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PHYSICAL REVIEW D VOLUME 4, NUMBER 3 1 AUGUST 1971

# Antibaryon-Baryon Scattering Problem*

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(Received 22 February 1971)

The implications of the failure of duality in $\bar{B}B$ scattering are discussed in the framework of a quark model in which all quarks of the target interact simultaneously with all quarks in the projectile through two-body quark-antiquark interactions. It is assumed that the peculiarity of $\bar{B}B$ interactions, which at the particle level manifests itself through annihilation channels, is due to the strong $q\bar{q}$ interaction at the quark level which inhibits the presence of nonactive quarks (spectators) in the scattering process. This leads to the replacement of the additivity approach of quark amplitudes by factorization of quark amplitudes. Factorization of quark amplitudes implies, in general, nonfactorization of particle amplitudes and appearance of effects similar to cuts (dips versus peaks in the forward direction) due to the simultaneous exchange of natural and unnatural parity at the quark level. Exchange of exotic quantum numbers appears as a natural consequence of the model, without assuming the existence of exotic particles. It is, however, inhibited by the smallness of charge- and strangeness-exchange cross sections. This model should apply both for large- and small-angle scattering, the difference in these two regions manifesting itself only through different relations between the helicity amplitudes at the quark level. Predictions are made for quasielastic $\bar{B}B$ scattering at small and large momentum transfer. For the reactions where data exist, the agreement between the relations predicted by the model and experiment is satisfactory.

## I. INTRODUCTION: DUALITY, $\bar{B}B$ SCATTERING, AND EXOTIC STATES

The assumptions of (i) the nonexistence of exotic states of first or second kind,[1] and (ii) duality have led to remarkable results for $MB$ scattering[2] ($M$ and $B$ represent a meson and baryon, respectively), constituting the starting point for many interesting developments in hadron phenomenology. However, it was soon realized that antibaryon-baryon scattering represented a singular case with respect to the compatibility between duality and nonexotic states, giving rise to a series of puzzles.

Indeed, Lipkin[3] has pointed out that hypotheses (i) and (ii) lead, e.g., to such paradoxes as the vanishing of the $\bar{\Delta}\Delta$ scattering amplitude. Another failure of duality and nonexotic states in $\bar{B}B$ scattering can be visualized in quite a simple way by contemplating duality quark diagrams.[4,5] While for $MM$ and $MB$ scattering the drawing of such diagrams is possible without invoking exotic states, for antibaryon-baryon scattering the consideration of exotic quantum numbers is unavoidable at least in one channel. Finally, the exchange degeneracy, which is a consequence of (i) and (ii), does not work in $\bar{B}B$ scattering. This was shown by Rosner[6] in connection with the $s$ dependence of $\bar{B}B$ scattering amplitudes and is supported also by the recent experimental data of the CERN $\bar{p}p \to \bar{Y}^*Y$ experiment.[7]

The most obvious way to get out of this puzzling situation is to assume that either assumption (i) or assumption (ii) does not hold for $\bar{B}B$ scattering.

The first kind of approach has been chosen by

Rosner,[6] who assumes the existence of exotic mesons of the first kind coupled only to $\bar{B}B$. While the validity of this assumption can be checked ultimately only by experiment, at least the following two questions have then to be answered by the theory:

(a) Why is the $\bar{B}B$ vertex so privileged?

(b) Why are exotic channels so rare?

In fact, up to the present, the only more or less reliable evidence for exotic channels comes from the reaction

$$\bar{p}p \to \bar{\Sigma}^- \Sigma^-$$

and the corresponding cross section is very small[8] (1.3 $\mu$b at 5.7 GeV/$c$).

The second kind of approach to this problem intends to avoid the assumption of existence of exotic particles and is due to Pinsky[9] and more recently to Kugler.[10] Pinsky suggests that the above-mentioned difficulties can be avoided if one takes into account Regge cuts in the exchange, while, more specifically, Kugler proposes to abandon the factorization of Regge amplitudes in $\bar{B}B$ scattering for the baryonic octet and the decoupling of $\bar{B}B$ from the leading trajectories (natural parity) in the case of the decuplet. The suggestions of Refs. 9 and 10 are related, but not identical. The assumption of cuts implies in general the failure of factorization of amplitudes, but has also other implications. This can be seen among others from the fact that, as pointed out by Kugler,[10] the abandoning of factorization alone is not enough to prevent the $\bar{\Delta}\Delta$ amplitude from vanishing. (That is why Kugler makes the supplementary assumption of the decoupling of leading trajectories in the decuplet case.) On the other hand, the cut assumption by itself is strong enough to avoid the above-described puzzles both for the octet and the decuplet.

These last two solutions remind us again, however, of the tantalizing question (a), and if we want indeed to solve the $\bar{B}B$-duality problem, we have to explain why it is just $\bar{B}B$ scattering where, e.g., factorization of amplitudes fails.

On the other hand, it is important to emphasize that question (b) is of great interest independent of its link to the duality problem. Indeed, even if exotic particles do not exist, there is no theoretical reason why double or multiple exchange of nonexotic states should not exist. But multiple exchange of nonexotic quantum numbers is equivalent to exchange of exotic quantum numbers and question (b) immediately emerges.

It is the purpose of this paper to suggest an answer to both questions (a) and (b). We construct for this purpose a model which takes into account the specific features of $\bar{B}B$ scattering, and we investigate its consequences. It turns out that cuts necessarily appear in antibaryon-baryon scattering, although the quark-quark scattering amplitude corresponds to one particle (pole) exchange and factorizes, and it turns out also that the exchange of exotic quantum numbers is very rare, indeed, independent of the assumption of the existence or nonexistence of exotic states, but just as a consequence of the fact that charge exchange and strangeness exchange are much rarer than elastic scattering. Assuming an evasive Regge exchange at quark level, new sum rules for $\bar{B}B$ scattering amplitudes are obtained which are in satisfactory agreement with experiment. A by-product of this model is the possibility of a simple explanation of the occurrence of dips and peaks in the forward direction for $\bar{B}B$ scattering in the framework of multiple parity exchange in the quark-quark scattering amplitude.

Most recently, it was pointed out by Finkelstein[11] that for $MM$ scattering too, duality implies the necessity of exotic exchange, unless there is parity doubling for leading trajectories at $t=0$. It is argued below that this result is closely related to the $\bar{B}B$ puzzle, and, in particular, that parity doubling might occur also in $\bar{B}B$ scattering. Furthermore, the questions of the specificity of $\bar{B}B$ and $MM$ scattering might have the same answer.

## II. EXOTIC PARTICLES AND THE QUARK MODEL

To find an answer to the question of the specific role of exotic states in $\bar{B}B$ scattering, it is most natural to go back to the origin of the "exotic" concept. This origin lies in the quark model,[12] which, in its "naive" form, states that baryons can be built only from three quarks and mesons from one quark and one antiquark. Given the conventional[12] quantum numbers for the three fundamental quarks $\mathcal{P}$, $\mathfrak{N}$, $\lambda$ it follows from the above statement that:

(1) There do not exist strange baryons with $Q$ (charge) $>1$, $S$ (strangeness) $>0$, $S<-3$, $I$ (isospin) $>1$, $B$ (baryonic quantum number) $>1$, nonstrange baryons with $Q>2$, $I>\frac{3}{2}$, $B>1$ and mesons with $Q>1$, $S>1$, $I>1$. These "forbidden" particles are called exotic particles of the first kind.

(2) There do not exist mesons of natural parity $P=(-)^J$ with $PC=-1$. ($P$ and $C$ are the internal parity and charge conjugation, respectively, and $J$ is the spin.) These are exotic particles of the second kind.

Now, it is remarkable that no exotic particles, either of the first kind or of the second kind, have been found up to now in production experiments – i.e., in the $s$ channel. Moreover, the duality approach in $MB$ scattering leads to results in agreement with experiment if one assumes that neither exotic particles of the first kind nor exotic particles of the second kind can be exchanged in the

$t$ channel.[13]

Since the quark model is the only model, up to now, which can explain these facts and the only model which establishes a link between exotic particles of the first and second kind, it is most natural to look for a solution to the problem of the specific role of exotic states in $\bar{B}B$ scattering in the quark model.

