Letters 27B, 657 (1968).

[6]PCAC breakdown is defined as follows: $\partial_\mu A_a^\mu = F_{ab}\mu_b{}^2 S_b + \mathfrak{F}_a$. A_a^μ is the axial-vector current of the ath type, and μ_b is the mass of the bth pseudoscalar-meson field, S_b. F_{ab} are the axial-vector current, pseudoscalar-meson coupling strengths. The first term on the right-hand side is the usual pole term, $\mathfrak{F}_a$ represents any additional (breaking) terms added to PCAC.

[7]M. G. Miller, preceding paper, Phys. Rev. D 4, 757 (1971), hereafter referred to as I.

[8]F_π is experimentally (97 ± 2) MeV in the Cabibbo theory of $\pi^\pm\beta$ decay. The Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation $g_\rho{}^2 = 2m_\rho{}^2 F_\pi{}^2$ is used to evaluate g_ρ.

[9]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[10]Particle Data Group, Rev. Mod. Phys. 42, 87 (1970).

[11]States are normalized such that $N(q) = [2\omega_q(2\pi)^3]^{-1/2}$, where $\omega_q = (\vec{q}^2 + m^2)^{1/2}$. The metric used is $-g_{00} = g_{11} = g_{22} = g_{33} = 1$.

[12]In terms of λ', the ϕ-π-ρ coupling constant is $g_{\pi\rho\phi} = 4\lambda'/F_\pi$.

[13]J. E. Augustin *et al.*, Phys. Letters 28B, 503 (1969).

[14]λ and λ' are selected from a fit of $\Gamma(\omega \to \pi^0 + \gamma)$ and $\Gamma(\phi \to \pi^0 + \rho^0)$, respectively. See Refs. 5 and 7.

[15]R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. 174, 2008 (1968).

[16]Alexander Donnachie and Graham Shaw, Ann. Phys. (N.Y.) 37, 333 (1966).

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Once-Subtracted Dispersion Relations, Current Algebra, and Nonleptonic Weak Decays of Hyperons*

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Nonleptonic weak decays of hyperons and the $K_1 \to 2\pi$ decay are carefully analyzed using current algebra, partial conservation of axial-vector current, and once-subtracted dispersion relations, a method suggested by Okubo, Mathur, and Marshak. We have calculated the dispersion integral by assuming it to be saturated by the low-mass intermediate states of the nucleon octet, of the $\frac{3}{2}^+$ decuplet of $\Delta(1236)$, and of the $\frac{1}{2}^-$ SU_3 singlet $Y_0^*(1405)$, in the case of hyperon decays, and by the intermediate states of vector mesons and a possible 0^+ scalar-meson nonet, in the case of $K_1 \to 2\pi$ decay. The matrix element of the parity-violating weak Hamiltonian density between two baryons (which is required to evaluate the hyperon decay amplitudes) is related to the $K_1 \to 2\pi$ decay amplitude by the K_1 tadpole mechanism. With pseudovector SU_3-symmetric coupling among the nucleon and pseudoscalar-meson octets, we are able to obtain a good fitting of all the hyperon decay amplitudes in terms of four parameters. We also find that the corrections to the soft-pion values are very important not only for P-wave hyperon decay amplitudes, but also for S-wave amplitudes. They are also extremely important for $K_1 \to 2\pi$ decays. Furthermore, we find that, contrary to the findings of Hara and Nambu, there is no evidence for the concept of a "universal parity-conserving spurion" in hyperon and $K_1 \to 2\pi$ weak decays.

I. INTRODUCTION

Based on a current-current theory of weak interactions, current algebra, partial conservation of axial-vector current (PCAC), and a baryon pole model, nonleptonic weak decays of hyperons have been discussed by several authors.[1-9] Our work in this article is along the general lines of these previous works, but is intended to be more complete than any of these. We get several new interesting results, which to the best of our knowledge have not been reported in the literature.

We will start by discussing the work of Kumar and Pati (KP),[6] which is probably one of the most improved forms of all previous works. In their work, KP[6,10] demonstrated that it is possible to fit the eight independent (if the $|\Delta I| = \frac{1}{2}$ property is already built into the theory, as is the case here) decay amplitudes in hyperon decays within about (60–70)%, in terms of essentially only two parameters. This was considerably better than the previous analyses, in which there appeared to be discrepancies with factors of 2 to 3, especially in the P-wave decay amplitudes.

Their work is based on the following assumptions:

(1) The current-current theory is used to the extent that the equal-time commutators involving parity-conserving (P.C.) and parity-violating (P.V.) weak Hamiltonian densities satisfy the simple relations

$$[\mathcal{H}_W^{\mathrm{P.V.}}, F_5^i] = [\mathcal{H}_W^{\mathrm{P.C.}}, F^i], \tag{1}$$

$$[\mathcal{H}_W^{\mathrm{P.C.}}, F_5^i] = [\mathcal{H}_W^{\mathrm{P.V.}}, F^i], \tag{2}$$

where F^i and F_5^i $(i=1,2,3)$ are the charges of the vector and axial-vector currents V_μ^i and A_μ^i, i being the Gell-Mann[11] SU_3 index.

(2) The soft-pion limits $(q=0)$ of the decay amplitudes (calculable through the use of current algebra and PCAC) are very close to the physical values, except for the contributions from the $\frac{1}{2}^+$ nucleon-octet pole diagrams.

(3) Even though in the current-current theory there is an equal mixture of octet and $\underline{27}$ parts in the nonleptonic weak Hamiltonian, somehow, due to some strong-interaction effect, the matrix elements of $\mathcal{H}_W^{\text{P.C.}}$ and $\mathcal{H}_W^{\text{P.V.}}$ between two members of a nucleon octet predominantly contain only the octet part, the so-called octet dominance of matrix elements.[12]

(4) The effects of the breaking of SU_3 symmetry are important to those entities which vanish in the SU_3-symmetry limit and are, hopefully, negligible in those which do not. Thus, KP used physical masses for the particles involved and assumed a nonzero value for the matrix element of $\mathcal{H}_W^{\text{P.V.}}$ between two octet baryon states, while using SU_3-symmetric values for the couplings of the pseudoscalar-octet mesons and the weak parity-conserving spurion ($\mathcal{H}_W^{\text{P.C.}}$) to the nucleon octet. In the current-current theory, the matrix element of $\mathcal{H}_W^{\text{P.V.}}$ between two octet baryons and the $K_1 \to 2\pi$ amplitude are zero[13] in the SU_3 limit. But the latter, instead of being zero, is rather large. Using the K_1 tadpole mechanism, the above two matrix elements can be related.[14] Using this mechanism, $\langle B_f^8 | \mathcal{H}_W^{\text{P.V.}} | B_i^8 \rangle$ is proportional to the product of the $K_1 B_f^8 B_i^8$ vertex and a parameter f_K defined by

$$(2\pi)^{3/2}\sqrt{2h^0}\,\langle 0 | \mathcal{H}_W^{\text{P.V.}} | \bar{K}^0(h)\rangle = \tfrac{1}{2} i f_K m_K^{\,2}\,. \qquad (3)$$

f_K was calculated from the $K_1 \to 2\pi$ decay rate by assuming its decay amplitude is proportional to the appropriate extrapolation of the $KK\pi\pi$ vertex times f_K. The $KK\pi\pi$ vertex itself was calculated in a pole model.[14,15] KP found[14,15]

$$f_K \simeq (1-3)\times 10^{-9}\ \text{BeV}\,. \qquad (4)$$

With this value of f_K, they concluded that the effect of the nonvanishing of $\langle B_f^8 | \mathcal{H}_W^{\text{P.V.}} | B_i^8 \rangle$ is negligible in hyperon decays. In particular, the corrections to the soft-pion values of the S-wave decay amplitudes were negligible. It is to be emphasized that these results, as well as the good fitting of experimental amplitudes they obtained, depend crucially on the range of values [$(1-3)\times 10^{-9}$ BeV] assumed for f_K.

It is possible, at present, to make a much more reliable estimate of the parameter f_K from the known $K_1 \to 2\pi$ decay rate, using current algebra, PCAC, and once-subtracted dispersion relations. We have calculated the $K_1 \to 2\pi$ decay amplitude, in the $p_\pi^{\,2}=0$ limit, using the above techniques, and found (see Sec. IV)

$$f_K \simeq 0.5\times 10^{-7}\ \text{BeV}. \qquad (5)$$

This is about 25 times larger[15] *than KP's estimate* [Eq. (4)]. With this value of f_K, *the effects of the nonvanishing of* $\langle B_f^8 | \mathcal{H}_W^{\text{P.V.}} | B_i^8 \rangle$ are not negligible; on the contrary, *they are very important*. It is found that if we use only the nucleon-octet pole diagrams, the S-wave decay amplitude of $\Sigma^+ \to n\pi^+$ (which is almost zero experimentally) is not small, and the observed Lee-Sugawara relation for S waves does not hold well. Furthermore, the over-all fit is much worse than that obtained by KP.

