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Hard-Meson Current Algebra and the Breakdown of Partial Conservation of Axial-Vector Current*

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The Arnowitt-Friedman-Nath model of PCAC (partially conserved axial-vector current) breakdown which permits the two-body photon decays of π^0, η, and vector mesons is extended to the η' meson and to higher order. The model is required to satisfy a set of conditions, the most important of which is that it be consistent with $SU(3)\times SU(3)$ hard-meson current algebra. It is shown that if PCAC is no longer pole-dominated, then the general σ commutator, $[A_a^0, \partial_\mu A_b^\mu]$, cannot be pole-dominated either. In addition, the variation of the Lagrangian under chiral transformations will no longer be proportional to the divergence of the current. The model is used to calculate decay widths for the processes $\eta'\to\rho^0+\gamma$, $\eta'\to 2\gamma$, $\omega\to 3\pi$, $\varphi\to 3\pi$, and $\eta\to 2\pi+\gamma$ with good success.

I. INTRODUCTION

In recent years, hard-meson techniques using $SU(2)\times SU(2)$ current algebra have been successfully applied to a number of processes.[1] These techniques have also been extended to the algebra involving strangeness-changing currents (which can then be applied to the K_{l3} decay),[2] $SU(3)\times SU(3)$ three-point functions,[3] and $SU(3)\times SU(3)$ four-point functions for the case of channels involving strange mesons.[4] Very recently, the hard-meson method has been further extended in order to compute an arbitrary N-point function using chiral $SU(3)\times SU(3)$ current algebra.[5] However, it has been known for some time that the hypothesis of partially conserved axial-vector currents (PCAC) forbids photon decays such as $\pi^0\to 2\gamma$ and $\omega\to\pi^0+\gamma$ in the soft-pion approximation.[6] This result holds true in the hard-meson method as well.[7-10] A variety of solutions to this problem have been proposed. Very briefly, they are as follows. Perrin[7] accommodates the photon decays by rejecting the field-current identities and invoking the space-space quark current commutation relations, while Brown and West[8] and Riazuddin and Sarker[9] add higher-order derivative couplings which lead to rapidly varying off-mass-shell momentum dependence. Adler[11] and others[12,13] include spinor electrodynamic interactions which lead to anomalous terms proportional to the electromagnetic tensor in the neutral PCAC equation. Brown, Munczek, and Singer[14] and Gounaris[15] add strong-interaction terms obeying certain symmetry requirements to the Lagrangian which lead to extra terms in the PCAC equations. Baracca and Bramon[16] account for these decays by use of a vector-meson-dominated model, including ω-φ mixing. Finally, Arnowitt, Friedman, and Nath[10] add higher-order strong-interaction terms directly to the PCAC equation. In all of these models, the analysis is limited to a study of three-point functions (i.e., two-body photon decays), although several[7-9,14,16] have considered four-point functions such as those corresponding to $\omega\to 3\pi$ and $\eta\to 2\pi+\gamma$, but only within the approximation that any possible seagull terms may be neglected.

The approach taken in the analysis presented here is that of II, i.e., higher-order strong-interaction terms are added directly to the PCAC equation with no *a priori* assumption made about the chiral symmetry of the Lagrangian. The analysis of II is extended in two ways. First, the three-point theory is enlarged so that the decays $\eta'\to 2\gamma$ and $\eta'\to\rho^0+\gamma$ can be included. Second, higher-order contributions are developed in a manner consistent with I so that four-point processes such as $\omega\to 3\pi$, $\varphi\to 3\pi$, and $\eta\to 2\pi+\gamma$ may be considered, including any seagull terms which may be required by the theory.

An outline of the remainder of this paper is as follows. In Sec. II the hard-meson method as formulated in I is reviewed and a general form of PCAC is introduced. In Secs. III and IV the effects of this general PCAC on the σ commutator and chiral transformations of the Lagrangian are studied. In Secs. V and VI, restrictions on the PCAC breaking terms are formulated and a specific model is proposed. The final two sections of the paper, Secs. VII and VIII, deal with the application of the theory to the processes mentioned above and discussions of this and other models.

II. THE HARD-MESON METHOD

Essential to the hard-meson technique are the assumptions that (1) intermediate sums may be saturated by low-lying single-meson states and (2) the particle vertex functions may be approximated by a low-order polynomial in momenta. The results of these assumptions are that

(a) currents are defined by field-current identities;

(b) an "effective" Lagrangian is defined by

$$\mathfrak{L}=\mathfrak{L}_{\text{free}}+g\mathfrak{L}_3+g^2\mathfrak{L}_4+g^3\mathfrak{L}_5+\cdots,$$

where g is a coupling constant, $\mathfrak{L}_3$ is cubic in fields, $\mathfrak{L}_4$ is quartic in fields, etc.;

(c) the vacuum expectation value of the T product of N currents may be calculated from the field-current identities using $\mathfrak{L}$ to order $(N-2)$ in perturbation theory.

The assumption that the currents obey the canonical-commutation relations of the chiral current algebra (CCR) along with the postulates of conserved nonstrange vector currents (CVC), partially conserved strange vector currents (PCVC), and PCAC can now be imposed on the theory (CVC, PCVC, and PCAC will be noted collectively as PCC), resulting in a set of constraint equations on the Lagrangian coupling constants.

At this point one additional assumption is made. This is that the time components of the currents are quadratic functions of the canonical fields (QCF). The physical significance of QCF can be better understood by considering the σ commutator defined by

$$\sigma_{ab}=[\partial_\mu A_a^\mu(x),A_b^0(y)]\delta(x_0-y_0), \tag{1}$$

where A_a^μ is the axial-vector current of the ath type. If pole dominance of PCAC is assumed ($\partial_\mu A_a^\mu \sim$pseudoscalar field), then QCF requires the σ commutator to be pole-dominated.[17]

For the $SU(3)\times SU(3)$ algebra, the Lagrangian is assumed in I to have the form

$$\mathfrak{L}=\mathfrak{L}(S_a,\partial^\mu S_a,v_b^\mu,\partial^\mu v_a^\nu), \tag{2}$$

where S_a is the pseudoscalar-scalar field and v_a^μ is the axial-vector–vector field. Second-order Lagrangian formalism is used and $\mathfrak{L}$ has been restricted to a single derivative per field. The index a runs over the values 1–9 and $\bar{1}$–$\bar{9}$. (The bar indicates a pseudoscalar or axial-vector field. Absence of a bar indicates a scalar or vector field.) The subscripts i, j, and k will be restricted to the values 1–9 only. If unnatural-parity states are meant i, j, and k will be barred. All other Latin subscripts run over the full range, i.e., 1–9 and $\bar{1}$–$\bar{9}$. Hence S_i refers to the 0^+ nonet, $S_{\bar{j}}$ refers to the 0^- nonet, and S_t refers to both nonets. The four nonets are assumed to be

$$\begin{aligned}&0^+=(\delta,\kappa,\sigma,\eta_{0^+}),\quad 0^-=(\pi,K,\eta,\eta'),\\&1^+=(A_1,K_A,D,E),\ 1^-=(\rho,K^*,\omega,\phi).\end{aligned} \tag{3}$$

The field-current identities take the form

$$V_a^\mu=g_{ab}v_b^\mu+F_{ab}\partial^\mu S_b. \tag{4}$$

PCC is given by the equation[18]

$$\partial_\mu V_a^\mu=F_{ab}\,\mu_b{}^2S_b. \tag{5}$$

The Gell-Mann form of CCR is assumed:

$$\begin{aligned}&[V_a^0(x),V_b^\mu(y)]\delta(x_0-y_0)\\&\qquad=iC_{abc}V_c^\mu\delta^4(x-y)+c\text{-number S.T.}\end{aligned} \tag{6}$$

The Schwinger terms (S.T.) are restricted to c-numbers only. The result of this restriction is a generalized first Weinberg sum rule relating various components of g_{ab} and F_{ab}.

The assumption of QCF is satisfied if

$$\mathfrak{L}=-\tfrac{1}{2}V_a^\mu W_{ab}{}^{-1}V_{\mu b}+\mathfrak{L}'(\gamma^\mu,H^{\mu\nu},S), \tag{7}$$

where

$$W_{ab}=g_{at}g_{bt}/m_t{}^2+F_{at}F_{bt}, \tag{8}$$

$$\begin{aligned}\gamma_a^\mu=&(\delta_{ab}-F_{ca}W_{cd}{}^{-1}F_{db})\partial^\mu S_b\\&-F_{ca}W_{cd}{}^{-1}g_{db}v_b^\mu-Z_{dab}^1W_{dc}{}^{-1}S_bV_c^\mu,\end{aligned} \tag{9}$$

and

$$H_a^{\mu\nu}=\partial^\mu v_a^\nu-\partial^\nu v_a^\mu+g_{ad}{}^{-1}W_{de}{}^{-1}C_{ebc}V_b^\mu V_c^\nu. \tag{10}$$

In order that CCR have no q-number Schwinger terms, W_{ab} must have the form

$$\begin{aligned}W_{ab}&=(g_\rho/m_\rho)^2\delta_{ab}\text{ for }a,b=1\text{–}8,\text{ and }\bar{1}\text{–}\bar{8},\\&\neq0\text{ for }a=b=9\text{ and }a=b=\bar{9}\\&=0\text{ for all other components.}\end{aligned} \tag{11}$$

C_{abc} is the $SU(3)\times SU(3)$ antisymmetric structure constant,

$$C_{ijk}=C_{\bar{i}\bar{j}k}=C_{i\bar{j}\bar{k}}=C_{\bar{i}j\bar{k}}=f_{ijk}. \tag{12}$$

The second term of Eq. (7), $\mathfrak{L}'$, can be any arbitrary function of the indicated variables.

