yield the identical result. However, this value would lead to large seagull contributions to this and other decays and so it is eliminated.

[27]The constants h, B_1, and B_2 of Eq. (70) are complicated functions of the various components of g_{ab} and F_{ab}. In deriving the results given in Eqs. (72)–(77), $\eta - \eta'$ has been neglected due to the smallness of the mixing angle ($\approx 10°$). In this approximation $h = \lambda/\sqrt{3}F_\eta$, $B_1 = -2h(g_\rho{}^{-1} + \frac{1}{4}k)$, and $B_2 = 4(\lambda/g_\rho F_\pi\sqrt{3})[1 - \frac{1}{2}(F_\eta/F_\pi)^2 - (F_{\bar{9}\bar{8}}{}^2/W) \times (1 + F_{\eta'}/F_\eta)]$. The value $kg_\rho = -4$ has also been assumed. W is the $(\bar{9}, \bar{9})$ component of W_{ab}.

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Hard-Meson Current-Algebra Calculation of the γ-3π Amplitude

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The $\gamma + \pi \rightarrow \pi + \pi$ interaction is considered within the context of hard-meson current algebra and a model of PCAC (partially conserved axial-vector current) breakdown previously proposed. The form factor for this process is calculated and the γ-3π coupling constant evaluated. The results are in agreement with experimental data.

In the past several years a number of authors[1–4] have used various techniques to evaluate the coupling constant for the process $\gamma + \pi \rightarrow \pi + \pi$. This interaction is of interest since it is known to contribute to the process $\pi + N \rightarrow \pi + N + \gamma$. Arnowitt, Friedman, and Nath[5] have proposed a hard-meson model of the breakdown[6] of partial conservation of axial-vector current (PCAC) which allows the two-body photon decays of π^0, η, and vector mesons. Very recently[7] this model has been extended to higher order and successfully applied to the four-point processes $\omega \rightarrow 3\pi$, $\phi \rightarrow 3\pi$, and $\eta \rightarrow 2\pi + \gamma$. These same interactions will also contribute to $\gamma + \pi \rightarrow \pi + \pi$.

Assuming that the isoscalar part of the electromagnetic current is dominated by the ω and ϕ mesons, the amplitude for $\gamma + \pi \rightarrow \pi + \pi$ is related to the following matrix element:

$$\langle \pi k_1 a : \pi k_2 b | J^\beta_{I=0}(0) | \pi k_3 c\rangle = (e/\sqrt{3})[g_{88}\langle \pi k_1 a : \pi k_2 b | \omega^\beta(0) | \pi k_3 c\rangle + g_{89}\langle \pi k_1 a : \pi k_2 b | \phi^\beta(0) | \pi k_3 c\rangle]. \tag{1}$$

k_1, k_2, and k_3 and a, b, and c are the momenta and isotopic spins of the pions. $J^\beta_{I=0}$ is the isoscalar part of the electromagnetic current. ω^β and ϕ^β are the ω and ϕ fields, respectively. g_{88} and g_{89} are the field-current coupling strengths of the ω and ϕ mesons to the eighth vector current. Hence the matrix element for $\gamma + \pi \rightarrow \pi + \pi$ is related to the matrix elements for $\omega \rightarrow 3\pi$ and $\phi \rightarrow 3\pi$.

The interaction Lagrangian for $\omega \rightarrow 3\pi$ is given in I as

$$\begin{aligned}\mathcal{L}(\omega \rightarrow 3\pi) = &-\epsilon_{abc} g_\rho{}^{-1}[m_\rho{}^2 \pi_a \partial^\mu \pi_b \rho_{\mu c} + \tfrac{1}{2}\lambda_A \partial_\mu \pi_a \partial_\nu \pi_b \partial^\mu \rho^\nu_c] \\ &- (2\lambda/\sqrt{3}F_\pi)\epsilon_{\mu\nu\alpha\beta}[4\pi_a \partial^\mu \rho^\nu_a \partial^\alpha \omega^\beta + \epsilon_{abc}(g_\rho{}^{-1} + \tfrac{1}{2}k)\pi_a \partial^\mu \pi_b \partial^\nu \pi_c \partial^\alpha \omega^\beta],\end{aligned} \tag{2}$$