## III. $\bar{B}B$ SCATTERING IN THE ADDITIVITY QUARK MODEL

$\bar{B}B$ scattering is an "enfant terrible" not only with respect to duality, but also in its relation to the quark model. The conventional quark-model approach to scattering processes assumes additivity of quark amplitudes and considers that in a given process only one quark of the projectile interacts with only one quark of the target. This model leads to good results (i.e., in agreement with experiment) for $MB$ scattering, but encounters difficulties when applied to $\bar{B}B$ scattering. This is true even for reactions which can be considered as one-particle-exchange reactions and where the additivity quark model is expected to work. This has been interpreted by Kokkedee and Van Hove[14] as being due to the annihilation channels which are present in the last case but absent in meson-baryon scattering, and indeed, when one subtracts from the corresponding cross sections the annihilation contribution, the agreement with data is much improved.[12,14] The fact that the annihilation part of the scattering cross section cannot be accounted for by the additivity quark model is easy to realize since annihilation channels involve baryon exchange, i.e., triple quark exchange.

Annihilation channels, however, are not the only weakness of the additivity model. As mentioned above, the additivity assumption denies the existence of multiple exchange and thus this model cannot deal also with processes of the type

$$\pi^- p - \pi^+ \Delta^- , \quad (1)$$

where no annihilation is present. Furthermore, the additivity approach is limited to small scattering angles because at large $t$, multiple quark scattering is expected to become very important. Now it is remarkable that these two facts which are not taken into account by the additivity model, namely, (a) annihilation contributions and (b) multiple scattering, are not independent. Indeed, we have seen above that the annihilation channels correspond in some sense to triple quark exchange. It is thus not surprising that the additivity approach does not apply to both categories of processes. On the other hand, this fact suggests a way to generalize the additivity model in order to take into account features (a) and (b).

The new approach must replace the additivity assumption corresponding to single quark scattering by a more general one which allows also for multiple scattering. But multiple scattering is not a new problem and has been studied in connection with nuclear scattering along the lines of the theory developed by Glauber.[15] It would thus be tempting to apply the Glauber formalism to quark scattering (in fact this has been done[16]) in order to generalize the additivity model to large-angle scattering and multiple-exchange processes. At a first look, such a treatment would appear very promising also for the duality-$\bar{B}B$ puzzle because multiple scattering leads to cuts and that, as mentioned in Sec. I, might be the feature needed in this context. However, the same multiple-scattering formalism leads also to cuts in $MB$ scattering, and it is not clear at all why these cuts do not spoil the good results of the duality approach in this last case, too. In other words, question (a) in Sec. I remains unanswered. Furthermore, there are also practical reasons why the Glauber theory cannot constitute a satisfactory solution, at least at present, for our problem.

Indeed, in the multiple-scattering approach one has to know the radial dependence of the quark wave function, i.e., the quark-quark interaction, and that is obviously impossible as long as no quarks could be produced. One must therefore use supplementary assumptions and thus, even if all multiple scattering terms would be taken into account (which is so far not the case because of computational difficulties), we would not know how reliable the result would be.

## IV. THE FACTORIZATION QUARK MODEL

### A. A New Approach to $\bar{B}B$ Scattering

From the arguments invoked above, it follows that we have to look for a solution of the problem of $\bar{B}B$ scattering in the quark model which should take into account the peculiar features of this process, i.e., annihilation channels, and at the same time be sufficiently general to include also the possibility of multiple exchange without, however, reducing itself to the multiple-scattering theory. It seems to us that such a solution exists and has already been used by the author in the special case of *large-angle* $\bar{B}B$ scattering.[17]

Kokkedee and Van Hove suggested that for $\bar{B}B$ annihilation, the additivity of quark amplitudes should be replaced by the factorization of quark amplitudes. In Ref. 17 we suggested that factorization[18] should replace additivity not only for annihilation but also for large-momentum-transfer $\bar{B}B$ scattering. That additivity does not hold for large-angle scattering in general is not surprising in view of the one-quark-interaction assumption implied by additivity, which precludes multiple scattering; at large angles multiple scattering is, however, expected to be the dominant mechanism.[15,16] The factorization model, although containing some multiple-scattering features, is quite different from the conventional multiple-scattering approach because it is designed to take into account the peculiarities of the $\bar{B}B$ case. We shall show in the following how the factorization model can be applied to small-angle scattering, too.[19,20]

### B. Physical Motivation and Mathematical Formulation of the Factorization Quark Model

We consider that because of the annihilation channels, the quark-antiquark interaction is much stronger than the quark-quark interaction. Thus, for $MB$ or $BB$ scattering (where no annihilation is possible), the interaction can be attributed in a first approximation to the interaction of a pair of quarks, one from the projectile and one from the target, the other quarks remaining spectators. On the other hand, in $\bar{B}B$ scattering all antiquarks from the projectile tend to "saturate" simultaneously all antiquarks from the projectile in such a way that it is not possible for even one single quark to remain "inactive," i.e., a spectator. This assumption is supported by the observation that there exist bound $\bar{q}q$ states, but no bound $qq$ states.

The fact that there are no spectators in the factorization quark model is important for the spin treatment since this avoids the problem of Wigner rotations.[21,22] Furthermore, since for the reactions in which we are interested, mass differences are relatively small at least at sufficiently high energies, form factors of the type discussed in Ref. 21, which might be important at large angles, are expected to be nearly the same in all reactions and should not influence our conclusions.

We work in the helicity formalism and write[23] the wave function of a baryon in a given helicity state $h$:

$$\psi^h = f\sum_\alpha a_{\alpha_i\alpha_j\alpha_k} q_i^{\alpha_i} q_j^{\alpha_j} q_k^{\alpha_k}, \tag{2}$$

where the $q$ are the wave functions of the quarks, $f$ is the form factor mentioned above which describes the space variables of $q$, and the summation extends over all helicity states $\alpha$ of $q$ compatible with $h$. Because of our assumption that $f$ is the same for all baryons, we shall put $f=1$ in the following.

We adopt the "naive" version of the quark model in which a baryon is composed of three quarks. The coefficients $\alpha$ in (2) depend on the symmetry scheme adopted for the classification of particles in the quark model. We use two different, but related, symmetry schemes and compare the predictions which follow from each of these schemes.

The helicity amplitude of a process

$$A+B\to C+D \tag{3}$$

reads then

$$A_{CDAB}^{h_C h_D h_A h_B} \equiv \langle \psi_C^{h_C}\psi_D^{h_D} | T | \psi_A^{h_A}\psi_B^{h_B}\rangle$$

$$=\sum_\alpha a_{\alpha_i(A)\alpha_j(A)\alpha_k(A)} a_{\alpha_i(B)\alpha_j(B)\alpha_k(B)} a^*_{\alpha_i(C)\alpha_j(C)\alpha_k(C)} a^*_{\alpha_i(D)\alpha_j(D)\alpha_k(D)} Q^\alpha, \tag{4}$$

where

$$Q^\alpha \equiv \langle q_{i(C)}^{\alpha_i(C)} q_{j(C)}^{\alpha_j(C)} q_{k(C)}^{\alpha_k(C)} q_{i(D)}^{\alpha_i(D)} q_{j(D)}^{\alpha_j(D)} q_{k(D)}^{\alpha_k(D)} | T | q_{i(A)}^{\alpha_i(A)} q_{j(A)}^{\alpha_j(A)} q_{k(A)}^{\alpha_k(A)} q_{i(B)}^{\alpha_i(B)} q_{j(B)}^{\alpha_j(B)} q_{k(B)}^{\alpha_k(B)} \rangle. \tag{5}$$

The coefficients $a_\alpha$ are normalized:

$$\sum_\alpha |a_\alpha|^2 = 1. \tag{6}$$

The assumption of factorization means that $Q^\alpha$ is a sum of all possible three-quark amplitudes. Here "possible" stands for "compatible with the conservation laws of strong interactions." A three-quark amplitude is a product of three single-quark scattering amplitudes, where by single-quark amplitude we understand the scattering amplitude of a quark in the projectile with a quark in the target. This computation scheme is explained in greater detail in Appendix A.

The factorization model defined by Eqs. (4)–(6) has indeed the properties we looked for in connection with the puzzle of duality, $B\bar{B}$ scattering, and exotic states. It contains the "good" properties of the multiple-scattering approach such as the possibility of multiple exchange, which is forbidden in the additivity quark model, and the exchange of exotic quantum numbers as suggested by the duality diagrams. This is obvious from Eq. (5) because the product of three quark amplitudes can carry among others exotic quantum numbers. It does not imply factorization of vertices in particle (pole) exchange, since in fact it contains cuts, generated by the multiple exchange. Finally, it interprets the specificity of the $\bar{B}B$ case by the assumption that there are no spectators and that all three quarks of the projectile interact simultaneously with all three quarks of the target, differing in that respect from the conventional multiple-scattering approach where the amplitude is a sum of terms, among which there are both single- and multiple- scattering quark amplitudes.[24] From this point of view the factorization model constitutes probably the simplest possible solution for our problem, given the constraints defined above.

