Wojtaszek, R. E. Marshak, and Riazuddin, *ibid.* 136, B1053 (1964).

[14]Essentially we are multiplying (3.13) of paper III by the general infinitesimal rotation

$$\begin{pmatrix} 1 & \psi_1 & \psi_3 \\ -\psi_1 & 1 & \psi_2 \\ -\psi_3 & -\psi_2 & 1 \end{pmatrix}$$

and absorbing ψ_2 in the angle θ_p without loss of generality.

[15]The value $W=1.7$ corresponds to fitting the η' mass exactly with our formulas. This is not very different from the value $2F_K/F_\pi - 1 \simeq 1.56$ predicted from weak interactions. The value of $\alpha \simeq \frac{1}{2}\pi^0$ corresponds to $\alpha = \frac{1}{2}F_\pi$. See paper I.

[16]Particle Data Group, Rev. Mod. Phys. 42, 87 (1970).

[17]D. G. Sutherland, Phys. Letters 23, 384 (1966).

[18]Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. 161, 1612 (1967).

[19]S. Bose and A. Zimmerman, Nuovo Cimento 47, 669 (1967). See also D. G. Sutherland, Nucl. Phys. B2, 433 (1967), and H. Osborn and D. J. Wallace, Nucl. Phys. B20, 23 (1970).

[20]It should be stressed that our "tadpole" model is actually quite different in detail from the one of Coleman and Glashow. Thus our treatment is different from earlier approaches using the "tadpole" idea. See, for example, S. K. Yun, Phys. Rev. 173, 1764 (1968); L. Clavelli, Nuovo Cimento 52A, 73 (1967).

[21]Experimentalists parametrize the decay amplitude as being proportional to

$$1+\beta y\,,$$

where $y=3T_0/Q-1$. T_0 is the kinetic energy $\omega_0 - \pi^0$ of the π^0. The form $1-2\omega_0/\eta$ corresponds to $\beta=-0.48$ while the experimental value [Columbia-Berkeley-Purdue-Wisconsin-Yale collaboration, Phys. Rev. 149, 1049 (1966)] is $\beta=-0.478$.

[22]S. Weinberg, Phys. Rev. Letters 18, 188 (1967). See also Ref. 9.

[23]If the T amplitude for $\eta\to\pi^+\pi^-\pi^0$ is expanded as $T=\tau_0 + \tau_1(\omega_0-\pi^0)+\cdots$, the width in the nonrelativistic limit is given by

$$\Gamma(\eta\to\pi^+\pi^-\pi^0) = \frac{1}{64\eta(3.14)^3}\iint d\omega_+ d\omega_0 |T|^2 = \frac{\sqrt{3}Q^2}{1152(3.14)^2\eta}\left[\left(\tau_0+\frac{Q\tau_1}{3}\right)^2 + \frac{Q^2\tau_1^{\,2}}{36}\right],$$

where Q is the decay Q value.

[24]For large $\epsilon_+^{\,2}$ we have, from (3.6) and (3.8), the result

$$\delta R^2 = \frac{1+W}{1-W}\delta\tilde{K}^2 \simeq -3.86\delta\tilde{K}^2\,.$$

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Nonleading Regge Behavior in νW_2 and the Possibility of Fixed Poles with Polynomial Residues*

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We discuss some of the theoretical arguments for the existence in Compton scattering of right-signatured fixed poles with polynomial residues. We show that if one could "switch off" the strong interactions, then a fixed pole with residue linear in q^2 (the photon mass squared) would be necessary for the consistency of the fixed-q^2 dispersion relation for νT_2 (whose absorptive part νW_2 is measured in inelastic electroproduction). We show that if the above conjecture is correct, then there must be some energy dependence in νW_2 over and above the conventional leading Regge form (Pomeranchukon plus f-A_2). Evidence is presented for the presence of such "nonleading behavior" in a similar process. In addition we show why the on-shell $\sigma_{\rm tot}(\gamma p)$ could be compatible with the neglect of such a nonleading term. We find that a fixed pole with polynomial residue and the correct $q^2\to 0$ Thomson limit can be accommodated by the present data on νW_2 at large q^2. With the above assumptions on the fixed-pole behavior, we predict the high-energy behavior of νW_2 and find that asymptotically it must fall to a value substantially less than its present maximum magnitude.

I. INTRODUCTION

The amplitude for forward scattering of off-mass-shell photons on spin-averaged nucleons can be written in terms of

$$T_{\mu\nu} = 4\pi^2\alpha\left[\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)T_1 + \left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right)\frac{T_2}{M^2}\right], \tag{1.1}$$

which defines the structure functions $T_{1,2}(\nu, q^2)$ whose absorptive parts are $W_{1,2}$. Here P_μ, q_μ are the momenta of the nucleon and photon, respectively, $\nu = P\cdot q/M$ and M is the nucleon mass.

In traditional Regge language T_2 is considered to be described in terms of the Pomeranchukon P, and ordinary exchange-degenerate Regge trajectories f, A_2 with intercepts at $t=0$ of about $\frac{1}{2}$. However, as is well known, there may also be a right-signatured fixed pole at $J=0$ which contributes to the real part of T_2.[1] For such a situation the following sum rule holds[2]:

$$\left(1+\frac{q^2}{4M^2}\right)^{-1}\left[G_E{}^2(q^2)+\frac{q^2}{4M^2}G_M{}^2(q^2)\right]+\frac{2M}{q^2}\int_{\nu_{\rm TH}}^{\infty} d\nu\, \nu W_2^p(\nu,q^2)-\int_0^\infty \sum_{\alpha_i>0}\beta_i(q^2)\nu^{\alpha_i-1}d\nu\frac{2M}{q^2}=\frac{\pi M R_P(q^2)}{q^2}, \tag{1.2}$$

where $\nu_{\rm TH} = (2Mm_\pi + m_\pi{}^2 + q^2)/2M$ ($q^2 > 0$ spacelike), and where $R_P(q^2)$ is the residue of the fixed pole. Damashek and Gilman and also Dominguez, Ferro Fontan, and Suaya[3] have separately examined this relation at $q^2=0$ assuming that the proton's total photoabsorption cross section is well described at high energies by only a Pomeranchukon- and f-A_2-type term, and concluded that such a $J=0$ fixed pole exists in the on-mass-shell Compton amplitude. It is therefore not improbable that there is a $J=0$ fixed pole in the off-mass-shell amplitude, i.e., that $R_P(q^2)\neq 0$.

Were we to take the limit of Eq. (1.2) as $q^2\to\infty$ and suppose that $\nu W_2(\nu,q^2)\to F_2(\omega,\infty)$ in this domain,[4] with $\omega = 2M\nu/q^2$, then the following sum rule would result:

$$\int_0^\infty [F_2^p(\omega,\infty)-\sum_i \gamma_i(\infty)\omega^{\alpha_i-1}]d\omega = \pi M \lim_{q^2\to\infty} R_p(q^2)/q^2 \equiv R_p \pi M \tag{1.3}$$

(p, n subscripts and superscripts will refer to proton and neutron, respectively; whereas the combination pn will refer to proton-neutron difference data), where

$$\gamma_i(\infty)\equiv \lim_{q^2\to\infty}\beta_i(q^2)(q^2/2M)^{\alpha_i-1}$$

and the Born term is assumed to be negligible in the large-q^2 limit. Empirically the data[5] appear to show that to a good approximation $\nu W_2^p(\nu,q^2)$ has become a function of the single variable ω even for values of q^2 greater than about 1.5 $(\mathrm{GeV}/c)^2$. For such values of q^2, the Born term in (1.2) is negligible in size, and so one might then write (1.3) to a good approximation even for q^2 in the range $1.5 \leq q^2 \leq \infty$ (neglecting also the small q^2 dependence in ω threshold). Thus to the extent that one believes that $F_2^p(\omega, q^2)$, or at least the combination of integrals in (1.2), has in fact become independent of q^2 by $q^2 = 1.5$ $(\mathrm{GeV}/c)^2$, one can write $R_p(q^2)\simeq q^2 R_p$, i.e., $R_p(q^2)$ must be (nearly) linear in q^2 for the entire range $1.5\leq q^2\leq\infty$.

Without committing ourselves as to whether or not $F_2^p(\omega,q^2)$ has in fact become a function of only ω for $q^2 \geq 1.5$ $(\mathrm{GeV}/c)^2$, we can still examine (1.2) in that range assuming that the νW_2 "scaling" data[5] are those appropriate for at least *one* value of q^2 for which both the Born term and the errors invoked by approximating the true ω threshold by unity are negligible.

In evaluating the residue of the fixed pole $R_p(q^2)/q^2 \sim R_p$ in the range $1.5\leq q^2 \leq\infty$, we will for convenience refer to Eq. (1.4), below, in which (and in what follows) $F_2^p(\omega)$ is specifically taken to be the "scaling" data of Ref. 5 and the γ_i are the Regge residues appropriate to this data:

$$\int_0^\infty [F_2^p(\omega)-\sum\gamma_i\omega^{\alpha_i-1}]d\omega = R_p\pi M\,. \tag{1.4}$$

R_p in (1.4) could be either positive, negative, or zero. However, returning to the sum rule (1.2) and considering the limit $q^2\to 0$ while noting that

$$\lim_{q^2\to 0}\nu W_2(\nu,q^2)/q^2 = \sigma(\nu)/4\pi^2\alpha\,, \tag{1.5}$$

with $\sigma(\nu)$ the total photoabsorption cross section for on-shell photons of energy ν, we obtain

$$1+\frac{M}{2\pi^2\alpha}\left[\int_{\nu_{\rm TH}}^\infty \sigma(\nu)d\nu - \int_0^\infty\sum_{\alpha_i>0}\beta_i(0)\nu^{\alpha_i-1}d\nu\right]=\pi M\lim_{q^2\to 0}R_p(q^2)/q^2\,, \tag{1.6}$$

where $\nu_{\rm TH}$ is the threshold for pion production and the 1 on the left-hand side results from the Born term. The authors in Ref. 3 found that the left-hand side of (1.6) is consistent with unity and so we shall assume that it is indeed 1. Hence we deduce that

$$\lim_{q^2\to 0}R_p(q^2)/q^2 = 1/M\pi\,, \tag{1.7}$$

which implies that for small q^2, $R_p(q^2)=q^2\mathcal{R}_p + O(q^4)$, where $\mathcal{R}_p = 1/M\pi$.

