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$\Delta I = 1$ Mass Differences and $\eta \to 3\pi$ Decay*

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The octet baryon $\Delta I = 1$ mass shifts, kaon electromagnetic mass shift, and $\eta \to 3\pi$ decay amplitude are all correlated in the framework of the linear $SU(3)$ σ model. It is shown how this model provides a natural realization of the "tadpole" idea of Coleman and Glashow. An attempt is made to resolve the long-standing mystery associated with the $\eta \to 3\pi$ process, and the rare mode $\eta' \to 3\pi$ is also calculated. Finally, it is shown that the chiral-symmetry-breaking and isospin-nonconserving coefficients in the Lagrangian have the same order of magnitude.

I. INTRODUCTION

In this paper, we study $\Delta I = 1$ electromagnetic effects in the linear $SU(3)$ σ model of pseudoscalar and scalar mesons. The formalism to be used was developed in several previous papers[1-3] for the purpose of studying chiral-symmetry breaking and $SU(3)$ breaking in this model. However, we are attempting to make the present paper self-contained enough that the reader may get the *general idea* without constantly having to refer back.

We shall limit the mesons in the model to the spin-0 ones to avoid the usual maze of alternatives that present themselves when many additional kinds of particles are treated together with symmetry breaking. It was found that in the isospin limit with the simplest "[(3, 3*)+(3*, 3)]" type of symmetry-breaking term,[4] the present model gave quite a reasonable amount of the mass spectrum. Here we shall see that the model is able to correlate successfully the baryon electromagnetic mass shifts with the kaon electromagnetic mass shift and the $\eta \to 3\pi$ decay process. Furthermore, the mass of the isovector scalar meson is constrained (from electromagnetic considerations) to be around a physically reasonable value. It turns out, in addition, that the fit to the mass spectrum requires the coefficient in the Lagrangian which violates isospin invariance to be of the same order of magnitude as the chiral-symmetry-breaking coefficient. This might indicate a fundamental connection between electromagnetism and chiral-symmetry breaking.[5]

Before going on to the formalism, we shall briefly discuss the historical background. In the problem of electromagnetic mass splittings and associated second-order processes (such as $\eta \to 3\pi$), it has been recognized that it is necessary to treat the $\Delta I = 1$ and $\Delta I = 2$ effects separately. From a dispersion-theory viewpoint, as pointed out by Harari[6] and others, this amounts to a statement that we can calculate the $\Delta I = 2$ processes by ordinary Feynman diagrams involving photon exchange, since the amplitudes for these processes satisfy *unsubtracted* dispersion relations, implying that high-energy contributions can be safely neglected. On the other hand, the $\Delta I = 1$ processes are said to satisfy *subtracted* dispersion relations so that a knowledge of the subtraction constant (which does not come from the lowest-order Feynman diagram) is required. This general argument is borne out by the fact that the $\Delta I = 2$ π^+-π^0 mass difference has been calculated successfully from essentially second-order perturbation theory by many workers.[7] On the other hand, all the $\Delta I = 1$ mass differences and the $\Delta I = 1$ $\eta \to 3\pi$ processes have not been explained by analogous calculations.

One way to explain the $\Delta I = 1$ mass differences is to use the "tadpole" approach of Coleman and Glashow.[8] By postulating scalar mesons having nonzero vacuum expectation values, they were able to correlate neatly the octet-baryon electromagnetic mass differences with the kaon electromagnetic mass shift. The apparent drawback to their model is that the scalar mesons were introduced on a rather *ad hoc* basis.

Slightly more recently it has become apparent that the (essentially equivalent) "current-algebra" or "phenomenological-Lagrangian" approaches to strong interactions, which are based on a chiral $SU(3)\times SU(3)$ symmetry, are successful in explaining a fairly large body of low-energy phenomena. It is noteworthy that rather broad scalar mesons (some of which have nonzero vacuum expectation values) are *intrinsically contained*[9] in these models. Thus it would seem that the same type of model which gives all the well-known (and not so well-known) "current-algebra" results should also provide an explicit realization of the "tadpole" mechanism and explain the $\Delta I=1$ electromagnetic effects. In what follows we attempt to substantiate this claim.

Even though the basic mechanism is similar to that of Coleman and Glashow, the actual details of the calculation, as might be expected, are very different. From a physical viewpoint it may be helpful to think of the present model in the following terms: In order to apply electromagnetic perturbation theory it is necessary to find the *stable* ground state of the system under consideration. However, the electromagnetic interaction plays a part in establishing the ground state. This manifests itself in the appearance of an effective "tadpole"-type interaction.

A brief review of the model and a discussion of the symmetry properties of the ground state is given in Sec. II. Section III contains the treatment of the spin-zero-meson and octet-baryon electromagnetic mass splittings. In Sec. IV it is shown that $\eta\to 3\pi$ can be explained in the present model, while an analogous treatment for the rare mode $\eta'\to 3\pi$ is presented in Sec. V.

II. MODEL LAGRANGIAN

We start from the following well-known Lagrangian density of the linear $SU(3)$ σ model:

$$\mathcal{L}=-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu\phi\partial_\mu\phi)-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu S\partial_\mu S)-V_0-V_{SB}, \tag{2.1}$$

where ϕ and S are, respectively, the 3×3 matrices of pseudoscalar and scalar fields. V_0 is the most general nonderivative chiral invariant and V_{SB} is a symmetry-breaking term which will be taken to include the second-order effects of electromagnetism as well as the so-called "medium-strong" effects.