Once we have convinced ourselves that our model can satisfy these constraints, we must test whether or not the other consequences which emerge from the factorization model are in agreement with experiment. To do this we compute with this model scattering amplitudes for various $\bar{B}B$ processes in terms of quark amplitudes which are the same for all processes and which are the parameters of our model. By eliminating these common parameters we get relations between particle scattering amplitudes (cross sections) which can be compared with experimental data.

### C. The Parameters of the Model

The number of parameters in our problem – i.e., the number of independent quark amplitudes – is determined by two factors:

($\alpha$) The different channels available in quark-quark two-body scattering, corresponding to the channels of two-body $\bar{B}B$ scattering, i.e., elastic scattering, charge exchange, and strangeness exchange.

($\beta$) The different helicity configurations at quark level which correspond to the different helicity particle amplitudes.

With regard to ($\alpha$), by assuming $SU(2)$ symmetry (i.e., isospin conservation), the number of parameters can be reduced substantially. This simplification can be pushed even further by taking into account some empirical facts, as shown below.

On the other hand, factor ($\beta$) introduces serious computational difficulties because we are faced with a system of equations of third degree (this is a consequence of the factorization principle) in which the number of unknowns (parameters) exceeds, in general, the number of knowns. Indeed, up to the present there are no data for helicity amplitudes, and what one measures is, in the best case, the spin density matrix. In particular, for the $\bar{B}B$ reactions in which we are interested, there are very few spin-density-matrix measurements; that is why, in order to compare our predictions with data, we must limit ourselves in the following to spin-averaged cross sections. Now, by averaging over spin, and moreover by measuring only the modulus of the amplitude and not its phase, much information is lost, and that means that the number of knowns in our equations is much smaller, the number of unknowns remaining essentially the same. To be able, nevertheless, to learn something at this stage, we have to reduce the number of unknown parameters. This can be done by assuming an exchange mechanism at the quark level.

#### 1. *Charge and Strangeness Exchange versus Elastic Scattering*

Assuming $SU(2)$ but not $SU(3)$, for each helicity configuration there are three independent single-quark amplitudes in terms of which all particle amplitudes can be expressed:

$$x \equiv \langle \bar{\mathcal{P}}\mathcal{P} | \bar{\mathcal{P}}\mathcal{P} \rangle, \quad y \equiv \langle \bar{\mathfrak{N}}\mathfrak{N} | \bar{\mathcal{P}}\mathcal{P} \rangle, \quad z \equiv \langle \bar{\lambda}\lambda | \bar{\mathcal{P}}\mathcal{P} \rangle . \tag{7}$$

The other quark amplitudes can be expressed in terms of $x$, $y$, $z$. Thus, e.g., using $SU(2)$ invariance,

$$r \equiv \langle \bar{\mathfrak{N}}\mathcal{P} | \bar{\mathfrak{N}}\mathcal{P} \rangle = x + y . \tag{8}$$

Now we take into account the empirical fact that charge- and strangeness-exchange two-body reactions have much smaller cross sections than elastic reactions. Thus, e.g.,

$$\frac{\sigma(\bar{p}p \to \bar{n}n)}{\sigma(\bar{p}p \to \bar{p}p)} = \frac{0.58 \text{ mb}}{16.3 \text{ mb}} \simeq 3 \times 10^{-2} \tag{9}$$

at 5.7 GeV/$c$, and

$$\frac{\sigma(\bar{p}p \to \bar{\Lambda}\Lambda)}{\sigma(\bar{p}p \to \bar{p}p)} = \frac{0.04 \text{ mb}}{16.3 \text{ mb}} \simeq 3 \times 10^{-3} \tag{10}$$

at 5.7 GeV/$c$. (The experimental data for $\bar{p}p \to \bar{n}n$, $\bar{p}p \to \bar{p}p$, $\bar{p}p \to \bar{\Lambda}\Lambda$ are taken from Refs. 25, 26, and 8, respectively.) The smallness of these ratios is observed at all energies available at present. Since in the quark model, the amplitudes for charge- and strangeness-exchange reactions are proportional to the quark amplitudes $y$ and $z$, respectively, defined above, it follows from Eqs. (7)–(10) that in a very good approximation,

$$y \ll x, r \tag{11}$$

and

$$z \ll x, r . \tag{12}$$

We shall make use of these inequalities in the following, neglecting whenever possible $y$, $z$ in comparison with $x$, $r$.

#### 2. *Large-Angle versus Small-Angle Scattering*

The spin treatment in the quark model is intimately related to the problem of the wave functions used in order to express the particle states in terms of quark states [cf. Eq. (2)]. Until now, in almost all applications of the quark model where the spin was taken into account, $SU(6)$ wave functions were used, i.e., $SU(6)$ was used as a classification scheme. (This does not mean, of course, that $SU(6)$ was considered to be a good symmetry for the amplitudes.[27]) There exists, however, another possibility connected with the "hidden spin" formalism of Franklin.[28] We shall compare the predictions of the factorization model for these two cases. To compensate for the proliferation of parameters due to the spin, we shall consider two extreme cases (large-angle and small-angle scattering) and make use of some relations which are expected to hold between the helicity amplitudes in these extreme situations. These relations are quite different in the two kinematical cases, and therefore our results will also be quite different for large momentum transfer and forward scattering.

As will be seen below, this different behavior of cross sections at small and large angles seems to be supported by the data available until now.

*Large-angle scattering.* In this case we assume that interferences between three-quark amplitudes are negligible, i.e., that the amplitudes add incoherently (the coherence is destroyed, e.g., by the average over an angle interval), and that all helicity states have equal probabilities so that the squares of the quark helicity amplitudes are equal. These two assumptions are equivalent to the neglect of spin and correspond to a statistical approach which is expected to work at large momentum transfer. We have thus three parameters $x$, $r$, $z$ [or four if we also keep $y$ for convenience, cf. Eq. (8)] and obtain a series of relations between cross sections (because of our incoherence assumption we deal directly with cross sections and not with amplitudes) which were given in Ref. 17 (cf. the second entry in Ref. 17).

*Small-angle scattering.* Here interference effects are expected to be important so that we add the three-quark amplitudes coherently. As concerns the helicity states, however, an important simplification can be achieved if we consider forward scattering ($t \simeq 0$). If we consider quarks as genuine spin-$\frac{1}{2}$ particles, and use parity conservation and time-reversal invariance, there are in general six independent helicity amplitudes[29-31]

$$A_1 \equiv \langle ++|++\rangle,\quad A_2 \equiv \langle ++|--\rangle,\quad A_3 \equiv \langle +-|+-\rangle,$$
$$A_4 \equiv \langle +-|-+\rangle,\quad A_5 \equiv \langle ++|+-\rangle,\quad A_6 \equiv \langle -+|++\rangle. \tag{13}$$

If $t \cong 0$, because of angular momentum conservation

$$A_4 = A_5 = A_6 = 0 \tag{14}$$

and we are left with only three amplitudes, $A_1$ and $A_3$ (nonflip amplitudes) and $A_2$.

### D. Evasive Exchange in Quark-Quark Scattering

Now we make the assumption that the quark-quark scattering process (in our case the quark-antiquark two-body scattering) is due to the exchange of an object with given spin $J$ and parity $P$ (this exchanged object can be either a particle or a Regge pole). This means, among other things that the quark scattering amplitude itself factorizes, while the particle amplitude does not. It has been shown by Cohen-Tannoudji, Salin, and Morel[32] that for large $\cos\theta_t$, the following relations hold between the $s$-channel helicity amplitudes of a two-body scattering process:

$$A(\lambda_3, \lambda_4; \lambda_1, \lambda_2) \simeq P\zeta P_4 P_2 (-)^{S_4 - S_2} (-)^{\lambda_4 - \lambda_2} A(\lambda_3, -\lambda_4; \lambda_1, -\lambda_2). \tag{15}$$

Here $P_i$, $S_i$, $\lambda_i$ are the intrinsic parities, spins, and helicities of particles $i$, respectively, and $\zeta$ is the signature of the exchanged pole [for particle exchange $\zeta = (-)^J$].

$$\cos\theta_t = \frac{2st - 2t(m_1^2 + m_2^2) + [t + (m_1^2 - m_3^2)][t + (m_2^2 - m_4^2)]}{\{[t - (m_1 + m_3)^2][t - (m_1 - m_3)^2][t - (m_2 + m_4)^2][t - (m_2 - m_4)^2]\}^{1/2}}. \tag{16}$$

We are interested in the limit of small $t$. From the above expression for $\cos\theta_t$ it follows that the condition $\cos\theta_t \gg 1$ can be achieved in this case provided $s$ is sufficiently large, which we shall assume in the following. An exact estimate of "sufficiently large" depends on the masses $m_i$ of the quarks.