There are at least two possibilities for improving the model of KP. (1) The partial use of SU_3-symmetric coupling constants may not be justified, and we should try to calculate the effects of SU_3 breaking in these also. (2) Another possibility is to examine the extrapolation properties (with respect to the pion four-momentum q) of contributions of higher intermediate states to the decay amplitudes, and include them appropriately in the calculation. Since there do not seem[16] to be reliable ways of calculating the effects of SU_3 breaking in coupling constants, we propose to consider the second possibility[17] in this paper.

We deal with the extrapolation properties of contributions of higher intermediate states to the hyperon decay amplitudes, according to a method suggested by Okubo.[18] According to this method, once-subtracted dispersion relations are written in a suitable variable, keeping $q^2=0$, for certain functions (see Sec. II) whose appropriate limits give the S-wave and P-wave decay amplitudes (which are actually constants, not functions) of hyperons. The subtraction point is chosen at the soft-pion limit, so that the subtraction constant is given by current algebra and PCAC. Then the dispersion integral is the change in S- and P-wave amplitudes in going from the $q=0$ limit to a much more physical limit of $q^2=0$. We calculate the contributions of higher intermediate states according to a dispersion theory rather than a Feynman-diagram approach, mainly because the perturbation expansion which underlies a Feynman-diagram calculation has no basis here, since strong interaction is also involved, and also because there is some ambiguity regarding propagators of particles of spin $\frac{3}{2}$ and higher.

In this article, we calculate the subtracted dispersion integral (mentioned above) by assuming it to be saturated by the intermediate states of the nucleon octet, of the $\frac{3}{2}^+$ decuplet of $\Delta(1236)$, and of the $\frac{1}{2}^-$ SU_3 singlet $Y_0^*(1405)$. We will *retain the assumptions (1), (3), and (4) of KP*[6] except that we will use a more reliable estimate of f_K. Also, differing from KP, we will use an SU_3-symmetric pseudovector coupling for the $B^8B^8P^8$ vertex. It is found that the fitting of the experimental amplitudes in hyperon decays is much worse, if we use pseudoscalar SU_3-symmetric values for the above vertex. As will be discussed in the concluding section, this is a nice feature of our model, as there seems to be more justification[19] for the former assumption. We calculate the matrix element of $\mathcal{H}_W^{\text{P.V.}}$ between any two baryons by assuming that it satisfies an unsubtracted dispersion relation in the appropriate variable and that the dispersion integral is saturated by the lowest-mass intermediate state, namely the K meson. This is the dispersion version of the K_1 tadpole mechanism.[14] So, all the two-body vertices of $\mathcal{H}_W^{\text{P.V.}}$ are expressed in terms of f_K. This approximation also automatically brings about the octet dominance in the matrix elements of $\mathcal{H}_W^{\text{P.V.}}$, whatever may be its primary nature.

With the above assumptions, there are four free parameters in the theory, in terms of which we are able to fit the eight independent decay amplitudes (four S wave and four P wave) within 20% to 30%. It is found, with the present value of f_K [Eq. (5)], that the dispersion-integral corrections (corrections in going from the $q=0$ limit to the $q^2=0$ limit) are important for both S-wave and P-wave amplitudes.

The remaining part of the article is divided into four sections. In Sec. II we formulate the problem for the use of once-subtracted dispersion relations. In Sec. III we discuss the calculation of the dispersion integral and give the final expressions for the decay amplitudes. In Sec. IV we give the outline of the calculation of the $K_1^0 \to 2\pi$ amplitude in the $p_\pi{}^2=0$ limit, using current algebra and once-subtracted dispersion relations. Section V contains the numerical comparison of our results with experiment. Finally, Sec. VI is a collection of concluding remarks. Several remarks with respect to the concept of the universal parity-conserving spurion,[20] the $|\Delta I|=\frac{1}{2}$ rules, the validity of using SU_3 coupling constants, the test of current-current theory in nonleptonic weak decays, etc. are made here.

II. FORMULATION OF THE PROBLEM FOR THE USE OF ONCE-SUBTRACTED DISPERSION RELATIONS

Consider a typical decay of interest:

$$N_\alpha(p) \to N_\beta(p') + \pi_j(q)\,,$$

where α, β, and j are Gell-Mann's SU_3 indices. The transition matrix element for the above decay due to the weak-interaction Hamiltonian density $\mathcal{H}_W(x)$ can be written

$$(2\pi)^{9/2}\left(\frac{2p_0p_0'q_0}{M_\alpha M_\beta}\right)^{1/2} {}_{\text{out}}\langle N_\beta(p')\pi_j(q)|\mathcal{H}_W(0)|N_\alpha(p)\rangle_{\text{in}} = i\bar{u}_\beta(p')[S+\gamma_5 P]u_\alpha(p)\,, \tag{6}$$

where S and P are the S-wave (parity-violating) and the P-wave (parity-conserving) decay amplitudes, respectively. The S- and P-wave amplitudes defined here (namely, S and P) are numbers because, by means of four-momentum conservation, all the relativistically invariant variables in the problem are fixed. By means of the reduction formula we can consider this matrix element, when the pion is off the mass-shell, and the energy-momentum-conservation requirement is relaxed. Using the reduction formula, the left-hand side of Eq. (1) can be written (when the pion is off the mass shell) as

$$M^j_{\beta\alpha}(q) = i\left(\frac{p_0p_0'}{M_\alpha M_\beta}\right)^{1/2}(-q^2+m_\pi{}^2)(2\pi)^3\int d^4x\, e^{iq\cdot x}\langle N_\beta(p')|T\{\varphi_j(x)\mathcal{H}_W(0)\}|N_\alpha(p)\rangle\,. \tag{7}$$

Following Okubo,[18] we can treat $M^j_{\beta\alpha}(q)$ defined by the above equation as the matrix element of a scattering process of the type

$$N_\alpha(p) + S_i(h) \to N_\beta(p') + \pi_j(q)\,,$$

where $S_i(h)$ is a hypothetical massless particle called spurion with four-momentum h and carrying unit strangeness and isotopic spin $\frac{1}{2}$. The subscript i is the SU_3 index, which can take the values 4, 5, 6, or 7. The matrix element $M^j_{\beta\alpha}$ in Eq. (7), when $h\to 0$ and $q^2\to m_\pi{}^2$, represents the physical nonleptonic weak decay generated by the weak Hamiltonian density $\mathcal{H}_W(x)$. In general, $M^j_{\beta\alpha}$ when $h\neq 0$ can be written as

$$\begin{aligned} M^j_{\beta\alpha}(q,u) &= i\bar{u}_\beta(p')[(F_1+\gamma_5F_2)+\tfrac{1}{2}(\not{h}+\not{q})(G_1+\gamma_5G_2)]\,u_\alpha(p) \\ &= i\bar{u}_\beta(p')\{(H_1+\gamma_5H_2)+[\not{h},\not{q}](K_1+\gamma_5K_2)\}\,u_\alpha(p)\,. \end{aligned} \tag{8}$$

The H_i's and K_i's $(i=1,2)$ can be expressed in terms of the F_i's and G_i's $(i=1,2)$ using the Dirac equation. F_i, G_i, H_i, and K_i $(i=1,2)$ are, in general, functions of the invariants s, t, u, q^2, and h^2, where

$$s=(p+h)^2=(p'+q)^2\,, \tag{9a}$$

$$t=(p-p')^2=(q-h)^2\,, \tag{9b}$$

$$u=(p-q)^2=(p'-h)^2\,. \tag{9c}$$

Consider first the physical limit $h\to 0$ and $q^2\to m_\pi{}^2$. Since there are no singularities in K_1 and K_2 as $h\to 0$, the terms involving K_1 and K_2 in Eq. (8) will vanish in this limit. Then, by comparing this limit of Eq. (8) with Eq. (6), we obtain

$$H_1(s=M_\alpha{}^2,\, t=m_\pi{}^2,\, q^2=m_\pi{}^2)=S\,, \tag{10a}$$

$$H_2(s=M_\alpha{}^2,\, t=m_\pi{}^2,\, q^2=m_\pi{}^2)=P\,. \tag{10b}$$

In the approximation of neglecting the variation of the functions H_1 and H_2 in the range $0\leqslant t,\ q^2\leqslant m_\pi{}^2$,

$$H_1(s=M_\alpha{}^2,\, t=q^2=0)\simeq S\,, \tag{11a}$$

$$H_2(s=M_\alpha{}^2,\, t=q^2=0)\simeq P\,. \tag{11b}$$

Next, let us take the $q\to 0$ limit of $M^j_{\beta\,\alpha}(q,h)$. Then $s\to M_\beta{}^2$, $u\to M_\alpha{}^2$, and $t, q^2\to 0$. At first sight there may appear to be some problems in taking this limit. In this case, when the masses of the intermediate-state baryons in the s and u channels equal, respectively, the external masses M_α and M_β, the poles due to these intermediate states occur exactly at the $q\to 0$ limit. Because of this, the contributions to K_1 and K_2 from the Born diagrams, namely K_1^B and K_2^B, have singularities as $q\to 0$ and $\bar u_\beta(p')[\not h,\not q](K_1^B+\gamma_5K_2^B)u_\alpha(p)$ does not vanish in the $q\to 0$ limit. In fact it is even ambiguous. On the other hand, the contributions to H_1 and H_2 from Born diagrams have definite and finite limits as $q\to 0$. This is because in this case the residues, as well as the denominators of the pole contributions to H_1 and H_2 from the Born diagrams, vanish in the $q\to 0$ limit, and the limits of these contributions are well defined. In other words, H_1 and H_2 do not have pole singularities due to degenerate intermediate states at the $q\to 0$ limits of H_1 and H_2, namely $H_1(s=M_\beta{}^2, t=q^2=0)$ and $H_2(s=M_\beta{}^2, t=q^2=0)$, have definite and finite values in all cases. In order to find these values, let us proceed in the following manner. Using Eqs. (7) and (8), the $q\to 0$ limit of $M^j_{\beta\alpha}$ is