At this stage there is no reason to suppose that $R_p = \mathcal{R}_p$ or that terms of $O(q^4)$ are not present in

$R_p(q^2)$ near $q^2=0$. Indeed if one *assumes* that the data for $F_2^p(\omega)$ have already ($\omega > 12$, say) attained a maximum value and that $F_2^p(\omega)$ is asymptotically falling off to $\omega=\infty$ with an $A+B\omega^{-1/2}$ behavior,[6] then it is clear that the left-hand side of (1.4) is negative and that $R_p<0$. This implies that $R_p(q^2)$ at $q^2=0$ and $q^2=1.5$ ($q^2\to\infty$) have opposite signs. In Ref. 7 the sum rule (1.2) was tested directly for fixed q^2 of 1.5 $(\mathrm{GeV}/c)^2$ and, with the assumption that the ν ($\omega=2M\nu/q^2$) behavior was of the form $a+b\nu^{-1/2}$, the fixed-pole residue was indeed shown to have an opposite sign to that found in on-shell scattering. This would imply that the fixed-pole residue changes sign between $q^2=0$ and $q^2=1.5$ $(\mathrm{GeV}/c)^2$, and scaling by $q^2=1.5$ would say that at this point the residue has become nearly proportional to q^2. Whereas such a state of affairs is, in principle, not impossible, a dynamical origin for such a perverse behavior is difficult to imagine.

Theoretical arguments have been made[8] which suggest that the residue of the fixed pole may be a polynomial in q^2. However some of the arguments of Ref. 8 would fail in the presence of fixed poles in photoproduction amplitudes (as has been pointed out in Refs. 1 and 9, it appears that charged-pion photoproduction over a broad range of t, has an energy dependence from 2 to 16 GeV, which is compatible with fixed $J=0$ poles playing the dominant role in the t channel). However, the interesting observation was made in Ref. 1, and reiterated by Harari,[10] that, in the absence of strong interactions and to lowest order in α, one would expect only the Thomson term to survive in (1.6), so that

$$\lim_{q^2\to 0}\frac{R_p(q^2)}{q^2}=\frac{1}{M\pi}\,. \tag{1.7}$$

This argument can be extended to all values of q^2.[11] If we "turn off" the strong interactions in (1.2), we expect that $G_M=\mu G_E=1$ for all q^2 with $\mu=1$ (there being no anomalous moment to this order in α). This procedure would then require $R_p(q^2)\equiv q^2/\pi M$ for *all* q^2 in the absence of strong interactions. If the fixed pole is purely electromagnetic in origin, then this would be true even in the presence of strong interactions.

In any event, restricting the residue to be a polynomial in (1.7) and using the fact that the limit in (1.3) must be finite as $q^2\to\infty$, we are forced to conclude that

$$R_p\equiv\mathcal{R}_p=1/M\pi\,, \tag{1.8}$$

which is the result obtained immediately in the above argument. The sum rule (1.4) would then become

$$\int_0^\infty d\omega[F_2^p(\omega)-F_2^{p\ \mathrm{Regge}}(\omega)]=1\,. \tag{1.9}$$

If it is the case that the behavior of $F_2^p(\omega)$ for $\omega\gtrsim 12$ is simply an asymptotic decay of form $A+B/\sqrt{\omega}$, then the sum rule (1.9) is not satisfied and the residue of the fixed pole cannot be a polynomial in q^2. Conversely, if the residue is to be a polynomial in q^2, then the functional form of $F_2^p(\omega)$ for $\omega>12$ must be more complicated than the above. At present it is not possible to say which of these alternatives is nature's choice. We show in this paper that the latter situation, as represented by (1.9), is a viable possibility. In particular, we will find that an effective trajectory of intercept $\alpha\sim-\frac{1}{2}$ contributing to $F_2^p(\omega)$ enables one to satisfy (1.9) and that there is evidence elsewhere, namely in pp and $p\bar{p}$ scattering, for the importance of such a term.[12]

We put additional constraints on such a trajectory's contribution to $F_2(\omega)$ by examining its role in the difference of proton and neutron data, $F_2^p(\omega)-F_2^n(\omega)$, and by demanding that

(1) $F_2^p(\omega)-F_2^n(\omega)$ satisfy the well-known quark charge sum rule,[13] and

(2) the neutron fixed-pole residue R_n (defined analogously to R_p) be one of three fixed numbers 0, $\frac{2}{3}$, or 1. An attractive choice might be zero at $q^2=0$ which would imply that $R_n(q^2)=0$ for all q^2 if $R_n(q^2)$ is a polynomial.[14]

The resulting universal curves for $F_2^p(\omega)$ are then considered.

For comparison to the assumption that $F_2^p(\omega)$ has, in fact, scaled in the data of Ref. 5, we examine an alternative viewpoint, namely, we consider in Appendix C one of the possibilities proposed by suri and Yennie.[15] They propose that the νW_2 data presently available might not be close to the scaling limit but that $\tilde{F}_2$ (νW_2 minus a specified vector-dominance contribution which is diffractive in nature) might be. That is to say that in (1.4), $F_2^p(\omega)$ and γ_i separately possess important q^2 dependence. Nonetheless, the polynomial-residue assumption still demands that the integrated combination in (1.4) does *not*. The familiar $\nu W_2^p(\omega,q^2)$ data and their $\tilde{F}_2^p(\omega)$ "data" are plotted in Figs. 1 and 2, respectively. The $\alpha\sim-\frac{1}{2}$ contribution plays a very important role regardless of what one believes the scaling function to be. It remains impossible to obtain a positive or zero proton fixed pole in either of the scaling functions F_2 or $\tilde{F}_2$ without it.

II. THE NONLEADING TRAJECTORY

In this section we will establish the importance of a nonleading trajectory[16] of intercept $\alpha(0)$ near $-\frac{1}{2}$ in a process similar to γ-p scattering. In par-

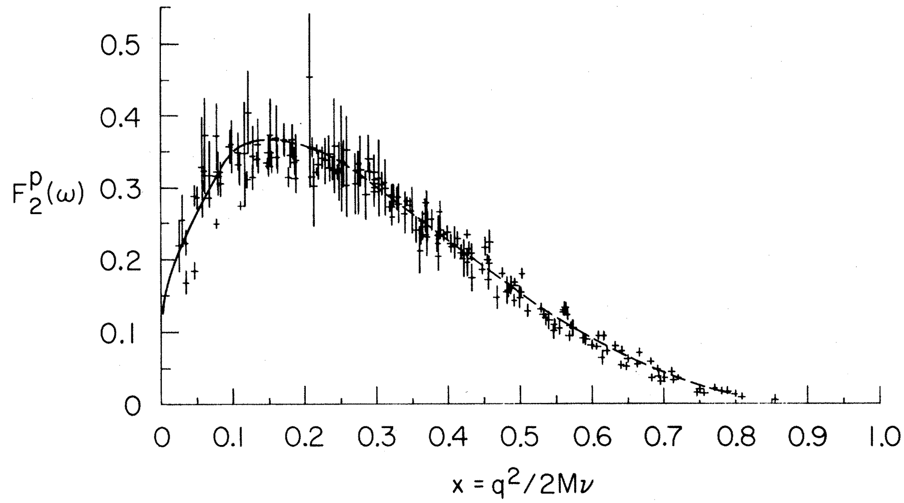

FIG. 1. Data for νW_2 plotted against the reciprocal of $\omega\,(=2M\nu/q^2)$ assuming $R\,(=\sigma_S/\sigma_T)=0$. The solid curve for $\omega > 12$ ($x < 0.08$) is the curve $0.12 + 0.462\omega^{-1/2} + 4.02\omega^{-3/2}$. The dashed curve below $\omega = 12$ (above $x = 0.08$) is an arbitrary hand-drawn curve through the data which encloses an area (I_p) of about 3.38 between $\omega = 1$ and $\omega = 12$.

ticular, we will look at $\sigma_{\text{tot}}(pp) + \sigma_{\text{tot}}(\bar{p}p) = \Sigma_T$, which is related by the optical theorem to an amplitude with the same crossing properties and presumably the same leading Regge contributions as the corresponding amplitude for $\sigma_{\text{tot}}(\gamma p)$.

Both pp and $p\bar{p}$ scattering may be considered to have contributions from a Pomeranchuk-type term, a ρ-ω-f-A_2 trajectory term, and other nonleading terms. In particular, a possible model for nonleading terms might be that of Regge cuts for which there would be an RR cut, an RRR cut, and so forth, with intercepts 0, $-\frac{1}{2}, \ldots$. Duality and exchange degeneracy put additional constraints on these terms; for example, the single Regge contribution to pp scattering must be purely real, which for a ρ-ω-f-A_2 intercept of $\frac{1}{2}$ constrains the single Regge contribution in $p\bar{p}$ to be purely imaginary. Moreover, duality requires that the single Regge exchange yield a *positive* imaginary contribution to $p\bar{p}$, which in turn tells us that the single Regge exchange contribution to pp is real and *negative*. The resulting approximate multi-Regge-cut phases in the forward direction are given in Table I.[17] It is apparent that in such a model Σ_T (which is proportional to $1/\nu$ times an amplitude) might be adequately described by[17]

$$\Sigma_T = 2a + b/\sqrt{\nu} + c/\nu^{3/2}\,, \tag{2.1}$$

where the $1/\nu$ imaginary contribution from $\sigma_{\text{tot}}(pp)$ and $\sigma_{\text{tot}}(\bar{p}p)$ have cancelled in the sum Σ_T since