PCC will be satisfied if $\mathfrak{L}$ satisfies the equation

$$F_{ab}\left(\frac{\delta\mathfrak{L}}{\delta S_b}\right)=-Z_{abc}^1\left[\frac{\delta\mathfrak{L}}{\delta S_b}S_c+\frac{\delta\mathfrak{L}}{\delta\gamma_b^\mu}\gamma_c^\mu\right]-C_{abc}\left[\frac{\delta\mathfrak{L}}{\delta V_b^\mu}V_c^\mu+g_{db}{}^{-1}g_{ce}\frac{\delta\mathfrak{L}}{\delta H_d^{\mu\nu}}H_e^{\mu\nu}\right]-F_{ab}\mu_b{}^2S_b. \tag{13}$$

The theory has been formulated in terms of the variables V^μ, γ^μ, $H^{\mu\nu}$, and S in order that the integrability of Eqs. (7) and (13) may be more easily proven. The details of the proof are given in I.

The time components of the currents are given by

$$V_a^0 = (g_{ab}/m_b^2)\partial_i G_{0ib} - F_{ab}S_{0b} - Z^1_{acb}S_b S_{0c} + Z^2_{abc}v_{ib}G_{0ic} + Z^3_{abc}S_b\partial_i G_{0ic} + Z^4_{abc}\partial_i S_b G_{0ic}\,, \tag{14}$$

where S_0 and G_{0i} are the canonical momenta for the pseudoscalar-scalar and axial-vector–vector fields, respectively. The three constants Z^2_{abc}, Z^3_{abc}, and Z^4_{abc} are completely determined by CCR,

$$Z^2_{abc} = C_{ade}\,g_{db}\,g_{ce}^{-1}, \tag{15}$$

$$Z^3_{abc} = C_{ade}\,F_{db}\,g_{ce}^{-1}, \tag{16}$$

$$Z^4_{abc} = -Z^1_{aeb}\,g_{cd}^{-1}F_{de}\,. \tag{17}$$

Z^1_{abc} is partially determined by CCR and PCC as given in I.

In order to overcome the difficulties discussed in the Introduction, the PCC conditions will now be taken to have the more general form

$$\partial_\mu V_a^\mu = F_{ab}\mu_b^2 S_b + \mathfrak{F}_a\,, \tag{18}$$

where $\mathfrak{F}_a$ is as yet completely arbitrary. In order to satisfy QCF it is only necessary to assume that $\mathfrak{F}_a$ is a function of γ^μ, $H^{\mu\nu}$, and S. The reason for this is that the Lagrangian terms corresponding to $\mathfrak{F}_a$ will then be functions of γ^μ, $H^{\mu\nu}$, and S also. Hence this part of the Lagrangian will come from the second term of Eq. (7) and thus QCF will be maintained. This form will not change any of the results of CCR or the form of the time components of the currents. The PCC equation will have one additional term:

$$F_{ab}\left(\frac{\delta\mathfrak{L}}{\delta S_b}\right) = -Z^1_{abc}\left[\frac{\delta\mathfrak{L}}{\delta S_b}S_c + \frac{\delta\mathfrak{L}}{\delta\gamma_b^\mu}\gamma_c^\mu\right] - C_{abc}\left[\frac{\delta\mathfrak{L}}{\delta V_b^\mu}V_c^\mu + g_{db}^{-1}g_{ce}\frac{\delta\mathfrak{L}}{\delta H_d^{\mu\nu}}H_c^{\mu\nu}\right] - F_{ab}\mu_b^2 S_b - \mathfrak{F}_a(\gamma, H, S). \tag{19}$$

Now consider the integrability of Eqs. (7) and (19). Since the new form of PCC does not change Eq. (7) or the form of the currents, it is only necessary to consider the effects of $\mathfrak{F}_a$ on the integrability of PCC. Assume that the Lagrangian may be written as the sum of two terms,

$$\mathfrak{L} = \mathfrak{L}_{\rm NB} + \mathfrak{L}_{\rm B}\,. \tag{20}$$

$\mathfrak{L}_{\rm NB}$ is the Lagrangian required by the theory when PCC is not broken ($\mathfrak{F}_a = 0$) and $\mathfrak{L}_{\rm B}$ represents the extra terms needed in the Lagrangian when the PCC-breaking terms are present. Then from Eq. (19)

$$F_{ab}\left(\frac{\delta\mathfrak{L}_{\rm NB}}{\delta S_b}\right) = -Z^1_{abc}\left[\frac{\delta\mathfrak{L}_{\rm NB}}{\delta S_b}S_c + \frac{\delta\mathfrak{L}_{\rm NB}}{\delta\gamma_b^\mu}\gamma_c^\mu\right] - C_{abc}\left[\frac{\delta\mathfrak{L}_{\rm NB}}{\delta V_b^\mu}V_c^\mu + g_{db}^{-1}g_{ce}\frac{\delta\mathfrak{L}_{\rm NB}}{\delta H_d^{\mu\nu}}H_c^{\mu\nu}\right] - F_{ab}\mu_b^2 S_b \tag{21}$$

and

$$F_{ab}\left(\frac{\delta\mathfrak{L}_{\rm B}}{\delta S_b}\right) = -Z^1_{abc}\left[\frac{\delta\mathfrak{L}_{\rm B}}{\delta S_b}S_c + \frac{\delta\mathfrak{L}_{\rm B}}{\delta\gamma_b^\mu}\gamma_c^\mu\right] - C_{abc}\left[\frac{\delta\mathfrak{L}_{\rm B}}{\delta V_b^\mu}V_c^\mu + g_{db}^{-1}g_{ce}\frac{\delta\mathfrak{L}_{\rm B}}{\delta H_d^{\mu\nu}}H_c^{\mu\nu}\right] - \mathfrak{F}_a(\gamma, H, S). \tag{22}$$

Equation (21) is identical to Eq. (13) and so $\mathfrak{L}_{\rm NB}$ is guaranteed to be integrable by the proof given in I. Thus it is only necessary to consider the integrability of the "breaking Lagrangian," $\mathfrak{L}_{\rm B}$. Since a specific form for $\mathfrak{F}_a$ has not yet been chosen, it is not possible to establish a general proof of the integrability of $\mathfrak{L}_{\rm B}$ at this point. Once a form is chosen, Eq. (22) can then be used to determine the required $\mathfrak{L}_{\rm B}$. However, it should be noted that because of symmetry requirements on the coefficients of $\mathfrak{L}_{\rm B}$, there does not exist complete freedom in picking the form of the PCC-breaking terms. The inclusion of certain terms in $\mathfrak{F}_a$ requires the inclusion of other higher-order terms in order to satisfy PCC [Eq. (22)]. This problem will be discussed further in Sec. VI, when a specific model of PCC breaking is proposed.

III. THE σ COMMUTATOR

As has already been pointed out, the σ commutator takes on a particularly simple form when QCF is assumed and PCC is pole-dominated. Since PCC enters directly into this commutator, it is of interest to consider it for a general PCC.

When the Schwinger terms are explicitly calculated, CCR takes the form

$$[V_a^0(x), V_b^\mu(y)]\delta(x_0 - y_0) = iC_{abc}V_c^\mu(x)\delta^4(x-y) - iW_{ab}g_i^\mu\partial^i(x)\delta^4(x-y)\,. \tag{23}$$

Taking the total divergence with respect to y of Eq. (23), changing the arguments of the derivatives of the δ functions from y to x, and using the antisymmetry of C_{abc} and the symmetry of W_{ab} yields

$$[V_a^0(x), \partial_\mu V_b^\mu(y)]\delta(x_0 - y_0) - [V_b^0(y), \partial_\mu V_a^\mu(x)]\delta(y_0 - x_0) = iC_{abc}\partial_\mu V_c^\mu\delta^4(x-y). \tag{24}$$

From this equation it can be seen that if PCC is not pole-dominated, then the σ commutator can no longer be pole-dominated either. Hence the assumption of QCF loses some of the motivation that it originally had. However, the commutator $[V_a^0, S_b]$ will remain pole-dominated as a result of QCF, and therefore, QCF is still attractive. In addition, QCF simplifies the form of the currents and leads to results that agree well with experiment.

One final comment about the σ commutator can be made. In the case when PCC is pole-dominated, Eq. (24) (i.e., CCR) forces the antisymmetric part [in both $SU(3)$ and spatial arguments] of the σ commutator to be pole-dominated. Therefore QCF limits only the symmetric part.