Equation (15) corresponds to *evasive* exchange as opposed to the *conspiracy* solution which assumes the exchange of two trajectories with opposite parities. Experimental data strongly favor the evasive solution (associated with cuts[33,34]).

### E. The Leading Trajectories

We recall now that in $MB$ scattering the hypothesis of exchange degeneracy between the two leading nonets (isovector and isotensor) leads to results in fair agreement with data. Now since in these cases the additivity model works quite well, and since in the additivity approach the scattering process is due to two-body quark-quark scattering, it follows that in quark-quark scattering, at least in $MB$ scattering, the exchanged objects are the leading trajectories which have natural parity

$$P=(-)^J=\zeta\,.$$

We make now the assumption that this holds also in $\bar{B}B$ scattering at the *quark level* whenever natural-parity exchange does not decouple. This is in accordance with our philosophy that the peculiarity of $\bar{B}B$ scattering lies in the fact that all quarks interact simultaneously, and not in the manner in which two-body quark scattering takes place.

In the following we shall discuss separately the implications of the assumptions outlined above for two kinds of $\bar{B}B$ scattering:

$$\bar{B}B\to\bar{B}'B'\,, \tag{17}$$

where both $B$ and $B'$ are members of the baryonic octet, and

$$\begin{aligned}&\bar{B}B\to\bar{B}^*B'\,,\\ &\bar{B}B\to\bar{B}^*B^*\,,\end{aligned} \tag{18}$$

where $B^*$ makes part of the baryonic decuplet.

## V. PREDICTIONS FOR QUASIELASTIC OCTET SCATTERING

### A. Forward Scattering

The assumption of leading trajectory (i.e., natural-parity exchange) leads, with the evasive solution (15), to the following relations between the quark amplitudes (12):

$$A_1=A_3\,, \tag{19}$$

$$A_2=-A_4\,. \tag{20}$$

From Eq. (14), it follows then that

$$A_2=0 \tag{21}$$

and we are left with one single-quark helicity amplitude $A_1=A_3$. This means that if (as we shall assume) in all three of the single-quark scattering processes $\bar{\mathcal{P}}\mathcal{P}\to\bar{\mathcal{P}}\mathcal{P}$, $\bar{\mathcal{P}}\mathcal{P}\to\bar{\mathfrak{N}}\mathfrak{N}$, and $\bar{\mathcal{P}}\mathcal{P}\to\bar{\lambda}\lambda$ only natural parity is exchanged, then we have only one helicity configuration for all three [four] parameters defined by Eq. (7) [Eqs. (7) and (8)], and it is easy now, using the approximations (11) and (12), to express all particle amplitudes in terms of these parameters. This is done in Appendix B.

By eliminating the common quark amplitudes $x$, $y$, $z$, $r$, we get the following relations. (The choice of processes considered is dictated by the availability of experimental data. Wherever possible we have written these relations in terms of spin-averaged cross sections $\bar{\sigma}$, in the other cases the amplitudes were used.[35])

$$\epsilon_1\equiv\frac{\bar{\sigma}(\bar{p}p\to\bar{\Lambda}\Lambda)}{\bar{\sigma}(\bar{p}p\to\bar{\Lambda}\Sigma^0)}=3\,, \tag{22}$$

$$\epsilon_2\equiv\frac{\bar{\sigma}(\bar{p}p\to\bar{\Lambda}\Lambda)}{\bar{\sigma}(\bar{p}p\to\bar{\Sigma}^0\Sigma^0)}=9\,, \tag{23}$$

$$\frac{\bar{\sigma}(\bar{p}p\to\bar{\Xi}^-\Xi^-)}{\sigma(\bar{p}p\to\bar{\Sigma}^-\Sigma^-)}=\frac{16}{9}\,\frac{\bar{\sigma}(\bar{p}p\to\bar{\Lambda}\Lambda)}{\bar{\sigma}(\bar{p}p\to\bar{n}n)}\,\rho^2\,, \tag{24}$$

$$\delta\equiv\frac{\bar{\sigma}(\bar{p}p\to\bar{\Xi}^0\Xi^0)}{\bar{\sigma}(\bar{p}p\to\bar{\Xi}^-\Xi^-)}=4\,, \tag{25}$$

$$A(\bar{p}p\to\bar{\Lambda}\Lambda)\rho=\tfrac{3}{8}A(\bar{p}p\to\bar{\Sigma}^+\Sigma^+)\left[1+\tfrac{4}{3}\left(\frac{A(\bar{p}n\to\bar{\Sigma}^-\Lambda)}{A(\bar{p}p\to\bar{\Sigma}^+\Sigma^+)}\right)^2\rho\right]\,, \tag{26}$$

$$A(\bar{p}p\to\bar{\Sigma}^-\Sigma^-)=\{\tfrac{1}{3}A(\bar{p}p\to\bar{\Sigma}^+\Sigma^+)[2A(\bar{p}p\to\bar{\Lambda}\Lambda)\rho-\tfrac{3}{4}A(\bar{p}p\to\bar{\Sigma}^+\Sigma^+)]\}^{1/2}-\tfrac{1}{2}A(\bar{p}p\to\bar{\Sigma}^+\Sigma^+)\,. \tag{27}$$

The parameter $\rho$ takes on the following two values:

$$\rho=\begin{cases}1 \text{ for Franklin's[28] wave functions}\\ 2 \text{ for } SU(6) \text{ wave functions.}\end{cases}$$

As was to be expected, the difference between $SU(6)$ and Franklin's wave functions appears only in those relations which involve $\Lambda$ or $\Sigma^0$ (but not in the $\Lambda/\Sigma^0$ ratio). This is due to the fact that the relative assignment of $\Lambda$ and $\Sigma^0$ in the hidden-spin formalism[28] is not *a priori* known. We have obtained relations between exotic and nonexotic exchange cross sections [Eqs. (24), (27)] as well as relations between purely nonexotic exchange cross-sections [Eqs. (22), (23), (26)] and purely exotic exchange cross sections (25).

### B. Large-Momentum-Transfer (l.m.t.) Scattering

For the sake of comparison, we list also some relations[36] which should hold at l.m.t. in the factorization quark model, within the assumption discussed in Sec. IV C2 and Ref. 17:

$$\epsilon_1^{\text{l.m.t.}}=1\,, \tag{22$'$}$$

$$\epsilon_2^{\text{l.m.t.}}=1\,. \tag{23$'$}$$

These two relations were given in Ref. 17, too.

$$\frac{\bar{\sigma}(\bar{p}p\to\bar{\Xi}^-\Xi^-)}{\bar{\sigma}(\bar{p}p\to\bar{\Sigma}^-\Sigma^-)}=2\frac{\bar{\sigma}(\bar{p}p\to\bar{\Lambda}\Lambda)}{\bar{\sigma}(\bar{p}p\to\bar{n}n)}\,, \tag{24$'$}$$

$$\delta^{\text{l.m.t.}}=2\,, \tag{25$'$}$$

$$\bar\sigma(\bar p p\to\bar\Lambda\Lambda)=\tfrac{1}{2}\bar\sigma(\bar p p\to\overline{\Sigma^+}\Sigma^+)\left[1+\left(\frac{\bar\sigma(\bar p n\to\overline{\Sigma^-}\Lambda)}{\bar\sigma(\bar p p\to\overline{\Sigma^+}\Sigma^+)}\right)^2\right]. \qquad (26')$$

## VI. PREDICTIONS FOR QUASIELASTIC DECUPLET PRODUCTION

### A. The Decoupling of Leading Trajectories at $t=0$

Let us now consider processes of the type (18), i.e., decuplet production in two-body $\bar B B$ scattering. It turns out that when one makes the same assumptions about the evasive one-"object" exchange at quark level as for relations (17) for *all* three single-quark amplitudes $x$, $y$, $z$, the amplitudes for

$$\bar B B\to\bar B^* B$$

and

$$\bar B B\to\bar B^* B^*.$$

vanish.