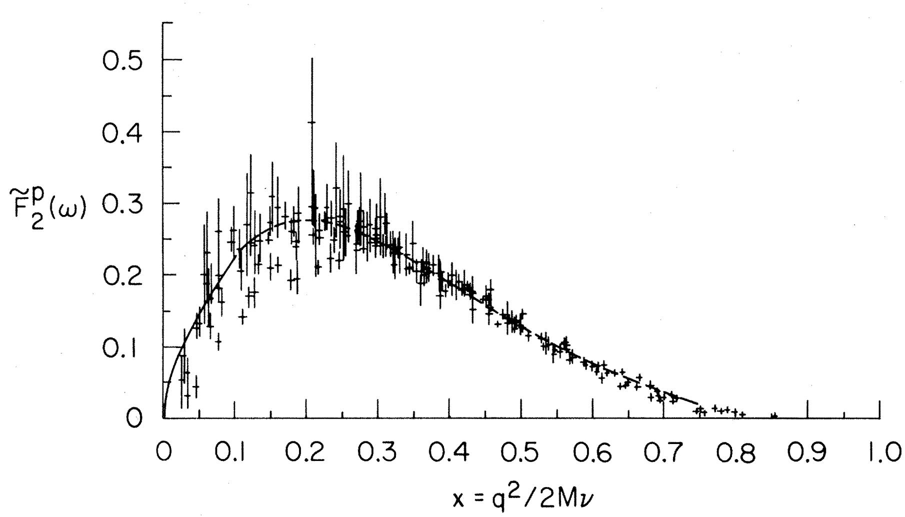

FIG. 2. The nondiffractive contribution to νW_2 in the suri-Yennie model (Ref. 15). The variable $x = 1/\omega$. The solid curve is the solution of Table VIII with $A = 0$, $3B = 0.56$, $C_p = 2.01$. The dashed curve is hand-drawn through the data below $\omega = 12$ (above $x = 0.08$).

TABLE I. Approximate multi-Regge-cut phases in the forward direction.

	Pomeranchukon	R	$R\otimes R$	$R\otimes R\otimes R$
pp	i	-1	i	1
$p\bar{p}$	i	i	$-i$	i
$\alpha_{\text{eff}}(0)$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$

they have opposite signs but the same magnitude.

To explore the validity of the hypothesis of an effective trajectory we have examined Σ_T in the region $2.00 \leq p_{\text{lab}} \leq 50$ GeV/c (using a smooth extrapolation of the pp data above 25 GeV/c.[18] We have attempted to fit Σ_T with the form

$$2a + b/\sqrt{\nu} + c/\nu^d. \tag{2.2}$$

Taking a to be 38.35 mb (from the pp data), we determine the best values of b and c for various values of d and plot the resulting χ^2 of the fits in Fig. 3.

There is a clear minimum in the χ^2 curve at $d \sim 1.5$. For a different value of a, say $a = 38.6$, the best value of d was found to be about 1.45. For any magnitude of a between 38 and 39 the same general features emerge; namely, a term c/ν^d is required with d between 1.3 and 1.7. A value of a larger than 39 is manifestly inconsistent with the trend of the higher-energy pp cross sections unless there is some significant leading Regge contributions with negative imaginary part (such an object would be inconsistent with present theoretical ideas of duality, etc.).

The low-energy cutoff used in making the above fits was p_{lab} of 2.75 GeV/c. In Table II we show the effect of varying the low-energy cutoff in the data fitting. At first sight it appears that the higher cutoffs give better χ^2 per degree of freedom, leading to higher-power behaviors for our additional nonleading object. However, this is not really the case because these "better fits" do not extrapolate at all well into the lower p_{lab} region. Our intent is to show that there is nonleading behavior in Σ_T, and therefore, we are looking for a term that is less significant at high energies due to its $1/\nu^d$ behavior. Therefore, in order to allow such a term to manifest itself, one should look for as low a cutoff as possible while avoiding threshold and/or resonance regions. Table II shows that one can reach a cutoff as low as 2.75 GeV/c before the solutions become unstable due to the nearby presence of the threshold region. [Certainly there is no evidence of a need for a $1/\nu$-type behavior, so that if nonleading terms are generated by multi-Regge iteration, the large $\ln(\nu/\nu_0)$ approximation must be largely valid.]

Therefore, at the very least, we see that there is definitely *something* in Σ_T over and above the naive $2a + b/\sqrt{\nu}$ leading behavior, and that it very probably has a $1/\nu^{3/2}$ dependence.

To exhibit the quality of the parametrization (2.2) of Σ_T, we plot in Fig. 4 the solution with the minimum χ^2 with

$$a = 38.35, \quad b = 41.3, \quad c = 121.1,$$

where ν is in GeV and Σ_T is in mb. The curve in Fig. 4 is seen to be extremely faithful to the data points. Further consideration of the effects of nonleading contributions such as we have investigated here in pp and $p\bar{p}$ will be discussed elsewhere.

Thus it seems that a nonleading term of the form $1/\nu^d$, with d in the region of 1.5, could be of importance in a process such as γp scattering to which it should also couple. Such a term should also appear in $\sigma_{\text{tot}}(K^+p) + \sigma_{\text{tot}}(K^-p)$ and also $\sigma_{\text{tot}}(\pi^+p) + \sigma_{\text{tot}}(\pi^-p)$. Unfortunately, resonance effects at low energies in these processes are more important than in pp and $p\bar{p}$ so that nonleading contributions such as we discussed become obscured.

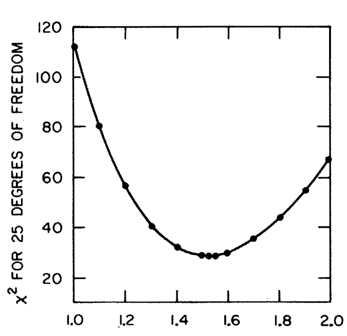

FIG. 3. Plot of χ^2 versus d for fits to the $pp + p\bar{p}$ total cross-section data (Fig. 4) with form $76.7 + b\nu^{-1/2} + c\nu^{-d}$ for $p_{\text{lab}} \geq 2.75$ GeV/c.

TABLE II. Variation of the minimum χ^2 and power for the additional object with the low-energy cutoffs. The "increasing goodness" of the χ^2 with higher cutoffs is misleading since the resulting fits do not extrapolate well in the lower p_{lab} region. The table shows a definite jump in χ^2 at $p_{\text{lab}} = 2.7$ GeV/c indicative of additional structure (e.g., resonant or threshold effects). $a = 38.35$.

p_{lab} cutoff	Degrees of freedom	min χ^2	d at min χ^2
2.6	28	54	1.35
2.65	27	49	1.4
2.7	26	47	1.4
2.75	25	28.3	1.5
2.8	24	26	1.5
2.85	23	23	1.6
2.95	21	16.2	1.7
3.15	17	10.3	1.9

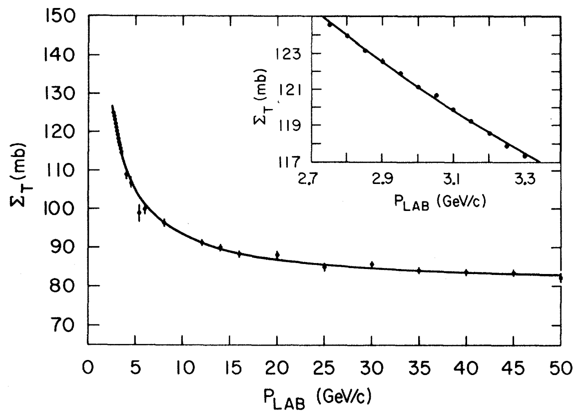

FIG. 4. $\Sigma_T = \sigma_{tot}(pp) + \sigma_{tot}(\bar{p}p)$ from 2.75 to 50 GeV/c incident laboratory momentum. The data are compiled from Ref. 16. The solid curve is a best fit to the data given by $76.7 + 41.3\,\nu^{-1/2} + 121.2\,\nu^{-1.52}$. The insert is an enlarged version of the region $2.75 \le p_{1ab} \le 3.5$ GeV/c. (Σ_T is in mb, ν is in GeV, and other quantities are in appropriate units.)

III. NONLEADING BEHAVIOR IN ELECTROPRODUCTION

In this section we wish to consider the effect of an additional nonleading term of the form C_p/ω^d in the asymptotic behavior of $F_2^p(\omega)$ for $\omega >$ some ω^*. In Sec. I we pointed out that $F_2(\omega)$ for $\omega > 12$ cannot be simply $A + B/\sqrt{\omega}$ if one is to obtain a non-negative fixed-pole residue in virtual Compton scattering. The addition of such a nonleading term makes it quite easy to obtain a positive fixed-pole residue and the problem becomes one of applying sensible constraints to restrict its contribution. With this in mind we make use of the inelastic e-p scattering data and the inelastic e-p- and e-n- difference data for $1 \le \omega \le \omega^*$ (ω^* we take to be 12 in this paper[19]), by demanding that any curve representing the asymptotic behavior of the scaling function $F_2(\omega)$ shall be consistent with the known data at $\omega = \omega^*$ and above. Hence, we require

$$F_2^p(\omega) = A + 3B/\sqrt{\omega} + C_p/\omega^d \tag{3.1}$$

and

$$F_2^p(\omega) - F_2^n(\omega) = B/\sqrt{\omega} + (C_p - C_n)/\omega^d\,, \tag{3.2}$$

for $\omega \ge \omega^*$, where A and B are the coupling strengths of the Pomeranchukon and the f-A_2 trajectory to the virtual photon. We have assumed pure F coupling for the f-A_2 trajectory in writing (3.1) and (3.2), this assumption being supported by the on-shell data[20] and favored by exchange degeneracy and universality arguments. We write $C_n = xC_p$, where x is related to the effective F value of the nonleading term by

$$F = (x-4)/10(x-1)\,, \tag{3.3}$$

and so for $F = 1$, we have $x = \frac{2}{3}$ (F is the antisymmetric octet coupling coefficient defined so that $F + D = 1$).

If we believed our $1/\omega^d$ object to be a triple Regge cut then d would be approximately $\frac{3}{2}$. Support for such an intercept was found in Sec. II.[21] The effective x of the exchanged object would then be $(\frac{2}{3})^3 = \frac{8}{27}$ which corresponds to $F \sim 0.5$, in the absence of coupled channels. Coupled channels might tend to increase the value of x.