IV. CHIRAL ROTATION OF THE LAGRANGIAN

Consider the effect on the Lagrangian of the transformations generated by the time components of the currents:

$$\mathcal{L}(t) \to U(t)\mathcal{L}(t)U^{-1}(t), \tag{25}$$

where

$$U(t) = \exp[i\lambda_a F_a(t)]. \tag{26}$$

λ_a are constants and

$$F_a(t) = \int d^3x\, V_a^0(\vec{x}, t). \tag{27}$$

If the transformation is infinitesimal, then the variation of the Lagrangian under this transformation is given by

$$\frac{\delta\mathcal{L}}{\delta\lambda_a} = i\frac{\delta\mathcal{L}}{\delta S_b}[F_a, S_b] + i\frac{\delta\mathcal{L}}{\delta\partial^\mu S_b}[F_a, \partial^\mu S_b] + i\frac{\delta\mathcal{L}}{\delta v_b^\mu}[F_a, v_b^\mu] + i\frac{\delta\mathcal{L}}{\delta\partial^\mu v_b^\nu}[F_a, \partial^\mu v_b^\nu]. \tag{28}$$

Bringing the derivatives outside the commutators and using the equations of motion results in

$$\frac{\delta\mathcal{L}}{\delta\lambda_a} = i\partial^\mu\left(\frac{\delta\mathcal{L}}{\delta\partial^\mu S_b}[F_a, S_b] + \frac{\delta\mathcal{L}}{\delta\partial^\mu v_b^\nu}[F_a, v_b^\nu]\right) - i\left(\frac{\delta\mathcal{L}}{\delta\partial^0 S_b}[\partial^0 F_a, S_b] + \frac{\delta\mathcal{L}}{\delta\partial^0 v_b^i}[\partial^0 F_a, v_b^i]\right). \tag{29}$$

The last term is due to the fact that F_a is a function of time. The derivatives of $\mathcal{L}$ in this term are just the canonical momenta. Furthermore,

$$\partial^0 F_a(t) = -\int d^3x\, \partial_0 V_a^0(\vec{x}, t) = -\int d^3x\, \partial_\mu V_a^\mu(\vec{x}, t). \tag{30}$$

Substitution of Eqs. (18) and (30) into Eq. (29) yields

$$\frac{\delta\mathcal{L}}{\delta\lambda_a} = i\partial^\mu\left(\frac{\delta\mathcal{L}}{\delta\partial^\mu S_b}[F_a, S_b] + \frac{\delta\mathcal{L}}{\delta\partial^\mu v_b^\nu}[F_a, v_b^\nu]\right) - i\int d^3y \{S_{0b}(\vec{x}, t)[\mathfrak{F}_a(\vec{y}, t), S_b(\vec{x}, t)] + G_{0ib}(\vec{x}, t)[\mathfrak{F}_a(\vec{y}, t), v_b^i(\vec{x}, t)]\}. \tag{31}$$

From the field-current identities, the definition of V_a^μ, the solutions for Z_{abc}^2, Z_{abc}^3, and Z_{abc}^4, the antisymmetry of $(\delta\mathcal{L}/\delta\partial^\mu v_a^\nu)$, and the general form of PCC,

$$i\partial^\mu\left(\frac{\delta\mathcal{L}}{\delta\partial^\mu S_b}[F_a, S_b] + \frac{\delta\mathcal{L}}{\delta\partial^\mu v_b^\nu}[F_a, v_b^\nu]\right) = \partial_\mu V_a^\mu - ig_{bc}{}^{-1}F_{cd}\partial^i(x)\int d^3y\, G_{0ib}(\vec{x}, t)[\mathfrak{F}_a(\vec{y}, t), S_d(\vec{x}, t)]. \tag{32}$$

Hence the variation of the Lagrangian under infintesimal transformations is given by

$$\frac{\delta\mathcal{L}}{\delta\lambda_a} = \partial_\mu V_a^\mu - ig_{bc}{}^{-1}F_{cd}\partial^i(x)\int d^3y\, G_{0ib}(\vec{x}, t)[\mathfrak{F}_a(\vec{y}, t), S_d(\vec{x}, t)]$$
$$-i\int d^3y\{S_{0b}(\vec{x}, t)[\mathfrak{F}_a(\vec{y}, t), S_b(\vec{x}, t)] + G_{0ib}(\vec{x}, t)[\mathfrak{F}_a(\vec{y}, t), v_b^i(\vec{x}, t)]\}. \tag{33}$$

From this result several things are apparent. First, the variation of the Lagrangian is no longer necessarily equal to the divergence of the current. However, if PCC is not broken (i.e., $\mathfrak{F}_a = 0$), then the usual equality does hold. Second, the variation and hence the transformed Lagrangian may not be a Lorentz scalar as a result of the last three terms on the right-hand side of Eq. (33).

V. RESTRICTIONS ON MODEL

Having considered the effects of a general PCC on CCR, the σ commutator, and chiral transformations of the Lagrangian, it is now possible to place some restrictions on the PCC-breaking term $\mathfrak{F}_a$. It has been shown that changing PCC has no effect on CCR and only requires a minor modification of the general PCC equation, provided $\mathfrak{F}_a$ is a function of γ^μ, $H^{\mu\nu}$, and S only. All that the inclusion of $\mathfrak{F}_a$ in the theory demands is that additional pieces be added to the Lagrangian to account for the breaking terms in Eq. (19). The important thing to recognize is that CCR in no way limits the form or coupling constants of $\mathfrak{F}_a$. As was mentioned in Sec. II, the form of $\mathfrak{F}_a$ is restricted by PCC but these restrictions are easily satisfied.

If PCC is no longer pole-dominated, the σ commutator cannot be either. Hence this property of the previous model[5] shall have to be abandoned.

The existence of $\mathfrak{F}_a$ leads to a complicated variation of the Lagrangian under chiral transformations. If the rotation of the Lagrangian under these transformations is to contain any physics, the rotated Lagrangian must be a Lorentz scalar. In the pole-dominated PCC model, the variation is proportional to $F_{ab}\mu_b{}^2 S_b$, where S_b are renormalized fields (since the Lagrangian is phenomenological). Thus the conservation breaking is related to a set of pseudoscalar, scalar particles. When a general PCC is assumed, it is possible to have contributions to the variation which are not related to this set of fields. Thus if the Lagrangian variation is required to be related to the above set of fields for a general PCC, restrictions on $\mathfrak{F}_a$ and hence on the theory will result.

The three-point theory was successful in explaining a number of decays. Therefore it will be included in the general theory. In addition this model had a large amount of $SU(3)$ symmetry. This characteristic will be maintained for the general theory.

A characteristic of all current-algebra theory is that CCR and PCC only determine a portion of the possible Lagrangian couplings. It is reasonable to try to limit the undetermined part as much as possible. Hence a minimal Lagrangian which includes only those terms necessary to satisfy PCC will be chosen.

Finally, since the breakdown in the original theory appears to occur only for interactions involving an odd number of unnatural-parity fields, it is reasonable to try to limit the theory to interactions of this type. Unfortunately it is not possible to satisfy this restriction and maintain the condition that the transformed Lagrangian under chiral rotation be a Lorentz scalar simultaneously. Since the addition of terms to the breaking that involve an even number of unnatural-parity fields leads to a great many interactions which do not seem to be necessary to fit the experimental situation, it will be assumed that such terms will be added only to the extent necessary to satisfy the conditions on the Lagrangian under chiral rotations.

These restrictions can be summarized as follows:

(1) The breaking will be restricted to functions of γ^μ, $H^{\mu\nu}$, and S only. This will ensure the requirements of CCR and QCF, and the proof of integrability.

(2) The only Lagrangian couplings included will be those necessary to satisfy PCC. This will result in a minimal-interaction Lagrangian.

(3) (a) The transformed Lagrangian under the chiral rotation

$$U(t) = \exp[i\lambda_a F_a(t)], \quad F_a(t) = \int d^3x\, V_a^0(\vec{x}, t)$$

will be required to be a Lorentz scalar.

(b) The variation of the Lagrangian under this rotation will be required to have the form

$$\frac{\delta \mathcal{L}}{\delta \lambda_a} = K_{ab} S_b, \tag{34}$$

where K_{ab} may be a q number and is nonzero for a = unconserved index, i.e., 4, 5, 6, 7, $\overline{1}$–$\overline{9}$.

(4) The three-point theory of II will be included in the general theory.

(5) The breaking terms will be chosen to have as much $SU(3)\times SU(3)$ chiral symmetry as possible, consistent with experimental data.

(6) The only PCC breaking terms involving an even number of unnatural-parity fields allowed will be those necessary to satisfy condition (3a).

VI. SPECIFIC MODEL OF PCC BREAKDOWN

Now that a set of conditions on the PCC-breaking theory have been formulated, it is possible to develop a specific model consistent with these restrictions. First, the exact form chosen for PCC and the breaking Lagrangian will be given, followed by a discussion of the various terms appearing in the model.

PCC is given by

$$\begin{aligned} \partial_\mu V_a^\mu = {} & F_{ab}\mu_b{}^2 S_b + \epsilon_{\mu\nu\alpha\beta}(\lambda_{abc} + K_{adbc} S_d) H_b^{\mu\nu} H_c^{\alpha\beta} + \epsilon_{\mu\nu\alpha\beta}(J_{abcd} + L_{aebcd} S_e)\gamma_b^\mu \gamma_c^\nu H_d^{\alpha\beta} \\ & - [2(\lambda_{atd} h_{bct} + \lambda_{atc} h_{bdt}) + Q_{aebcd} S_e] S_b H_{\mu\nu c} H_d^{\mu\nu}, \end{aligned} \tag{35}$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita symbol with $\epsilon^{0123}=1=-\epsilon_{0123}$. The first term on the right-hand side of Eq. (35) is the usual PCC pole term. All other terms correspond to the PCC-breaking term, $\mathfrak{F}_a$, of Eq. (18).