In the treatment of (2.1), it is essential to take account of symmetry breaking in the vacuum or ground state. This can be conveniently handled by a technique analogous to the introduction of normal coordinates in small-oscillation theory. An extensive discussion has been given, for example, in our previous papers. It is necessary to introduce as parameters in the theory the three vacuum expectation values:

$$\alpha_a=\langle S_a^a\rangle_0 \quad (a=1,2,3). \tag{2.2}$$

Then, the breakdown of symmetry arises not only from explicit terms in V_{SB} but also from nonzero values of the α_a. Furthermore, the ground state is required to satisfy a "minimum" condition

$$\left\langle\frac{\partial V_0}{\partial S_b^a}\right\rangle_0+\left\langle\frac{\partial V_{SB}}{\partial S_b^a}\right\rangle_0=0,$$

which leads to the three equations

$$\alpha_a[V_1+2V_2\alpha_a{}^2+3V_3\alpha_a{}^4]+6\,\frac{\alpha_1\alpha_2\alpha_3}{\alpha_a}\,V_4=-\tfrac{1}{2}\left\langle\frac{\partial V_{SB}}{\partial S_a^a}\right\rangle_0 \quad (a=1,2,3), \tag{2.3}$$

where the V_i are the coefficients $\langle\partial V_0/\partial I_i\rangle_0$, the I_i being the four independent chiral invariants which can be made from ϕ and S [see Eq. (22) of paper I].

As an explicit symmetry-breaking term we shall, as before, use the expression

$$V_{SB}=-2(A_1S_1^1+A_2S_2^2+A_3S_3^3)+d_\pi\phi_1^2\phi_2^1+d_K\phi_1^3\phi_3^1+\cdots. \tag{2.4}$$

The three coefficients A_i are analogous to the quark "masses" in Gell-Mann's quark model.[4] It seems fair to expect that the origin of the difference between A_1 and A_2 is due to a (second-order) electromagnetic "tadpole" diagram [see Fig. 1(a)]. However, it is not necessary for us to specify the origin of the A_i in order to proceed with the calculation. We only mention that a number of authors[10-12] have recently proposed interesting speculations on this matter.

The quantities d_π and d_K are expected to represent contributions to the charged pion and charged kaon masses from Feynman diagrams of the "self-energy" type [see Fig. 1(b)]. These have been estimated by many workers. It is found[7] that d_π essen-

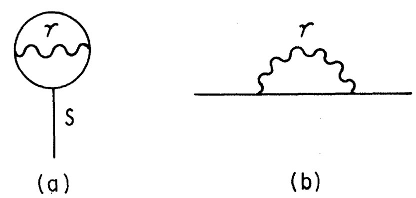

FIG. 1. Diagrams for electromagnetic perturbations.

tially explains the entire π^+-π^0 mass difference and is therefore

$$d_\pi \simeq 0.069, \tag{2.5}$$

in units of squared π^0 masses. For d_K the value[13]

$$d_K \simeq 0.15, \tag{2.6}$$

in squared π^0 mass units [corresponding to $(K^+\text{-}K^0)_{\text{self-energy}} \simeq 2.8$ MeV], is generally agreed upon. This quantity has the *wrong* sign to explain the K^+-K^0 mass difference by itself.

We will first show that the ground state of our model must violate $SU(2)$ invariance. With the symmetry-breaking term of (2.4) the first two equations of (2.3) become

$$\begin{aligned}&\alpha_1[V_1+2V_2(\alpha_1)^2+3V_3(\alpha_1)^4]+6\alpha_2\alpha_3V_4=A_1,\\&\alpha_2[V_1+2V_2(\alpha_2)^2+3V_3(\alpha_2)^4]+6\alpha_1\alpha_3V_4=A_2.\end{aligned} \tag{2.7}$$

Subtracting these two gives

$$\begin{aligned}(\alpha_1-\alpha_2)[V_1&+2V_2(\alpha_1{}^2+\alpha_2{}^2+\alpha_1\alpha_2)\\&+3V_3(\alpha_1{}^4+\alpha_2{}^4+\alpha_1{}^2\alpha_2{}^2+\alpha_1{}^3\alpha_2+\alpha_2{}^3\alpha_1)\\&-6V_4\alpha_3]=A_1-A_2.\end{aligned} \tag{2.8}$$

Thus if $\alpha_1=\alpha_2$ we must have $A_1=A_2$ if the V_i's are all finite. In other words, when V_{SB} contains an intrinsic symmetry-breaking term such that $A_1\neq A_2$, the ground state of this model *cannot* be $SU(2)$-invariant unless we take a limit of the theory in which the V_i's become infinite. In a similar way we see that in the $SU(2)$ limit, the ground state cannot be $SU(3)$-invariant. We remark that most previous work has assumed the $SU(3)$ invariance of the ground state ($\alpha_1=\alpha_2=\alpha_3$). In our model this assumption is inconsistent. However, it is allowed if forms of V_{SB} different from (2.4) are adopted (see Ref. 11 of paper III).