In addition to the constraints (3.1) and (3.2), we invoke two sum rules. The first relates to the fixed pole [Eq. (1.9)]. With the asymptotic form (3.1), we obtain from (1.9)[22]

$$\begin{aligned} 1 &= I_p(\omega^*) - A\omega^* - 6B\sqrt{\omega^*} + C_p/(d-1)\omega^{*(d-1)}\,, \\ I_p(\omega^*) &\equiv \int_1^{\omega^*} F_2^p(\omega)d\omega\,. \end{aligned} \tag{3.4}$$

The second sum rule assumes the validity of the quark-model result[13]

$$\int_1^\infty \frac{d\omega}{\omega}[F_2^p(\omega) - F_2^n(\omega)] = \frac{1}{3}\,. \tag{3.5}$$

Using (3.2) for $\omega > \omega^*$ and substituting into (3.5) yields

$$H_{pn}(\omega^*) + 2B(\omega^*)^{-1/2} + C_p(1-x)/d\omega^{*d} = \tfrac{1}{3}\,, \tag{3.6}$$

where

$$H_{pn}(\omega^*) \equiv \int_1^{\omega^*} \frac{d\omega}{\omega}[F_2^p(\omega) - F_2^n(\omega)]$$

can be computed from the known data[5] (see Fig. 5). Hereafter, when referring to Eqs. (3.1) and (3.2), we assume them to be written for $\omega = \omega^*$. We take $I_p(\omega^*) = 3.32$ and $F_p(\omega^*) = 0.35$ for $\omega^* = 12$.

Finally we write the equation for the residue, $R_n(q^2)$, of the fixed pole in the electron-neutron inelastic scaling function, $F_2^n(\omega)$:

$$\begin{aligned} R_n &= \lim_{q^2 \to \infty} \frac{R_n(q^2)}{q^2} \\ &= I_p(\omega^*) - I_{pn}(\omega^*) - A\omega^* \\ &\quad - 4B(\omega^*)^{1/2} + C_p/(d-1)\omega^{*(d-1)}\,, \end{aligned} \tag{3.7}$$

where

$$I_{pn}(\omega^*) = \int_1^{\omega^*} [F_2^p(\omega) - F_2^n(\omega)]d\omega\,.$$

If one believes that the charge of the particle determines the fixed-pole residue then one should find $R_n = 0$. There are, however, other "reasonable" possibilities.[14] Hence we also explore the possibility of the values $R_n = \frac{2}{3}$ and $R_n = 1$.

Unfortunately there is considerable leeway in the proton-neutron difference experimental data.

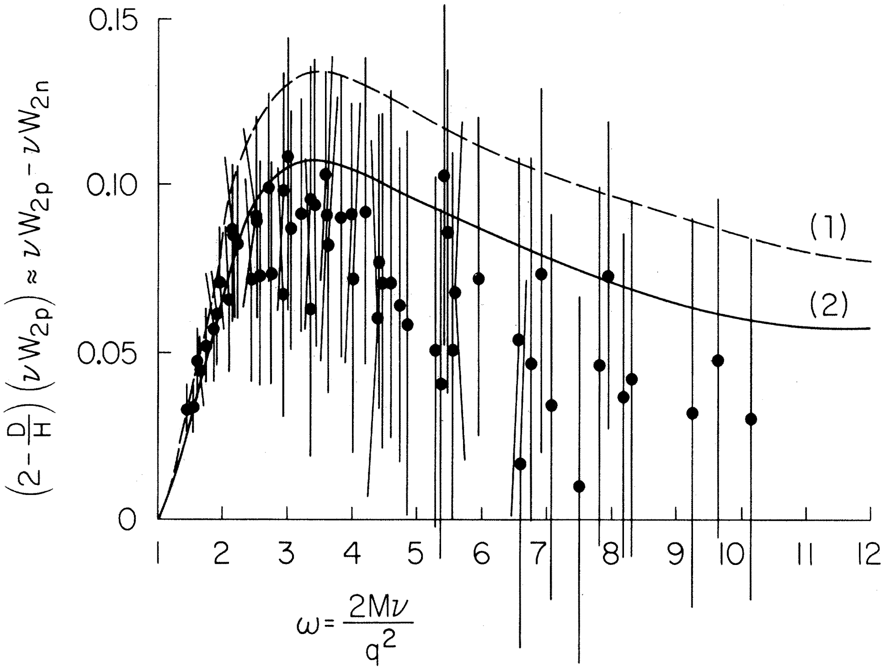

FIG. 5. Data for $(\nu W_2^{\text{proton}} - \nu W_2^{\text{neutron}})$ plotted against ω. The two hand-drawn curves are included in order to indicate the relationships between the magnitudes of H_{pn}, I_{pn}, F_2^{pn}, and the data. The curve labeled (1) has $I_{pn}=1.11$, $H_{pn}=0.23$, $F_2^{pn}=0.08$. Curve (2) has $I_{pn}=0.85$, $H_{pn}=0.17$, $F_2^{pn}=0.06$.

It is obvious from (3.6) and (3.2) (evaluated at ω^*) that $F_2^{pn}(\omega^*)$ and $H_{pn}(\omega^*)$ determine the quantities B and $C_p(1-x)$. However the experimental error bars for F_2^{pn} are quite large. It is easily possible to imagine that H_{pn} (Ref. 12) might be as small as 0.13 or as large as 0.23 with similar sorts of variation allowed for I_{pn} and F_2^{pn}. We have thus investigated the solutions as a function of $H_{pn}(\omega^*)$, $I_{pn}(\omega^*)$, and $F_2^{pn}(\omega^*)$ within the possible extremes of variation. Once $C_p(1-x)$ and B have been determined, we may use Eqs. (3.1) and (3.4) to determine C_p (and hence the effective x) and A.

In Tables III–VI we list the values of A, B, and C_p, which result from a sampling of values of H_{pn}, F_2^{pn}, and I_{pn} (all assumed evaluated for $\omega^*=12$) all within the extremes of the neutron-proton data. Each table corresponds to a different choice for the neutron and proton pole residue. In Appendix A we discuss the sensitivity of the results obtained to the value of d as well as the trends exhibited in the tables, and explain in more detail the criteria for choosing the preferred solutions which we shall present shortly. Basic to the choice of solutions is the observation that H_{pn}, F_2^{pn}, and I_{pn} are not really independent quantities, e.g., if one draws reasonable curves through the error bars, one obtains the following typical correlations corresponding to the two curves in Fig. 5:

H_{pn}	I_{pn}	F_2^{pn}
0.17	0.85	0.06
0.23	1.11	0.08

We note that in Tables III–VI there are no solutions with $H_{pn}(\omega^*) < 0.18$ for a proton residue $R_p=1$; and $R_p=0$ allow us only to reach $H_{pn}(\omega^*)=0.17$. Thus if one believes in our version of the proton fixed pole and in the constraints we impose, then the neutron fixed pole is "reasonable" in size only so long as $F_2^{pn}(\omega)$ lies above the central data-point values shown in Fig. 5.

The necessity for stretching the p-n data arises from the presence of the quark charge sum rule as a constraint. To illustrate the effect of altering the quark charge sum rule from a value of $\frac{1}{3}$ to, say, 0.28 (which is not to say that any theory predicts 0.28, this number was chosen merely for purposes of illustration) we exhibit in Figs. 6 and 7 the quantities A, B, C_p, and R_n as a function of H_{pn} [F_2^{pn} and I_{pn} are assumed to vary linearly with H_{pn} (see figure captions)]. It will be noticed that a solution with $R_n=0$, for example, is obtainable with A and $B>0$ for either case, but that the magnitude of H_{pn} required is far more reasonable in the latter case. Thus, if future experiments show that p-n-difference data are smaller than we have allowed for here, it will still be possible to retain polynomial residues for both neutron and

TABLE III. $R_p=1$, $R_n=0$ (neutron residue proportional to charge). H_{pn}, F_2^{pn}, and I_{pn} refer to the values at $\omega=12$ of $H_{pn}(\omega^*)$ etc. The table gives the values of A, B, C_p, F, I_{pn}, H_{pn}, and F_2^{pn} which are solutions to the constraint equations of Sec. III with the indicated choices for R_p and R_n. Values of H_{pn} and F_2^{pn} with their corresponding A, B, C_p, I_{pn}, and F, satisfying the constraint equations, are not given if they fall outside of reasonable limits on the p-n data.

A	B	C_p	H_{pn}	F_2^{pn}	I_{pn}	F
0.09	0.179	4.32	0.194	0.1	1.08	0.75
			0.1975	0.095	1.21	0.824
0.1	0.171	4.225	0.1975	0.1	0.97	0.705
			0.2005	0.095	1.09	0.77
			0.204	0.09	1.2	0.853
0.11	0.162	4.115	0.201	0.1	0.844	0.658
			0.204	0.095	0.976	0.72
			0.2075	0.09	1.09	0.79
			0.211	0.085	1.21	0.88
0.12	0.154	4.02	0.211[a]	0.09	0.97	0.735
			0.214[a]	0.085	1.096	0.816
			0.2175	0.08	1.21	0.913
0.13	0.1455	3.915	0.2175[a]	0.085	0.97	0.758
			0.221[a]	0.08	1.09	0.84
			0.224	0.075	1.22	0.956
0.14	0.1365	3.81	0.224[b]	0.08	0.976	0.779
			0.2275[b]	0.075	1.09	0.875
			0.231	0.07	1.204	0.99
0.15	0.128	3.7	0.231	0.075	0.964	0.8
			0.234	0.07	1.21	1.9

[a]A "preferred" solution.
[b]A possible but unlikely combination of H_{pn}, F_2^{pn}, and I_{pn}.

TABLE IV. $R_p=1$, $R_n=\frac{2}{3}$ (quark-model possibility). Format as in Table III.