The minimal-breaking Lagrangian is given by

$$\mathcal{L}_B=\epsilon_{\mu\nu\alpha\beta}[h_{abc}S_a H_b^{\mu\nu}H_c^{\alpha\beta}+A_{abcd}S_a\gamma_b^\mu\gamma_c^\nu H_d^{\alpha\beta}]-(h_{ard}h_{bcr}+h_{arc}h_{bdr})S_aS_bH_{\mu\nu c}H_d^{\mu\nu}. \tag{36}$$

PCC [Eq. (22)] requires h_{abc} and A_{abcd} to have the following form:

$$h_{abc}=\begin{cases}0 \text{ for } a=1-9\\ -F_{at}^{-1}\lambda_{tbc} \text{ for } a=\bar{1}-\bar{9}\end{cases} \tag{37}$$

and

$$A_{abcd}=\begin{cases}0 \text{ for } a=1-9\\ -F_{at}^{-1}J_{tbcd} \text{ for } a=\bar{1}-\bar{9}.\end{cases} \tag{38}$$

The constants λ_{abc}, K_{abcd}, J_{abcd}, L_{abcde}, and Q_{abcde} have the following values:

$$\lambda_{abc}=\lambda_{acb}, \tag{39a}$$

$$\lambda_{\bar{i}bc}=\lambda_{a\bar{i}c}=\lambda_{ab\bar{i}}=0, \tag{39b}$$

$$\lambda_{\bar{i}jk}=\begin{cases}\lambda d_{ijk} & \text{for } i,j,k=1-8\\ \tfrac{1}{2}\lambda'\delta_{ij} & \text{for } i=1-8 \text{ and } k=9\\ \lambda''\delta_{jk} & \text{for } i=9,\end{cases} \tag{39c}$$

$$K_{arst}=-Z^1_{abr}h_{bst}-C_{abc}g_{db}^{-1}(g_{ct}h_{rds}+g_{cs}h_{rdt}), \tag{40}$$

$$J_{abcd}=\begin{cases}0 \text{ for } a=1-9\\ \tfrac{1}{2}k[(\lambda_{adt}C_{tbc}-\lambda_{adt}C_{tcb})-(\lambda_{bdt}C_{tac}+\lambda_{cdt}C_{tba})-(\lambda_{abt}C_{tdc}+\lambda_{act}C_{tbd})] \text{ for } a=\bar{1}-\bar{9},\end{cases} \tag{41}$$

$$L_{arstu}=-Z^1_{abr}A_{bstu}-(Z^1_{abs}A_{rbtu}-Z^1_{abt}A_{rbsu})-C_{abc}g_{db}^{-1}g_{cu}A_{rstd}, \tag{42}$$

$$Q_{arstu}=-Z^1_{abs}(h_{bqu}h_{rtq}+h_{bqt}h_{ruq})-Z^1_{abr}(h_{bqu}h_{stq}+h_{bqt}h_{suq}) - C_{abc}g_{db}^{-1}[g_{cu}(h_{rqd}h_{stq}+h_{rqt}h_{sdq})+g_{ct}(h_{rqd}h_{suq}+h_{rqu}h_{sdq})]. \tag{43}$$

d_{ijk} is the $SU(3)$-symmetric structure constant. λ, λ', λ'', and k are constants to be determined from experimental data. Since λ_{abc} and J_{abcd} have been set to zero for $a=1-9$, this model does not break PCVC (or CVC) through the second, fourth, or sixth terms on the right-hand side of Eq. (35). However, as a result of Eqs. (40), (42), and (43), PCVC (but not CVC) is broken at third and fourth order. If upon study of the data it appears necessary to break PCVC through the λ_{abc} and J_{abcd} terms as well, this model can be extended to the strange vector currents in a straightforward manner.

The origin of the various terms appearing in Eq. (35) is as follows. The first term of Eq. (35) is just the usual PCC result. The second term is necessary to account for the two-body photon decays. The three-point theory is included in the definition of λ_{abc} [Eq. (39)]. The two constants, λ and λ', have been determined in II. The constant λ'' is included to extend the theory to the photon decays of the η' meson.

The K_{abcd} term of Eq. (35) is necessary to satisfy the general PCC equation [Eq. (22)]. This equation cannot be satisfied by merely retaining the h_{abc} term of Eq. (36) and compensating for it by adding to the Lagrangian a term of the type $G_{abcd}\epsilon_{\mu\nu\alpha\beta}S_a \times S_b H_c^{\mu\nu}H_d^{\alpha\beta}$. Symmetry forces G_{abcd} to be symmetric on a and b, and hence this term is of no help in satisfying Eq. (22). Thus the K_{abcd} must be introduced. Under these circumstances, the G_{abcd} term is discarded in the interest of obtaining a minimal Lagrangian.

The form of J_{abcd} [Eq. (41)] was chosen for several reasons. First, when the h_{abc} term of the Lagrangian is expanded in particle fields, it contributes fourth-order couplings identical in form to those generated by the A_{abcd} term. The coupling constants of these terms are of the form of a product of λ_{abc} and C_{abc}. Thus it is reasonable to make A_{abcd} have the same form. Second, since λ_{abc} is proportional to the $SU(3)$-symmetric structure constant, this form of J_{abcd} displays a large amount of $SU(3)\times SU(3)$ chiral symmetry. Finally, there is *a posteriori* justification for this form since it gives agreement with experimental data.

The L_{abcde} term of Eq. (35) is necessary for reasons identical to those requiring the introduction of the K_{abcd} term. Just as the K_{abcd} term was needed to cancel out higher-order contributions of the h_{abc} terms in Eq. (22), the L_{abcde} term is necessary to cancel out higher-order contributions of the A_{abcd} term in Eq. (22).

The second-to-last term of Eq. (35) is necessary in order to satisfy the condition that the Lagrangian variation under chiral rotation be a Lorentz scalar. This term [as well as the last term of Eq. (35)] involves an even number of unnatural-parity fields. The variation of the Lagrangian under chiral rotation is given by

$$\frac{\delta \mathcal{L}}{\delta \lambda_a} = (F_{ar}\mu_r^{\,2} + 8\,\lambda_{abc}\,h_{rsb}\,H_{\mu\nu s}\,H_c^{\mu\nu})S_r\,. \tag{44}$$

Note that the additional piece due to the breaking is only a function of breaking constants. The result given by this equation [Eq. (44)] is good only to third order in particle fields. The reason for this is that at present, only four-point functions are of interest and hence it is only required that the total Lagrangian be known to fourth order.

Finally, the last terms of Eqs. (35) and (36) are needed to satisfy PCC [Eq. (22)]. The $h_{ard}\,h_{bcr}$ term of Eq. (36) is needed to cancel out the $\lambda_{atd}\,h_{bct}$ term of Eq. (35). The Q_{abcde} term of Eq. (35) is needed to cancel out higher-order contributions of the last term of Eq. (36), just as the K_{abcd} term is needed to cancel out higher-order contributions of the h_{abc} term.

With the PCC-breaking structures of Eq. (35) and the breaking Lagrangian of Eq. (36), the PCC conditions on the breaking part of the theory [Eq. (22)] have been explicitly satisfied. Thus the general PCC equation [Eq. (19)] has been reduced to the PCC equation for the nonbreaking part of the theory [Eq. (21)]. As was pointed out in Sec. II, Eq. (21) is identical to the PCC equation of the original theory [Eq. (13)] and hence the theory presented here is guaranteed to be integrable by the proof given in I.

Integrability of the theory provides an additional reason for choosing the minimal-breaking Lagrangian given by Eq. (36). For example, if the term $G_{abcd}\,\epsilon_{\mu\nu\alpha\beta}S_a\,S_b\,H_c^{\mu\nu}\,H_d^{\alpha\beta}$ discussed in conjunction with the K_{abcd} term were introduced, then additional fourth-order contributions would be generated in the PCC equation due to the first through fourth terms on the right-hand side of Eq. (22). In order to satisfy PCC it would then be necessary to add a fifth-order term of the form $G'_{abcde}\,\epsilon_{\mu\nu\alpha\beta}S_a\,S_b\,S_c\,H_d^{\mu\nu} \times H_e^{\alpha\beta}$ to $\mathcal{L}_B$ [which would contribute a fourth-order term to the left-hand side of Eq. (22)] and/or another fourth-order term of the form $H_{abcde}\,\epsilon_{\mu\nu\alpha\beta}S_b \times S_c\,H_d^{\mu\nu}\,H_e^{\alpha\beta}$ to $\mathfrak{F}_a$. In the case of the addition of the G'_{abcde} term, Eq. (22) would then require a sixth-order Lagrangian term and/or a fifth-order $\mathfrak{F}_a$ term. If this procedure were continued, it can be seen that terms of all orders would be needed in the Lagrangian and $\mathfrak{F}_a$. General integrability of the theory would then be very difficult to establish. In addition, a number of constants would be added with no new physical guidance for limiting their form or value. Hence it is desirable to truncate the breaking Lagrangian at fourth order, resulting in a theory that is definitely integrable. This is also a reasonable procedure since at present, only four-point functions are to be calculated and so the Lagrangian need only be known through fourth order. However, it should be stressed that the model given in Eqs. (35) and (36) is integrable to all orders, and thus could be used to calculate an arbitary N-point function is a manner consistent with all restrictions on the theory.

