lations by a factor of 10^{-3}.

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Violation of the Pomeranchuk-Theorem Conditions and the Odd-Signature Amplitude

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We discuss the properties of the odd-signature amplitude in the case that the total cross sections of the particle and antiparticle tend to constant but different limits at high energy. The imaginary and real part of the odd-signature amplitude must be correlated by a simple relation and also be subjected to various stringent conditions. Sum rules are obtained that can be used as a test of the violation of the Pomeranchuk theorem. A specific model is proposed satisfying these conditions, and some consequences are discussed.

There have recently been wide theoretical investigations[1-9] motivated by the Serpukhov experiment.[10] In the case where the total cross sections of the particle σ^p and antiparticle σ^a tend to constant but different limits, it necessarily follows from analyticity, crossing, and polynomial boundedness that the real parts of both amplitudes $F^{p,a}(s,t=0)$ behave as $\pm s\ln s$ as $s\to\infty$.[11] In the original work of Pomeranchuk, this behavior was rejected on physical grounds, and $\lim\sigma^p=\lim\sigma^a$ followed. Although in some cases or by some assumptions this physical assumption can be replaced by unitarity,[12] we cannot reject the $\pm s\ln s$ behavior in general by the fundamental requirement of the field theory. Indeed, experimental confirmation of the high-energy behavior of the real part of the scattering amplitudes in the forward direction was urged in Refs. 1 and 2. We recall here that even in the case where $\Delta\sigma(s) \equiv \sigma^a(s) - \sigma^p(s)$ tends to zero as $s \to \infty$, $\mathrm{Re}F^{p,a}(s,0)$ may dominate over $\mathrm{Im}F^{p,a}(s,0)$. This happens, roughly, when $\Delta\sigma(s)$ tends to zero at least as slowly as $1/\ln s$ [$1/\ln s$ corresponds to $\mathrm{Re}F^{p,a}(s,0) \sim \pm s \ln s$]. Moreover we have the case where $\mathrm{Re}F^{p,a}(s,0)$ grows arbitrarily closely to $s \ln s$. Thus, whether or not $\Delta\sigma(\infty)$ is actually zero, which is difficult in practice to decide experimentally, the experimental study of the correlation between the slowness of $\Delta\sigma(s) \to 0$ and the growth of $\mathrm{Re}F^{p,a}(s,0)$ may in principle serve as a test of our fundamental understanding of the whole theory. But the experimental situation in this direction is not so hopeful. The circumstances are such that it is desirable to investigate further the general properties of the amplitudes under assumptions familiar to us, and to confront them with experiment if possible. Motivated by these situations both in experiment and in theory, in this paper we study some of the properties of the odd-signature amplitude in the case where $\sigma^p(s)$ and $\sigma^a(s)$ tend to finite but different limits.

Introducing the impact parameter $b(s) = 2l\sqrt{s}$, we can neglect the contribution above $b(s) = c \ln s$, as was shown by Froissart. Then, first of all, using the Schwarz inequality and unitarity, it is easy to show that all the waves below $b(s) = c \ln s$ contribute, and we conclude that the radius of the "odd-signature particle" grows with energy as $c \ln s$. (We compare this with the "even-signature particle" with the usual Pomeranchukon pole which grows as $c\sqrt{\ln s}$.) The elastic cross sections $\sigma_{\mathrm{el}}^{p,a}$ tend to finite but nonzero constants,[4]

$$\sigma_{\mathrm{el}}^{p,a} \geq \frac{(\sigma^p - \sigma^a)^2}{4\pi^3 c^2}. \tag{1}$$

The equality holds when all the real parts of the partial-wave amplitudes become equal at $s \to \infty$:

$$\mathrm{Re}F_l^{p,a}(s) \sim \frac{1}{\ln s}, \quad \frac{\mathrm{Im}F_l^{p,a}(s)}{\mathrm{Re}F_l^{p,a}(s)} \sim 0. \tag{2}$$

When we consider odd-signature amplitudes, we have to somehow eliminate the appearance of the massless vector meson; it was shown in Ref. 4 that in the "scale-invariant" case this is automatically guaranteed. Thus we seek a simple form of $F^-(s,t)$ that satisfies the following requirements.