The masses of the pseudoscalar mesons and some of the scalar mesons in this model can be computed from chiral symmetry alone. This was done in paper II. For the convenience of the reader we reproduce the results here. [The pseudoscalar nonet is designated by (π, K, η, η') and the scalar nonet by $(\epsilon, \kappa, \sigma, \sigma')$. The particle symbol will also stand for its mass.] Then

$$\begin{aligned}\pi_+{}^2&=2\left(\frac{A_1+A_2}{\alpha_1+\alpha_2}\right)+d_\pi,\\K_+{}^2&=2\left(\frac{A_1+A_3}{\alpha_1+\alpha_3}\right)+d_K,\\K_0{}^2&=2\left(\frac{A_2+A_3}{\alpha_2+\alpha_3}\right);\end{aligned} \tag{2.9}$$

$$\begin{aligned}\epsilon_+{}^2&=2\left(\frac{A_1-A_2}{\alpha_1-\alpha_2}\right),\\\kappa_+{}^2&=2\left(\frac{A_1-A_3}{\alpha_1-\alpha_3}\right),\\\kappa_0{}^2&=2\left(\frac{A_2-A_3}{\alpha_2-\alpha_3}\right).\end{aligned} \tag{2.10}$$

Finally, the squared masses of the π^0, η, and η' particles are given by the roots of the secular equation of the following matrix whose (ab) element is $\langle \partial^2V_0/\partial\phi_a^a\partial\phi_b^b\rangle_0$:

$$\begin{pmatrix}2A_1/\alpha_1-12V_4\alpha_2\alpha_3/\alpha_1 & -12V_4\alpha_3 & -12V_4\alpha_2\\ -12V_4\alpha_3 & 2A_2/\alpha_2-12V_4\alpha_1\alpha_3/\alpha_2 & -12V_4\alpha_1\\ -12V_4\alpha_2 & -12V_4\alpha_1 & 2A_3/\alpha_3-12V_4\alpha_1\alpha_2/\alpha_3\end{pmatrix}. \tag{2.11}$$

III. ELECTROMAGNETIC MASS DIFFERENCES AND MIXINGS

The comparison of (2.9), (2.10), and (2.11) with experiment was done in paper I for the isospin-invariant limit ($\alpha_1=\alpha_2$ and $A_1=A_2$). Here we will allow $\alpha_1\neq\alpha_2$ and $A_1\neq A_2$; the basic assumption in this approach will be that solutions are only *slightly* different from the ones in the isospin-invariant case. Thus deviations from isospin invariance will be treated to first order. Actually, this was already done in paper II; here we will reach similar conclusions to paper II by a much simpler and clearer method and, in addition, extract more information from the system.

In order to diagonalize the matrix (2.11) we write the physical π^0, η, and η' fields as the most general linear combinations of the fields ϕ_1^1, ϕ_2^2, ϕ_3^3 which differ only to first order from the appropriate combinations in the isospin-invariant limit [Eq. (3.13) of paper III]. Thus we write[14]

$$\begin{pmatrix}\pi^0\\ \eta\\ \eta'\end{pmatrix}=\begin{pmatrix}(\frac{1}{2})^{1/2}+\psi_1b+\psi_3a & -(\frac{1}{2})^{1/2}+\psi_1b+\psi_3a & \sqrt{2}(-\psi_1a+\psi_3b)\\ -(\frac{1}{2})^{1/2}\psi_1+b & (\frac{1}{2})^{1/2}\psi_1+b & -\sqrt{2}\,a\\ -(\frac{1}{2})^{1/2}\psi_3+a & (\frac{1}{2})^{1/2}\psi_3+a & \sqrt{2}\,b\end{pmatrix}\begin{pmatrix}\phi_1^1\\ \phi_2^2\\ \phi_3^3\end{pmatrix}, \tag{3.1}$$

where ψ_1 is the $\pi^0\eta$ mixing angle, ψ_3 is the $\pi^0\eta'$ mixing angle, and

$$a=(\tfrac{1}{6})^{1/2}(\sin\theta_p+\sqrt{2}\cos\theta_p)\,,\qquad b=(\tfrac{1}{6})^{1/2}(\cos\theta_p-\sqrt{2}\sin\theta_p)\,, \tag{3.2}$$

θ_p being the η-η' mixing angle. ψ_1 and ψ_3 are quantities of first order.

It is convenient to give the matrix elements of the mass matrix (2.11) in terms of the quantities just introduced, and the squared masses of the physical particles π_0, η, and η'. We have, by transforming the mass matrix with the matrix of (3.1),