At this point, one additional result should be mentioned. When this model of PCC breakdown is used, the σ commutator is not only nonpole-dominated, but is no longer a Lorentz scalar as well. The only way this property of the σ commutator can be maintained is to make λ_{abc} identically zero. Thus if the two-body decays are to be included in the theory, this commutator cannot be a Lorentz scalar.

VII. APPLICATIONS

As a first application of this model, consider the decays $\eta' \to \rho^0 + \gamma$ and $\eta' \to 2\gamma$. When the Lagrangian of Eq. (36) is specialized to the interactions of an η' with two neutral nonstrange vector mesons, the result is

$$\mathcal{L} = 4\epsilon_{\mu\nu\alpha\beta}\,\eta'[h_1\partial^\mu\rho_3^\nu\,\partial^\alpha\rho_3^\beta + h_2\partial^\mu\omega^\nu\,\partial^\alpha\omega^\beta + h_3\partial^\mu\varphi^\nu\,\partial^\alpha\varphi^\beta + 2h_4\partial^\mu\omega^\nu\,\partial^\alpha\varphi^\beta]\,, \tag{45}$$

where η' is the η' field, ρ^μ is the ρ field, ω^μ is the ω field, and φ^μ is the φ field;

$$h_1 = \frac{(F_{\overline{9}\overline{8}}\,\lambda/\sqrt{3}) - F_{\overline{8}\overline{8}}\,\lambda''}{F_{\overline{8}\overline{8}}F_{\overline{9}\overline{9}} - F_{\overline{8}\overline{9}}\,F_{\overline{9}\overline{8}}}\,, \tag{46}$$

$$h_2 = \frac{-(F_{\overline{9}\overline{8}}\,\lambda/\sqrt{3} + F_{\overline{8}\overline{8}}\,\lambda'')}{F_{\overline{8}\overline{8}}F_{\overline{9}\overline{9}} - F_{\overline{8}\overline{9}}F_{\overline{9}\overline{8}}}\,, \tag{47}$$

$$h_3=\frac{-F_{\overline{8}\overline{8}}\lambda''}{F_{\overline{8}\overline{8}}F_{\overline{9}\overline{9}}-F_{\overline{8}\overline{9}}F_{\overline{9}\overline{8}}}\,, \tag{48}$$

$$h_4=\frac{F_{\overline{9}\overline{8}}\lambda'}{2(F_{\overline{8}\overline{8}}F_{\overline{9}\overline{9}}-F_{\overline{8}\overline{9}}F_{\overline{9}\overline{8}})}\,. \tag{49}$$

$\eta'\to\rho^0+\gamma$ depends only on the first term, while $\eta'\to2\gamma$ depends on all four terms. The matrix elements for these decays are given by [19]

$$\langle\gamma q\sigma;\rho_3 k\lambda|\eta'p\rangle=-8i(2\pi)^4\delta^4(p-k-q)(eg_\rho/m_\rho^2)h_1N_{\eta'}(p)N_\rho(k)N_\gamma(q)\epsilon^{\nu\mu\alpha\beta}p_\nu k_\alpha\epsilon_\mu^{(\sigma)*}(q)\epsilon_\beta^{(\lambda)*}(k)\,, \tag{50}$$

where q, k, and p are the momenta of the γ, ρ, and η', respectively, and σ and λ are the polarizations of the γ and ρ;

$$\begin{aligned}\langle\gamma q_1\sigma;\gamma q_2\lambda|\eta'p\rangle=&-ie^2(2\pi)^4\delta^4(p-q_1-q_2)N_{\eta'}(p)N_\gamma(q_1)N_\gamma(q_2)\epsilon_\mu^{(\sigma)*}(q_1)\epsilon_\beta^{(\lambda)*}(q_2)\epsilon^{\nu\mu\alpha\beta}p_\nu q_{2\alpha}\\&\times[(8h_1g_\rho^2/m_\rho^4)+(8h_2g_{88}^2/3m_\omega^4)+(8h_3g_{89}^2/3m_\varphi^4)+(16h_4g_{88}g_{89}/3m_\omega^2m_\varphi^2)]\,,\end{aligned} \tag{51}$$

where q_1 and q_2 are the momenta and σ and λ are the polarizations of the photons. p is the momentum of the η' meson. Forming the decay widths and performing the necessary sums and integrations yields

$$\Gamma(\eta'\to\rho^0+\gamma)=8\alpha m_{\eta'}{}^3h_1^2(g_\rho/m_\rho^2)(1-m_\rho^2/m_{\eta'}{}^2)^3 \tag{52}$$

and

$$\begin{aligned}\Gamma(\eta'\to2\gamma)=&\,16\pi\alpha^2m_{\eta'}{}^3h_1^2(g_\rho/m_\rho^2)^4\\&\times[1+(g_{88}{}^2m_\rho^4h_2/3g_\rho^2m_\omega^4h_1)+(g_{89}{}^2m_\rho^4h_3/3g_\rho^2m_\varphi^4h_1)+(2g_{88}g_{89}m_\rho^4h_4/3g_\rho^2m_\varphi^2m_\omega^2h_1)]^2.\end{aligned} \tag{53}$$

Experimentally[20] the branching ratio for these decays is

$$\Gamma(\eta'\to2\gamma)/\Gamma(\eta'\to\rho^0+\gamma)=0.16\pm0.11. \tag{54}$$

Since $\eta'\to\rho^0+\gamma$ depends only on h_1 while $\eta'\to2\gamma$ depends on h_1, h_2, h_3, and h_4, it is possible to determine a numerical value of λ''. In terms of the ω-φ mixing model of Augustin *et al.*,[21] $g_{88}=-m_\omega^2/f_\omega$ and $g_{89}=-m_\varphi^2/f_\varphi$, where $f_\omega^2/4\pi=14.8\pm2.8$ and $f_\varphi^2/4\pi=11.0\pm1.6$. (The ω-φ mixing angle found in this model is $\theta_y=40.8°\pm3.5°$.) λ and λ' have the values $\lambda=0.348\pm0.024$ and $\lambda'=0.026\pm0.005$.[22] The KSRF relation is used to evaluate g_ρ. The masses of the ρ and η' are taken as 765 MeV and 958 MeV, respectively. These numerical values yield

$$\lambda''/\lambda=(F_{\overline{9}\overline{8}}/\sqrt{3}\,F_{\overline{8}\overline{8}})(1+\delta),\quad \delta=0.14 \text{ or } -0.06. \tag{55}$$

Due to the large uncertainties in the experimental data, no attempt has been made to establish an error range for these values of δ. At the present time such a range would be large and of questionable significance.

The (8,9) components of F_{ab} are given in I in terms of m_η, $m_{\eta'}$, m_π, m_K, m_κ, F_π, F_K, F_κ, and an undetermined angle. Using the first-order symmetry-breaking result $3F_\eta=4F_K-F_\pi$ ($F_\eta=F_{\overline{8}\overline{8}}$), and the numerical values[24] $F_\pi/F_K=0.885$ and $F_\kappa/F_\pi=0.379$, yields numerical values for these components of F_{ab}. These values and the value of λ'' given by Eq. (55), when substituted into Eqs. (52) and (53), give

$$\Gamma(\eta'\to\rho^0+\gamma)=\begin{cases}114\pm36\text{ keV for }\delta=0.14\\16\pm5\text{ keV for }\delta=-0.06,\end{cases} \tag{56}$$

$$\Gamma(\eta'\to2\gamma)=\begin{cases}18\pm3\text{ keV for }\delta=0.14\\2.6\pm0.5\text{ keV for }\delta=-0.06.\end{cases} \tag{57}$$

Experimentally[20]

$$\Gamma(\eta'\to\rho^0+\gamma)<1.3\text{ MeV}, \tag{58}$$

$$\Gamma(\eta'\to2\gamma)<0.3\text{ MeV}. \tag{59}$$

Consider the decay $\omega\to3\pi$. The Feynman diagrams for this process are given in Fig. 1. The intermediate state of the tree diagram is restricted by G parity and isospin conservation to be only a ρ meson. Specializing the Lagrangian of Eq. (36) to this process results in

$$\mathcal{L}=\mathcal{L}(\rho\to2\pi)-(2\lambda/\sqrt{3}\,F_\pi)\epsilon_{\mu\nu\alpha\beta}[4\pi_a\partial^\mu\rho_a^\nu\partial^\alpha\omega^\beta+\epsilon_{abc}(g_\rho^{-1}+\tfrac{1}{2}k)\pi_a\partial^\mu\pi_b\partial^\nu\pi_c\partial^\alpha\omega^\beta]\,. \tag{60}$$

k is defined in Eq. (41) and $\mathcal{L}(\rho\to 2\pi)$ is given by[25]

$$\mathcal{L}(\rho\to 2\pi)=-\epsilon_{abc}\,g_\rho^{-1}(m_\rho^2\pi_a\,\partial^\mu\pi_b\,\rho_{\mu c}+\tfrac{1}{2}\lambda_A\partial_\mu\pi_a\,\partial_\nu\pi_b\,\partial^\mu\rho_c^\nu)\ . \tag{61}$$

π_a is the pion field and λ_A is the anomalous magnetic moment of the A_1 meson.