(i) $\mathrm{Im}F^-(s,0) \to \dfrac{s}{8\pi}\Delta\sigma(\infty)$,

$\mathrm{Re}F^-(s,0) \to \dfrac{-s\ln s}{4\pi^2}\Delta\sigma(\infty)$, as $s \to \infty$;

(ii) $\dfrac{1}{s^2}\displaystyle\int_{-\infty}^{0} |F^-(s,t)|^2 dt \to \text{const}$, as $s \to \infty$;

(iii) $\mathrm{Im}F_l^-(s) \sim \dfrac{1}{\ln^2 s}\, a(x)$,

$\mathrm{Re}F^-(s) \sim \dfrac{1}{\ln s}\, d(x)$, as $s \to \infty$

with

$$x \equiv \frac{b(x)}{\ln s} \quad \text{(scale invariance)}.$$

We consider the following form for $F^-(s,t)$ at high energy[13]:

$$\begin{aligned} \mathrm{Im}F^-(s,t) &= \frac{s}{8\pi} f(T), \\ \mathrm{Re}F^-(s,t) &= \frac{1}{4\pi^2}\frac{s}{\sqrt{-t}}\, Tg(T), \end{aligned} \tag{3}$$

where $T \equiv \sqrt{-t}\,\ln s$.

These forms follow from a combination of the impact-parameter projection and the dispersion relation. $f(T)$ and $g(T)$ must satisfy the various restrictions

$$f(0) = -g(0) = \Delta\sigma(\infty) \quad \text{from (i)}, \tag{4}$$

$$\int_0^\infty |g(T)|^2 T\, dT \quad \text{from (ii)}, \tag{5}$$

$$\int_0^\infty f(T) T\, dT < \infty, \quad \int_0^\infty g(T) T\, dT < \infty \quad \text{from (iii)}. \tag{6}$$

$a(x)$ and $d(x)$ are given by

$$a(x) = \frac{1}{16\pi}\int_0^\infty T f(T) J_0(xT)\, dT. \tag{7}$$

$$d(x) = \frac{1}{8\pi^2}\int_0^\infty T g(T) J_0(xT)\, dT. \tag{8}$$

$f(T)$ and $g(T)$ are related through the dispersion relation. Explicit evaluation of $\mathrm{Re}F^-$ using $\mathrm{Im}F^-$ shows that in order to get rid of the $1/\sqrt{-t}$ singularity at $t = 0$, we must have

$$\int_0^\infty f(T)\, dT = 0. \tag{9}$$

Thus $f(T)$ oscillates as a function of T. Then, because of Eq. (6), we get as, $s \to \infty$,[14]

$$\mathrm{Re}F^-(s,t) \sim -\frac{s\ln s}{4\pi^2}\,\frac{1}{T}\int_0^T f(T)\, dT.$$

Thus we obtain a simple relation between $f(T)$ and $g(T)$ by a simple argument:

$$f(T) = -\frac{d}{dT}[Tg(T)], \tag{10}$$

i.e.,

$$f^{(n)}(0) = -(n+1) g^{(n)}(0). \tag{11}$$

This automatically ensures the validity of Eq. (9) because of Eqs. (4) and (6). When transferred to $a(x)$ and $d(x)$, Eq. (10) reads

$$a(x) = \tfrac{1}{2}\pi\,\frac{d(x\,d(x))}{dx}.$$

This is the condition obtained in Ref. 4 which guarantees the absence of the massless vector particle. Indeed, the t-channel partial-wave amplitude $f_j^-(t)$ is given by the Froissart-Gribov projection

$$f_j^-(t)=\frac{1}{\pi}\int_{z_0}^{\infty} Q_j(z)\mathrm{Im}F^-(s,t)dz, \quad z=1+\frac{2s}{(t-4\mu^2)},$$
$$\sim\frac{1}{48\pi}\frac{t-4\mu^2}{\sqrt{-t}}\int e^{-T(j-1)/\sqrt{-t}}f(T)dT, \quad \text{near } j=1. \qquad (12)$$