$$
\begin{aligned}
&i\bar u_\beta(p')[H_1(s=M_\beta{}^2, t=q^2=0)+\gamma_5H_2(s=M_\beta{}^2, t=q^2=0)]\,u_\alpha(p)+\lim_{q\to 0} i\bar u_\beta(p')[\not h,\not q](K_1^B+\gamma_5K_2^B)u_\alpha(p)\\
&\qquad=-i(2\pi)^3\left(\frac{\sqrt2}{F_\pi}\right)\left(\frac{p_0p_0'}{M_\alpha M_\beta}\right)^{1/2}\langle N_\beta(p')|[F_j^5(0),\mathcal{H}_W(0)]|N_\alpha(p)\rangle\\
&\qquad\quad+(2\pi)^3\left(\frac{\sqrt2}{F_\pi}\right)\left(\frac{p_0p_0'}{M_\alpha M_\beta}\right)^{1/2}\lim_{q\to 0}q_\mu\int d^4x\,e^{iq\cdot x}\langle N_\beta(p')|T\{A_j^\mu(x)\mathcal{H}_W(0)\}|N_\alpha(p)\rangle\ .
\end{aligned} \tag{12}
$$

In deriving Eq. (12) we have substituted for the pion interpolating field $\varphi^j(x)$ in Eq. (7) the divergence of the axial-vector current by the relation

$$\partial^\mu A^j_\mu(x)=(F_\pi/\sqrt2)m_\pi{}^2\varphi^j(x)\,. \tag{13}$$

Ordinarily one would expect the second term on the right-hand side of Eq. (12) to vanish in the $q\to 0$ limit. But in the cases when M_α or M_β is equal to the mass of the intermediate state that can be inserted between $A_j^\mu(x)$ and $H_W(0)$, the contributions from these intermediate states to the above term do not vanish in the $q\to 0$ limit. In fact, this limit is ambiguous, and a careful calculation of the limit shows that it has precisely the same form as

$$\lim_{q\to 0}\bar u_\beta(p')[\not h,\not q](K_1^B+\gamma_5K_2^B)u_\alpha(p)\,.$$

So they cancel exactly in Eq. (13) and we get the results

$$\bar u_\beta(p')H_1(s=M_\beta{}^2, t=q^2=0)u_\alpha(p)=\frac{\sqrt2}{F_\pi}\left(\frac{p_0p_0'}{M_\alpha M_\beta}\right)^{1/2}\langle N_\beta(p')|[F_j^5(x_0=0),\mathcal{H}_W^{\text{P.V.}}(0)]|N_\alpha(p)\rangle\ , \tag{14}$$

$$\bar u_\beta(p')\gamma_5H_2(s=M_\beta{}^2, t=q^2=0)u_\alpha(p)=\frac{\sqrt2}{F_\pi}\left(\frac{p_0p_0'}{M_\alpha M_\beta}\right)^{1/2}\langle N_\beta(p')|[F_j^5(x_0=0),\mathcal{H}_W^{\text{P.C.}}(0)]|N_\alpha(p)\rangle\ . \tag{15}$$

F_j^5 is, of course, the axial charge $(j=1,2,3)$.

In the case of nondegenerate intermediate-state masses, the derivation of Eqs. (14) and (15) is trivial. In fact, another way to arrive at the same result, in the case of degenerate intermediate states, is to pretend there is a small mass difference ϵ between the intermediate state and the external state N_α or N_β, and take the limit $\epsilon \to 0$ only after the $q \to 0$ limit is taken. This will make the second terms on both sides of Eq. (12) vanish and we will get Eqs. (14) and (15) as before. Now we make use of Eqs. (1) and (2) to evaluate the matrix elements on the right-hand sides of Eqs. (14) and (15).

Following Okubo,[18] we now assume once-subtracted dispersion relations for the functions H_1 and H_2 in the variable s, keeping t, q^2, and h^2 zero. p^2 and p'^2 are, of course, kept on the mass shell. We do not inquire here into the validity of the dispersion relations. This is taken to be one of the assumptions of the model considered. Choosing the subtraction point at the soft-pion limit, we write[18]

$$S \simeq H_1(s=M_\alpha^2, t=q^2=h^2=0)$$
$$= H_1(s=M_\beta^2, t=q^2=h^2=0)+\frac{(M_\alpha^2-M_\beta^2)}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}_{s'}H_1(s', t=q^2=h^2=0)ds'}{(s'-M_\beta^2)(s'-M_\alpha^2)}\,, \tag{16}$$

$$P \simeq H_2(s=M_\alpha^2, t=q^2=h^2=0)$$
$$= H_2(s=M_\beta^2, t=q^2=h^2=0)+\frac{(M_\alpha^2-M_\beta^2)}{\pi}\int\frac{\mathrm{Im}_{s'}H_2(s', t=q^2=h^2=0)ds'}{(s'-M_\beta^2)(s'-M_\alpha^2)}\,. \tag{17}$$

III. CALCULATION OF THE DISPERSION INTEGRALS

The intermediate states in the s and u channels will contribute to the above dispersion integrals. Also, only intermediate states with baryon number 1, and odd half-integral angular momentum, can contribute in these channels. It can be shown (proof given in Appendix) that

(the contribution of a spin-$\frac{1}{2}$ single-baryon intermediate state to the once-subtracted dispersion integrals of H_1 and H_2, with subtraction point at the soft-pion limit) = (the difference in the contributions of the corresponding pole diagram with physical pion four-momentum and zero pion four-momentum). (18)

In general, it can also be shown that the above equality does not hold for spin-$\frac{3}{2}$ and higher-spin single-baryon intermediate states (see Appendix). Thus, if we include only the contribution to the dispersion integral of baryons belonging to the nucleon octet, our results should be identical with those obtained by KP,[6] except for the different value of f_K used by us.

As explained in the Introduction, we will calculate the dispersion integral by saturating it by the $\frac{1}{2}^+$ nucleon octet, the $\frac{3}{2}^+$ decuplet of $\Delta^+(1236)$, and the $\frac{1}{2}^-$ SU_3 singlet $Y_0^*(1405)$.

We deal with the broken SU_3 symmetry in our calculations in the spirit of KP[6] (see Introduction). Thus, SU_3 symmetry is assumed for the strong coupling of psuedoscalar mesons and the weak coupling of the P.C. spurion to the nucleon octet and to the system of the nucleon octet and the $\frac{3}{2}^+$ decuplet baryons. Most of the effects of SU_3 breaking are assumed to be taken into account by using physical masses and a nonzero value for the matrix element $\langle B_f^8|\mathcal{H}_W^{\mathrm{P.V.}}|B_i^8\rangle$.

The SU_3-symmetric *pseudovector* coupling of the pseudoscalar-meson octet with the nucleon octet is given by

$$\left(\frac{2M_N}{i\sqrt{2}G}\right)\mathcal{H}_I = -(1-\alpha)\mathrm{Tr}([\bar{B},\gamma_\mu\gamma_5 B]\partial^\mu P)+\alpha\mathrm{Tr}(\{\bar{B},\gamma_\mu\gamma_5 B\}\partial^\mu P)\,. \tag{19}$$

B, $\bar{B}$, and P are the usual (3×3) SU_3 matrices[21] of the octet baryons, antibaryons, and pseudoscalar meson fields, respectively. From pion-nucleon scattering data, the coupling constant

$$G^2/4\pi \simeq 14.6\,. \tag{20}$$

The SU_3 parameter α, denoting the fraction of d-type coupling, can be determined from the SU_3 analysis of semileptonic weak decays of hyperons, if we assume the generalized Goldberger-Treiman relations [Sec. V, Eq. (53)].