where π_a is the pion field, ρ^μ_c is the ρ-meson field, and ω^β is the ω field. $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Cevita symbol with $\epsilon^{0123} = 1 = -\epsilon_{0123}$. g_ρ and F_π are the field-current coupling strengths for the ρ and π mesons.[8] λ_A is the anomalous magnetic moment of the A_1 meson. The π-ρ-ω coupling constant is given by $g_{\pi\rho\omega} = 8\lambda/\sqrt{3}F_\pi$. The first three terms of Eq. (2) lead to the Gell-Mann–Sharp–Wagner[9] ρ-dominated tree diagram. The last term leads to a direct seagull contribution. The value of kg_ρ was determined in I to be -4 by a fit to the experimental value of 9.3 ± 2.1 for the branching ratio $\Gamma(\omega \rightarrow 3\pi)/\Gamma(\omega \rightarrow \pi^0\gamma)$.[10]

A straightforward calculation using the Lagrangian of Eq. (2) gives[11]

$$\begin{aligned}\langle \pi k_1 a : \pi k_2 b | \omega^\beta(0) | \pi k_3 c\rangle = &-i\left(\frac{16\lambda}{\sqrt{3}F_\pi}\right)\epsilon^{\mu\nu\alpha\beta}\epsilon_{abc} N_\pi(k_1)N_\pi(k_2)N_\pi(k_3)\frac{k_{1\mu}k_{2\nu}k_{3\alpha}}{(k_1 + k_2 - k_3)^2 + m_\omega{}^2} \\ &\times\left[\frac{m_\rho{}^2}{g_\rho}(1 - \tfrac{1}{4}\lambda_A)\left(\frac{1}{m_\rho{}^2 - s} + \frac{1}{m_\rho{}^2 - u} + \frac{1}{m_\rho{}^2 - t}\right) + \frac{3}{4}\left(\frac{\lambda_A + 1}{g_\rho}\right)\right],\end{aligned} \tag{3}$$

where $s = -(k_1 + k_2)^2$, $t = -(k_1 - k_3)^2$, and $u = -(k_2 - k_3)^2$. Since Eq. (1) requires this matrix element to be evaluated on the photon mass shell, $s + t + u = 3m_\pi{}^2$. The ϕ matrix element will be identical to Eq. (3) except that m_ω is replaced by m_ϕ and λ is replaced by $\frac{1}{2}\sqrt{3}\,\lambda'$.[12]

Substitution of these matrix elements into Eq. (1) and evaluation at the photon mass shell results in

$$\langle \pi k_1 a : \pi k_2 b | J^\beta_{I=0}(0) | \pi k_3 c\rangle = -iN_\pi(k_1)N_\pi(k_2)N_\pi(k_3)\left(\frac{16e\lambda}{3F_\pi}\right)\left(\frac{g_{88}}{m_\omega^2}+\frac{\sqrt{3}\,\lambda'}{2\lambda}\frac{g_{89}}{m_\phi^2}\right)$$
$$\times\epsilon_{abc}\epsilon^{\beta\mu\nu\alpha}k_{1\mu}k_{2\nu}k_{3\alpha}\left[\frac{m_\rho^2}{g_\rho}(1-\tfrac{1}{4}\lambda_A)\left(\frac{1}{m_\rho^2-s}+\frac{1}{m_\rho^2-t}+\frac{1}{m_\rho^2-u}\right)+\frac{3}{4}\left(\frac{\lambda_A+1}{g_\rho}\right)\right]. \quad (4)$$