Thus $f_1^-(t)$ has no singularity in t due to Eq. (9). Equation (10) is written at fixed t and s, respectively, as

$$\frac{\mathrm{Im}F^-(s,t)}{s^2}=-\tfrac{1}{2}\pi\frac{\partial}{\partial s}\left(\frac{\mathrm{Re}F^-(s,t)}{s}\right), \qquad (13)$$

$$\frac{\ln s\,\mathrm{Im}F^-(s,t)}{\sqrt{-t}}=\pi\frac{\partial}{\partial t}\left[\sqrt{-t}\,\mathrm{Re}F^-(s,t)\right]. \qquad (14)$$

These relations hold for any $t<0$ and large s. Integrating them, we obtain new s and t sum rules which hold at high energy:

$$\sqrt{-t}\int_{s_0}^{\infty}\frac{\mathrm{Im}F^p(s,t)}{s^2}ds =\sqrt{-t}\int_{s_0}^{\infty}\frac{\mathrm{Im}F^a(s,t)}{s^2}ds+\tfrac{1}{2}\pi\sqrt{-t}\,\frac{\mathrm{Re}F^-(s_0,t)}{s_0} \qquad (15)$$

$$\int_{-\infty}^{0}\frac{\mathrm{Im}F^p(s,t)}{\sqrt{-t}}dt=\int_{-\infty}^{0}\frac{\mathrm{Im}F^a(s,t)}{\sqrt{-t}}dt. \qquad (16)$$

Here, s_0 is some finite constant. Equation (15) is consistent with Eq. (9) because of Eq. (4), and it corresponds to the sum rule obtained in Ref. 4 for $t<0$. Now Eq. (15) holds at $t=0$ in the limiting sense. Thus the imaginary part of $F^-(s,t)$ oscillates both in s and in t. Equations (13) – (16) can in principle be tested experimentally. The conditions (4), (5), and (6) and the requirement of the Froissart bound on $F_l^-(s)$, which states that as $s\to\infty$

$$|F_l^-(s)|<As^N\exp[-b(s)/b_0], \qquad (17)$$

with A and b_0 some constant and $N\leq 2$,[15] rather severely restrict the form of $f(T)$ or $g(T)$, and hence these functions necessarily become complicated. There do not seem to exist models that satisfy these conditions. Take, for example, Finkelstein's model corresponding to $f(T)\propto(1-\cos T)/T^2$. It does not satisfy the condition (6) and hence, when the partial waves are summed over, the total cross section and elastic cross section grow infinitely as $s\to\infty$:

$$\mathrm{Im}F_l^-(s)\sim\frac{\ln\ln s}{\ln^2 s}\quad \text{for } b(s)<c\ln s$$
$$\sim\frac{1}{\ln^2 s},\quad \text{for } b(s)=c\ln s$$

and similarly for $\mathrm{Re}F^-(s)$ with $\ln^2 s$ replaced by $\ln s$. Accordingly, $\sigma^{p,a}$, for example, grows as

$$\sigma^{p,a}=\frac{16\pi}{s}\sum_{l=0}^{c\sqrt{s}\ln s}(2l+1)\mathrm{Im}F_l^-(s)$$
$$>16\pi c^2\sqrt{\ln\ln s}.$$

The same argument applies also to the model proposed in Ref. 7. Thus in the model theory the t dependence of the amplitude must be chosen carefully. We consider here the simplest possible model that satisfies all the requirements given above:

$$f(T)=\frac{1}{T}[J_1(aT)-J_1(bT)],$$

with

$$a-b=2\Delta\sigma(\infty),\quad a>b>0,$$

$$g(T)-\frac{1}{T}\int_0^T f(T')dT',$$

$$a(x)=\frac{1}{16\pi}\left[\frac{1}{a}\theta(a-x)-\frac{1}{b}\theta(b-x)\right],$$

$$d(x)=\frac{1}{8\pi^2}\times\begin{cases}\left(\frac{1}{a}-\frac{1}{b}\right), & x<b\\ \left(\frac{1}{a}-\frac{1}{x}\right), & b<x<a\\ 0 & a<x.\end{cases}$$

The j-plane singularity can be deduced from Eq. (12):

$$f_j^-(t)\sim\frac{1}{48\pi}\frac{(t-4\mu^2)}{\sqrt{-t}}\left[\left(1+\frac{(j-1)^2}{-ta^2}\right)^{1/2}-\frac{j-1}{a\sqrt{-t}}\right]-(a\to b). \qquad (18)$$