The matrix element is given by

$$\begin{aligned}\langle\pi a k_1;\pi b k_2;\pi c k_3|\omega\sigma q\rangle=&(2\pi)^4\delta^4(q-k_1-k_2-k_3)N_\pi(k_1)N_\pi(k_2)N_\pi(k_3)N_\omega(q)(16\lambda/\sqrt{3}\,F_\pi)\epsilon_{abc}\epsilon^{\mu\nu\alpha\beta}k_{1\mu}k_{2\nu}k_{3\alpha}\epsilon_\beta^{(\sigma)}(q)\\&\times\{m_\rho^2g_\rho^{-1}(1-\tfrac{1}{4}\lambda_A)[((k_1+k_2)^2+m_\rho^2)^{-1}+((k_1+k_3)^2+m_\rho^2)^{-1}\\&+((k_2+k_3)^2+m_\rho^2)^{-1}]+\tfrac{3}{4}[(\lambda_A-1)/g_\rho-\tfrac{1}{2}k]\}\,,\end{aligned} \tag{62}$$

where k_1, k_2, and k_3 are the momenta and a, b, and c are the isotopic spins of the three pions. q is the momentum and σ is the polarization of the ω.

Forming the decay width and performing the necessary sums and integrations results in

$$\Gamma(\omega\to 3\pi)=(16\lambda/\sqrt{3}\,F_\pi)^2m_\omega^3m_\rho^4[3\pi^3(2)^6g_\rho^2]^{-1}(1-\tfrac{1}{4}\lambda_A)^2(I_1+2CI_2+C^2I_3). \tag{63}$$

I_1, I_2, and I_3 are phase-space factors and C is a constant:

$$C=\tfrac{3}{4}(m_\omega/m_\rho)^2(1-\tfrac{1}{4}\lambda_A)^{-1}(\lambda_A-1-\tfrac{1}{2}kg_\rho). \tag{64}$$

$$I_1=3.227\times10^{-3},\quad I_2=6.076\times10^{-4},\quad I_3=1.145\times10^{-4}. \tag{65}$$

Rather than calculating the decay width it is more convenient to evaluate the branching ratio $\Gamma(\omega\to 3\pi)/\Gamma(\omega\to\pi^0\gamma)$ since this quantity is independent of λ. Using just the second term of Eq. (60) results in

$$\Gamma(\omega\to\pi^0\gamma)=(16\lambda/\sqrt{3}\,F_\pi)^2\alpha g_\rho^2[3(2)^5m_\rho^4m_\omega^3]^{-1}(m_\omega^2-m_\pi^2)^3. \tag{66}$$

Thus the branching ratio is given by

$$\Gamma(\omega\to 3\pi)/\Gamma(\omega\to\pi^0\gamma)=(8\pi^3\alpha)^{-1}(m_\rho/F_\pi)^4(1-\tfrac{1}{4}\lambda_A)(1-m_\pi^2/m_\omega^2)^{-3}(I_1+2CI_2+C^2I_3). \tag{67}$$

Assuming the values $\lambda_A=0.4\pm0.3$ (see Ref. 25) and $kg_\rho=-4$ (see Ref. 26) yields

$$\Gamma(\omega\to 3\pi)/\Gamma(\omega\to\pi^0\gamma)|_{\mathrm{TH}}=9.3\pm2.8. \tag{68}$$

Experimentally[20]

$$\Gamma(\omega\to 3\pi)/\Gamma(\omega\to\pi^0\gamma)|_{\mathrm{TH}}=9.3\pm2.1. \tag{69}$$

Consider the decay $\eta\to 2\pi+\gamma$. The Feynman diagrams for this process are given in Fig. 2. The intermediate states are restricted to ρ mesons by G parity and isospin conservation. The strong-interaction Lagrangian is given by

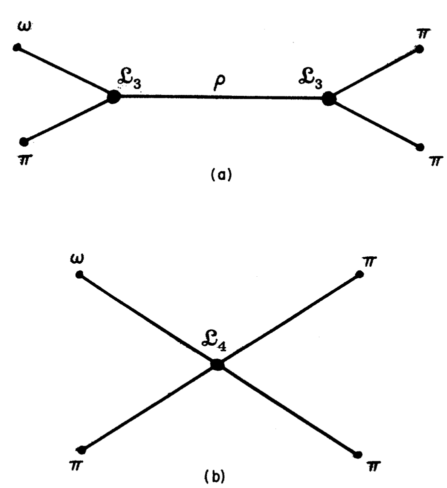

FIG. 1. Feynman diagrams for the decay $\omega\to 3\pi$. (a) Tree contribution. (b) Seagull contribution.

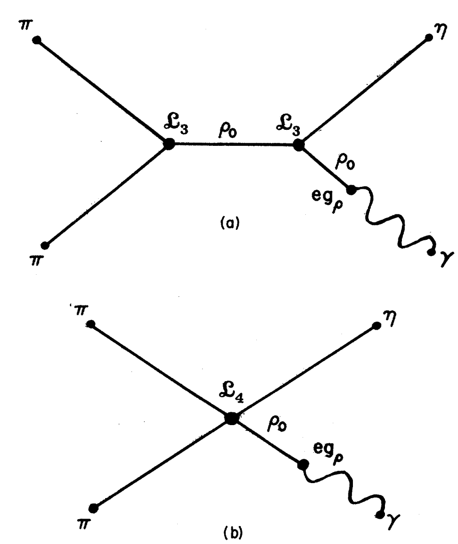

FIG. 2. Feynman diagrams for the decay $\eta\to 2\pi+\gamma$. (a) Tree contribution. (b) Seagull contribution.

$$\mathcal{L} = \mathcal{L}(\rho \to 2\pi) - 4h\epsilon_{\mu\nu\alpha\beta}\eta\partial^{\mu}\rho_a^{\nu}\partial^{\alpha}\rho_a^{\beta} + \epsilon_{abc}\epsilon_{\mu\nu\alpha\beta}(B_1\eta\partial^{\mu}\pi_a\partial^{\nu}\pi_b + B_2\pi_a\partial^{\mu}\pi_b\partial^{\nu}\eta)\partial^{\alpha}\rho_c^{\beta}. \tag{70}$$

The matrix element is given by

$$\langle \pi a k_1; \pi b k_2; \gamma\sigma q|\eta p\rangle = 16eh(1-\tfrac{1}{4}\lambda_A)N_\pi(k_1)N_\pi(k_2)N_\eta(p)N_\gamma(q)(2\pi)^4\delta^4(p-q-k_1-k_2)\epsilon_{ab3}\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu^{(\sigma)*}(q)p_\nu k_{1\alpha}k_{2\beta}$$
$$\times\{[(k_1+k_2)^2+m_\rho^2]^{-1}+[4m_\rho^2(1-\tfrac{1}{4}\lambda_A)]^{-1}[\lambda_A+(B_1+B_2)g_\rho/2h]\}, \tag{71}$$

where k_1 and k_2 are the momenta and a and b are the isotopic spins of the pions. q is the momentum and σ is the polarization of the photon. p is the momentum of the η. The decay rate resulting from this matrix element is

$$\Gamma(\eta \to 2\pi+\gamma) = 4e^2h^2(1-\tfrac{1}{4}\lambda_A)^2 m_\eta^3\pi^{-3}(J_1+2DJ_2+D^2J_3), \tag{72}$$

where J_1, J_2, and J_3 are phase-space factors and D is a constant,

$$J_1 = 3.994\times10^{-5},\quad J_2 = 5.479\times10^{-5},\quad J_3 = 7.685\times10^{-5}, \tag{73}$$

$$D = (m_\eta/2m_\rho)^2(1-\tfrac{1}{4}\lambda_A)^{-1}\{\lambda_A + (2F_\eta/F_\pi)[1-(F_\eta/\sqrt{2}\,F_\pi)^2-(F_{\overline{98}}{}^2/W)(1+F_{\eta'}/F_\eta)]\}, \tag{74}$$

$$W = (g_{\overline{98}}/m_D)^2 + (g_{\overline{99}}/m_E)^2 + F_{\overline{98}}{}^2 + F_{\overline{99}}{}^2, \tag{75}$$

$$F_\eta = F_{\overline{88}},\quad \text{and}\quad F_{\eta'} = F_{\overline{99}}. \tag{76}$$

The decay width for the process $\eta \to 2\gamma$ is given by[27]

$$\Gamma(\eta \to 2\gamma) = 16\pi\alpha^2 m_\eta^3 h^2(g_\rho/m_\rho^2)^4[1-(g_{88}{}^2m_\rho^4/3g_\rho^2m_\omega^4)+(\lambda' g_{89}g_{88}m_\rho^4/\sqrt{3}\,\lambda g_\rho^2 m_\omega^2 m_\varphi^2)]^2. \tag{77}$$

Thus the branching ratio $\Gamma(\eta\to 2\pi+\gamma)/\Gamma(\eta\to2\gamma)$ has the value

$$\Gamma(\eta \to 2\pi+\gamma)/\Gamma(\eta\to 2\gamma) = (4\pi^3\alpha)^{-1}(m_\rho/F_\pi)^4(1-\tfrac{1}{4}\lambda_A)^2(J_1+2DJ_2+D^2J_3)$$
$$\times[1-(g_{88}{}^2m_\rho^4/3g_\rho^2m_\omega^4)+(\lambda' g_{89}g_{88}m_\rho^4/\sqrt{3}\,\lambda g_\rho^2m_\omega^2m_\varphi^2)]^{-2}. \tag{78}$$

If either of the values $D=1.4$ or 0.006 is used, the theory matches the experimental result[20]

$$\Gamma(\eta\to 2\pi+\gamma)/\Gamma(\eta\to2\gamma)|_{\text{EX}} = 0.14\pm0.03. \tag{79}$$

The value $D=-1.4$ can be eliminated since it would require W to be negative. From Eq. (75) it can be seen that this is not physically acceptable since the components of g_{ab} and F_{ab} must be real. Thus by requiring the branching ratio of Eq. (78) to match the experimental value, it is possible to select a unique value for W.