The form factor for $\gamma+\pi\to\pi+\pi$, $F(s,t,u)$ is defined by[1]

$$\langle \pi k_1 a : \pi k_2 b | J^\beta_{I=0}(0) | \pi k_3 c\rangle = i(2\pi)^{-9/2}(8\omega_1\omega_2\omega_3)^{-1/2}\epsilon_{abc}\epsilon^{\beta\mu\nu\alpha}k_{1\mu}k_{2\nu}k_{3\alpha}F(s,t,u)\,. \quad (5)$$

Thus,

$$F(s,t,u) = -\left(\frac{16e\lambda}{3F_\pi}\right)\left(\frac{g_{88}}{m_\omega^2}+\frac{\sqrt{3}}{2}\frac{\lambda'}{\lambda}\frac{g_{89}}{m_\phi^2}\right)\left[\frac{m_\rho^2}{g_\rho}(1-\tfrac{1}{4}\lambda_A)\left(\frac{1}{m_\rho^2-s}+\frac{1}{m_\rho^2-t}+\frac{1}{m_\rho^2-u}\right)+\frac{3}{4}\left(\frac{\lambda_A+1}{g_\rho}\right)\right]. \quad (6)$$

The γ-3π coupling constant[1] Λ is related to $F(s,t,u)$ by

$$e\Lambda/m_\pi^3 = F(s=t=u=m_\pi^2)\,. \quad (7)$$

Evaluating the form factor at the symmetric point gives the following expression for Λ:

$$\Lambda = -\left(\frac{16\lambda m_\pi^3}{\sqrt{2}\,m_\rho F_\pi^2}\right)\left(\frac{g_{88}}{m_\omega^2}+\frac{\sqrt{3}\,\lambda' g_{89}}{2\lambda m_\phi^2}\right)\left[(1-\tfrac{1}{4}\lambda_A)\left(1-\frac{m_\pi^2}{m_\rho^2}\right)^{-1}+\tfrac{1}{4}(\lambda_A+1)\right]. \quad (8)$$

In terms of the ω-ϕ mixing model of Augustin *et al.*,[13] $g_{88}=-m_\omega^2/f_\omega$ and $g_{89}=-m_\phi^2/f_\phi$, where $f_\omega^2/4\pi = 14.8\pm2.8$ and $f_\phi^2/4\pi=11.0\pm1.6$. λ and λ' have the values[14] $\lambda=0.348\pm0.024$ and $\lambda'=0.026\pm0.005$. The value of λ_A is taken as 0.4 ± 0.3.[15] These numerical values result in

$$\Lambda = 0.15\pm0.03\,, \quad (9)$$

which is in agreement with the value obtained by Donnachie and Shaw[16] of

$$\Lambda = 0.04\pm0.15\,, \quad (10)$$

from the analysis of photoproduction data.

It is interesting to note that if Λ is calculated using only the Gell-Mann–Sharp–Wagner ρ-dominated tree diagram, the result is

$$\Lambda_{\rm GSW} = 0.12\pm0.02\,, \quad (11)$$

which also agrees with Eq. (10). However, neglecting the seagull term of Eq. (2) for the process $\omega\to3\pi$ leads to a value of $\Gamma(\omega\to3\pi)$ which is a factor of approximately 2 smaller than the experimental value.

It is also of interest to calculate that portion of the total coupling constant that is due to the ϕ meson. Using the numerical values given above, it is found that the g_{89} term of Eq. (8) is approximately 8% of the total coupling constant. This is not surprising since it is well known[10] that the ϕ decays are quite suppressed relative to the ω decays.