The strong coupling of the pseudoscalar-meson octet with the nucleon octet and the $\frac{3}{2}^+$ decuplet is taken to be

$$\mathcal{H}_I = \frac{\sqrt{2}G_D}{M_\pi}(\bar{B}_j^l\partial^\mu P_j^m\epsilon_{klm})(\mathrm{T}^{ijk})_\mu + \mathrm{H.c.} \tag{21}$$

T^{ijk} $(i,j,k,l=1,2,3)$ is a completely symmetric SU_3 tensor,[22] which represents the decuplet particle fields, except for normalization factors. The $\bar{B}_i^l$ are, for example, the elements of the (3×3) SU_3 matrix $\bar{B}$ of octet antibaryon fields. Using the experimental value[23]

$$\Gamma(\Delta^{++} \to p\pi^+) \simeq 120 \text{ MeV}, \tag{22}$$

we get

$$|G_D/m_\pi| \simeq 11 . \tag{23}$$

The effective Hamiltonian for the $Y_0^*(1405)\Sigma\pi$ interaction is

$$\mathcal{H}_I = G_{\bar{Y}_0^*\Sigma\pi} \bar{Y}_0^* \vec{\Sigma} \cdot \vec{\pi} + \text{H.c.} \tag{24}$$

Using the experimental value[23]

$$\Gamma(Y_0^* \to \Sigma\pi) \simeq 40 \text{ MeV}, \tag{25}$$

we obtain

$$|G_{\bar{Y}_0^*\Sigma\pi}| \simeq 0.85 . \tag{26}$$

Under octet dominance and SU_3 symmetry[13] the coupling of the P.C. spurion to the nucleon octet is defined in terms of two parameters D and F:

$$(2\pi)^3 \left(\frac{p_0 p_0'}{M_\nu M_\alpha}\right)^{1/2} \langle N_\nu(p')|\mathcal{H}_W^{\text{P.C.}}(0)|N_\alpha(p)\rangle = \bar{u}_\nu(p')[D d_{\nu 6 \alpha} + iF f_{\nu 6 \alpha}] u_\alpha(p), \tag{27}$$

where $d_{\nu 6 \alpha}$ and $f_{\nu 6 \alpha}$ are Gell-Mann's[11] SU_3 coefficients. Similarly, the two-body coupling of the P.C. spurion to the nucleon octet and the $\frac{3}{2}^+$ decuplet baryons is given in terms of a parameter g_2 by the effective Hamiltonian

$$\mathcal{H}_W^{\text{P.C.}} = i g_2 [\partial^\mu \bar{B}_i^l \epsilon_{kl2} \gamma_5 (T^{i3k})_\mu + \partial^\mu \bar{B}_i^l \epsilon_{kl3} \gamma_5 (T^{i2k})_\mu] + \text{H.c.} \tag{28}$$

The coupling of the P.C. spurion to the $Y_0^*(1405)$ and neutron is defined in terms of a parameter $b_{nY_0^*}$:

$$\langle n(p')|\mathcal{H}_W^{\text{P.C.}}(0)|Y_0^*(p)\rangle = \frac{i}{(2\pi)^3}\left(\frac{M_{Y_0^*} M_n}{p_0 p_0'}\right)^{1/2} \bar{u}_n(p') b_{nY_0^*} \gamma_5 u_{Y_0^*}(p) . \tag{29}$$

Since Y_0^* is an SU_3 singlet, it is clear that only the octet part of $\mathcal{H}_W$ will contribute to the above matrix element, whatever may be its primary nature.

By T invariance, the coupling constants G, G_D/m_π, $G_{\bar{Y}_0^*\Sigma\pi}$, D, F, g_2, and f_K are real.

Using the above assumptions, the final expressions for S- and P-wave amplitudes are given in terms of the unknown parameters D, F, g_2, and $b_{nY_0^*}$. As typical expressions, we give the decay amplitudes of $\Sigma^- \to n\pi^-$:

$$\begin{aligned} S(\Sigma^- \to n\pi^-) = &-\frac{1}{2F_\pi}(D-F) - \tfrac{1}{6} f_K \left(\frac{G_D}{m_\pi}\right)^2 (M_\Sigma - M_N)\frac{(M_\Delta + M_N)(M_\Delta + M_\Sigma)}{M_\Delta^{\,2}} + G^2 f_K (1-\alpha)(2\alpha - 1)\frac{(M_\Sigma - M_N)}{4M_N^{\,2}} \\ &+ \tfrac{1}{3} G^2 f_K \alpha (3 - 2\alpha)\frac{(M_\Sigma - M_N)}{4M_N^{\,2}} - \tfrac{1}{36} f_K \left(\frac{G_D}{m_\pi}\right)^2 (M_\Sigma - M_N)\frac{(M_{Y_1^*} + M_N)(M_{Y_1^*} + M_\Sigma)}{M_{Y_1^*}^{\,2}} \\ &- \frac{1}{\sqrt{6}} f_K \frac{(G_{\bar{Y}_0^*\Sigma\pi})^2}{(M_{Y_0^*} - M_\Sigma)} \frac{(M_\Sigma - M_N)}{(M_{Y_0^*} - M_N)} , \end{aligned} \tag{30}$$

$$\begin{aligned} P(\Sigma^- \to n\pi^-) = &-\frac{1}{\sqrt{2}}\frac{f_K}{F_\pi} G(2\alpha - 1)\frac{(M_\Sigma + M_N)}{2M_N} - \frac{\sqrt{2}}{6} g_2 \left(\frac{G_D}{m_\pi}\right)\frac{(M_\Sigma + M_N)(M_\Delta + M_N)}{M_\Delta^{\,2}}(M_\Delta - M_\Sigma) \\ &+ \frac{1}{\sqrt{2}} G (1-\alpha)(D-F)\frac{(M_\Sigma + M_N)}{2M_N(M_\Sigma - M_N)} + \frac{1}{\sqrt{18}} G\alpha(D+3F)\frac{(M_\Sigma + M_N)}{2M_N(M_\Lambda - M_N)} \\ &- \frac{1}{6\sqrt{18}} g_2 \left(\frac{G_D}{m_\pi}\right)\frac{(M_{Y_1^*} + M_\Sigma)(M_\Sigma + M_N)}{M_{Y_1^*}^{\,2}}(M_{Y_1^*} - M_N) + G_{\bar{Y}_0^*\Sigma\pi} b_{nY_0^*} \frac{(M_\Sigma + M_N)}{(M_{Y_0^*} + M_N)(M_{Y_0^*} - M_\Sigma)} . \end{aligned} \tag{31}$$

In Eqs. (30) and (31) the first term on the right-hand side is the soft-pion limit or the subtraction constant, while the rest of the expression is the dispersion-integral correction, coming from the various intermediate states. The dispersion-integral correction for the S-wave amplitudes as well as the soft-pion limit of the P-wave amplitudes are proportional to f_K.

IV. ESTIMATE OF f_K FROM THE $K_1^0 \to \pi^+\pi^-$ DECAY RATE

Consider the decay

$$K_1^0(p_K) \to \pi^+(p_+) + \pi^-(p_-) .$$

The matrix element for this process can be written

$$_{\text{out}}\langle \pi^+(p_+)\pi^-(p_-)|\mathcal{H}_W^{\text{P.V.}}(0)|K_1^0(p_K)\rangle_{\text{in}} = \frac{1}{(2\pi)^{9/2}}\frac{1}{(8p_K^0 p_+^0 p_-^0)^{1/2}} M, \tag{32}$$

where M is the relativistically invariant decay amplitude. As before, we can reduce either one of the pions π^+ or π^- in the final state using Lehmann-Symanzik-Zimmermann (LSZ) techniques, and consider this matrix element when either of the pions is off the mass shell and the requirement of energy-momentum conservation is relaxed. Reducing π^+,

$$\frac{-p_+^2+m_\pi^2}{(2\pi)^{3/2}\sqrt{2p_+^0}}\int e^{ip_+\cdot x}\, d^4x \langle \pi^-(p_-)|T\{\varphi_{\pi^+}(x)\mathcal{H}_W^{\text{P.V.}}(0)\}|K_1^0(p_K)\rangle = \frac{1}{(2\pi)^{9/2}}\frac{1}{(8p_K^0 p_+^0 p_-^0)^{1/2}}\, T . \tag{33}$$

The left-hand side of Eq. (33) can be treated as the matrix element of the reaction

$$K_1^0(p_K) + S(h) \to \pi^+(p_+) + \pi^-(p_-) ,$$

where $S(h)$ is a *massless* spurion with four-momentum h. T is a function of $s, t, u,$ and p_+^2,

$$s = (p_K + h)^2 = (p_+ + p_-)^2 , \tag{34a}$$

$$t = (p_K - p_+)^2 = (p_- - h)^2 , \tag{34b}$$

$$u = (p_K - p_-)^2 = (p_+ - h)^2 . \tag{34c}$$

When $h \to 0$ and $p_+^2 \to m_\pi^2$, we get back the physical decay whose amplitude is M. That is,

$$T(s = m_K^2, t = m_\pi^2, u = m_\pi^2, p_+^2 = m_\pi^2) = M . \tag{35}$$

If we make the approximation that the change in T in going from $u = p_+^2 = 0$ to $u = p_+^2 = m_\pi^2$ is negligible (on account of the smallness of m_π^2 on the hadronic scale),

$$M \simeq T(s = m_K^2, t = m_\pi^2, u = 0, p_+^2 = 0) . \tag{36}$$

The soft-pion limit $(p_+ \to 0)$ is given by

$$\lim_{p_+\to 0} T = T(s = m_\pi^2, t = m_K^2, u = 0, p_+^2 = 0) . \tag{37}$$

Now we write a once-subtracted dispersion relation for T in the variable s, keeping $u = 0$ and $p_+^2 = 0$, and choosing the subtraction point at the soft-pion limit, i.e.,

$$\begin{aligned} M &\simeq T(s = m_K^2, t = m_\pi^2, u = 0, p_+^2 = 0) \\ &= T(s = m_\pi^2, t = m_K^2, u = 0, p_+^2 = 0) + \frac{m_K^2 - m_\pi^2}{\pi}\int_{-\infty}^{\infty} \frac{\operatorname{Im} T(s', u = 0, p_+^2 = 0)ds'}{(s' - m_\pi^2)(s' - m_K^2)} . \end{aligned} \tag{38}$$