An analogous situation has also been found[37] in the additivity quark model where, however, the natural-parity exchange is obviously restricted to a single quark-quark interaction, and where this decoupling of natural parity exchange is related to $W$-spin conservation at $t=0$.

On the other hand, if one mixes in the factorization model $P=(-)^J$ exchange with $P=(-)^{J+1}$ exchange in the three-quark amplitude, the situation is changed and the particle amplitude can be nonzero. By mixing, we understand the assumption that in the single-quark scattering process $\bar{\mathcal{P}}\mathcal{P}\to\bar{\mathcal{P}}\mathcal{P}$, e.g., natural parity is exchanged while in the other two single-quark processes $\bar{\mathcal{P}}\mathcal{P}\to\bar{\mathcal{N}}\mathcal{N}$ and $\bar{\mathcal{P}}\mathcal{P}\to\bar\lambda\lambda$ an object with $P=(-)^{J+1}$ is exchanged. Various other "mixing configurations" are of course possible and are discussed in Appendix C.

From Eq. (15) it follows that for $P=(-)^{J+1}$ exchange relations (19) and (20) become, respectively,

$$A_1=-A_3, \qquad (28)$$

$$A_2=A_4, \qquad (29)$$

while at $\theta_S=0$, this leads to the same result for $A_2$, i.e.,

$$A_2=A_4=0.$$

Equation (28) introduces essential differences for $A_1$, $A_3$, i.e., for $x_1$, $x_3$, $z_1$, $z_3$, $r_1$, $r_3$. In particular, e.g., while $x_1=x_3$, $z_1=z_3$, $r_1=r_3$, the amplitude for $\bar p p\to\bar Y^*\Lambda$ vanishes; for $x_1=x_3$, $z_1=z_3$, $r_1=-r_3$, it does not. Because of this parity mixing, which is a necessary condition for a nonvanishing amplitude at $t=0$, relations between cross sections for processes (18), analogous to the relations (22)–(27) deduced for processes (17), can be obtained only with supplementary assumptions about the relative importance of the different possible exchanges. This problem, as well as decuplet production at large momentum transfer, for which no data yet exist, might be discussed elsewhere.

### B. Dips versus Peaks; One-Pole Exchange Versus Cuts

The decoupling of the scattering amplitude from natural-parity exchange leads, assuming pure parity exchange, to a dip in the differential cross section as a function of $t$, at $t=0$, since the unnatural-parity contribution is generally dominated at small $t$ by the pion, which also vanishes at $t=0$.[34] In this case we are in the same situation as, e.g., in the photoproduction of pions, where natural-parity exchange is forbidden because of angular momentum and parity conservation,[38] and where pion exchange is believed to dominate at $t=0$. However, as is well known,[34] the experimental data present a peak at $t=0$ instead of the expected dip and this was interpreted at first as evidence for a conspiracy mechanism between an *ad hoc* invented natural-parity trajectory and the pion. However, this natural-parity conspirator failed to be seen in other effects and was soon abandoned in favor of cuts. This last resolution of the discrepancy between one-pole-exchange predictions and data is the generally accepted one at present.[34] In any case, what both the conspiracy and the cut mechanisms stand for is a mixture of natural- and unnatural-parity exchange. On the other hand, no unique and simple prescription for cuts exists as yet , and to quote from the recent review of Collins[39] along these lines: "The absence of a reliable method of estimating cut magnitudes is the principal problem of Regge phenomenology at present."

One of the challenging results of this paper is that it provides a possible simple frame for the computation of cuts. Indeed, as pointed out in Sec. VI A, since in the factorization quark model we have triple exchange, there always exists the possibility of mixing natural- and unnatural-parity exchange. The question of the relation between the exchanged objects at quark level and the exchanged objects at particle level, which is connected with (among other things) the energy dependence of the scattering amplitudes, is presently under investigation.

## VII. COMPARISON WITH EXPERIMENTAL DATA

The predictions obtained in Sec. V refer to relations between different cross sections at the same energy of the incoming particle. This restricts the possibility of comparison with experiment, since as yet only some of the processes considered have been studied at the same energy. Furthermore, the experimental errors are relatively large (up to $\simeq 20\%$ for nonexotic exchange and 30% for the exotic process $\bar{p}p \to \bar{\Sigma}^-\Sigma^-$). Finally in general there are no data for $(d\sigma/dt)_{t=0}$ for which the predictions of Sec. V A apply, but only for the "small-$t$ region" where the cross sections are in general peaked. It follows that at this stage the comparison with data can be only a qualitative one.

### A. Small Momentum Transfer

We assume that the relations (22)–(27) derived for $t=0$ are valid also for $t \neq 0$ but small. This is a reasonable assumption in view of the sharp forward peak which most of the cross sections present (with the possible exception of $\bar{p}p \to \bar{\Sigma}^-\Sigma^-$), and which means that the largest contribution to $d\sigma/dt$ comes from $t=0$. For relation (22), we use the data of Ref. 8, where a compilation for different energies is given. The agreement between the predictions of our model and experiment in the range 3–7 GeV/$c$ is good (within the experimental errors). For relations (26) and (27), which involve amplitudes, we have to make a supplementary assumption about the relative phases, in order to be able to use the data which are given for cross sections. We make the assumption that all amplitudes involved have equal phases. This assumption seems reasonable since the processes considered have approximately the same $s$ dependence, which in a Regge model means they have the same $\alpha$ and thus the same phase at a given parity. From exchange degeneracy of leading trajectories, this equality of phases would also follow, but exchange degeneracy is a stronger assumption since it implies, besides the equality of trajectories $\alpha$, equality of coupling constants also. With that assumption, the agreement between predictions and data is again satisfactory and seems to favor Franklin's[28] wave functions. It is remarkable that we get the correct order of magnitude (and even the correct figure, with Franklin's wave functions) for the cross section of the exotic process

$$\bar{p}p \to \bar{\Sigma}^-\Sigma^- . \tag{30}$$

As we see, the small value of this cross section (1.3 $\mu$b) turns out in our model to be a consequence of the smallness of the parameters $y$ and $z$ defined in Eqs. (7) and (8), because the amplitude for (30) is proportional to $xyz$. Table I (see Refs. 40 and 41) summarizes the comparison of the theoretical predictions with experiment. Furthermore, relation (24) permits us to predict the order of magnitude for the as yet unmeasured cross section of the exotic process

$$\bar{p}p \to \bar{\Xi}^-\Xi^- . \tag{31}$$

At 5.7 GeV/$c$, using the data of Refs. 8 and 42, we get $\bar{\sigma}(\bar{p}p \to \bar{\Xi}^-\Xi^-) \simeq 0.18$ $\mu$b with Franklin's[28] wave functions and $\bar{\sigma}(\bar{p}p \to \bar{\Xi}^-\Xi^-) \simeq 0.72$ $\mu$b with $SU(6)$ wave functions. The experimental upper limit[43] for (31) is 0.45 $\mu$b, again in qualitative agreement with the theoretical predictions and apparently again favoring Franklin's model.