As a final application consider the decay $\varphi\to3\pi$. Since both the ω and the φ are neutral nonstrange vector mesons, this process is identical to the decay $\omega\to3\pi$. Thus the matrix element is given by Eq. (62) with all ω-dependent parameters replaced by φ-dependent ones. Since the mass of the φ is 1020 MeV, the major contribution to this decay will arise when the intermediate ρ meson of the tree diagram is on its mass shell. Considering only this contribution and the seagull term results in

$$\Gamma(\varphi\to3\pi) = 4\pi^{-1}(\lambda'/F_\pi)^2(a^2-m_\pi^2)^{3/2}\{1+[3m_\varphi^7I(1+\lambda_A)^2/128\pi^2m_\rho^2F_\pi^2(a^2-m_\pi^2)^{3/2}]\}, \tag{80}$$

where $a=(m_\varphi^2+m_\pi^2-m_\rho^2)/2m_\varphi$ and $I=2.22\times10^{-4}$ is a phase-space factor. Using the corrected value of λ' yields

$$\Gamma(\varphi\to3\pi)|_{\text{TH}} = (0.064\pm0.25)\text{ MeV}. \tag{81}$$

This value compares favorably with the experimental value[20]

$$\Gamma(\varphi\to3\pi)|_{\text{EX}} = (0.71\pm0.26)\text{ MeV}. \tag{82}$$

VIII. DISCUSSION

In the preceding sections, a hard-meson current-algebra model has been developed which allows processes involving an odd number of unnatural-parity particles. These processes, examples of which are the photon decays of π^0 and η and the four-point decays $\omega\to3\pi$ and $\eta\to2\pi+\gamma$, were totally prohibited in the previous theory.[5] The difficulties are traceable to the hypothesis of PCAC and hence the starting point of this analysis was the modification of PCAC to a more general form (Eq. 18). This modification was made in such a way as not to disturb the previous successes of PCAC. This analysis was actually carried out for three-point functions in II. The analysis presented here includes the three-point model and in addition extends the theory to the η' meson and four-point functions.

Due to this modification of PCAC, several interesting results have occurred. First it is found (Eq. 24) that if PCAC is not pole-dominated, then the σ commutator cannot be either. It should be emphasized that this result is due to the assumption that the currents obey the $SU(3)\times SU(3)$ current algebra and is thus model-independent [with the exception that the Schwinger terms are required to be c numbers and symmetric in $SU(3)\times SU(3)$ indices]. Hence if pole dominance of PCAC is to be abandoned, pole dominance of the σ commutator must be also.

Second it is found that nonpole dominance of PCAC leads to a Lagrangian variation under chiral transformation which is not proportional to the divergence of the current (Eq. 33). The extra terms appearing in the variation are functions of the nonpole parts of PCAC and may lead to contributions which are not Lorentz scalars. If it is demanded that the variation be a Lorentz scalar (as is done in this work), then definite restrictions on the nonpole parts of PCAC result. It is interesting to note that the equality between the Lagrangian variation and the divergence of the current can be reestablished by demanding that the PCAC-breaking terms ($\mathfrak{F}_a$) commute with the canononical variables s and v^i. However, it is not possible to do this and maintain the three-point model of II simultaneously.

Now consider the numerical predictions of the theory. With the inclusion of the η' meson, the three-point theory has now been applied to a number of decays involving the three neutral pseudoscalar mesons (π^0, η, and η') and the three neutral vector mesons (ρ^0, ω, and φ). All of these predictions are in good agreement with experiment, where such data exist. The models discussed in the Introduction also predict some of these same decay widths with varying success. In particular, Adler[11] fits $\Gamma(\pi^0\to 2\gamma)$ but has difficulty predicting the experimental value for $\Gamma(\eta\to 2\gamma)$. This model, as extended by Glashow, Jackiw, and Shei[13] uses the experimental values $\Gamma(\pi^0\to 2\gamma)$ and $\Gamma(\eta\to 2\gamma)$ as input and predicts values for $\Gamma(\eta'\to 2\gamma)$. The numerical results of Gounaris[15] are considerably different from those presented here and in II. With respect to the η' decays, all models that consider these decays predict numerical values that are less than the experimental upper limits[20] of 300 keV for $\Gamma(\eta'\to 2\gamma)$ and 1.32 MeV for $\Gamma(\eta'\to\rho^0+\gamma)$. However, all of these predictions are substantially different as can be seen from Table I. Thus an experimental determination of these widths, particularly $\Gamma(\eta'\to 2\gamma)$, would be of great interest.

When the theory is extended to four-point functions, it has been found necessary to add three third-order and two fourth-order terms to the PCC equation (Eq. 35) in order to satisfy the restrictions formulated in Sec. V. However only one new free parameter (k) has been added [Eq. (41)]. Although these higher-order terms will contribute to a variety of processes, experimental data exist for only a few, notably $\omega\to 3\pi$, $\varphi\to 3\pi$, and $\eta\to 2\pi+\gamma$. Thus numerical results have been calculated for these processes. The value of the constant k has been determined by using the experimental value[20] 9.3 ± 2.1 for the branching ratio $\Gamma(\omega\to 3\pi)/\Gamma(\omega\to\pi^0\gamma)$. Based on this determination, the theory predicts the value (0.64 ± 0.25) MeV for $\Gamma(\varphi\to 3\pi)$, in good agreement with the experimental value[20] of (0.71 ± 0.26) MeV. Owing to the dependence of $\Gamma(\eta\to 2\pi+\gamma)$ on the field-current coupling strengths of the D and E meson [Eqs. (72)–(76)], a unique prediction for this width is not possible until these coupling strengths are known. Several other models also consider these four-point decays. Brown, Munczek, and Singer[14] consider all three, using $\Gamma(\varphi\to 3\pi)$ as input, predicting a value for $\Gamma(\omega\to 3\pi)$, and determining $\Gamma(\eta\to 2\pi+\gamma)$ to within two free parameters which

TABLE I. Decay rates for $\eta'\to 2\gamma$ and $\eta'\to\rho^0+\gamma$ as predicted by various models.

Model	$\Gamma(\eta'\to 2\gamma)$ (keV)	$\Gamma(\eta'\to\rho^0+\gamma)$ (keV)
Experiment [a]	<300	<1320
This analysis [b]	18 ±3	114 ±36
This analysis [b]	2.6 ±0.5	16 ±5
Baracca and Bramon [c]	50 ±30	810 ±500
Riazuddin and Sarker [d]	15.1 ±3.3	130
Riazuddin and Sarker [e]	31.4	300
Glashow, Jackiw, and Shei [f]	≥80	...
Glashow, Jackiw, and Shei [g]	350 ±80	...
Glashow, Jackiw, and Shei [h]	120 ±30	...
Gounaris [i]	6 ±1	...

[a] Reference 20.

[b] Due to the quadratic dependence of $\Gamma(\eta'\to 2\gamma)/\Gamma(\eta'\to\rho^0+\gamma)$ on the constant λ'', two solutions exist. See Eqs. (52)–(57).

[c] Reference 16.

[d] Based on the assumption that η-η' mixing is predicted by vector dominance without symmetry breaking and $\tilde{U}(12)$ or quark model. See Ref. 9.

[e] Inputs are used to fix η-η' mixing and predict the symmetry breaking in $P\to V+\gamma$. See Ref. 9.

[f] Based on a consistency requirement for a sum rule involving $\Gamma(\pi^0\to 2\gamma)$, $\Gamma(\eta\to 2\gamma)$, and $\Gamma(\eta'\to 2\gamma)$. See Ref. 13.

[g] Based on the assumption of a quark model. See Ref. 13.

[h] Based on the assumption of an integrally charged triplet model. See Ref. 13.