The intermediate states in the s and t channels will contribute to the above dispersion integral. We saturate it by the intermediate states of the 1^- vector meson ρ, of the 0^+ scalar mesons $\epsilon(750\text{ MeV})$[24,25] and $\sigma(1080\text{ MeV})$[23,26] in the s channel, and those of 1^- K^* and 0^+ $\kappa(1080\text{ MeV})$[27-29] in the t channel. The matrix element of $\mathcal{H}_W$ between any of the above intermediate states and $K_1^0(p_K)$ or $\pi^-(p_-)$ is calculated by assuming that this vertex satisfies an unsubtracted dispersion relation in the relevant variable h^2, and that the dispersion integral is saturated by the lowest-mass state, which in this case is the K-meson state. The next higher state is a $(K+2\pi)$ state, whose contribution is assumed to be negligible at the point $h^2=0$ compared to that of the single K-meson state. So all the matrix elements of $\mathcal{H}_W$ between two single-particle states are proportional to the parameter f_K [defined by Eq. (3)] times a strong vertex involving the two particles and a K meson.

Using Eq. (33), the soft-pion limit $(p_+ \to 0)$ is

$$T(s = m_\pi^2, u = 0, p_+^2 = 0) = (2\pi)^3(4p_K^0 p_-^0)^{1/2}\frac{i}{F_\pi}\langle \pi^-(p_-)|[\mathcal{H}_W^{\text{P.V.}}(0), F_5^{\pi^+}(0)]|K_1^0(p_K)\rangle|_{u=0} . \tag{39}$$

The matrix element on the right-hand side of Eq. (39) is a function $\tau(u, p_-^2 = m_\pi^2)$ of $u = (p_K - p_-)^2$. We want to evaluate it at $u=0$. If π_- is reduced in this matrix element and the $p_-\to 0$ limit is taken, we get (using PCAC)

$$\tau(u=m_K^2, p_-^2=0)=(2\pi)^{3/2}\sqrt{2p_K^0}\,\frac{(-1)}{F_\pi^2}\langle 0|[[\mathcal{H}_W^{\text{P.V.}}(0),F_5^{\pi^+}(0)],F_5^{\pi^-}(0)]|K_1^0(p_K)\rangle \tag{40}$$

$$=(2\pi)^{3/2}\sqrt{2p_K^0}\,\frac{1}{F_\pi^2}\langle 0|[[\mathcal{H}_W^{\text{P.V.}}(0),F^{\pi^+}(0)],F^{\pi^-}(0)]|K_1^0(p_K)\rangle \tag{41}$$

$$\simeq -\frac{1}{2\sqrt{2}}f_K\frac{m_K^2}{F_\pi^2}\,. \tag{42}$$

Equation (41) follows from Eq. (40) because of Eqs. (1) and (2). Equation (42) follows from Eq. (41) because of Eq. (3). If we write a once-subtracted dispersion relation for $\tau(u,p_-^2=0)$ in the variable u, keeping $p_-^2=0$ and choosing the subtraction point at $u=m_K^2$ [so that the subtraction constant is given by Eq. (42)], we can find $\tau(u=0,p_-^2=0)$ by calculating the dispersion integral. $\tau(u=0,p_-^2=0)$ should be close to $\tau(u=0,p_-^2=m_\pi^2)$ required to evaluate Eq. (39). The contribution of the lowest-mass single-particle intermediate state, namely, that of a possible 0^+ κ(1080 MeV) to the dispersion integral, is found to be negligibly small, unless the value[30] of the matrix element $\langle 0|[\mathcal{H}_W^{\text{P.V.}}(0),F_5^{\pi^+}(0)]|\kappa^+(p')\rangle$ is unreasonably large. Then, neglecting this dispersion integral, we write the approximate expression for M, in the limit of $p_+^2=0$, as

$$\begin{aligned}M &\simeq T(s=m_K^2,t=m_\pi^2,u=0,p_+^2=0)\\ &\simeq -\frac{1}{2\sqrt{2}}f_Km_K^2-\frac{1}{\sqrt{2}}\frac{G_{\sigma\pi\pi}G_{\sigma K\bar{K}}(m_K^2-m_\pi^2)}{(m_\sigma^2-m_\pi^2)(m_\sigma^2-m_K^2)}f_K-\frac{1}{\sqrt{2}}\frac{G_{\epsilon\pi\pi}G_{\epsilon K\bar{K}}(m_K^2-m_\pi^2)}{(m_\epsilon^2-m_\pi^2)(m_\epsilon^2-m_K^2)}f_K\\ &\quad-\frac{1}{\sqrt{2}}\frac{(G_{K^*K\pi})^2(m_K^2-m_\pi^2)}{(m_{K^*}^2-m_\pi^2)}f_K+\frac{1}{\sqrt{2}}\frac{(G_{\kappa K\pi})^2(m_K^2-m_\pi^2)}{(m_\kappa^2-m_\pi^2)(m_\kappa^2-m_K^2)}f_K\,.\end{aligned} \tag{43}$$

Equation (43) is the approximate expression for T in the limit $p_+^2=0$ and keeping p_-^2 on the mass shell. Since pions are bosons, we should symmetrize between π_+ and π_- and so we should also calculate T in the limit $p_-^2=0$, keeping p_+^2 on the mass shell, and take the mean of these two expressions as representing the approximate value of M. It is found that these two limits are the same and thus Eq. (43) indeed represents approximately the physical value of M.

The coupling constants, $G_{\sigma\pi\pi}$, $G_{\sigma K\bar{K}}$, $G_{K^*K\pi}$, etc. in Eq. (43) are defined by the effective Hamiltonian densities,

$$\begin{aligned}\mathcal{H}_I^{\sigma\pi\pi}&=-G_{\sigma\pi\pi}\sigma(\vec{\pi}\cdot\vec{\pi})\,,\\ \mathcal{H}_I^{\epsilon\pi\pi}&=-G_{\epsilon\pi\pi}\epsilon(\vec{\pi}\cdot\vec{\pi})\,,\\ \mathcal{H}_I^{\kappa K\pi}&=-G_{\kappa K\pi}(\bar{K}\vec{\tau}\kappa)\cdot\vec{\pi}+\text{H.c.}\,,\\ \mathcal{H}_I^{K^*K\pi}&=-iG_{K^*K\pi}[\partial^\mu\vec{\pi}\bar{K}-\vec{\pi}\partial^\mu\bar{K}]\cdot\vec{\tau}K_\mu^*+\text{H.c.}\end{aligned} \tag{44}$$

In Eq. (43), the first term on the right-hand side represents approximately the subtraction constant and the other terms are the dispersion-integral corrections in going from

$$T(s=m_\pi^2,t=m_K^2,u=0,p_+^2=0)$$

to

$$T(s=m_K^2,t=m_\pi^2,u=0,p_+^2=0)\,.$$

In order to evaluate them, we should know the various coupling constants $G_{\epsilon\pi\pi}$, $G_{K^*K\pi}$, etc. $G_{K^*K\pi}$ is known from the known width,[23] $\Gamma(K^*\to K\pi)\simeq 50$ MeV. The scalar mesons are not yet completely well established, although their existence now seems to be fairly certain. The existence of an $I=0$, $s=0$, scalar meson ϵ[23-25] around 750 MeV, and $I=1$, $s=0$, scalar meson π_s[23,26] around 1000 MeV, and an $I=0$, $s=0$, scalar meson σ[23,26] around 1000 MeV seems to be most certain. We also assume the existence of an $I=\frac{1}{2}$, strangeness-carrying scalar meson κ around 1080 MeV. Its existence is supported by a phase-shift analysis[27] of $K\pi$ S-wave scattering and a study[28,29] of Kl_3 decays using current algebra and once-subtracted dispersion relations. We take $m_\epsilon\simeq 750$ MeV,[24,25] $m_\sigma\simeq 1070$ MeV,[23,26] $m_{\pi_s}\simeq 1016$ MeV,[23,26] and $m_\kappa\simeq 1000$ MeV.[27,28] We assume $\Gamma(\epsilon)\simeq 400$ MeV,[24,25] $\Gamma(\sigma)\simeq 80$ MeV,[23,26] and $\Gamma(\kappa\to K\pi)\simeq 200$ MeV.[28,27] In order to satisfy the Gell-Mann–Okubo mass formula with mixing, we should take $\theta\simeq 60°$ and then, fitting the above widths with an SU_3 analysis, we find[31]

$$\begin{aligned}G_{\epsilon\pi\pi}&\simeq +2.3\,,\\ G_{\epsilon K\bar{K}}&\simeq +1.2\,,\\ G_{\sigma\pi\pi}&\simeq +0.3\,,\\ G_{\sigma K\bar{K}}&\simeq -1.5\,,\\ G_{\kappa K\pi}&\simeq +1.76\,.\end{aligned} \tag{45}$$