### B. Large Momentum Transfer (l.m.t.)

The only relation for which l.m.t. data exist until now is Eq. (22′). At 5.7 GeV/$c$ we get from Ref. 8

$$\epsilon_1^{\text{l.m.t.}} = \frac{d\bar{\sigma}}{dt}(\bar{p}p \to \bar{\Lambda}\Lambda) \Big/ \frac{d\bar{\sigma}}{dt}(\bar{p}p \to \bar{\Lambda}\Sigma^0) = \frac{10 \pm 2}{8.5 \pm 2}, \tag{32}$$

in good agreement with the prediction of Eq. (22′) (cf. also Ref. 17 where this prediction was made for the first time),

TABLE I. Comparison of relations between cross sections (amplitudes) predicted by the factorization quark model at $t=0$ and experimental data.

| Relation | Type of wave function | Theory versus experiment | | Incoming momentum (GeV/$c$) | References for experimental data |
|---|---|---|---|---|---|
| | | Theory | Exp. | | |
| (22) | Franklin and $SU(6)$ | $\epsilon_1 = 3$ | $\epsilon_1 = \frac{42 \pm 4}{13 \pm 3}$ | 5.7 | Atherton *et al.* (Ref. 8) |
| (26) | Franklin | $10.6 \simeq 8.3$ [a] | | 2.8 | Bacon *et al.* (Ref. 40) Fisher *et al.* (Ref. 41) |
| (26) | $SU(6)$ | $21.2 \simeq 14.5$ [a] | | 2.8 | Bacon *et al.* (Ref. 40) Fisher *et al.* (Ref. 41) |
| (27) | Franklin | $1.1 \simeq 1.5$ [a] | | 5.7 | Atherton *et al.* (Ref. 8) |
| (27) | $SU(6)$ | $1.1 \simeq 3.5$ [a] | | 5.7 | Atherton *et al.* (Ref. 8) |

[a] This equality is obtained by introducing the experimental data into the left- and right-hand sides of the theoretical relation and is based on the supplementary assumption of phase equality (cf. Sec. VIII A).

$$\epsilon_1^{\text{l.m.t.}} = 1\,. \tag{22'}$$

Hagedorn's[44] statistical model gives for $\epsilon_1^{\text{l.m.t.}}$ the value

$$\epsilon_1^{\text{l.m.t.}} = \sqrt{3}\,. \tag{33}$$

This agreement between the predictions of the factorization quark model and data, if confirmed by further data, suggests that the model can explain in a consistent way the differences between small-angle and large-angle scattering. [Compare the different predictions of Eqs. (22) and (22′) for $t=0$ and l.m.t.]

An interesting prediction which reflects also the differences between these two $t$ regions is that of Eq. (25′), which differs again significantly from Eq. (25) at $t=0$.[45,46]

## VIII. CONCLUSIONS

The factorization quark model is based on the assumption that the peculiarity of the $\bar{B}B$ scattering process, which consists of the presence of annihilation channels, can be taken into account by abandoning additivity of quark amplitudes and replacing it by factorization of quark amplitudes. In this way one gets a formalism in which exotic and multiple exchange can be treated with the same methods as single-particle exchange. Moreover, some of the effects for which Regge cuts are invoked, such as peaks in the forward amplitude where dips are expected, can be explained in a natural and simple way along these lines.

The small cross sections for exotic exchange are, in this picture, a consequence of the smallness of change and strangeness exchange, and the predictions for other various scattering cross sections are so far in satisfactory agreement with data. The mechanism of factorization of quark amplitudes implies that in $\bar{B}B$ scattering, exotic exchange is unavoidable[47] (although in general small); this could explain why the usual nonexotic duality approach fails in $\bar{B}B$ scattering. New support for this assumption comes from the fact that duality encounters similar difficulties in $MM$ scattering,[11] where again annihilation channels are present. We consider that in this case factorization of quark amplitudes should apply too, and thus exotic exchange should emerge.

If this is so, the conclusion arises that it is not $\bar{B}B$ scattering that is exceptional with respect to the duality (and additivity quark model), but, rather, $MB$ scattering. In any case, we feel that the $\bar{B}B$ puzzle begins to lose some of its mystery.

## ACKNOWLEDGMENTS

The comments of Dr. G. Cohen-Tannoudji, Dr. L. Jones, and Dr. B. Sadoulet were very helpful in elucidating many of the problems discussed here. The author has also benefited from instructive discussions with his colleagues at the Physics Department at Indiana University, and especially with Dr. P. Chylek, Dr. S. Y. Chu, Dr. A. W. Hendry, and Dr. R. G. Newton.

## APPENDIX A: FACTORIZATION OF QUARK AMPLITUDES IN $\bar{B}B$ SCATTERING

From Eq. (5) it follows that a typical three-quark amplitude $Q$ can be written under the form

$$Q \equiv \langle abc\ \bar{a}\bar{b}\bar{c} \,|\, T \,|\, def\ \bar{d}\bar{e}\bar{f} \rangle\,. \tag{A1}$$

Here $a, b, c\ldots$ denotes a quark with given helicity. This amplitude is factorized into two-body quark scattering amplitudes in the following way:

At first we consider all possible $q\bar{q}$ configurations both in the final and the initial state. This leads to

$$\begin{aligned}
\langle abc\ \bar{a}\bar{b}\bar{c} \,|\, T \,|\, def\ \bar{d}\bar{e}\bar{f} \rangle = {} & \langle a\bar{a}(b\bar{b}\ c\bar{c} + b\bar{c}\ c\bar{b}) + a\bar{b}(b\bar{a}\ c\bar{c} + b\bar{c}\ c\bar{a}) \\
& + a\bar{c}(b\bar{a}\ c\bar{b} + b\bar{b}\ c\bar{a}) \,|\, T \,|\, d\bar{d}(e\bar{e}\ f\bar{f} + e\bar{f}\ f\bar{e}) + d\bar{e}(e\bar{d}\ f\bar{f} + e\bar{f}\ f\bar{d}) + d\bar{f}(e\bar{d}\ f\bar{e} + e\bar{e}\ f\bar{d})\rangle \\
= {} & \langle a\bar{a}\ b\bar{b}\ c\bar{c} \,|\, d\bar{d}\ e\bar{e}\ f\bar{f} \rangle + \langle a\bar{a}\ b\bar{b}\ c\bar{c} \,|\, d\bar{d}\ e\bar{f}\ f\bar{e} \rangle + \langle a\bar{a}\ b\bar{b}\ c\bar{c} \,|\, d\bar{e}\ e\bar{d}\ f\bar{f} \rangle \\
& + \langle a\bar{a}\ b\bar{b}\ c\bar{c} \,|\, d\bar{e}\ e\bar{f}\ f\bar{d} \rangle + \langle a\bar{a}\ b\bar{b}\ c\bar{c} \,|\, d\bar{f}\ e\bar{d}\ f\bar{e} \rangle + \langle a\bar{a}\ b\bar{b}\ c\bar{c} \,|\, d\bar{f}\ e\bar{e}\ f\bar{d} \rangle \\
& + \langle a\bar{a}\ b\bar{c}\ c\bar{b} \,|\, d\bar{d}\ e\bar{e}\ f\bar{f} \rangle + \langle a\bar{a}\ b\bar{c}\ c\bar{b} \,|\, d\bar{d}\ e\bar{f}\ f\bar{e} \rangle + \langle a\bar{a}\ b\bar{c}\ c\bar{b} \,|\, d\bar{e}\ e\bar{d}\ f\bar{f} \rangle \\
& + \langle a\bar{a}\ b\bar{c}\ c\bar{b} \,|\, d\bar{e}\ e\bar{f}\ f\bar{d} \rangle + \langle a\bar{a}\ b\bar{c}\ c\bar{b} \,|\, d\bar{f}\ e\bar{e}\ f\bar{d} \rangle + \langle a\bar{a}\ b\bar{c}\ c\bar{b} \,|\, d\bar{f}\ e\bar{d}\ f\bar{e} \rangle \\
& + \langle a\bar{b}\ b\bar{a}\ c\bar{c} \,|\, d\bar{d}\ e\bar{e}\ f\bar{f} \rangle + \langle a\bar{b}\ b\bar{a}\ c\bar{c} \,|\, d\bar{d}\ e\bar{f}\ f\bar{e} \rangle + \langle a\bar{b}\ b\bar{a}\ c\bar{c} \,|\, d\bar{e}\ e\bar{d}\ f\bar{f} \rangle \\
& + \langle a\bar{b}\ b\bar{a}\ c\bar{c} \,|\, d\bar{e}\ e\bar{f}\ f\bar{d} \rangle + \langle a\bar{b}\ b\bar{a}\ c\bar{c} \,|\, d\bar{f}\ e\bar{d}\ f\bar{e} \rangle + \langle a\bar{b}\ b\bar{a}\ c\bar{c} \,|\, d\bar{f}\ e\bar{e}\ f\bar{d} \rangle \\
& + \langle a\bar{c}\ b\bar{a}\ c\bar{b} \,|\, d\bar{d}\ e\bar{e}\ f\bar{f} \rangle + \langle a\bar{c}\ b\bar{a}\ c\bar{b} \,|\, d\bar{d}\ e\bar{f}\ f\bar{e} \rangle + \langle a\bar{c}\ b\bar{a}\ c\bar{b} \,|\, d\bar{e}\ e\bar{d}\ f\bar{f} \rangle \\
& + \langle a\bar{c}\ b\bar{a}\ c\bar{b} \,|\, d\bar{e}\ e\bar{f}\ f\bar{d} \rangle + \langle a\bar{c}\ b\bar{a}\ c\bar{b} \,|\, d\bar{f}\ e\bar{d}\ f\bar{e} \rangle + \langle a\bar{c}\ b\bar{a}\ c\bar{b} \,|\, d\bar{f}\ e\bar{e}\ f\bar{d} \rangle
\end{aligned}$$