$$\begin{aligned}
\left\langle\frac{\partial^2 V_0}{\partial\phi_1^1\partial\phi_1^1}\right\rangle_0 &= \pi_0{}^2(\tfrac{1}{2}+\sqrt{2}\,\psi_1 b+\sqrt{2}\,\psi_3 a)+\eta^2(b^2-\sqrt{2}\,b\psi_1)+\eta'^2(a^2-\sqrt{2}\,a\psi_3)\,,\\
\left\langle\frac{\partial^2 V_0}{\partial\phi_2^2\partial\phi_2^2}\right\rangle_0 &= \pi_0{}^2(\tfrac{1}{2}-\sqrt{2}\,\psi_1 b-\sqrt{2}\,\psi_3 a)+\eta^2(b^2+\sqrt{2}\,b\psi_1)+\eta'^2(a^2+\sqrt{2}\,a\psi_3)\,,\\
\left\langle\frac{\partial^2 V_0}{\partial\phi_3^3\partial\phi_3^3}\right\rangle_0 &= 2a^2\eta^2+2b^2\eta'^2\,,\\
\left\langle\frac{\partial^2 V_0}{\partial\phi_1^1\partial\phi_2^2}\right\rangle_0 &= -\tfrac{1}{2}\pi_0{}^2+b^2\eta^2+a^2\eta'^2\,,\\
\left\langle\frac{\partial^2 V_0}{\partial\phi_1^1\partial\phi_3^3}\right\rangle_0 &= \pi_0{}^2(-\psi_1 a+\psi_3 b)+\eta^2(-\sqrt{2}\,ab+\psi_1 a)+\eta'^2(\sqrt{2}\,ab-\psi_3 b)\,,\\
\left\langle\frac{\partial^2 V_0}{\partial\phi_2^2\partial\phi_3^3}\right\rangle_0 &= \pi_0{}^2(\psi_1 a-\psi_3 b)+\eta^2(-\sqrt{2}\,ab-\psi_1 a)+\eta'^2(\sqrt{2}\,ab+\psi_3 b)\,.
\end{aligned} \tag{3.3}$$

Equating (3.3) and (2.11) permits identification of the symmetry-breaking parameters in terms of "physical" quantities.

The formulas (3.1)–(3.3) also hold for mixing in the $(\epsilon^0,\sigma,\sigma')$ system when we make the replacements

$$\psi_1\to\chi_1\,,\quad \theta_p\to\theta_s\,,\quad b\to b'\,,\quad \psi_3\to\chi_3\,,\quad a\to a'\,. \tag{3.4}$$

Now let us assume that $\alpha_1-\alpha_2$ and A_1-A_2 are objects of first order, and express electromagnetic quantities in terms of them. For the "tadpole" part of the kaon mass shift $\delta\tilde{K}^2$, which is defined by subtracting the self-energy contribution d_K from the physical difference $K_+{}^2-K_0{}^2$, we have from (2.9)

$$\begin{aligned}\delta\tilde{K}^2 &\equiv K_+{}^2-K_0{}^2-d_K\\ &=\frac{2}{(\alpha_1+\alpha_3)(\alpha_2+\alpha_3)}\{(\alpha_2-\alpha_1)[\tfrac{1}{2}(A_1+A_2)+A_3]+(A_1-A_2)[\tfrac{1}{2}(\alpha_1+\alpha_2)+\alpha_3]\}\,,\end{aligned} \tag{3.5}$$

where no approximation has as yet been made. Assuming that A_1-A_2 is negligible compared to A_3 and that $\alpha_1-\alpha_2$ is negligible compared to α_3 enables us to write (3.5) as

$$\delta\tilde{K}^2=\frac{2}{(\alpha_1+\alpha_3)^2}[(\alpha_2-\alpha_1)(A_1+A_3)+(A_1-A_2)(\alpha_1+\alpha_3)]\,.$$

Using the formulas for $\epsilon_+{}^2$ and $K_+{}^2$, we can put this in the compact form

$$\delta\tilde{K}^2=\frac{\alpha_1-\alpha_2}{\alpha(1+W)}(\epsilon^2-K^2)\,, \tag{3.6}$$

where α is the value of α_1 in the isospin limit and

$$W=\alpha_3/\alpha\,. \tag{3.7}$$

We have omitted the charge indices from ϵ^2 and K^2 on the right-hand side of (3.6) because these objects may evidently be evaluated in the isospin limit.

It is presumably of only formal interest to note that the analogous formula to (3.6) for the "tadpole" part of the κ mass shift is

$$\delta\tilde{\kappa}^2=\frac{\alpha_1-\alpha_2}{\alpha(1-W)}(\epsilon^2-\kappa^2)\,. \tag{3.8}$$

To evaluate the "tadpole" part of the pion mass shift we first note from (3.3) that

$$\pi_0{}^2=\frac{1}{2}\left(\left\langle\frac{\partial^2 V_0}{\partial\phi_1^1\partial\phi_1^1}\right\rangle_0+\left\langle\frac{\partial^2 V_0}{\partial\phi_2^2\partial\phi_2^2}\right\rangle_0-2\left\langle\frac{\partial^2 V_0}{\partial\phi_1^1\partial\phi_2^2}\right\rangle_0\right).$$

Substituting the matrix elements from (2.11) into this gives

$$\pi_0{}^2=\frac{-6V_4\alpha_3}{\alpha_1\alpha_2}(\alpha_1-\alpha_2)^2+\left(\frac{A_1}{\alpha_1}+\frac{A_2}{\alpha_2}\right),$$

which may be combined with the expression for π_+^2 in (2.9) to reach the desired result

$$\delta\bar{\pi}^2=\pi_+^2-\pi_0^2-d_\pi$$
$$=\frac{(\alpha_1-\alpha_2)^2}{4\alpha_1\alpha_2}(24V_4\alpha_3+\epsilon_+^2-\pi_+^2)\,, \qquad (3.9)$$

where no approximation has been made. We see from (3.9) that to *first order* in $\alpha_1-\alpha_2$,

$$\delta\bar{\pi}^2=0\,. \qquad (3.10)$$

Thus, in this model, all the π^+-π^0 mass splitting is due to the ordinary self-energy diagrams. This agrees nicely with the many successful calculations on this basis. Equation (3.10) is, of course, not really surprising, since the "tadpole" electromagnetic perturbation is a pure $\Delta I=1$ object which can not contribute to the π^+-π^0 mass difference to first order since it must have negative G parity. From the same general argument we may conclude that the "tadpole" part of the ϵ^+-ϵ^0 mass difference, $\delta\bar{\epsilon}^2$, also vanishes. [This may be explicitly verified by using Eqs. (25), (26), and (22′) of paper I.]