With these values for the coupling constants of scalar mesons, we find that the dispersion-integral correction is of the same sign and has approximately the same magnitude as the soft-pion value. We find

$$M \simeq 11.5 f_K/\sqrt{2}\,. \tag{46}$$

Now, fitting with the decay rate[23]

$$\Gamma(K_1^0 \to \pi^+\pi^-) \simeq 1.85\times 10^{10}\ \mathrm{sec}^{-1}\,, \tag{47}$$

we get[15]

$$f_K \simeq \pm 0.5\times 10^{-7}\ \mathrm{BeV}\,. \tag{48}$$

If we had neglected the dispersion-integral correction, we would have obtained

$$f_K \simeq \pm 1\times 10^{-7}\ \mathrm{BeV}\,. \tag{49}$$

The bulk of the contribution to the dispersion integral, because of the comparatively lower masses and larger widths involved, comes from ϵ and K^* mesons. In Eq. (43) the contribution from ϵ is about $-3.7 f_K/\sqrt{2}$ (with the ϵ parameters we have used) and the K^* contribution is about $-3 f_K/\sqrt{2}$, while the combined contribution of σ and κ mesons is only about $+1 f_K/\sqrt{2}$. Because of this, the precise values of σ- and κ-meson parameters have only little effect on the value of f_K. The crucial thing, besides the K^* meson, which affects the value of f_K, is the existence of a low-mass $I=0$ scalar meson with a large width. It is also interesting to observe that the contributions of ϵ and K^* are of the same sign.

It should also be mentioned that once-subtracted dispersion relations were first used by Okubo, Marshak,[32] and Mathur in connection with $K_1 \to 2\pi$ decays. However, there are important differences[33] in the calculation of the dispersion integral, and in particular they have not included the contributions of scalar mesons. They concluded[32] that the dispersion-integral correction in going from $p_\pi = 0$ to $p_\pi^2 = 0$ is only of the order of 20% of the soft-pion limit, whereas we find it is of the order of 100%.

V. NUMERICAL ESTIMATE OF HYPERON DECAY AMPLITUDES

For the numerical estimate of the hyperon decay amplitudes [given in Eqs. (30) and (31) and their analogs], we use $\alpha \simeq 0.6$ given by the SU_3 Cabibbo analysis[19] of hyperon semileptonic weak decays. For the parity-violating spurion parameter f_K, we take the positive value[34] given by Eq. (48). The relative sign of G and F_π needed for the numerical estimate is given by the Goldberger-Treiman relation[35] connecting G, F_π, and g_A/g_V of the neutron β decay. Then there are eight independent amplitudes (only eight because the $|\Delta I| = \frac{1}{2}$ property of $\mathcal{H}_W$ is already built into the model) in terms of four unknown parameters D, F, g_2, and $b_{nY_0^*}$. We are able to get a good fitting. A possible good fitting is given in Table I. The agreement with experiment[36] is within (20–30)%.

Several remarks are now in order about Table I:

(a) First, it is to be emphasized that with the new value of f_K, we will get violent disagreement with experimental values if we retain the contribution of only the nucleon octet to the dispersion integral. Apart from the fact that in this case we can not attain a good fitting of P-wave amplitudes, we can not satisfy the Lee-Sugawara triangular relation for S waves and the experimentally satisfied relation $S(\Sigma^+ \to n\pi^+) \simeq 0$. In this case, $S(\Sigma^+ \to n\pi^+)\times 10^6 = 0.16$, comparable to other S-wave amplitudes.

(b) With the values of D and F obtained from the fitting of S-wave amplitudes, we have to assume $g_2 \simeq 2\times 10^{-6}\ \mathrm{BeV}^{-1}$ to obtain a good fitting of $P(\Lambda \to P\pi^-)$ and $P(\Xi^- \to \Lambda\pi^-)$. With this value of g_2, it is seen that the coupling of the P.C. spurion to the decuplet-octet baryon system is about five times stronger than that of the P.V. spurion. Now, if we want to fit the amplitude $P(\Sigma^+ \to n\pi^+)$, which is very large, and the amplitude $P(\Sigma^- \to n\pi^-)$, which is nearly zero, we have to assume a rather large value for $b_{nY_0^*}$ ($b_{nY_0^*} \simeq 0.7\times 10^{-6}$ BeV), the coupling constant of the P.C. spurion to the $Y_0^* n$ system. It is interesting to note that the Y_0^*, which contributes only to $\Sigma^+ \to n\pi^+$ and $\Sigma^- \to n\pi^-$, is responsible for the vanishing of $P(\Sigma^- \to n\pi^-)$ and the large value of $P(\Sigma^+ \to n\pi^+)$. The fitted value of $b_{nY_0^*}$ suggests that the coupling of the P.C. spurion to the $Y_0^* n$ system is about 20 times stronger than that of the P.V. spurion (assumed to be given by the K_1 tadpole mechanism). Probably, we should interpret the contribution from Y_0^* as an effective contribution, coming from all the SU_3 singlet Y_0^*'s including the higher-mass Y_0^*'s [$Y_0^*(1670)$ etc.].

(c) With the present value of f_K, the dispersion-integral corrections to both S- and P-wave amplitudes are considerable, as seen from Table I. The individual contributions to S-wave amplitudes (except for the soft-pion limit) do not satisfy the Lee-Sugawara (LS) relation even approximately, but the total dispersion-integral correction satisfies the LS relation approximately. Since the current-commutator terms satisfy the LS relation exactly, the total S-wave amplitudes satisfy the relation approximately, within 20%. For the P-wave amplitudes, none of the individual contributions satisfies the LS relation exactly, but the total contribution, because of the special values of D, F, and g_2 we have chosen, satisfies the relation approximately within about 30%.

(d) Finally, it should be noted that if we had assumed an SU_3-symmetric pseudoscalar coupling for the interaction of the nucleon octet with the pseudoscalar-meson octet, we would not have been able to get as good a fitting as we had obtained in

TABLE I. Fitting of the eight independent S- and P-wave amplitudes. As discussed in the text, we have $f_K \simeq 0.5\times10^{-7}$ (BeV), $\alpha \simeq 0.6$. The values of the parameters which have entered into the fitting in this table are: $D \simeq -0.1\times10^{-6}$ (BeV), $F \simeq +0.24\times10^{-6}$ (BeV), $g_2 \simeq 2\times10^{-6}$ (BeV^{-1}), and $b_{nY_0^*} \simeq 0.68\times10^{-6}$ (BeV).

(a) Fitting of S-wave amplitudes

S-wave amplitude	Soft-pion limit or current-commutator term	Contribution of the nucleon octet in the s channel $(B_8)_s$	Contribution of the nucleon octet in the u channel $(B_8)_u$	Contribution of the $\frac{3}{2}^+$ decuplet in the s channel $(B_{10})_s$	Contribution of the $\frac{3}{2}^+$ decuplet in the u channel $(B_{10})_u$	Contribution of the Y_0^* in the u channel $(Y_0^*)_u$	Total (prediction)	Experimental value[a]
$S(\Lambda\to p\pi^-)\times10^6$	0.95	−0.34	−0.05	0	−0.22	0	0.34	0.335 ± 0.004
$S(\Xi^-\to\Lambda\pi^-)\times10^6$	−1.25	+0.26	−0.026	+0.3	+0.27	0	−0.35	−0.44 ± 0.006
$S(\Sigma^+\to n\pi^+)\times10^6$	0	+0.132	+0.027	−0.3	+0.136	−0.03	−0.04	0.001 ± 0.006
$S(\Sigma^-\to n\pi^-)\times10^6$	1.27	0	+0.29	−0.9	−0.136	−0.03	+0.49	0.405 ± 0.003
$\dfrac{S(\Lambda\to p\pi^-)+2S(\Xi^-\to\Lambda\pi^-)}{\sqrt{3}S(\Sigma^+\to p\pi^0)}$	1						1.20	~1

(b) Fitting of P-wave amplitudes

P-wave amplitude	Current-commutator term	Contribution of the nucleon octet in the s channel $(B_8)_s$	Contribution of the nucleon octet in the u channel $(B_8)_u$	Contribution of the $\frac{3}{2}^+$ decuplet in the s channel $(B_{10})_s$	Contribution of the $\frac{3}{2}^+$ decuplet in the u channel $(B_{10})_u$	Contribution of the Y_0^* in the u channel $(Y_0^*)_u$	Total (prediction)	Experimental value[a]
$P(\Lambda\to p\pi^-)\times10^6$	−2.9	14.8	−6.8	0	−2.6	0	2.5	2.3 ± 0.1
$P(\Xi^-\to\Lambda\pi^-)\times10^6$	+1.15	−6.71	4.06	0.445	2.55	0	1.5	1.47 ± 0.12
$P(\Sigma^+\to n\pi^+)\times10^6$	0	14.4	−13.96	−0.228	1.12	2.52	3.7	4.2 ± 0.08
$P(\Sigma^-\to n\pi^-)\times10^6$	0.825	0	−1.08	−0.68	−1.12	2.52	0.38	−0.034 ± 0.085
$\dfrac{P(\Lambda\to p\pi^-)+2P(\Xi^-\to\Lambda\pi^-)}{\sqrt{3}P(\Sigma^+\to p\pi^0)}$							1.32	~1

[a] See Ref. 36.

the pseudovector case. Apart from the fact that the over-all fitting is worse, we can not satisfy the Lee-Sugawara relation for S waves. In this case we find

$$\frac{S(\Lambda\to p\pi^-)+2S(\Xi^-\to\Lambda\pi^-)}{\sqrt{3}\,S(\Sigma^+\to p\pi^0)}\simeq 2\,. \tag{50}$$

The discrepancy is because, in this case, the total dispersion-integral corrections to the S-wave amplitudes do not satisfy the LS relation even approximately. Also, we find $S(\Sigma^+\to n\pi^+)$ to be larger than the value found in the pseudovector-coupling case.