$$+\langle a\bar{c}\; b\bar{b}\; c\bar{a}\,|\,d\bar{d}\; e\bar{e}\; f\bar{f}\rangle+\langle a\bar{c}\; b\bar{b}\; c\bar{a}\,|\,d\bar{d}\; e\bar{f}\; f\bar{e}\rangle+\langle a\bar{c}\; b\bar{b}\; c\bar{a}\,|\,d\bar{e}\; e\bar{d}\; f\bar{f}\rangle$$
$$+\langle a\bar{c}\; b\bar{b}\; c\bar{a}\,|\,d\bar{e}\; e\bar{f}\; f\bar{d}\rangle+\langle a\bar{c}\; b\bar{b}\; c\bar{a}\,|\,d\bar{f}\; e\bar{d}\; f\bar{e}\rangle+\langle a\bar{c}\; b\bar{b}\; c\bar{a}\,|\,d\bar{f}\; e\bar{e}\; f\bar{d}\rangle$$
$$+\langle a\bar{b}\; b\bar{c}\; c\bar{a}\,|\,d\bar{d}\; e\bar{f}\; f\bar{e}\rangle+\langle a\bar{b}\; b\bar{c}\; c\bar{a}\,|\,d\bar{d}\; e\bar{e}\; f\bar{f}\rangle+\langle a\bar{b}\; b\bar{c}\; c\bar{a}\,|\,d\bar{e}\; e\bar{d}\; f\bar{f}\rangle$$
$$+\langle a\bar{b}\; b\bar{c}\; c\bar{a}\,|\,d\bar{e}\; e\bar{f}\; f\bar{d}\rangle+\langle a\bar{b}\; b\bar{c}\; c\bar{a}\,|\,d\bar{f}\; e\bar{d}\; f\bar{e}\rangle+\langle a\bar{b}\; b\bar{c}\; c\bar{a}\,|\,d\bar{f}\; e\bar{e}\; f\bar{d}\rangle\,. \qquad \text{(A2)}$$

Now each expression appearing in (A2) can be factorized into a sum of products of two-body quark amplitudes. Thus, e.g.,

$$\langle a\bar{a}\; b\bar{b}\; c\bar{c}\,|\,d\bar{d}\; e\bar{e}\; f\bar{f}\rangle=(a\bar{a}\to d\bar{d})[(b\bar{b}\to e\bar{e})(c\bar{c}\to f\bar{f})+(b\bar{b}\to f\bar{f})(c\bar{c}\to e\bar{e})]$$
$$+(a\bar{a}\to e\bar{e})[(b\bar{b}\to d\bar{d})(c\bar{c}\to f\bar{f})+(b\bar{b}\to f\bar{f})(c\bar{c}\to d\bar{d})]$$
$$+(a\bar{a}\to f\bar{f})[(b\bar{b}\to d\bar{d})(c\bar{c}\to e\bar{e})+(b\bar{b}\to e\bar{e})(c\bar{c}\to d\bar{d})]\,. \qquad \text{(A3)}$$

Any particle scattering amplitude can be expressed in terms of quark amplitudes $Q$ by using Eqs. (2)–(6). The three-quark helicity amplitudes for $\bar{B}B$ scattering have been computed in Ref. 48. As an example we give below the particle amplitude $\langle \overline{\Delta^{++}_{3/2}}\Delta^{++}_{3/2}|T|\bar{p}_{1/2}p_{1/2}\rangle$ in terms of the corresponding two-body quark amplitudes $x$ and $y$ (Eq. 7). Use has been made of the approximation (11). The ± helicity labels are defined in Eq. (13). The normalization corresponds to Franklin's[28] wave functions.

$$\langle \overline{\Delta^{++}_{3/2}}\Delta^{++}_{3/2}|T|\bar{p}_{1/2}p_{1/2}\rangle=\tfrac{2}{3}(a_1-a_2-a_3+a_4)\,,$$

$$a_1=\langle p\uparrow p\uparrow p\uparrow\; \bar{p}\uparrow\bar{p}\uparrow\bar{p}\uparrow\,|\,p\uparrow p\uparrow n\downarrow\; \bar{p}\uparrow\bar{p}\uparrow\bar{n}\downarrow\rangle=72x_{++++}{}^2y_{++--}\,,$$
$$a_2=\langle p\uparrow p\uparrow p\uparrow\; \bar{p}\uparrow\bar{p}\uparrow\bar{p}\uparrow\,|\,p\uparrow p\uparrow n\downarrow\; \bar{p}\downarrow\bar{p}\uparrow\bar{n}\uparrow\rangle=72x_{++++}\,x_{+++-}y_{++-+}\,,$$
$$a_3=\langle p\uparrow p\uparrow p\uparrow\; \bar{p}\uparrow\bar{p}\uparrow\bar{p}\uparrow\,|\,p\uparrow p\downarrow n\uparrow\; \bar{p}\uparrow\bar{p}\uparrow\bar{n}\downarrow\rangle=72x_{++++}\,x_{++-+}\,y_{+++-}\,,$$
$$a_4=\langle p\uparrow p\uparrow p\uparrow\; \bar{p}\uparrow\bar{p}\uparrow\bar{p}\uparrow\,|\,p\uparrow p\downarrow n\uparrow\; \bar{p}\uparrow\bar{p}\downarrow\bar{n}\uparrow\rangle=36(x_{++++}\,x_{++--}+x_{++-+}\,x_{+++-})y_{++++}\,. \qquad \text{(A4)}$$

## APPENDIX B: PARTICLE AMPLITUDES IN TERMS OF QUARK AMPLITUDES $x, y, z, r$ AT $t=0$ FOR $P=(-)^J$ EXCHANGE AT QUARK LEVEL

The normalization corresponds to Franklin's[28] prescription for wave functions. Because of Eqs. (19)–(21) we have a single-helicity configuration:

$$\langle \bar{p}p|T|\bar{p}p\rangle=8x(x^2+2r^2)\,,$$
$$\langle \bar{n}n|T|\bar{p}p\rangle=4y(x^2+r^2)\,,$$
$$\langle \bar{\Lambda}\Lambda|T|\bar{p}p\rangle=3z(x^2+r^2)\,,$$
$$\langle \bar{\Sigma}^0\Sigma^0|T|\bar{p}p\rangle=z(x^2+r^2)\,,$$
$$\langle \overline{\Sigma^+}\Sigma^+|T|\bar{p}p\rangle=8zx^2\,,$$
$$\langle \overline{\Sigma^-}\Sigma^-|T|\bar{p}p\rangle=4xyz\,,$$
$$\langle \overline{\Sigma^-}\Lambda|T|\bar{p}n\rangle=4\sqrt{3}xzr\,,$$
$$\langle \overline{\Xi^-}\Xi^-|T|\bar{p}p\rangle=4xz^2\,,$$
$$\langle \bar{\Xi}^0\Xi^0|T|\bar{p}p\rangle=8xz^2\,.$$

## APPENDIX C: PARITY MIXING IN DECUPLET PRODUCTION

Let us consider a process of the type

$$\bar{B}B\to\bar{B}^*B \qquad \text{(C1)}$$

and, in particular,

$$\bar{p}p\to\bar{Y}^{*0}\Lambda \qquad \text{(C2)}$$

at $t=0$. As pointed out in Sec. IV C, there are three nonvanishing two-body quark helicity amplitudes $A_1$, $A_2$, $A_3$ [cf. Eq. (13)].