A	B	C_p	H_{pn}	F_2^{pn}	I_{pn}	F
0.04	0.2235	4.85	0.1775	0.1	1.03	1.085
			0.181	0.095	1.14	1.24
0.05	0.2145	4.74	0.181	0.1	0.9	1.0
			0.184	0.095	1.02	1.13
			0.187	0.09	1.15	1.33
0.06	0.206	4.64	0.1875	0.095	0.904	1.04
			0.191[a]	0.09	1.02	1.19
			0.194	0.085	1.15	1.41
0.07	0.197	4.54	0.194[b]	0.09	0.909	1.09
			0.1975[a]	0.085	1.023	1.27
			0.201	0.08	1.14	1.51
0.08	0.1885	4.435	0.201[b]	0.085	0.9	1.145
			0.204[a]	0.08	1.03	1.35
			0.2075	0.075	1.14	1.66
0.09	0.18	4.33	0.2075[b]	0.08	0.91	1.21
			0.211[a]	0.075	1.02	1.45
			0.214	0.07	1.14	1.81
0.1	0.171	4.22	0.214	0.075	0.909	1.29
			0.2175[a]	0.07	1.023	1.58
			0.221	0.065	1.14	2.05
0.11	0.162	4.12	0.221	0.07	0.9	1.39
			0.224[b]	0.065	1.02	1.73
			0.2275	0.06	1.14	2.37
0.12	0.154	4.02	0.2275	0.065	0.904	1.51
			0.231	0.06	1.02	1.94

[a]A "preferred" solution.
[b]A possible but unlikely combination of H_{pn}, F_2^{pn}, and I_{pn}.

proton fixed poles so long as one is willing to modify the $\frac{1}{3}$ in the quark charge sum rule. (See Appendix B for further discussion of the difficulty in satisfying this sum rule in general.) Of course, the precise asymptotic form for $F_2^p(\omega)$ would be changed.

It should also be noted from the tables that for F (effective) to be <1, we must believe the value of R_n to be 0 and that $R_p=1$. Thus the only situation in which the solution F values correspond to those likely for a triple Regge cut even with a reasonable number of coupled channels mixed in would seem to be the case $R_n=0$, $R_p=1$. Nonetheless we will continue to allow the other possibilities.

For each of the four cases we pick the preferred solution with the lowest H_{pn}. These are given below:

(1) $R_p=1$, $R_n=0$: $F_2^p(\omega)=0.12+0.462\omega^{-1/2}+4.02\omega^{-3/2}$, $H_{pn}=0.21$;

(2) $R_p=1$, $R_n=\frac{2}{3}$: $F_2^p(\omega)=0.06+0.618\omega^{-1/2}+4.64\omega^{-3/2}$, $H_{pn}=0.19$;

(3) $R_p=1$, $R_n=1$: $F_2^p(\omega)=0.05+0.645\omega^{-1/2}+4.75\omega^{-3/2}$, $H_{pn}=0.195$;

(4) $R_p=0$, $R_n=0$: $F_2^p(\omega)=0.07+0.663\omega^{-1/2}+3.67\omega^{-3/2}$, $H_{pn}=0.19$.

There is some evidence that $F_2^p(\omega)$ would like to be as high as 0.25 or so in the vicinity of $\omega=25$. The values of $F_2^p(25)$ for the four cases are given below where the errors are estimated by taking into account the possibility of using other preferred solutions:

(1) $F_2^p(25)=0.245\pm0.015$,

(2) $F_2^p(25)=0.221\pm0.015$,

(3) $F_2^p(25)=0.217\pm0.015$,

TABLE V. $R_p=1$, $R_n=1$ (isoscalar fixed pole). Format as in Table III.

A	B	C_p	H_{pn}	F_2^{pn}	I_{pn}	F
0.02	0.24	5.05	0.174	0.095	1.04	1.52
			0.1775	0.09	1.18	1.9
0.03	0.231	4.95	0.1775[b]	0.095	0.94	1.38
			0.181[b]	0.09	1.05	1.65
			0.184	0.085	1.17	2.06
0.04	0.222	4.85	0.181	0.095	0.81	1.245
			0.184[b]	0.09	0.93	1.46
			0.1875[b]	0.085	1.06	1.81
			0.191	0.08	1.17	2.35
0.05	0.215	4.75	0.1875	0.09	0.815	1.32
			0.191	0.085	0.93	1.58
			0.194[a]	0.08	1.05	2.00
			0.1975	0.075	1.175	2.72
0.06	0.206	4.64	0.194	0.085	0.815	1.4
			0.1975[a]	0.08	0.935	1.74
			0.201[a]	0.075	1.05	2.25
			0.204	0.07	1.17	3.25
0.07	0.197	4.535	0.204[b]	0.075	0.93	1.91
			0.2075[a]	0.07	1.055	2.61
0.08	0.1888	4.435	0.2075	0.075	0.815	1.66
			0.211[b]	0.07	0.93	2.15
			0.214[b]	0.065	1.05	3.1
0.09	0.18	4.33	0.214	0.07	0.813	1.82
			0.2175[b]	0.065	0.935	2.5
			0.221	0.06	1.05	4.0
0.10	0.171	4.225	0.221	0.065	0.815	2.06
			0.224	0.06	0.93	3.0

[a] A "preferred" solution.
[b] A possible but unlikely combination of H_{pn}, F_2^{pn}, and I_{pn}.

TABLE VI. $R_p=0$, $R_n=0$ (no fixed poles). Format as in Table III.

A	B	C_p	H_{pn}	F_2^{pn}	I_{pn}	F
0.05	0.239	3.88	0.1715	0.1	0.91	1.0
			0.175	0.095	1.025	1.17
			0.178	0.09	1.14	1.36
0.06	0.23	3.71	0.178	0.095	0.9	1.05
			0.1815[b]	0.09	1.03	1.26
			0.185	0.085	1.15	1.75
0.07	0.221	3.67	0.185[b]	0.09	0.9	1.11
			0.188[a]	0.085	1.025	1.35
			0.1915	0.08	1.15	1.75
0.08	0.213	3.57	0.1915	0.085	0.91	1.19
			0.195[a]	0.08	1.03	1.48
			0.198	0.075	1.15	2.0
0.09	0.204	3.46	0.198[b]	0.08	0.91	1.28
			0.201[a]	0.075	1.03	1.66
			0.205	0.07	1.15	2.33
0.10	0.195	3.36	0.205[b]	0.075	0.91	1.4
			0.208[a]	0.07	1.08	1.88
			0.212	0.065	1.15	2.93
0.11	0.187	3.26	0.212	0.07	0.91	1.56
			0.215[b]	0.065	1.03	2.23
			0.218	0.06	1.15	4.0
0.12	0.178	3.15	0.218	0.065	0.91	1.77
			0.22[b]	0.06	1.03	2.76
0.13	0.170	3.05	0.221	0.065	0.79	1.47
			0.225	0.06	0.91	2.1

[a] A "preferred" solution.
[b] A possible but unlikely combination of H_{pn}, F_2^{pn}, and I_{pn}.

(4) $F_2^p(25)=0.232\pm0.015$.

It is clear that situation (1) is to be preferred on this basis. Certainly this aspect of the model will be tested in the not-too-distant future. If the data should refuse to fall, then the model as presented here is wrong. It would then have to be true that (assuming a positive residue for the proton fixed pole still) Regge behavior has not yet begun as low as ω of 12 [e.g., an extensive "quasielastic peak" may be present or equivalently the behavior of $F_2(\omega)$ down to ω of 12 may not be representable by a simple three-power fit, i.e., additional daughters or cuts would need inclusion]. We plot solution (1) on top of the F_2^p data in Fig. 1.

IV. CONCLUSIONS AND DISCUSSION

We have shown that inclusion of an effective trajectory in the description of the off-shell deep-inelastic proton scaling function $F_2^p(\omega)$ makes it possible for the fixed pole in νT_2 to have a polynomial residue with the on-shell magnitude found in Refs. (3). Evidence for such an effective trajectory in the crossing-even $pp+p\bar{p}$ scattering amplitude was presented. It was clear, however, that other fixed-pole residues were also possible. Interestingly enough we discovered that it was possible to have a polynomial residue for the proton fixed pole and simultaneously satisfy the proton-neutron difference quark charge sum rule, and in addition, to have a polynomial residue for the fixed pole in the νT_2 for the neutron (if for instance at $q^2=0$ the pole is not present, as might be indicated by the Thomson limit). As was also discussed, relaxation of the quark charge sum rule makes it easier, in the sense that the F_2^{pn} data need not be stretched to their limits, to satisfy the other constraints if the $\frac{1}{3}$ of Eq. (3.5) is reduced. Conversely, it is harder or nearly impossible to satisfy the other constraints if the quark charge sum rule is increased to 1. For the case with all constraints present and $R_n=0$, $R_p=1$, we obtained a solution for the univeral $F_2^p(\omega)$ curve of

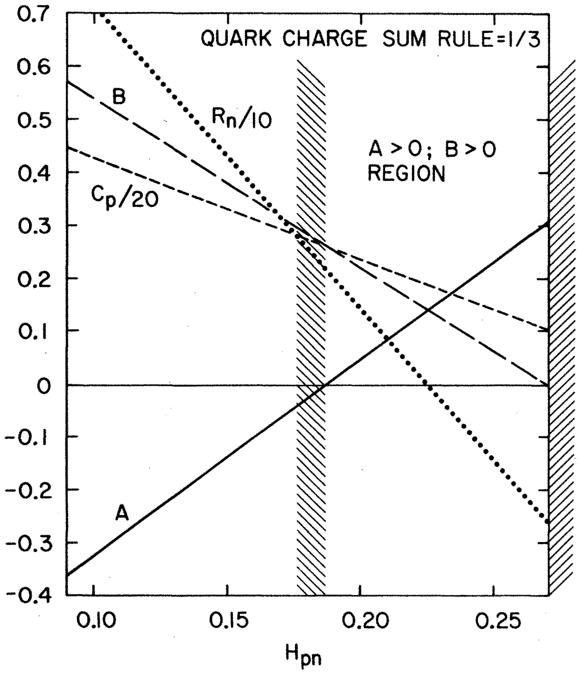

FIG. 6. For various values of H_{pn} [the area under the proton-neutron difference curve up to $\omega^* = 12$ (see Fig. 5)] we show the solutions for A, B, and C_p, defined in (3.1), which will yield a fixed pole in the proton data with residue +1, and which satisfy the p-n difference constraints. The magnitude of the fixed pole in the neutron data, for a given H_{pn}, is also shown (R_n). The demand that both A and B are non-negative restricts solutions to lie between the two cross-hatched vertical lines. Thus H_{pn} must be at least 0.18 if the quark charge sum rule has a value of $\frac{1}{3}$. F_2^{pn} and I_{pn} were taken to vary linearly from 0.06 and 0.94, respectively, as H_{pn} varies from 0.18.