Now consider the evaluation of this quantity by other techniques. Chatterjee[1] uses current algebra and invokes the soft-pion limit for two of the pions, predicting a value of 0.03. Sundaram[2] and Murtaza and Harun-Ar-Rashid[3] both use a Veneziano model, Sundaram predicting a value of 0.12, and Murtaza and Harun-Ar-Rashid predicting a value of order unity. Ali and Hussain[4] approach the problem from the point of view of the anomalous Ward identities. They predict a value of 0.16 in the Gell-Mann–Sharp–Wagner limit and values in the range 0.19–0.24 when the experimental value of $\Gamma(\omega\to3\pi)$ is used to determine the seagull contribution. This range of values is due to their using values of λ_A over the range 0 to 1 and evaluating their result with and without higher-order momentum-dependent contributions to the off-mass-shell $\omega\rho\pi$ vertex. All of these calculations ignore contributions due to the ϕ meson. The present calculation includes the ϕ contribution, although it is found to be much smaller than that due to the ω.

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[12]In terms of λ', the ϕ-π-ρ coupling constant is $g_{\pi\rho\phi} = 4\lambda'/F_\pi$.

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Once-Subtracted Dispersion Relations, Current Algebra, and Nonleptonic Weak Decays of Hyperons*

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Nonleptonic weak decays of hyperons and the $K_1 \to 2\pi$ decay are carefully analyzed using current algebra, partial conservation of axial-vector current, and once-subtracted dispersion relations, a method suggested by Okubo, Mathur, and Marshak. We have calculated the dispersion integral by assuming it to be saturated by the low-mass intermediate states of the nucleon octet, of the $\frac{3}{2}^+$ decuplet of $\Delta(1236)$, and of the $\frac{1}{2}^-$ SU_3 singlet $Y_0^*(1405)$, in the case of hyperon decays, and by the intermediate states of vector mesons and a possible 0^+ scalar-meson nonet, in the case of $K_1 \to 2\pi$ decay. The matrix element of the parity-violating weak Hamiltonian density between two baryons (which is required to evaluate the hyperon decay amplitudes) is related to the $K_1 \to 2\pi$ decay amplitude by the K_1 tadpole mechanism. With pseudovector SU_3-symmetric coupling among the nucleon and pseudoscalar-meson octets, we are able to obtain a good fitting of all the hyperon decay amplitudes in terms of four parameters. We also find that the corrections to the soft-pion values are very important not only for P-wave hyperon decay amplitudes, but also for S-wave amplitudes. They are also extremely important for $K_1 \to 2\pi$ decays. Furthermore, we find that, contrary to the findings of Hara and Nambu, there is no evidence for the concept of a "universal parity-conserving spurion" in hyperon and $K_1 \to 2\pi$ weak decays.

I. INTRODUCTION

Based on a current-current theory of weak interactions, current algebra, partial conservation of axial-vector current (PCAC), and a baryon pole model, nonleptonic weak decays of hyperons have been discussed by several authors.[1-9] Our work in this article is along the general lines of these previous works, but is intended to be more complete than any of these. We get several new interesting results, which to the best of our knowledge have not been reported in the literature.

We will start by discussing the work of Kumar and Pati (KP),[6] which is probably one of the most improved forms of all previous works. In their work, KP[6,10] demonstrated that it is possible to fit the eight independent (if the $|\Delta I| = \frac{1}{2}$ property is already built into the theory, as is the case here) decay amplitudes in hyperon decays within about (60–70)%, in terms of essentially only two parameters. This was considerably better than the previous analyses, in which there appeared to be discrepancies with factors of 2 to 3, especially in the P-wave decay amplitudes.

Their work is based on the following assumptions:

(1) The current-current theory is used to the extent that the equal-time commutators involving parity-conserving (P.C.) and parity-violating (P.V.) weak Hamiltonian densities satisfy the simple relations

$$[\mathcal{H}_W^{\text{P.V.}}, F_5^i] = [\mathcal{H}_W^{\text{P.C.}}, F^i], \tag{1}$$

$$[\mathcal{H}_W^{\text{P.C.}}, F_5^i] = [\mathcal{H}_W^{\text{P.V.}}, F^i], \tag{2}$$