From Eq. (15) it follows that

$$A_1=\pm A_3\,, \qquad \text{(C3)}$$

where the + sign corresponds to natural-parity exchange $[P=(-)^J]$ and the − sign to unnatural-parity

TABLE II. The particle scattering amplitude $A$ for the process $\bar{p}p\to\overline{Y^*}\Lambda$ for various parity exchanges at quark level, in accordance with Eq. (C5).

| | | | |
|---|---|---|---|
| $z_1=z_3$ [a] | $x_1=x_3$ [a] | $r_1=r_3$ [a] | $A=0$ |
| $z_1=z_3$ [a] | $x_1=x_3$ [a] | $r_1=-r_3$ [b] | $A=6zr^2$ |
| $z_1=z_3$ [a] | $x_1=-x_3$ [b] | $r_1=r_3$ [a] | $A=6zx^2$ |
| $z_1=z_3$ [a] | $x_1=-x_3$ [b] | $r_1=-r_3$ [b] | $A=6z(x^2+r^2)$ |
| $z_1=-z_3$ [b] | $x_1=x_3$ [a] | $r_1=r_3$ [a] | $A=6z(x^2+r^2)$ |
| $z_1=-z_3$ [b] | $x_1=x_3$ [a] | $r_1=-r_3$ [b] | $A=6zr^2$ |
| $z_1=-z_3$ [b] | $x_1=-x_3$ [b] | $r_1=r_3$ [a] | $A=6zr^2$ |
| $z_1=-z_3$ [b] | $x_1=-x_3$ [b] | $r_1=-r_3$ [b] | $A=0$ |

[a] Natural-parity exchange (leading trajectories: $K^*$, $K^{*\prime}$ for $z$; $\rho$, $A_2$, Pomeranchukon, $P'$, etc., for $x$, $r$).

[b] Unnatural-parity exchange ($K$ for $z$; $\pi$, $A_1$, etc., for $x$, $r$). Thus we see that the parity mixing at the quark level can transform a dip ($A=0$) into a peak ($A\neq 0$) and vice versa.

exchange $[P=(-)^{J+1}]$ and

$$A_2=0 \tag{C4}$$

independent of the parity of the exchanged object.

The amplitude for (C2) reads in the factorization model with Franklin's[28] normalization

$$A=\langle \overline{Y}^*\Lambda | T | \overline{p}p\rangle = 3z_1(x_1^2+r_3^2)-3z_3(x_1x_3+r_1r_3). \tag{C5}$$

Now, depending on the sign in (C3), $A$ takes on the values given in Table II.

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*Work supported in part by the National Science Foundation.

[1]A definition of exotic states of first and second kind is given in Sec. II.

[2]For a recent review along these lines cf. J. D. Jackson, Rev. Mod. Phys. 42, 12 (1970).

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[5]J. Rosner, Phys. Rev. Letters 22, 689 (1969).

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[17]R. Weiner, Nucl. Phys. B6, 226 (1968); B10, 191 (1969).

[18]The factorization of quark amplitudes is not to be confused with the factorization of one-*particle* (*-pole*) exchange amplitudes. In fact, as will be shown below, the factorization of quark amplitudes implies in general the abandon of factorization of particle exchange. In the following, if not stated otherwise, we understand by factorization the fact that the particle scattering amplitude is a sum of factorized quark amplitudes.

[19]In fact, we initially had in mind (see Ref. 20) to replace the additivity assumption by the factorization hypothesis not only for large momentum transfer scattering (l.m.t.) but also for scattering at any angle, including forward scattering; at that time, however, this idea was too shocking to be published and that is why we restricted the factorization model to l.m.t.

[20]R. M. Weiner, unpublished report, 1967, and private communication to H. J. Lipkin.

[21]H. J. Lipkin, Phys. Rev. 183, 1221 (1969), and Rehovoth report, 1970 (unpublished).

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[23]For the sake of clarity, we again sketch the essential points of the factorization model.[17]

[24]In the multiple-scattering formalism[15] there exists also a term analogous to Eq. (5), although in applications the Glauber series is stopped at an earlier stage, i.e., the scattering multiplicity taken into account is lower. The difference between the factorization and multiple-scattering formalism thus consists, from this point of view, of the fact that in the factorization quark model the triple-scattering term is the only contributing term. There are also, of course, other fundamental differences between these two approaches.

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[34]For a recent review of the Regge-pole theory, cf. P. D. B. Collins, University of Durham report, 1970 (unpublished).

[35]Equation (27) has been obtained by using Eq. (8) for the single-quark amplitudes, and that is why it is given for the particle amplitudes and not for the spin-aver-

aged cross sections.

[36]We do not give the l.m.t. analog of Eq. (27) because, as mentioned,[35] this equation is based on relation (8) between *single*-quark amplitudes, while in our l.m.t. treatment we add incoherently three-quark amplitudes. The neglect of interferences at single-quark amplitudes, which would read, in this case, $|y|^2=|x|^2+|r|^2$, is a different approximation.

[37]R. M. Weiner, Indiana University report, 1971 (unpublished).

[38]Cf., e.g., S. D. Drell and J. D. Sullivan, Phys. Rev. Letters 19, 268 (1967). I am indebted to Dr. J. D. Sullivan for an instructive correspondence on this subject.

[39]Reference 34, p. 185.

[40]T. Bacon *et al.*, in *Proceedings of the Twelfth International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat., Moscow, U.S.S.R., 1966), p. 697.

[41]G. Fisher *et al.*, Phys. Rev. 161, 1335 (1967).

[42]P. Astbury *et al.*, Phys. Letters 23, 160 (1966).

[43]B. Sadoulet (private communication).

[44]R. Hagedorn, Nuovo Cimento 35, 216 (1965).

[45]Relation (25′) can be obtained also in a multiple-scattering approach.[46] The numerical result of Ref. 46 ($\epsilon_2^{\text{l.m.t.}}=1$) is erroneous and has to be replaced by $\epsilon_2^{\text{l.m.t.}}=2$.

[46]N. Dean, Nucl. Phys. B7, 311 (1968).

[47]We recall that exotic exchange means only exchange of exotic quantum numbers. Whether this is due to "true" exotic particles or to multiple-particle exchange is an open question.

[48]R. Weiner, University of Bucharest reports, 1968 (unpublished).

PHYSICAL REVIEW D VOLUME 4, NUMBER 3 1 AUGUST 1971

# Sum Rules for *CP*-Nonconserving $BB\pi$ Amplitudes in Glashow's Model*

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(Received 29 April 1971)

Current algebra has been used to derive sum rules for the $CP$-nonconserving $\Delta S=1$ and $\Delta S=0$ nonleptonic transitions $B\to B'\pi$ in Glashow's model. Within the saturation scheme of Chiu, Schechter, and Ueda for the matrix elements of the weak current×current Hamiltonian between single baryon states, it is shown that the $CP$-nonconserving amplitudes do have octet dominance. The one-pion exchange contribution to the weak $P$- and $T$-violating nucleon-nucleon potential has been calculated by relating the weak $NN\pi$ vertex to hyperon decay amplitudes.

## I. INTRODUCTION

A number of theories have been put forward to explain the experimental observation of $CP$ nonconservation[1] in $K_L^0\to 2\pi$ decay. Among the models which embed the source of the $CP$ nonconservation in the current ×current form of the weak interactions,[2,3] Glashow's model[2] is unique in that it introduces no neutral currents and it predicts a violation of $CP$ invariance in the $\Delta S=0$ nuclear interactions. In this paper the predictions based on Glashow's model for $CP$, or equivalently $T$, violations in the $s$-wave parity-violating nonleptonic transitions $B\to B'\pi$ are investigated[4] by using current algebra and the soft-pion technique.[5] Using the saturation scheme of Chiu, Schechter, and Ueda[6] for the matrix elements of the current×current Hamiltonian between single baryon states, we show that the $CP$-violating parts of the amplitudes also exhibit octet dominance. These predictions are used to evaluate the one-pion exchange contribution to the weak $P$- and $T$-violating nucleon-nucleon potential.

In Glashow's model[2] the weak-interaction Hamiltonian is given by

$$H_w=(G/2\sqrt{2})\{J_\mu,J_\mu^\dagger\}, \tag{1}$$

where $G=1.02\times10^{-5}/M_p^2$ is the Fermi coupling constant, and the Cabibbo currents, in the $SU(3)$ tensor notation, are given by

$$J_\mu=J_\mu(\text{leptonic})+\cos\theta(V_\mu+e^{i\rho}A_\mu)_1{}^2+\sin\theta(V_\mu+e^{i\psi}A_\mu)_1{}^3. \tag{2}$$

The model allows for $CP$ nonconservation by the presence of the complex phases, with phase angles $\rho$ and $\psi$, between the hadronic vector and axial-vector currents. Since $CP$ violations have not been observed so far in nuclear $\beta$ decay,[7] the angle $\rho$ must be rather small. It has been shown by Pati[8] that the choice $\rho\cong\psi\le\frac{1}{600}$ ensures that the $\Delta T=\frac{1}{2}$ rule holds for the nonleptonic $CP$-conserving $K\to2\pi$ decays, and furthermore predicts an electric dipole moment for the neutron of the order of $10^{-23}$ $e$ cm, which is nearly outside the