Thus root-type singularities develop between $j=1\pm ia\sqrt{-t}$ and between $j=1\pm ib\sqrt{-t}$. In order to see the consistency of the form (3) and Eq. (10) with this Regge singularity, we reconstruct the amplitude from Eq. (18) using the odd-signature factor $(i+\tan\frac{1}{2}\pi j)\sim i+1/(j-1)$ near $j=1$,[16]

$$\mathrm{Im}F^-(s,t)=\frac{1}{2\pi i}\int(2j+1)f_j^-(t)P_j(z)dj$$
$$\sim\frac{1}{8\pi}as\int_0^1\cos(Tay)(1-y^2)^{1/2}dy-(a\to b)$$
$$=\frac{1}{8\pi}s\frac{1}{T}[J_1(a,T)-J_1(b,T)]$$
$$=\frac{1}{8\pi}sf(T),$$

$$\mathrm{Re}F^-(s,t)=\frac{1}{2\pi i}\int(2j+1)f_j^-(t)P_j(z)\tan\tfrac{1}{2}\pi j\,dj$$
$$\sim\frac{1}{4\pi^2}\frac{s}{\sqrt{-t}}\int_0^1\frac{\sin Tay}{y}(1-y^2)^{1/2}dy-(a\to b)$$
$$=-\frac{1}{4\pi^2}\frac{s}{\sqrt{-t}}f(T')dT'$$
$$=\frac{1}{4\pi^2}\frac{s}{\sqrt{-t}}Tg(T).$$

In this way we can reproduce the form (3) and the relation (10) from the Regge analysis also. The differential cross section is determined mainly by $\mathrm{Re}F^{-}(s,t)$ and shows a maximum at $t\neq 0$ near $-t\approx A/\ln^2 s$ with A determined by a and b, which follows from Eq. (11).

In the following we discuss briefly the zeros of the odd-signature amplitude. This problem has been discussed recently in many papers.[17] Here we want to give arguments based on the above discussions. The amplitude is an analytic function of t in the region independent of s. Thus at high energy, using Eq. (10), we conclude that both $f(T)$ and $g(T)$ are entire functions of T. Along the lines given by Casella, the zeros of F^- are shown to lie almost along the real positive axis, i.e., within the wedge of angle $-\epsilon<\arg T<\epsilon$. If these zeros are to be detected experimentally, they must lie on the positive real axis. In this case $F^-(T)$ displays interesting behavior near the zeros. If the zeros of $F^-(T)$ are on the real axis, $f(T)$ and $g(T)$ separately vanish at this point. On the other hand, Eq. (10) tells us that each zero of $f(T)$ is separated by zeros of $g(T)$. This is due to the theorem of Laguerre concerning the zeros of the entire functions.[18] We conclude therefore that if the zeros of F^- are on the real axis, $g(T)$ must have double zeros at these points. Denoting one of these zeros by T_0, the behavior of f or g near T_0 is given by

$$f(T)\sim C_1(T-T_0),$$

$$g(T)\sim C_2(T-T_0)^2.$$

Thus $F^-(T)$ is almost imaginary near $T=T_0$. If the experiment does not suggest this, then the zeros of $F^-(T)$ are in the complex region within $-\epsilon<\arg T<\epsilon$.[17] We must recall here that the even-signature amplitude must be added or subtracted before the comparison is made with experiment. In this connection the work by Eden and Kaiser is interesting. But it is the amplitude $F(s,t)/F(s,0)$ that has been proved to have a zero in a certain region. For the actual amplitude $F(s,t)$, the logarithmic growth of the real part may smear out the zeros of $F(s,t)$, and whether or not these zeros are detected by experiment depends on the models.

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[16]In the Regge language, dominance of the real or imaginary part at high energy follows from the signature factor $i+\tan\frac{1}{2}\pi j$ or $-i+\cot\frac{1}{2}\pi j$ for odd- or even-signature amplitudes, respectively, which are $i+1/(j-1)$ or $-i+(j-1)$ near $j=1$. We arrive at the same conclusion using dispersion relations.

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