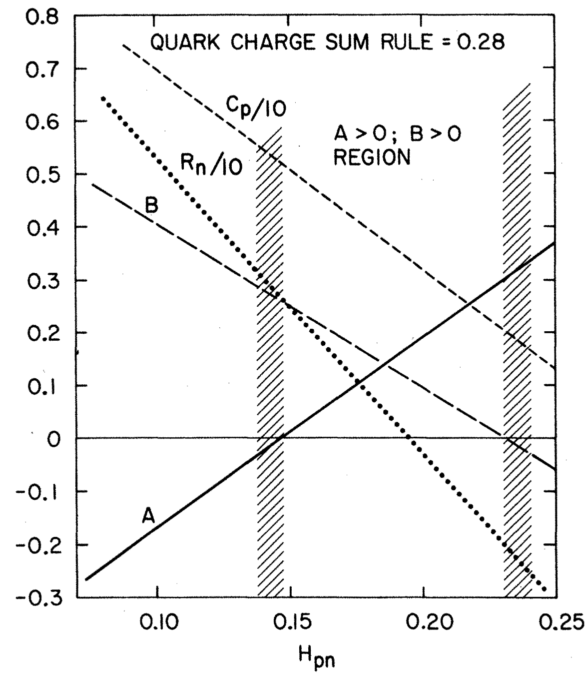

FIG. 7. As in Fig. 6, but showing the effect of changing the value of the quark charge sum rule, Eq. (3.5), from $\frac{1}{3}$ to 0.28. Note that the $A \geq 0$, $B \geq 0$ region has shifted to lower values of H_{pn} as compared to Fig. 6.

$$F_2^p(\omega) = 0.12 + 0.462\omega^{-1/2} + 4.02\omega^{-3/2}, \tag{4.1}$$

for $\omega > \omega^*$, which at an ω of 25 has fallen to about 0.25. For the solution to be correct the proton-neutron difference $F_2^{pn}(\omega)$ must actually be towards the outer edges of the error bars of the present data so long as we demand that the quark charge sum rule ($=\frac{1}{3}$) be satisfied. Thus higher ω data or more accurate proton-neutron data will be able to test the consistency of the theoretical quark charge sum rule value of $\frac{1}{3}$ with our other hypotheses.

It is also interesting to note the implications of (4.1) for on-shell Compton scattering. The current best fit to the world data with $\nu > 2$ GeV with a form $a + b\nu^{-1/2}$ gives $a \sim 100$ μb, $b \sim 62$ μb (GeV)$^{1/2}$ (clearly these values would not be greatly altered if a *small* $c\nu^{-3/2}$ term were allowed). It is clear that a very small mass must enter the problem, in some fashion, in going from the $q^2 = 0$ limit to the large q^2 region, since the relative ratios of the Pomeranchukon to the coefficient of the leading Regge term is considerably altered in the process. We use in what follows a specific ansatz for determining this mass. The Regge scaling assumption says $a \to AN$, $b \to 3BN(q^2/2M)^{1/2}$, $c \to C_pN(q^2/2M)^{3/2}$ for large q^2. If in fact the $q^2 \to 0$ limits were properly given by the following form for νW_2:

$$\nu W_2 = \frac{q^2}{q^2 + m^2}\left(A + \frac{3B}{\sqrt{\tilde\omega}} + \frac{C_p}{\tilde\omega^{3/2}}\right), \tag{4.2}$$

with $\tilde\omega = 2M\nu/(q^2 + \mathfrak{M}^2)$, then we will clearly also obtain the correct $q^2 \to \infty$ limit by the above expression (4.2). This form allows us to obtain an expression for the high-energy behavior of $\sigma(\nu)$ at $q^2 = 0$ using the facts that

$$\sigma(\nu) = 4\pi^2\alpha W_1/\nu \quad \text{at } q^2 = 0 \tag{4.3}$$

with

$$W_1 = \lim_{q^2 \to 0} W_2\nu^2/q^2, \tag{4.4}$$

which follows from the vanishing of the longitudinal cross section in this limit. The known values for A, B, and a, b give

$$\frac{0.12}{0.462} = \frac{A}{3B} \quad \text{and} \quad \frac{A}{3B}\frac{(2M)^{1/2}}{\mathfrak{M}} = \frac{100}{62}, \tag{4.5}$$

and hence, $\mathfrak{M} = 0.22$ GeV. Further,

$$c(q^2 = 0) = C_pN(\mathfrak{M}^2/2M)^{3/2} = 14.$$

Since $N=100/0.12$, and finally, since $N=4\pi^2\alpha/m^2$, we have that $m^2=0.134\ \text{GeV}^2$.

The implication of the above is that $\omega=100$ in the scaling data corresponds to $\nu=100\mathfrak{M}^2/2M=2.59$ in GeV. Therefore, we predict that the scaling curve at $\omega=100$ will be shaped like the on-shell data curve at $\nu=2.59$, i.e., the same relative contributions from the three terms will be present at these values of ω and ν. Thus at $\nu=1.68$ GeV the contribution from the effective trajectory to the on-shell cross section might be as much as 5 μb. This would require only a very small adjustment in the parameters a and b for the Damashek-Gilman analysis. With $a=100$, $b=62$, and the above value of c, we find $\sigma_{\gamma p}(\nu=1.68)=154\ \mu$b. They require that the Regge fit correspond to the low-energy data at $\nu=1.68$ with a value of $\sigma_{\gamma p}(1.68)=152\ \mu$b. Such a minor adjustment results in no change in their results as we have explicity verified. It might be argued that the small value of m which we obtain, which implies that scaling is good to about 90% above $q^2=1\ (\text{GeV}/c)^2$, was to some extent assumed, in that we assumed $F_2(\omega)$ had already scaled at $\omega=12$ and since the data points we used there came from relatively small q^2 measurements. Nonetheless, if the data as we assumed them hold up for large q^2, then an early onset of scaling is strongly indicated by the above considerations which suggested that the onset is characterized by a very small mass.

One could go even further, however, and assume that our ansatz is correct not only as a means for going from the $q^2=0$ region to the $q^2\to\infty$ region, but also as an interpolating formula. One should then find that $\omega\nu W_2$ is a universal function of the variable $\tilde{\omega}$ (and that scaling would be best seen by plotting everything as a function of this variable), were it not for the fact that $m\neq\mathfrak{M}$. It may, however, be that this inequality is only a result of the inaccuracies of this work.

In fact, if we look at solutions obtained by modifying the quark charge sum rule, in particular the one obtained in the example given in Sec. III ($R_n=0$), we find that it is possible to obtain a solution for which $m=\mathfrak{M}=0.44$ GeV. This solution is as follows:

$$A=0.17,\qquad B=0.113,\qquad C_p=3.42\,, \tag{4.6}$$

which was obtained with the $\frac{1}{3}$ in the quark charge sum rule replaced by 0.28. This solution gives an $F_2^p(25)=0.264$, which is perhaps more likely to be consistent with the future data at that point. We feel that this is a more likely solution than the previous one which yielded such a very small value for $\mathfrak{M}$. As we have seen requiring the above sort of value in the quark charge sum rule is far more consistent with the present p-n difference data (see also Appendix B).

The variable $\tilde{\omega}$ was, of course, found from considerations involving the large ν region. In going to small ν we might instead consider a variable $\tilde{W}=(2M\nu+M^2)/(q^2+\mathfrak{M}^2)$ the new analog of the older ω' of Bloom and Gilman.[23] Such a suggestion has independently been made by Rittenberg and Rubinstein[24] on the basis of very different considerations. They also come to the conclusion that the mass $\mathfrak{M}$ is quite small.

An indication that the masses m and $\mathfrak{M}$ must be quite small is seen in the large-ω small-q^2 data of Ref. 5. At $q^2=m^2$ our ansatz would imply that the value of νW_2 should have risen to 50% of its $q^2\to\infty$ value. This is in fact supported by the data of Ref. 5 (see Fig. 17 of that reference) if one assumes the limiting value to be of the order of 0.25 to 0.3 for the two higher ω bins. A substantially larger value of m^2, combined with the known values of νW_2 at smaller q^2, would require a final asymptotic value considerably higher than the 0.25 to 0.3 range, or else a more complicated interpolating ansatz. We remind the reader that the small value of m^2 in our analysis is directly a result of the relatively small contribution of the Pomeranchukon to νW_2 in the scaling region.

In the above discussion we have implicity assumed that the data of Ref. 5 are indeed truly representative of the $q^2\to\infty$ (scaling) limit. Should it turn out that this is not the case (see, e.g., Appendix C), much of what we have said in this section would need to be reconsidered. However, it remains true *in general* that fixed poles with polynomial residues are possible if, and only if, the ω dependence of νW_2 for $\omega\geq 12$ contains contributions additional to the usual leading Regge behavior.

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APPENDIX A

In Tables III–VI we listed the values of A, B, C_p which result from a sampling of values of I_{pn}, H_{pn}, and F_2^{pn} all assumed evaluated for $\omega^*=12$ within the extremes of the neutron-proton-difference data, for the various neutron pole residues and proton pole residues that we consider. Changing the value of d has little effect on these results unless d approaches unity, in which case, as referred to previously, equivalent solutions are obtained for slightly smaller values of H_{pn}, I_{pn}, and F_2^{pn}, i.e., for this smaller value of d it is not necessary to push the F_2^{pn} data to the limits of the error bars. The allowed range of A, and hence, B and C_p, etc., is not greatly altered by changing d. It is apparent that there is no lack of solutions provided one is liberal with the p-n difference data.