The $\pi^0\eta$ and $\pi^0\eta'$ mixing angles ψ_1 and ψ_3 may also be computed in terms of A_1-A_2 and $\alpha_1-\alpha_2$. To do this we first calculate

$$\left\langle\frac{\partial^2V_0}{\partial\phi_1^1\partial\phi_1^1}\right\rangle_0-\left\langle\frac{\partial^2V_0}{\partial\phi_2^2\partial\phi_2^2}\right\rangle_0$$

from both (2.11) and (3.3), i.e.,

$$2\left(\frac{A_1}{\alpha_1}-\frac{A_2}{\alpha_2}\right)-12V_4\alpha_3\left(\frac{\alpha_2}{\alpha_1}-\frac{\alpha_1}{\alpha_2}\right)$$
$$=2\sqrt{2}\,\pi_0^2(b\psi_1+a\psi_3)-2\sqrt{2}\,\eta^2b\psi_1-2\sqrt{2}\,\eta'^2a\psi_3 \qquad (3.11)$$

and do the same for

$$\left\langle\frac{\partial^2V_0}{\partial\phi_1^1\partial\phi_3^3}\right\rangle_0-\left\langle\frac{\partial^2V_0}{\partial\phi_2^2\partial\phi_3^3}\right\rangle_0.$$

This gives

$$12V_4(\alpha_1-\alpha_2)=-2\pi_0^2(a\psi_1-b\psi_3)+2\eta^2a\psi_1-2\eta'^2b\psi_3\,. \qquad (3.12)$$

Solving (3.11) and (3.12) simultaneously and keeping terms of first order in $\alpha_1-\alpha_2$ and A_1-A_2 gives the final results

$$\psi_1=\frac{b(\alpha_1-\alpha_2)}{\sqrt{2}\,\alpha(\eta^2-\pi^2)}\left[\pi^2-\epsilon^2+2a(\sqrt{2}\,bW-a)(\eta'^2-\eta^2)\right]\,,$$
$$(3.13)$$
$$\psi_3=\frac{a(\alpha_1-\alpha_2)}{\sqrt{2}\,\alpha(\eta'^2-\pi^2)}\left[\pi^2-\epsilon^2+2b(\sqrt{2}\,aW+b)(\eta'^2-\eta^2)\right].$$

Now all the electromagnetic "tadpole" contributions and pseudoscalar electromagnetic mixings have been expressed in terms of A_1-A_2 and $\alpha_1-\alpha_2$. The only objects which could not be so expressed were the $\epsilon^0\sigma$ and $\epsilon^0\sigma'$ mixing angles χ_1 and χ_3 [see Eq. (3.4)]. The reason for this is that, as noted in papers I and III, chiral symmetry alone does not relate the parameters of the $(\epsilon^0\sigma\sigma')$ mass matrix to other quantities.

Before studying the above equations we note that more information can be obtained from the baryon electromagnetic mass splittings. Previous analyses[8,13] have shown that the self-energy-type diagrams give relatively small contributions compared to "tadpole"-type diagrams for the baryons. To get an idea of what is happening we shall make the simplifying assumption that all baryon masses arise from the "tadpole" mechanism. In the present context this means that all octet baryon masses should arise from a nonderivative *chiral-invariant* baryon-meson interaction. In the Appendix of paper II we noted that the simplest chiral-invariant interaction which explains all eight different octet masses leads to the relation

$$\alpha_1-\alpha_2=\alpha(W-1)\frac{\Sigma^+-\Sigma^-}{\Xi-N}\,, \qquad (3.14)$$

where the particle symbol denotes the mass and wherein we have, for convenience of writing, used the experimentally accurate Coleman-Glashow formula.[8] Numerically, (3.13) yields

$$\alpha_1-\alpha_2\simeq-0.0075\pi_0^2\,, \qquad (3.15)$$

since $W\simeq1.7$ and $\alpha\simeq\frac{1}{2}\pi^0$ mass units.[15]

We shall adopt (3.15) for the purpose of making further numerical estimates. Then, once the value of A_1-A_2 is specified, all the electromagnetic parameters above are known. Equation (2.10) shows that choosing a value of A_1-A_2 is [assuming (3.15) to hold] the same as choosing the value of the ϵ^+ (isovector particle) mass. It is rather amusing that this "strong" mass comes out to be the ratio of two "electromagnetic" quantities in the present model. Unfortunately, the experimental existence of such a particle is not well established. There is some evidence[16] for the so-called $\delta(962)$ ($\epsilon_+^2\simeq50.8\pi_0^2$), but this is a narrow resonance and (see paper III) the ϵ in the present model is probably rather broad (although modifications of the model may change this).

