4.1)$$

which obey the following system of quasipotential equations[36]:

$$G = g + g \otimes G + h \otimes H, \qquad (4.2a)$$

$$H = h + h \otimes G + g \otimes H. \qquad (4.2b)$$

The symbol $\otimes$ in formulas (4.2) implies an integration in the sense of Eq. (1.1). The quantities g and h are the Fourier transforms of the "direct" and "exchange" parts of the quasipotential, respectively:

$$g(s,t) = (2\pi)^{-3} \int d\vec{r}\, e^{i\vec{p}\cdot\vec{r}}\, V(s,\vec{r}) e^{-i\vec{k}\cdot\vec{r}}, \quad t = -(\vec{p}-\vec{k})^2, \qquad (4.3a)$$

$$h(s,u) = (2\pi)^{-3} \int d\vec{r}\, e^{i\vec{p}\cdot\vec{r}}\, V_e(s,\vec{r}) \hat{P} e^{-i\vec{k}\cdot\vec{r}}, \quad u = -(\vec{p}+\vec{k})^2, \qquad (4.3b)$$

where $\hat{P}$ is the coordinate-exchange operator.

As a quasipotential of "direct" interaction we use the expression (1.6), or $g(s,t) = isg_0 e^{at}$.

The "exchange" part of the quasipotential is due to the crossed u-channel contributions.

Taking into account the condition

$$\left| \frac{h(s,0)}{sg_0} \right| \ll 1 \ \text{at} \ s \to \infty, \qquad (4.4)$$

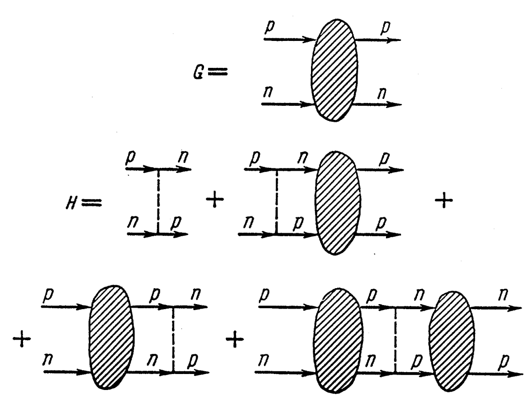

FIG. 4. Schematic presentation of the amplitudes G and H.

one can neglect the last term in Eq. (4.2a). Iterating the obtained system of equations, we get

$$H = h + h \otimes G + G \otimes h + G \otimes h \otimes G, \tag{4.5}$$

where G is determined by the solution of Eq. (1.1) with quasipotential (1.6). The expression (4.5) for the amplitude H is pictured symbolically in Fig. 4.

Let us assume now that the "exchange" quasipotential can be represented as a sum:

$$h(s,u) = \sum_i h_i(s) e^{b_i u}, \tag{4.6}$$

where $|h(s,u)/sg_0| \ll 1$ at high energies. For this case the amplitude H in the region $|u/s| \ll 1$ can be found in the following form:

$$H(\vec{\Delta}'^2, E) = \sum_i H_i(\Delta'^2, E), \quad \vec{\Delta}'^2 = -u \tag{4.7}$$

where

$$H_i(\vec{\Delta}'^2; E) = h_i(s) \sum_{n=0}^{\infty} \frac{e^{[ab_i/(a+nb_i)]u}}{(a+nb_i)n!} \left(-\frac{4\pi^2 g_0}{a}\right)^n. \tag{4.8}$$

These results were used for the analysis of the elastic np backward scattering at $p_L = 8.0$ GeV/c and $|u| < 0.6$ (GeV/c)^2.[37] Only two terms in expression (4.6) for the exchange quasipotential were taken into account. For the sake of simplicity the parameters h_1 and h_2 are assumed to be real; the cases of equal and opposite signs of h_i were considered.

The parameters a and g_0 entering the definition of the "direct" part of the quasipotential were determined from the experimental data on elastic proton-proton scattering at $p_L = 8.5$ GeV/c:

$$g_0 \approx 0.1\ (\text{GeV}/c)^{-2}, \quad a \approx 2.6\ (\text{GeV}/c)^{-2}.$$

The theoretical curves 1 and 2 on Fig. 5, which correspond to equal and opposite signs of the quantities h_1 and h_2, are calculated for the following values of the parameters h_i and b_i:

Equal signs (1)

$|h_1| = 0.07, \quad b_1 = 110.0\ (\text{GeV}/c)^{-2},$

$|h_2| = 0.30, \quad b_2 = 1.8\ (\text{GeV}/c)^{-2}.$

Opposite signs (2)

$|h_1| = 0.29, \quad b_1 = 34.0\ (\text{GeV}/c)^{-2},$

$|h_2| = 0.30, \quad b_2 = 1.8\ (\text{GeV}/c)^{-2}.$

On Fig. 6 the same theoretical curves are plotted for comparison together with the curve of the differential cross section of elastic pp scattering at $p_L = 8.5$ GeV/c, which is normalized to 1mb/(GeV/c)^2 at $t = 0$. One can see from Figs. 5 and 6 that the case of equal signs is, apparently, preferable.