VI. CONCLUDING REMARKS

(a) In the above analysis we assumed SU_3 symmetry for those couplings which do not vanish in the symmetry limit. In this connection it is to be mentioned that there may be some justification for assuming SU_3-symmetric pseudovector couplings for the $B^8B^8P^8$ interaction.

From an analysis[19] of the semileptonic hyperon decays, and also by certain broken-SU_3-symmetry calculations,[19] it has been found that the matrix elements of the axial-vector current between two baryons of the nucleon octet satisfy the SU_3-symmetric relations, to a good approximation. Now, if the axial-vector coupling constant $g_A^{\beta\alpha i}$ is defined by

$$\lim_{(p'-p)\to 0}\langle\beta(p')|A_\mu^i(0)|\alpha(p)\rangle(2\pi)^3\left(\frac{p_0p_0'}{M_\alpha M_\beta}\right)^{1/2}$$
$$=g_A^{\beta\alpha i}(0)\bar{u}_\beta(p')\gamma_\mu\gamma_5u_\alpha(p) \tag{51}$$

and the strong-coupling constant $G^{\beta\alpha i}$ by

$$\mathcal{H}_I^{\beta\alpha i}=+iG_{\beta\alpha i}\bar{N}_\beta(x)\gamma_5\gamma_\mu N_\alpha(x)\partial^\mu\varphi_i(x)\,, \tag{52}$$

we get, using the generalized PCAC, the generalized Goldberger-Treiman relation

$$G_{\beta\alpha i}=\frac{\sqrt{2}}{F_\pi}g_A^{\beta\alpha i}\,. \tag{53}$$

Equation (53) suggests that if SU_3 symmetry is good for relating $g_A^{\beta\alpha i}$, it is also good for relating $G_{\beta\alpha i}$ with the same d/f ratio. With pseudoscalar coupling, Eq. (53) will be changed into

$$G_{\beta\alpha i}=\frac{\sqrt{2}}{F_\pi}(M_\alpha+M_\beta)g_A^{\beta\alpha i}\,. \tag{54}$$

If we use physical masses for the baryons there is, of course, considerable difference between assuming SU_3 symmetry for $g_A^{\beta\alpha i}$ and $G_{\beta\alpha i}$. In deriving Eq. (53), besides using the well-tested PCAC for pions, we assumed PCAC for kaons. Even though, in general, the accuracy of PCAC for kaons is not known, there are indications[37] that it may be a good approximation (in this case), correct to within about 30%.

The coupling constants for the interaction among the nucleon octet, the $\frac{3}{2}^+$ decuplet baryons, and the pions can be determined from the known decay rates of the decuplet resonances. It is found that SU_3 symmetry is a good approximation[38] here (within 15%) except for the coupling constant $G_{\Xi^*\Xi\pi}$, for which there may be a deviation of more than 50% from the SU_3-symmetric value. Regarding the above coupling constants involving kaons, we cannot say anything from the decay rates since none of these resonances decays into a final state containing the kaon because of energy conservation. We simply hope that SU_3 symmetry is a good approximation there. It is encouraging to note that if we make a calculation using the experimental value of $G_{\Xi^*\Xi\pi}$ and the SU_3-symmetric values for all other coupling constants (this affects only the $\Xi^-\to\Lambda\pi^-$ decay), we can get even a slightly better over-all fit of the decay amplitudes, with approximately the same values for the parameters.

(b) It is to be noted that the octet dominance of the matrix element of $\mathcal{H}_W^{\text{P.C.}}$ between octet and decuplet baryon states is necessary, especially to explain the $|\Delta I|=\frac{1}{2}$ rules in P-wave Ξ and Λ decays, since for these amplitudes the contribution from the decuplet intermediate states to the dispersion integral is very considerable.

A comment also has to be made regarding the $|\Delta I|=\frac{1}{2}$ rules in K decays. Using once-subtracted dispersion relations and current algebra for $K\to 2\pi$ decays, we find that the soft-pion limit and the dispersion-integral correction have the $|\Delta I|=\frac{1}{2}$ property, even if the primary $\mathcal{H}_W$ has a $|\Delta I|=\frac{3}{2}$ part in it. The $|\Delta I|=\frac{1}{2}$ property of the dispersion-integral correction comes about because we assume the K_1 tadpole mechanism for the matrix elements of $\mathcal{H}_W^{\text{P.V.}}$. Since by current algebra and PCAC, $K\to 3\pi$ decays can be related to $K\to 2\pi$ decays, and since in this case the soft-pion approximation is good,[39] the above model of $K\to 2\pi$ decays successfully explains the $|\Delta I|=\frac{1}{2}$ rules in all nonleptonic K decays.

(c) It is to be mentioned that there is an ambiguity of a sign in the value of f_K obtained from the $K_1\to 2\pi$ decay rate, as seen from Eq. (48). We choose the positive value because with the negative value the fitting of the hyperon decay amplitudes is much worse.

(d) *Concept of "universal P.C. spurion."* With the fitted values of D and F we find $D/F\simeq -0.42$. The parameter D^M/F^M, which is the ratio of d-type to f-type coupling of the medium-strong SU_3-breaking spurion to the octet baryons, is $D^M/F^M=-0.3$.[40] The near equality of the two ratios may tempt one to assign the parity-conserving weak Hamiltonian to the same octet as the medium-strong SU_3-breaking Hamiltonian, the so-called concept[20] of "universal P.C. spurion." But this is not possible in our scheme, as can be seen by comparing D/D^M with D'/D'^M. D' is the strength of the coupling of the P.C. spurion to the $K\pi$ system and D'^M is the strength of the coupling of the medium-strong SU_3-breaking spurion to the pseudoscalar-meson octet. If the concept of universal P.C. spurion is correct, D/D^M should be equal to D'/D'^M. But we find, from our analysis of $K\to 2\pi$ decay and hyperon decays, that $D/D^M \simeq -0.3\times10^{-5}$, whereas $D'/D'^M \simeq +0.6\times10^{-6}$. This directly invalidates the concept of universal P.C. spurion, which is a nice thing because, if the above concept were correct, there would have been no[41] parity-conserving weak nonleptonic decays. It is to be mentioned that some other authors[20] have concluded that $D/D^M \simeq D'/D'^M$ because of an incomplete analysis of hyperon and $K\to 2\pi$ decays, which, of course, is a disastrous result.

(e) We assumed the current-current theory only to the extent that Eqs. (1) and (2) held and that $\mathcal{H}_W^{\text{P.C.}}$ transformed like λ_6 and not λ_7. It should be emphasized that if we assumed that $\mathcal{H}_W^{\text{P.C.}}$ transformed like λ_7, we would not have been able to get a good fitting of the S-wave amplitudes. In particular, in that case, the Lee-Sugawara relation for S waves can be satisfied only with $D\simeq 0$ and then $S(\Sigma^-\to n\pi^-)$ will be too small (nearly zero). This is an argument in favor of the current-current theory. On the other hand, the current-current theory predicts, in the SU_3-symmetry limit, vanishing matrix elements of $\mathcal{H}_W^{\text{P.V.}}$ between K_1 and 2π and between two nucleon-octet baryons. But in reality the $K_1\to 2\pi$ amplitude is not zero, and when we connect these two matrix elements through dispersion relations and current algebra, we predict a rather large value for $\langle B_f^8|\mathcal{H}_W^{\text{P.V.}}|B_i^8\rangle$ comparable to $\langle B_f^8|\mathcal{H}_W^{\text{P.C.}}|B_i^8\rangle$, the latter being *not* zero in the SU_3 limit. So, in the current-current theory, we should account[42] for an unusually large amount of symmetry breaking in the matrix elements of $\mathcal{H}_W^{\text{P.V.}}$.