In any event we shall proceed by choosing different values of ϵ_+^2 and computing the corresponding values of all the other parameters. These are shown in Table I. The values of the electromagnetic mixing angles ψ_1 and ψ_3 are useful in connection with other electromagnetic processes like $\eta\to3\pi$ decay while the values of $\delta\tilde{\kappa}^2$ are just of academic interest.

The predictions of present interest are in the columns for $\delta\tilde{K}^2$ and A_1-A_2. For comparison we

TABLE I. Electromagnetic parameters for $\alpha_1 - \alpha_2 = -0.0075\ \pi_0^2$ and different values of ϵ_+^2.

ϵ_+^2 (π_0^2 units)	$A_1 - A_2$ (π_0^3 units)	$\delta\tilde{\kappa}^2$ (π_0^2 units)	ψ_1	ψ_3	$\delta\tilde{K}^2$ (π_0^2 units)
25	−0.093	−0.52	2.2×10^{-3}	-3.2×10^{-3}	−0.063
50.8	−0.19	+0.025	9.3×10^{-3}	$\simeq 0$	−0.21
80.5	−0.30	+0.66	1.7×10^{-2}	3.7×10^{-3}	−0.37
100	−0.37	+1.08	2.3×10^{-2}	6.1×10^{-3}	−0.48
200	−0.75	+3.21	5.0×10^{-2}	1.8×10^{-2}	−1.03

note that the experimental kaon mass shift is $K_+^2 - K_0^2 = \delta\tilde{K}^2 + d_K \simeq -0.22\pi_0^2$. It is amusing to note that if we identify the ϵ with the $\delta(962)$, the entire kaon mass shift can be explained from the "tadpole" contribution alone. However, this may be just a coincidence. Another possibility would be to choose the nontadpole part from Eq. (2.6). Then the ϵ mass would be 1210 MeV ($\epsilon_+^2 \simeq 80.5\pi_0^2$).

Still another possibility would be to assume the ϵ to be a particle of even higher mass. Then it would be necessary to have a relatively large positive d_K to compensate for the large negative value of $\delta\tilde{K}^2$. We should stress that the calculations[13] leading to (2.6) are somewhat model dependent so (2.6) should probably not be accepted without question.

One amusing point which may potentially have deep significance is that for all the interesting values of ϵ_+^2 the magnitude $|A_1 - A_2|$ is always numerically comparable with $\frac{1}{2}(A_1+A_2)$ ($\simeq \frac{1}{2}\alpha\pi_+^2 \simeq \frac{1}{4}\pi_0^3$). This was one of the main conclusions of paper II. In our formulas above we neglected $|A_1 - A_2|$ compared with A_3 (which *is* valid since $A_3 \simeq 8.3\pi_0^3$) but never compared with $\frac{1}{2}(A_1+A_2)$. Thus, this result does not impair the validity of our approach. Since $|A_1 - A_2|$ is a measure of the strength of the "electromagnetic" part of V_{SB} while $\frac{1}{2}|A_1 + A_2|$ is a measure of the chiral-$SU(2)\times SU(2)$-breaking part of V_{SB}, this may indicate a close connection between electromagnetism and the breaking of chiral symmetry. However, we shall not speculate any further in the present paper.

IV. $\eta \to 3\pi$ DECAY

To explain $\eta \to 3\pi$ decay by either the current-algebra or phenomenological-Lagrangian technique has been regarded as a major puzzle. In this section we will give a possible solution of the puzzle.

It was noted by Sutherland[17] that the nontadpole contribution to this decay vanishes in the soft-pion limit. It is reasonable to neglect this type of contribution for physical pions too. Hence we are left with the "tadpole" kind of terms, or, from the standpoint of an effective Hamiltonian, a *scalar-density* electromagnetic perturbation. It is easy to see[18] that such a term gives the right spectrum shape when treated by the usual current-algebra techniques. However, the decay rate comes out to be considerably smaller[19] than the experimental value. We shall calculate the "tadpole" contribution in our model.[20] It will be noted that the model intrinsically contains a sufficient number of parameters to fit the experimental spectrum shape and decay rate. By taking a formal limit where the scalar masses go to infinity, the amplitude will be seen to reduce to the "current-algebra" one which gives the right spectrum shape but a decay rate smaller than the experimental value.

One may ask the question as to whether in picking up the tadpole contribution we are really introducing a basically new nonminimal electromagnetic interaction in nature. We feel (but cannot prove at present) that the tadpole actually arises from the usual minimal electromagnetic interaction among the *unphysical* scalar-meson fields. We cannot do perturbation theory with respect to the unphysical fields because they represent excitations around an unstable ground state or vacuum. In order to do perturbation theory it is necessary to define the physical fields (see paper III) which correspond

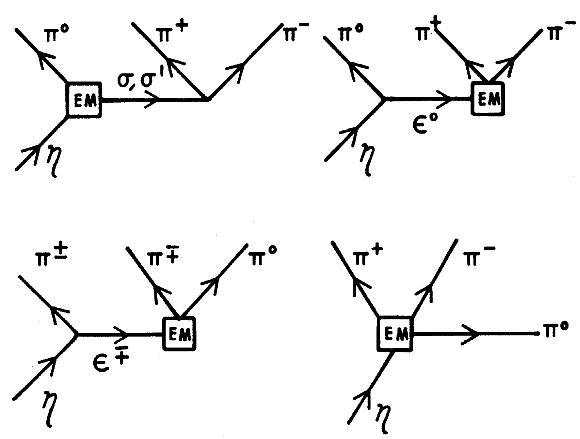

FIG. 2. Diagrams for $\eta \to 3\pi$ decay. EM denotes an effective electromagnetic vertex.