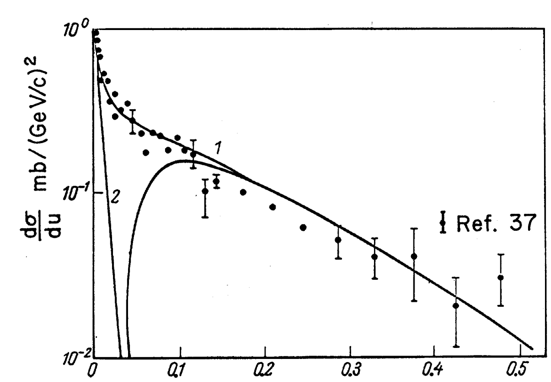

FIG. 5. np charge exchange at $p_L = 8.0$ GeV/c (data from Ref. 37). The solid curves are the results of our theoretical calculation.

V. CHARGE EXCHANGE AND INELASTIC SCATTERING

The process of charge exchange and inelastic scattering can be described in the framework of a multichannel quasipotential equation of the form

$$\hat{T} = \hat{V} + \hat{V} \otimes \hat{T}, \tag{5.1}$$

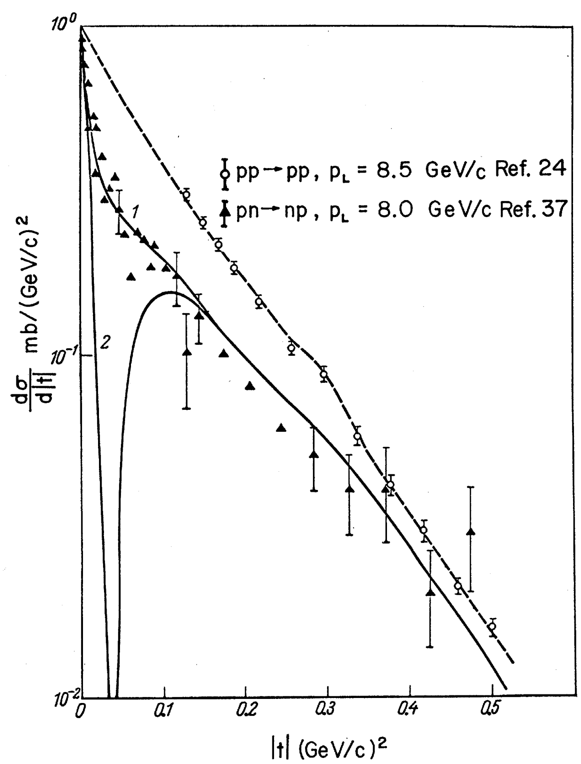

FIG. 6. np charge exchange at $p_L = 8.0$ GeV/c (data from Ref. 37), and elastic pp scattering at $p_L = 8.5$ GeV/c (data from Ref. 24). The solid (np charge exchange) and dashed (pp elastic scattering) curves are the results of our theoretical calculations. [The cross section for pp scattering is normalized to 1 mb/(GeV/c)^2 at $t = 0$.]

where $\hat{T}$ and $\hat{V}$ are some matrices and the symbol $\otimes$ denotes an integration as in Eq. (1.1).

The problem is simplified when inelastic effects are small as compared with elastic scattering. In this case we can write

$$\hat{T}=\hat{T}_0+\delta\hat{T},\quad \hat{V}=\hat{V}_0+\delta\hat{V}, \tag{5.2}$$

and Eq. (5.1) takes the form

$$\delta\hat{T}=\delta\hat{V}+\delta\hat{V}\otimes\hat{T}_0+\hat{V}_0\otimes\delta\hat{T}. \tag{5.3}$$

The quantities $\hat{V}_0$ and $\hat{T}_0$ are diagonal matrices and are related by the equation

$$\hat{T}_0=\hat{V}_0+\hat{V}_0\otimes\hat{T}_0. \tag{5.4}$$

Iterating Eq. (5.3) and using Eq. (5.4), we get the formal solution of the problem:

$$\delta\hat{T}=\delta\hat{V}+\hat{T}_0\otimes\delta\hat{V}+\delta\hat{V}\otimes\hat{T}_0+\hat{T}_0\otimes\delta\hat{V}\otimes\hat{T}_0. \tag{5.5}$$