We discuss here in detail the trends exhibited in the Tables and the preferred choices for correlated values of H_{pn}, F_2^{pn}, and I_{pn}. To do so we first rearrange the equations of Sec. III into the following more convenient format:

$$A=[(d-1)/d]\{[F_2^p(\omega^*)-2B(\omega^*)^{-1/2}]/(d-1)-[R_p-I_p+6B(\omega^*)^{+1/2}]/\omega^*\}\,, \tag{A1}$$

$$B\times(2-1/d)(\omega^*)^{-1/2}=\tfrac{1}{3}-H_{pn}(\omega^*)-F_2^{pn}(\omega^*)/d\,, \tag{A2}$$

$$C_p=(\omega^*)^d[(d-1)/d]\{F_2^p(\omega^*)-2B(\omega^*)^{-1/2}+[R_p-I_p+6B(\omega^*)^{+1/2}]/\omega^*\}\,, \tag{A3}$$

$$C_p(1-x)\times(2-1/d)(\omega^*)^{-d}=2F_2^{pn}(\omega^*)+H_{pn}(\omega^*)-\tfrac{1}{3}\,, \tag{A4}$$

where, again,

$$F_2^{pn}(\omega^*)=F_2^p(\omega^*)-F_2^n(\omega^*)\equiv F_2^{pn}\,,$$

$$I_{pn}(\omega^*)=\int_1^{\omega^*}d\omega F_2^{pn}(\omega)\quad\text{and}\quad H_{pn}(\omega^*)=\int_1^{\omega^*}\frac{d\omega}{\omega}F_2^{pn}(\omega)\,,$$

and R_p is the magnitude of the proton's fixed pole. Equation (3.7) for the neutron fixed-pole residue R_n can be rearranged to give $I_{pn}(\omega^*)$ as a function of A, B, C_p, and x for a given value of R_n.

We first note that there are no solutions for $H_{pn}(\omega^*)\leqslant 0.18$ when $R_p=1$. That is, if one believes in our version of the proton fixed pole, then the neutron fixed pole is only reasonable in size so long as the $F_2^{pn}(\omega)$ proton-neutron difference lies above the central data point values shown in Fig. 5. We note that relaxing the constraint that the proton pole be +1, e.g., allowing it to be zero, allows us to obtain solutions with $H_{pn}(\omega^*)$ as low as 0.17. If the data should eventually force one to accept a *maximum* value for $H_{pn}(\omega^*)$ which is less than 0.17 then one would be forced to accept a negative pole residue for the proton, or alternatively, if one's prejudices demanded a non-negative proton pole, then one could allow additional structure in the higher ω data or further, relax certain of our constraints – i.e., the quark sum rule.

Second, we note that for a fixed value of $I_{pn}(\omega^*)$, we can increase $H_{pn}(\omega^*)$ only if $F_2^{pn}(\omega^*)$ simultaneously falls. Since I_{pn} is only bounded, there is in actuality some leeway but the general trend remains valid. Thus, the boundedness of I_{pn} allows us to say that reasonable solutions can only fall in the ranges tabulated – high H_{pn} is inconsistent with too low a value of F_2^{pn} outside of the regions given, and for given values of H_{pn} and F_2^{pn}, I_{pn} can certainly only lie in the ranges which we give. Too high a value of F_2^{pn} (i.e., >0.1) we have also excluded as being manifestly inconsistent with the data. In actual fact, most of the solutions which we tabulate are not all that reasonable and only certain combinations correspond to acceptably shaped p-n difference curves for low ω. These particular solutions have been marked in the tables. Of course, there is a continuum of unlisted solutions with values of H_{pn}, F_2^{pn}, and I_{pn} in the immediate vicinities of the particular ones which we have recorded in our tables.

It is obvious that a given value of B can be obtained for a variety of combinations of H_{pn} and F_2^{pn} [Eq. (A2)], so long as H_{pn} and F_2^{pn} are inversely correlated. Once B has been determined, A and C_p follow for given values of the proton pole and proton data [Eqs. (A1) and (A3)]. Demanding a higher value for A clearly requires a lower value of B [Eq. (A1)], and a lower value of B only results if the combination of $H_{pn}+F_2^{pn}/d$ increases [Eq. (A2)]. This explains the general trends for increasing H_{pn} with increasing A. (One must of course keep in mind that we restrict F_2^{pn} to be <0.1.) The overlap results from allowing F_2^{pn} to vary within its allowed range.

For a given proton fixed pole once A, B, C_p, and x have all been determined, it only becomes a question of whether I_{pn} (which is the only remaining variable determining R_n) also lies within an acceptable range. This is the final limitation on the range of solutions accepted by us and given in the tables.

Graphically the residue of the fixed pole in either $F_2^p(\omega)$ or $F_2^n(\omega)$ is determined by the quantity $X+Y-Z$ in Fig. 8 where the data are to be taken either as F_2^n or F_2^p and the dashed curve represents the sum $A+2B/\sqrt{\omega}$ or $A+3B/\sqrt{\omega}$, respectively. The proton data, however, are fixed, i.e., I_p is fixed. At most we can imagine that I_{pn} can alter by about 30%, which would represent a change in the neutron I_n ($\equiv[I_p-I_{pn}]$) of about 10%.

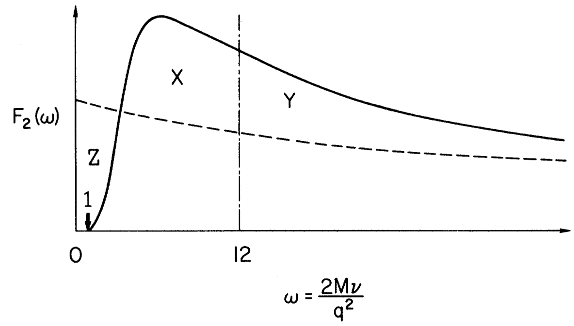

FIG. 8. The solid curve represents the scaling function $F_2(\omega)$. The dashed curve represents the low-ω extrapolation of the asymptotic "Regge" form $A+(\frac{3}{2})\times B\omega^{-1/2}$ (3 for F_2^p, 2 for F_2^n). The magnitude of the fixed pole is given by the combination of areas $X+Y-Z$.

This immediately tells us that requiring a significant increase in R_n will necessitate a decrease in the value of the area under $A+2B/\sqrt{\omega}$, while retaining the same area under $A+3B/\sqrt{\omega}$, which is equivalent to keeping the proton pole fixed. Since A and B are both >0, the only way to do this is to decrease A; recall that A and B are correlated in such a way that A decreasing causes B to increase [Eq. (A1)]. Where there is an overlap in the A, B, C_p solutions between two different R_n values, it occurs as a result of the adjustments possible in the p-n difference data. If the same values of A, B, C_p are consistent with two values of R_n, it is seen from Tables III, IV, and V that the H_{pn} corresponding to this solution is smaller, the smaller the value of R_n, and hence F_2^{pn} is larger. This is understood again by a graphical argument. Again, fixing I_{pn}, for a given value of A, B, C_p the difference $X-Z$ in Fig. 8 under the neutron curve is fixed (recall that I_p is always fixed and that $I_n \equiv I_p - I_{pn}$). Thus the only difference in the neutron fixed pole is that resulting from the area Y, which is larger, the higher the value $F_2^n(\omega^*)$ is. For a fixed-proton curve, a lower F_2^{pn} is needed to give a higher F_2^n; thus the higher the R_n value required, the lower F_2^{pn} wants to be. The I_{pn} fluctuation for a given B, of about 30%, in not capable alone of changing this correlation.

We wish to reiterate that we have been assuming in the preceding discussion that the proton data for $\omega < \omega^* = 12$ are well determined, i.e., that $I_p(\omega^*)$ and $F_2^p(\omega^*)$ are as we have given them. Decreasing $I_p(\omega^*)$ while keeping $F_2^p(\omega^*)$ fixed will cause a given solution (i.e., H_{pn}, F_2^{pn}, and I_{pn} fixed) to change as follows: F (effective) and C_p increase while A decreases {A and C_p are inversely correlated since B is fixed by the p-n difference [Eq. (A2)] and $F_2^p(\omega^*)$ remains fixed [Eq. (3.1)]}. The fact that C_p must increase can be seen from Eq. (A3). Consequently A will decrease. Conversely it follows that if $I_p(\omega^*)$ were to increase, then so will the value of A (again, holding the p-n solution unaltered). Similarly from (A1) and (A3) it is clear that if I_p is held fixed while F_2^p increases then A and C_p will increase, so the $\omega > 12$ curve will be somewhat higher than previously, in particular the value $F_2^p(25)$ will be slightly greater than 0.25.

APPENDIX B

We make a few comments here on the role of the quark charge sum rule in a Regge-type analysis of the p-n difference data. In the main body of the paper we discovered that for a three-pole Regge model of the high ω points, it is not possible to satisfy this sum rule unless the p-n difference data are larger than it presently appears to be. These same remarks apply in the case of a "conventional" two-pole model [Eqs. (3.1) and (3.2) with $C_p=0$]. In this event the sum rule becomes

$$\frac{1}{3} = \int_1^{\omega^*} \frac{d\omega\, F_2^{pn}(\omega)}{\omega} + 2F_2^{pn}(\omega^*)\,, \tag{B1}$$

and it is clear that this can only be satisfied if the data are pushed to their maximum values. Note that here, as before, increasing ω^* doesn't really help very much unless F_2^{pn} has some nonsmooth falloff, e.g., a rise and *then* a smooth asymptotic "Regge" fall.

TABLE VII. $R_p=1$, $R_n=0$. The input assumes the scaling function to be that obtained by suri and Yennie after their vector-dominance subtraction. Format as in Table III.