to excitations about the physical ground state as a zero point. The condition insuring the stability of the physical ground state is just Eq. (2.3). The crucial point is that the electromagnetic interaction of usual type goes into establishing the physical ground state (through the fact that $A_1 \neq A_2$). All the usual calculations, including the one of Sutherland,[17] deal with additional electromagnetic perturbations around the physical ground state, which already contains the effects of electromagnetism in our model (i.e., tadpoles). However, it should be stressed that the origin of the tadpole is irrelevant to the present work.

The actual computation is fairly straightforward. The relevant Feynman diagrams for $\eta\to\pi^+\pi^-\pi^0$ are shown in Fig. 2. Note that there are no diagrams with direct $\pi^0\eta$, $\epsilon^0\sigma$, etc., transitions. This is because we have diagonalized the $(\pi^0\eta\eta')$ system by (3.1) and the $(\epsilon^0\sigma\sigma')$ system by an equation analogous to (3.1) using the replacements given in (3.4). The ϕ^4 and some of the $S\phi\phi$ (effective) electromagnetic vertices can be related to other electromagnetic parameters of the model by using the chiral symmetry of V_0 in (2.1). Explicit formulas for doing this are given in paper III. We write the strong and electromagnetic Lagrangians which contribute to $\eta\to 3\pi$ below:

$$-\mathcal{L}_{\rm strong}(\eta\to 3\pi)=g_{\epsilon\pi\eta}\,\eta\vec{\pi}\cdot\vec{\epsilon}+\tfrac{1}{2}g_{\sigma\pi\pi}\sigma\vec{\pi}\cdot\vec{\pi}+\tfrac{1}{2}g_{\sigma'\pi\pi}\sigma'\vec{\pi}\cdot\vec{\pi}, \tag{4.1}$$

$$-\mathcal{L}_{\rm EM}(\eta\to 3\pi)=f_{\sigma\pi\eta}\,\sigma\pi^0\eta+f_{\sigma'\pi\eta}\,\sigma'\pi^0\eta+f^{(+)}_{\epsilon\pi\pi}(\epsilon^+\pi^0\pi^-+\epsilon^-\pi^0\pi^+)+f^{(0)}_{\epsilon\pi\pi}\,\epsilon^0\pi^+\pi^-+f^{(4)}\eta\pi^+\pi^-\pi^0. \tag{4.2}$$

In (4.1) the isospin notation was used. The "strong" coupling constants, $g_{\epsilon\pi\eta}$, $g_{\sigma\pi\pi}$, and $g_{\sigma'\pi\pi}$, are given in Eq. (4.7) of paper III. The electromagnetic couplings $f_{\sigma\pi\eta}$ and $f_{\sigma'\pi\eta}$ are not related to other things by chiral symmetry while the others are

$$f^{(+)}_{\epsilon\pi\pi}=\frac{1}{\alpha}(\psi_1 b+\psi_3 a)(\epsilon^2-\pi^2), \tag{4.3}$$

$$f^{(0)}_{\epsilon\pi\pi}=\frac{1}{\alpha}(\chi_1 b'+\chi_3 a')(\epsilon^2-\pi^2), \tag{4.4}$$

$$f^{(4)}=\frac{b}{\alpha}f^{(+)}_{\epsilon\pi\pi}+\frac{b'}{\alpha}f_{\sigma\pi\eta}+\frac{a'}{\alpha}f_{\sigma'\pi\eta}+\frac{1}{\alpha}[(\psi_1 b+\psi_3 a)+(\chi_1 b'+\chi_3 a')]g_{\epsilon\pi\eta}. \tag{4.5}$$

To see how (4.3) is derived, as an example, we note that $f^{(+)}_{\epsilon\pi\pi}$ can be written as

$$f^{(+)}_{\epsilon\pi\pi}=\left\langle\frac{\partial^3 V_0}{\partial S_1^2\partial\pi^0\partial\phi_2^1}\right\rangle_0=\sum_{a=1}^{3}\frac{\partial\phi_a^a}{\partial\pi^0}\left\langle\frac{\partial^3 V_0}{\partial S_1^2\partial\phi_a^a\partial\phi_2^1}\right\rangle_0.$$

The quantities $\partial\phi_a^a/\partial\pi^0$ can be calculated from (3.1), while the quantities

$$\left\langle\frac{\partial^3 V_0}{\partial S_1^2\partial\phi_a^a\partial\phi_2^1}\right\rangle_0$$

can be related to ϵ^2 and π^2 by the generalized "Goldberger-Treiman" formula (4.2) of paper III. Equations (4.4) and (4.5) are derived similarly.