We have demonstrated here that the consistent use of once-subtracted dispersion relations and current algebra to the nonleptonic weak decays of hyperons and $K\to 2\pi$ decays gives a satisfactory explanation of all the nonleptonic weak decays (except the Ω^- decays). In particular, the subtracted dispersion integrals in the case of hyperon decays seem to be saturated by the nucleon octet, the $\frac{3}{2}^+$ decuplet, and Y_0^*'s only. The $\frac{3}{2}^+$-decuplet contribution to the dispersion integral is especially necessary to achieve the vanishing of $S(\Sigma^+\to n\pi^+)$ and the Lee-Sugawara relation for S waves. Although the success of the model does not prove the validity of the model (especially in view of the fact that there are four parameters in the theory), it encourages hope in the correctness of the assumptions we have made.

There are also certain difficult questions to be answered. The most important are why H_1 and H_2 obey dispersion relations with only one subtraction and if so, why the subtracted dispersion integral is saturated by only the intermediate states we have taken into account, especially in view of the fact that there are so many baryon resonances. Okubo[18] has shown that H_1 and H_2 will, in general, require only one subtraction if we assume Regge asymptotics for the hyperon + spurion → hyperon + pion reaction amplitudes. It is interesting to speculate whether the Regge picture can be pushed further and the finite-energy sum rules can be used to calculate the high-energy part of the dispersion integral, and to show that indeed the high-energy contribution is small. Such investigations are currently in progress and we hope to report on them in the near future.

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APPENDIX

In this Appendix we prove two things:

(1) The contribution of the spin-$\frac{1}{2}$ single-baryon intermediate state to the once-subtracted dispersion integrals (in the s variable) of the function $H_1(s,t=0)$ or $H_2(s,t=0)$ [defined by Eq. (8)], with the subtraction point at the soft-pion limit, equals the difference in values of the corresponding Feynman pole diagrams with physical pion four-momentum and zero pion four-momentum.

(2) The statement in (1) is not, in general, true for spin-$\frac{3}{2}$ and higher-spin single-baryon intermediate states.

Consider the once-subtracted dispersion integrals for H_1 and H_2 defined by Eqs. (16) and (17). If $R_i(s)$ is the residue function corresponding to the intermediate state of mass M_I, the contributions to the once-subtracted dispersion integrals (O.D.I.) can be written

$$\Delta H_i(\text{O.D.I.}) = \frac{R_i(s=M_I^2)}{M_I^2-M_\alpha^2} - \frac{R_i(s=M_I^2)}{M_I^2-M_\beta^2}, \qquad \text{(A1)}$$

where $i=1,2$. The difference in values of the corresponding Feynman pole diagrams (F.P.D.) for H_i $(i=1,2)$ with physical pion four-momentum and zero pion four-momentum is

$$\Delta H_i(\text{F.P.D.}) = \frac{R_i(s=M_\alpha^2)}{M_I^2-M_\alpha^2} - \frac{R_i(s=M_\beta^2)}{M_I^2-M_\beta^2}. \qquad \text{(A2)}$$

It is clear that Eqs. (A1) and (A2) give the same values if $R_i(s)$ is a constant or of the form $R_i(s) = A_i + B_i s$, where A_i and B_i are independent of s. If it is a quadratic or higher-degree function of s, the two values differ.

Next we show that for $\frac{1}{2}^{\pm}$ single-baryon intermediate states $R_i(s)$ is a linear function of s, and for $\frac{3}{2}^{\pm}$ single-baryon intermediate states it is a quadratic function of s. For higher-spin intermediate states, $R_i(s)$ will be of higher degree in s.

Consider Eq. (8). Using the Dirac equation, we get

$$H_1 = F_1 + \frac{(s-u)G_1}{2(M_\alpha + M_\beta)}, \tag{A3}$$

$$H_2 = F_2 - \frac{(s-u)G_2}{2(M_\alpha - M_\beta)}. \tag{A4}$$

The residue functions for the poles of F_i and G_i $(i=1,2)$ coming from spin-$\frac{1}{2}$ single-baryon intermediate states are constants. For spin-$\frac{3}{2}$ single-baryon intermediate states the residue functions of F_i and G_i will be quadratic functions in s, if we take the propagation function for spin-$\frac{3}{2}$ particles as

$$S_F^{3/2}(p_I) = \frac{\not{p}_I + M_I}{M_I^2 - s}\left[g_{\mu\nu} - \tfrac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_{I\mu}p_{I\nu}}{3M_I^2} + \frac{p_{I\mu}\gamma_\nu - p_{I\nu}\gamma_\mu}{3M_I}\right]. \tag{A5}$$

This can be easily checked by an explicit calculation.

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*Part of the work was done while the author was at the University of Maryland.

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[13]In the current-current theory, $\mathcal{H}_W^{\text{P.C.}}$ and $\mathcal{H}_W^{\text{P.V.}}$ should transform like λ_6 under SU_3. See M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964).

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[15]It seems to be clear now that there is something wrong with the calculation of the $KK\pi\pi$ vertex in Refs. 6 and 14. The $KK\pi\pi$ vertex involved in the calculation of f_K in the K_1 tadpole model of these references is an off-mass-shell vertex where the four-momentum of one of the K mesons (the intermediate-state K meson in the tadpole diagram) is set equal to zero using $\mathcal{H}_W^{\text{P.V.}}$ as an interpolating field. It is implicitly assumed in them that this vertex, in the above case, smoothly extrapolates with respect to p_K^2 in the range $0 \le p_K^2 \le m_K^2$. But by kaon PCAC it has the same property also in the case when $\partial^\mu A_\mu^K$ is taken as an interpolating field for the K meson. These two assumptions together lead to the result that the $KK\pi\pi$ vertex to be used in the K_1 tadpole calculation of Refs. 6 and 14 must approximately vanish by the Adler consistency condition, arising from kaon PCAC. This should lead to an unreasonably large value for f_K, from the given rate for $K_1 \to 2\pi$ decay. But these references, by a pole-model calculation, produce a somewhat large value for the $KK\pi\pi$ vertex at the relevant point, and thus a fairly small (compared to ours) value for f_K. So, if kaon PCAC has at least approximate validity, their calculation should fail. Furthermore, since the K_1 tadpole diagram approximately vanishes according to the above considerations, we should also take into account other diagrams besides this one in the calculation of f_K from $K_1 \to 2\pi$ decay. In our method (see Sec. IV) the subtraction constant in the dispersion relation for T contains contributions from other diagrams, while the dispersion integral takes into account the bad extrapolation property of the K_1 tadpole diagram with respect to the four-momentum of one of the pions.

[16]There do exist broken-SU_3 sum rules for coupling constants involving pions, based on asymptotic SU_3 symmetry (symmetry at infinite momentum), current algebra, and PCAC. See, S. Matsuda and S. Oneda, Phys. Rev. 185, 1887 (1969); G. Fourez, *ibid.* 178, 2454 (1969). These sum rules are consistent with using pseudovector SU_3-symmetric couplings for the interaction of pseudoscalar mesons with the nucleon octet, and almost consistent with using pure SU_3-symmetric couplings among pseudoscalar mesons, the nucleon octet, and the $\frac{3}{2}^+$ decuplet, as we have assumed in this paper.

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[30]We can make a rough estimate of the matrix element by saturating the commutator by the lowest-mass intermediate states, namely K and π. Then this matrix element can also be related to f_K, and if we make a calculation in the frame of the infinite κ momentum ($|\vec{p}'|=\infty$), we find

$$\sqrt{2p_0'}\,(2\pi)^{3/2}\langle 0|[\mathcal{H}_W^{\mathrm{P.V.}}(0), F_5^{\pi^+}(0)]|\kappa^+(p')\rangle = \frac{1}{\sqrt{2}}\frac{f_K m_K^2 F_\pi G_{\kappa K\pi}}{m_\kappa^2 - m_K^2}.$$

With this value of the matrix element, the dispersion-integral correction in only about $\frac{1}{20}$ of the soft-pion limit. If the above estimate is correct at least by order of magnitude, we can completely neglect the κ contribution to the dispersion integral.

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[33]The authors of Ref. 32 made a mistake in calculating the matrix element $\langle\pi(q)|\mathcal{H}_W^{\mathrm{P.V.}}|K^*(p)\rangle$. They assumed that the form factor in this matrix element is zero at the soft-pion limit, whereas actually its value cannot be determined by current algebra and PCAC alone, since the matrix element is proportional to the pion four-momentum, q. Note that

$$\langle\pi(q)|\mathcal{H}_W^{\mathrm{P.V.}}|K^*(p)\rangle = g_{\pi K^*}(\epsilon_{K^*}\cdot q).$$

[34]See Sec. VI of this article.

[35]M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).

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[42]An alternative possibility is to renounce the assumption that the two-body vertices of $\mathcal{H}_W^{\mathrm{P.V.}}$ obey an *unsubtracted* dispersion relation. Without this assumption, we will not be able to relate the $K_1\to 2\pi$ amplitude with the matrix element of $\mathcal{H}_W^{\mathrm{P.V.}}$ between two nucleon-octet baryons.