[i] Reference 15.

can be determined by a fit to experimental data. Perrin[7] and Brown and West[8] predict values for $\Gamma(\omega \to 3\pi)$, while Baracca and Bramon[16] predict a value for $\Gamma(\eta \to 2\pi + \gamma)$. However all of these models ignore any seagull terms that may exist. As can be seen from Eqs. (60) and (70), the model presented here predicts definite nonzero seagull contributions to these decays.

There are a number of other interesting processes that will be effected by the PCAC breaking terms. The three-point interactions will contribute to the decays $\eta \to \pi + 2\gamma$, $\omega \to 2\pi + \gamma$, and $\varphi \to 2\pi + \gamma$ through tree diagrams. The first of these decays is known experimentally and upper limits have been established for the other two.[20] This model will also contribute to $\gamma + \pi \to \pi + \pi$ through both tree and seagull diagrams. This process is of interest since it is known to contribute to $\pi + N \to \pi + N + \gamma$. The application of the theory to some of these processes is now in progress.

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[18]g_{ab} and F_{ab} have the following nonzero values: $F_{ij} = F_K \delta_{ij}$ for $i,j = 4,5,6,7$; $F_{\bar{i}\bar{j}} = F_\pi \delta_{\bar{i}\bar{j}}$ for $i,j = 1,2,3$; $F_{\bar{i}\bar{j}} = F_K \delta_{\bar{i}\bar{j}}$ for $\bar{i},\bar{j} = \bar{4},\bar{5},\bar{6},\bar{7}$; $g_{ij} = g_\rho \delta_{ij}$ for $i,j = 1,2,3$; $g_{ij} = g_{K^*} \delta_{ij}$ for $i,j = 4,5,6,7$; $g_{\bar{i}\bar{j}} = g_A \delta_{\bar{i}\bar{j}}$ for $i,j = \bar{1},\bar{2},\bar{3}$; $g_{\bar{i}\bar{j}} = g_{K_A} \delta_{\bar{i}\bar{j}}$ for $\bar{i},\bar{j} = \bar{4},\bar{5},\bar{6},\bar{7}$. In addition $F_{\bar{i}\bar{j}}$, g_{ij}, and $g_{\bar{i}\bar{j}}$ are nonzero for $i,j = 8,9$. F_π is experimentally (97 ± 2) MeV in the Cabbibo theory of $\pi^\pm$ β-decay. The KSRF relation, $g_\rho{}^2 = 2m_\rho{}^2 F_\pi{}^2$, is used to evaluate g_ρ. μ_a and m_a are the masses of the S_a and $v_a{}^\mu$ fields, respectively. The metric used is $-g_{00} = g_{11} = g_{22} = g_{33} = 1$.

[19]States are normalized such that $N(q) = [2\omega_q (2\pi)^3]^{-1/2}$, where $\omega_q = (\vec{q}\,^2 + m^2)^{1/2}$. Polarization vectors are normalized such that $q_\mu \epsilon^{\mu(\sigma)}(q) = 0$ and $\epsilon^{\mu(\sigma)*} \epsilon_\mu{}^{(\sigma')} = \delta^{\sigma\sigma'}$.

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[26]Due to the quadratic dependence of the branching ratio on C, there exists another value (≈ 26) of kg_ρ that will

yield the identical result. However, this value would lead to large seagull contributions to this and other decays and so it is eliminated.

[27]The constants h, B_1, and B_2 of Eq. (70) are complicated functions of the various components of g_{ab} and F_{ab}. In deriving the results given in Eqs. (72)–(77), $\eta-\eta'$ has been neglected due to the smallness of the mixing angle ($\approx 10°$). In this approximation $h=\lambda/\sqrt{3}F_\eta$, $B_1=-2h(g_\rho{}^{-1}+\frac{1}{4}k)$, and $B_2=4(\lambda/g_\rho F_\pi\sqrt{3})[1-\frac{1}{2}(F_\eta/F_\pi)^2-(F_{\bar{9}\bar{8}}{}^2/W)\times(1+F_{\eta'}/F_\eta)]$. The value $kg_\rho=-4$ has also been assumed. W is the $(\bar{9},\bar{9})$ component of W_{ab}.

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Hard-Meson Current-Algebra Calculation of the γ-3π Amplitude

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The $\gamma+\pi\to\pi+\pi$ interaction is considered within the context of hard-meson current algebra and a model of PCAC (partially conserved axial-vector current) breakdown previously proposed. The form factor for this process is calculated and the γ-3π coupling constant evaluated. The results are in agreement with experimental data.

In the past several years a number of authors[1-4] have used various techniques to evaluate the coupling constant for the process $\gamma+\pi\to\pi+\pi$. This interaction is of interest since it is known to contribute to the process $\pi+N\to\pi+N+\gamma$. Arnowitt, Friedman, and Nath[5] have proposed a hard-meson model of the breakdown[6] of partial conservation of axial-vector current (PCAC) which allows the two-body photon decays of π^0, η, and vector mesons. Very recently[7] this model has been extended to higher order and successfully applied to the four-point processes $\omega\to3\pi$, $\phi\to3\pi$, and $\eta\to2\pi+\gamma$. These same interactions will also contribute to $\gamma+\pi\to\pi+\pi$.

Assuming that the isoscalar part of the electromagnetic current is dominated by the ω and ϕ mesons, the amplitude for $\gamma+\pi\to\pi+\pi$ is related to the following matrix element:

$$\langle\pi k_1 a:\pi k_2 b|J^\beta_{I=0}(0)|\pi k_3 c\rangle=(e/\sqrt{3})[g_{88}\langle\pi k_1 a:\pi k_2 b|\omega^\beta(0)|\pi k_3 c\rangle+g_{89}\langle\pi k_1 a:\pi k_2 b|\phi^\beta(0)|\pi k_3 c\rangle]\,. \tag{1}$$

k_1, k_2, and k_3 and a, b, and c are the momenta and isotopic spins of the pions. $J^\beta_{I=0}$ is the isoscalar part of the electromagnetic current. ω^β and ϕ^β are the ω and ϕ fields, respectively. g_{88} and g_{89} are the field-current coupling strengths of the ω and ϕ mesons to the eighth vector current. Hence the matrix element for $\gamma+\pi\to\pi+\pi$ is related to the matrix elements for $\omega\to3\pi$ and $\phi\to3\pi$.

The interaction Lagrangian for $\omega\to3\pi$ is given in I as

$$\begin{aligned}\mathcal{L}(\omega\to3\pi)=&-\epsilon_{abc}g_\rho{}^{-1}[m_\rho{}^2\pi_a\partial^\mu\pi_b\rho_{\mu c}+\tfrac{1}{2}\lambda_A\partial_\mu\pi_a\partial_\nu\pi_b\partial^\mu\rho_c^\nu]\\&-(2\lambda/\sqrt{3}F_\pi)\epsilon_{\mu\nu\alpha\beta}[4\pi_a\partial^\mu\rho_a^\nu\partial^\alpha\omega^\beta+\epsilon_{abc}(g_\rho{}^{-1}+\tfrac{1}{2}k)\pi_a\partial^\mu\pi_b\partial^\nu\pi_c\partial^\alpha\omega^\beta]\,,\end{aligned} \tag{2}$$

where π_a is the pion field, ρ_c^μ is the ρ-meson field, and ω^β is the ω field. $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Cevita symbol with $\epsilon^{0123}=1=-\epsilon_{0123}$. g_ρ and F_π are the field-current coupling strengths for the ρ and π mesons.[8] λ_A is the anomalous magnetic moment of the A_1 meson. The π-ρ-ω coupling constant is given by $g_{\pi\rho\omega}=8\lambda/\sqrt{3}F_\pi$. The first three terms of Eq. (2) lead to the Gell-Mann–Sharp–Wagner[9] ρ-dominated tree diagram. The last term leads to a direct seagull contribution. The value of kg_ρ was determined in I to be -4 by a fit to the experimental value of 9.3 ± 2.1 for the branching ratio $\Gamma(\omega\to3\pi)/\Gamma(\omega\to\pi^0\gamma)$.[10]

A straightforward calculation using the Lagrangian of Eq. (2) gives[11]

$$\begin{aligned}\langle\pi k_1 a:\pi k_2 b|\omega^\beta(0)|\pi k_3 c\rangle=&-i\left(\frac{16\lambda}{\sqrt{3}F_\pi}\right)\epsilon^{\mu\nu\alpha\beta}\epsilon_{abc}N_\pi(k_1)N_\pi(k_2)N_\pi(k_3)\frac{k_{1\mu}k_{2\nu}k_{3\alpha}}{(k_1+k_2-k_3)^2+m_\omega{}^2}\\&\times\left[\frac{m_\rho{}^2}{g_\rho}(1-\tfrac{1}{4}\lambda_A)\left(\frac{1}{m_\rho{}^2-s}+\frac{1}{m_\rho{}^2-u}+\frac{1}{m_\rho{}^2-t}\right)+\frac{3}{4}\left(\frac{\lambda_A+1}{g_\rho}\right)\right],\end{aligned} \tag{3}$$

where $s=-(k_1+k_2)^2$, $t=-(k_1-k_3)^2$, and $u=-(k_2-k_3)^2$. Since Eq. (1) requires this matrix element to be evaluated on the photon mass shell, $s+t+u=3m_\pi{}^2$. The ϕ matrix element will be identical to Eq. (3) except