Let us now consider the process of isobar production in high-energy proton-proton scattering. We will assume here that the amplitude of the elastic scattering of an isobar with isotopic spin $I=\frac{1}{2}$ is approximately equal, at high energies, to the proton-proton scattering amplitude T, and the amplitude of isobar production T^* is small as compared with T. Thus we have the equation[38]

$$T=V+V\otimes T, \tag{5.6a}$$

$$T^*=V^*+V\otimes T^*+V^*\otimes T. \tag{5.6b}$$

Here $V=isg_0e^{at}$ and V^* is an effective quasipotential of isobar production, which we will represent in the form

$$V^*=h(s)e^{bt}. \tag{5.7}$$

The solution of Eq. (5.6b) can be found easily by iteration procedure and has, as in the backward scattering case considered above, the following form:

$$T^*\approx h(s)\sum_{n=0}^{\infty}\frac{ae^{[ab/(a+nb)]t}}{(a+nb)n!}\left(\frac{4\pi^2g_0}{a}\right)^n. \tag{5.8}$$

We have applied this result to describe the production of the $N^*(1688)$ and $N^*(1450)$ isobars.

The theoretical results, as well as the experimental data[39,40] for these processes, are presented

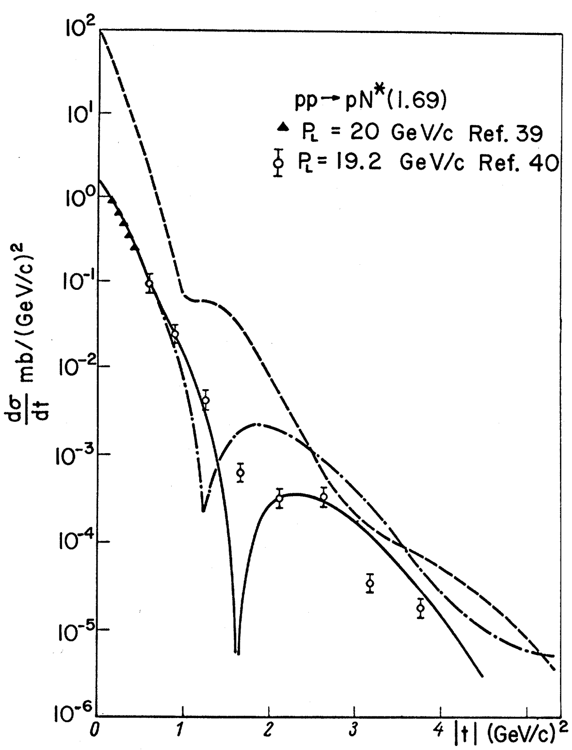

FIG. 7. Production of the $N^*(1.69)$ isobar in pp collisions at p_L =20 GeV/c (data from Ref. 39), and p_L =19.2 GeV/c (data from Ref. 40). The solid curve is the result of our theoretical calculation. The dashed-dotted curve is the result of the calculation of Frautschi, Kofoed-Hansen, and Margolis (Ref. 31). The dashed line is the result of our theoretical calculation on elastic pp scattering at p_L =20 GeV/c and is shown for comparison.

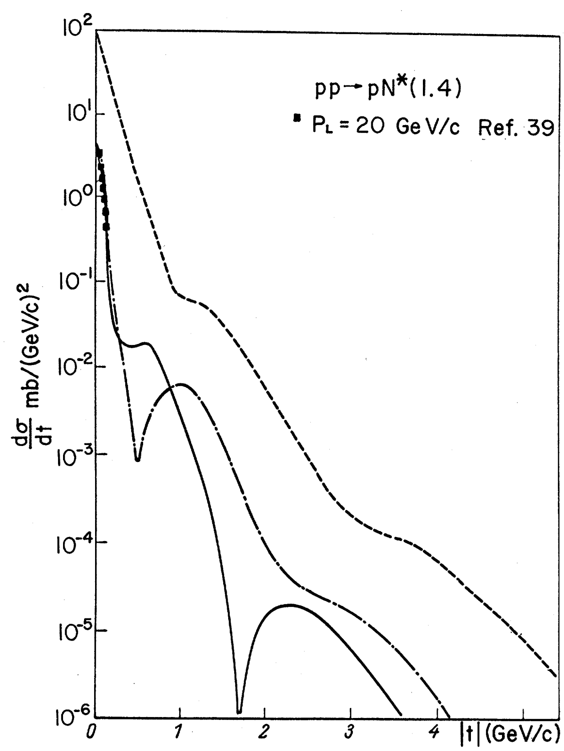

FIG. 8. Production of the $N^*(1.4)$ isobar in pp collisions at p_L =20 GeV/c (data from Ref. 39). The solid curve is the result of our theoretical calculation. The dashed-dotted curve is the result of the calculation of Frautschi, Kofoed-Hansen, and Margolis (Ref. 31). The dashed line has the same meaning as in Fig. 7.

in Figs. 7 and 8. One can see that the agreement is good enough.