A	B	C_p	H_{pn}	F_2^{pn}	I_{pn}	F
0.01	0.171	2.98	0.201	0.095	1.09	0.57
			0.204	0.09	1.21	0.63
0.00	0.162	2.87	0.204	0.095	0.97	0.53
			0.207[b]	0.09	1.1	0.58
			0.211	0.085	1.21	0.64
0.01	0.154	2.77	0.207	0.095	0.85	0.49
			0.211[b]	0.09	0.97	0.54
			0.214[a]	0.085	1.09	0.59
			0.217	0.08	1.22	0.665
0.02	0.045	2.67	0.217[a]	0.085	0.98	0.55
			0.221[a]	0.08	1.09	0.60
			0.224	0.075	1.2	0.68
0.003	0.136	2.56	0.224[b]	0.08	0.97	0.55
			0.227[b]	0.075	1.1	0.62
0.004	0.128	2.46	0.227	0.08	0.86	0.51
			0.231	0.075	0.97	0.56

[a] A "preferred" solution.

[b] A possible but unlikely combination of H_{pn}, F_2^{pn}, and I_{pn}.

These problems are totally independent of the considerations involving the residues of the fixed poles. Thus, if in fact the eventual p-n data restrict us to small H_{pn}, then it is clear that the quark charge sum rule must "fall," or alternatively, there must be some unusual structure in the higher ω regions of the p-n difference data.

APPENDIX C

We recall now the second possibility mentioned in Sec. I, namely, that the scaling function is not the full $F_2^p(\omega)$ but rather a subtracted $F_2^p(\omega)$ (which we called $\tilde{F}_2^{p(\omega)}$ as suggested by suri and Yennie.[15]

In Fig. 1 we exhibited the observed νW_2 data and in Fig. 2 we showed the resulting "data" for $\tilde{F}_2 = (\nu W_2 - \text{vector-dominance contribution})$. suri and Yennie propose that $\tilde{F}_2$ is the object which might already be scaling and that for $q^2 >$ some $q^2_{\min}$ the vector-dominance contribution is negligible, but that this region has not yet been reached. We can then write Eq. (1.3) with F_2 replaced by $\tilde{F}_2$ for $q^2 > q^2_{\min}$ and use the $\tilde{F}_2$ of Fig. 2 in the sum rule under the assumption that $\tilde{F}_2$ is already scaling.

In Tables VII and VIII we give the corresponding $R_p = 1$ and $R_n = 0$, $R_p = 0$ and $R_n = 0$ solution possibilities as they would occur in our scheme (we took $\tilde{F}_2^p = 0.21$ and $I_p = 2.72$ at $\omega^* = 12$). Quite clearly, if their model is correct, we would predict that most if not all of the Pomeranchukon has been subtracted by their vector-dominance subtraction. The true scaling function would contain very little constant component. Unfortunately it is still not possible to decide on any basis whether the vector-dominance subtraction has removed the fixed pole or not. Both $R_p = 1$ and $R_p = 0$ seem equally viable possibilities within our framework. Changing the quark sum rule would (as before) result in some modification of these results. In particular the 0.28 value in this sum rule would allow some additional Pomeranchuk contribution to remain in $\tilde{F}_2^p$.

TABLE VIII. $R_p = 0$, $R_n = 0$. Solutions are for the suri-Yennie subtraction case (see also Table VII.) Format as in Table III.

A	B	C_p	H_{pn}	F_2^{pn}	I_{pn}	F
−0.01	0.195	2.11	0.205[b]	0.075	0.91	0.92
			0.208[a]	0.07	1.02	1.2
0.00	0.187	2.01	0.208	0.075	0.79	0.79
			0.211[b]	0.07	0.91	1.0
			0.214[a]	0.065	1.03	1.4
+0.01	0.178	1.90	0.215	0.07	0.79	0.84
			0.218	0.065	0.91	1.1

[a] A "preferred" solution.
[b] A possible but unlikely combination of H_{pn}, F_2^{pn}, and I_{pn}.

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†NATO fellow, 1970–72.

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[4] J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[5] E. D. Bloom *et al.*, SLAC Report No. SLAC-PUB-796, 1970 (report presented at the Fifteenth International Conference on High Energy Physics, Kiev, U. S. S. R. 1970) (unpublished).

[6] H. Pagels, Phys. Letters 34B, 299 (1971).

[7] R. Suaya (unpublished).

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[9] G. C. Fox, in *Proceedings of the Third International Conference on High Energy Collisions, Stony Brook, 1969*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

[10] H. Harari, Scottish Universities Summer School, 1970 (unpublished).

[11] We thank Haim Harari for suggesting this possibility. This idea is also contained in Ref. 8 and motivates the work therein.

[12] Of course, the inclusion of an effective trajectory term in the scaling function implies that, in all probability, it should also appear in the on-shell scattering as will be discussed later. We will find that it is possible to guess at the coefficient of the effective trajectory which will appear in on-shell scattering from knowledge of its contribution to the scaling function. The coefficient in the on-shell scattering turns out to be quite small, nonetheless, it is reasonable to see what modifications in the magnitude of the on-shell fixed pole would result from its inclusion in the Damashek-Gilman analysis. We have done this and find that the fixed pole is still consistent with having a residue of about 3 μb GeV, the value obtained in Ref. 3, though there is a tendency for the residue to be slightly larger in magnitude depending upon the precise high-energy fit employed.

[13] J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969); J. D. Bjorken, in *Selected Topics in Particle Physics, International School of Physics "Enrico Fermi," Course XLI*, edited by J. Steinberger (Academic, New York, 1967); K. Gottfried, Phys. Rev. Letters 18, 1174 (1967).

[14] The authors in Ref. 3 found the magnitude of the $J = 0$ right-signatured fixed pole in on-shell Compton scattering to be consistent with the proton's Thomson amplitude,

$-\alpha/M$. This suggests that the magnitude of the analogous fixed pole in the case of neutron targets might be zero because the neutron's charge, and hence Thomson term, is zero. Alternatively, the fixed pole may have origin in the seagull diagrams for Compton scattering from the "partons" in the nucleon. For a conventional three-quark model $\sum Q_i^2 = 1$ for the proton and so the observed magnitude results, but for the neutron $\sum Q_i^2 = \frac{2}{3}$ and so, if this mechanism were correct, the magnitude of the neutron's fixed pole would be $\frac{2}{3}$. Finally, it is possible that the fixed pole is an isoscalar and if so the residue would be $+1$ for the neutron as well as for the proton. In principle, any value for the magnitude of the fixed pole is possible in the case of a neutron target, but we feel that, in the present state of ignorance, the above three possibilities are perhaps the most likely.

[15]A. suri and D. Yennie (private communication and unpublished report). We are indebted to Dr. suri and Dr. Yennie for permission to include a figure (Fig. 2) from their unpublished work.

[16]The investigation reported in this section was stimulated by H. Harari. See also H. Harari, Phys. Rev. Letters 26, 1079 (1971).

[17]The phases as given in Table I are true if one assumes the real and imaginary parts of the amplitude to have the same asymptotic pure power behavior with the alphas as given. In particular, a ν^0 term is impossible in the imaginary part of a crossing-symmetric amplitude. For multi-Regge cuts, the correct iteration form for a symmetric amplitude is

$$\frac{(\nu/\nu_0)^{\alpha_c^n}}{\ln^n(\nu/\nu_0)} + \frac{(-\nu/\nu_0)^{\alpha_c^n}}{[\ln(\nu/\nu_0) - i\pi]^n} ,$$

where $\alpha_c^n(0) = n\alpha(0) - n + 1$, n corresponding to the number of iterations. Clearly, as $\ln(\nu/\nu_0)$ becomes large we obtain

$$\left(\ln\frac{\nu}{\nu_0}\right)^{-n}\left(\frac{\nu}{\nu_0}\right)^{\alpha_c^n}[1+\exp(-i\pi\alpha_c^n)] ,$$

which again has no imaginary part for $\alpha_c^2 = 0$. This however is an approximation for large ν or small ν_0. We have chosen to adopt the simplest possible approximation of throwing away the relatively smaller imaginary part at $\alpha_c^2 = 0$ and of assuming the $[\ln(\nu/\nu_0)]^n$ in the term that remains to be slowly varying. The adequacy of this approach can only be tested by referring to the data as we shall do. Also note that consideration of Pomeranchuk-generated cuts is unnecessary in this slowly varying logarithm approach as they merely lead to a renormalization of the Regge and Regge-cut couplings.

[18]R. J. Abrams *et al.*, Phys. Rev. D 1, 1917 (1970); W. Galbraith *et al.*, Phys. Rev. 138, B913 (1965); J. V. Allaby *et al.*, Phys. Letters 30B, 500 (1969); D. V. Bugg *et al.*, Phys. Rev. 146, 980 (1966); K. J. Foley *et al.*, Phys. Rev. Letters 19, 857 (1967).

[19]It will be seen later that the resulting asymptotic form for $F_2(\omega)$ agrees with the data over an ω range of 2 about $\omega = 12$, so that our conclusions are not sensitive to the choice $\omega^* = 12$. It is possible that Regge behavior has not really begun by $\omega = 12$. In this case most of what we say will still remain valid, though the specific numerical results will change, so long as no unusual structure appears above $\omega = 12$.

[20]J. Ballam *et al.*, Phys. Rev. Letters 21, 1541 (1968); 23, 498 (1969); D. O. Caldwell *et al.*, *ibid.* 23, 1256 (1969); 25, 609 (1970); 25, 613 (1970); ABBHHM collaboration, Phys. Letters 27B, 474 (1968); H. Meyer *et al.*, DESY Report No. 70/17 (unpublished).

[21]It is difficult, from the consideration of the scaling region alone, to assign a given value to d although we will, in what follows, find that the constraints which we impose are slightly more easily satisfied with values of d just above 1, and are impossible to satisfy within reasonable limits of data for $d > 1.7$, say. However, it would be strange for such an object not to appear in the analogous amplitude for $pp + p\bar{p}$ which we considered in Sec. II. For this reason we restrict ourselves in what follows to $d = \frac{3}{2}$.

[22]Should we wish to test for the absence of a fixed pole in the scaling function F_2 we would want to reexamine our results with the 1 on the left-hand side changed to 0.

[23]E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

[24]V. Rittenberg and H. Rubinstein, Phys. Letters 35B, 50 (1971).