VI. CONCLUSION

We have tried to outline the methods and some applications of the quasipotential approach for high-energy hadron scattering. For simplicity we have ignored here spin complications, mass differences, etc. Moreover, we have considered here the simplest nonsingular quasipotential of Gaussian form, which is pure imaginary.

It is interesting, however, to investigate spin effects in high-energy hadron scattering and to try more refined quasipotentials. We shall consider these problems elsewhere. The comparison with experiment which we have made here is, in general, good enough. However, the main point we would like to state here is not the good agreement with experiment, but the fact that the quasipotential equation can serve as an effective tool in studying high-energy hadron scattering.

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*Present address: Joint Institute for Nuclear Research, Dubna, USSR.

†On leave of absence from the Joint Institute for Nuclear Research, Dubna, USSR.

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Parity Rule and Helicity Conservation in ρ Photoproduction

M. Barmawi*
International Centre for Theoretical Physics, Miramare-Trieste, Italy
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The parity rule for natural- and unnatural-parity exchange is proved on the basis of the L-S scheme. The parity rule is shown to be a general feature of t-channel exchange processes. Experimental consequences of the parity rule are discussed. The s-channel helicity conservation in ρ^0 photoproduction is considered on the basis of the Regge-pole model with L-S coupling and general arguments from the complex-angular-momentum theory. It is found that the s-channel helicity conservation in ρ^0 photoproduction can be described by the coupling $L=\alpha-2$, $S=2$ with a multiplicative fixed pole at the nonsense wrong-signature point $\alpha=1$.

I. INTRODUCTION

One of the difficulties encountered by the Regge-pole theory[1-4] is the description of the coupling between the exchanged Regge pole and the external particles with spin. Considerable progress has been made, among other things, by the introduction of the concept of parity-conserving amplitude[5] and the investigation of its kinematical singularities.[6]

From a phenomenological analysis of resonance-production data,[7] the author has earlier proposed a model[8] to describe the Regge-pole couplings, which is based on the L-S scheme. This model has been applied to simple processes.[9] One cannot hope to be able to explain detailed aspects, such as polarization, from such a simple model. For the explanation of polarization, additional correction terms to the Regge-pole contribution will be required.[10] As a first approximation, nevertheless, this model is still of interest, since the residues are characterized by the total spins of the external particles as a manifestation of the residue dependence on the external spins and quantum numbers. This might be useful, since one of the sources of ambiguities in Regge-pole models is the parametrization of the residues.

The L-S scheme itself has been shown to be useful in the study of the kinematical factors and the threshold condition by Jackson and Hite.[11]

Recently, more data suitable for the study of these couplings became available. These data are the data summarized in the Morrison empirical rule[12] and the data on the spin-density matrices for ρ^0 photoproduction,[13] which indicate s-channel helicity conservation.[14,15]

The Morrison rule[12] gives the condition for the appearance of the diffraction scattering stated in terms of spins and parities of the external particles involved in the vertices of the corresponding t-channel exchange process. The rule is stated in Sec. III. This rule is an attempt to classify the dynamics of the diffraction phenomena. The rule as it is stated suggests its intimate relationship with the nature of the coupling at the t-channel vertices. Leader[12,16] has shown that the rule for a spin-0-induced reaction is implied by the kinematics of the reaction. Considering that this rule has so far not been established for diffraction processes induced by particles with nonzero spin, it is possible that this rule has its origin in the kinematics alone. On the other hand, the s-channel helicity conservation might be related to the dynamics of the diffraction phenomenon, e.g., due to the nature of the above-mentioned coupling. In this case the Morrison rule would be valid only for a spin-0-induced reaction and the approximate s-channel helicity conservation is a characteristic of the diffraction phenomenon. These considerations are the motivation for the present investigation.[17,18]

Note that the evidence for s-channel helicity conservation in ρ photoproduction comes from the measurement of the spin-density matrix of the produced ρ^0.