Using the interactions of (4.1) and (4.2) we compute the T amplitude for $\eta\to\pi^+\pi^-\pi^0$ to be

$$\begin{aligned}T(\eta\to\pi^+\pi^-\pi^0)=&\frac{b'}{\alpha}f_{\sigma\pi\eta}\left(1-\frac{\sigma^2-\pi^2}{\sigma^2-\eta^2-\pi^2+2\eta\omega_0}\right)+\frac{a'}{\alpha}f_{\sigma'\pi\eta}\left(1-\frac{\sigma'^2-\pi^2}{\sigma'^2-\eta^2-\pi^2+2\eta\omega_0}\right)\\&+\frac{b}{\alpha^2}(\chi_1 b'+\chi_3 a')(\epsilon^2-\eta^2)\left(1-\frac{\epsilon^2-\pi^2}{\epsilon^2-\eta^2-\pi^2+2\eta\omega_0}\right)+\frac{b}{\alpha^2}(\psi_1 b+\psi_3 a)\\&\times\left[(2\epsilon^2-\pi^2-\eta^2)-(\epsilon^2-\eta^2)(\epsilon^2-\pi^2)\left(\frac{1}{\epsilon^2-\eta^2-\pi^2+2\eta\omega_+}+\frac{1}{\epsilon^2-\eta^2-\pi^2+2\eta\omega_-}\right)\right].\end{aligned} \tag{4.6}$$

Equation (4.6) was evaluated in the rest system of the η meson. The energies of the plus, minus, and neutral pions are denoted by ω_+, ω_-, and ω_0.

It is interesting to write (4.6) for the case (as actually seems to hold) where the scalar-meson squared masses ϵ^2, σ^2, and σ'^2 are large compared to π^2 and η^2:

$$T(\eta\to\pi^+\pi^-\pi^0)\simeq\frac{-\eta^2}{\alpha}\left(1-\frac{2\omega_0}{\eta}\right)\left\{\frac{b'f_{\sigma\pi\eta}}{\sigma^2}+\frac{a'f_{\sigma'\pi\eta}}{\sigma'^2}+\frac{b}{\alpha}[(b'\chi_1+a'\chi_3)-(b\psi_1+a\psi_3)]\right\}. \tag{4.7}$$

We immediately notice that the spectrum is predicted to be of the form $(1-2\omega_0/\eta)$. It has been previously observed[18] that this spectrum shape is in very good agreement with the experimental[21] one.

From (4.7) we can easily see why the "current-algebra" calculation gave the right spectrum shape but too small a decay rate. It was pointed out by Weinberg[22] that the prescription for getting the "current-algebra" results from the δ-model results is to let the scalar masses go to infinity. In this limit the $f_{\sigma\pi\eta}$ and $f_{\sigma'\pi\eta}$ terms in (4.7) vanish since these coupling constants are *not* related to other objects by generalized Goldberger-Treiman relations and hence remain finite. We shall also impose the prescription of setting the electromagnetic scalar mixing angles χ_1 and χ_3 to zero as we let the scalar masses become indefinitely large in order to get the "current-algebra" formula. Then (4.7) becomes

$$\frac{b}{\alpha^2}(\psi_1 b+\psi_3 a)\,\eta^2\left(1-\frac{2\omega_0}{\eta}\right).$$

Using (3.13) and (3.6) we may finally express the amplitude in the "current-algebra" limit as

$$T_{\rm CA}(\eta\to\pi^+\pi^-\pi^0)=\frac{-(1+W)\delta\tilde{K}^2}{12\sqrt{3}\,\alpha^2}\left[1+2\left(\frac{\eta}{\eta'}\right)^2\right]\left(1-\frac{2\omega_0}{\eta}\right), \tag{4.8}$$

where we have set $a=(\frac{1}{3})^{1/2}$ and $b=(\frac{1}{6})^{1/2}$, corresponding to negligible $\eta\eta'$ mixing (see paper III). From (4.8) the numerical width[23] in eV is given by $\Gamma=450(\delta\tilde{K}^2)^2$, where $\delta\tilde{K}^2$ is expressed in $\pi_0{}^2$ units. When $\delta\tilde{K}^2=-0.22\pi_0{}^2$ (corresponding to all the kaon mass splitting coming from the "tadpole"), $\Gamma\simeq 21$ eV, while for $\delta\tilde{K}^2=-0.37\pi_0{}^2$ [corresponding to the choice (2.6)], $\Gamma\simeq 62$ eV. Both of these values are considerably smaller than the experimental[16] one:

$$\Gamma_{\rm exp}(\eta\to\pi^+\pi^-\pi^0)=605\pm150\ \text{eV}. \tag{4.9}$$

We could explain the experimental rate in the "current-algebra" limit if we were willing to accept a sizeable value of d_K, but this would imply that the ordinary method[13] of estimating the self-energy diagrams leading to (2.6) is drastically wrong.

In the present model, of course, it is not necessary to go to the "current-algebra" limit. Since (4.7) shows that the spectrum shape is correctly predicted for all choices of parameters there are many ways of fitting the decay rate. To make an estimate we shall consider a simple model where $f_{\sigma\pi\eta}=f_{\sigma'\pi\eta}=0$. The experimental amplitude is

$$T_{\rm exp}(\eta\to\pi^+\pi^-\pi^0)\simeq\pm0.98(1-2\omega_0/\eta). \tag{4.10}$$

If (2.6) is adopted we find from Table I that $b\psi_1+a\psi_3\simeq0.92\times10^{-2}$. Then, in order to fit (4.10), we must have for the combination of $\epsilon^0\sigma$ and $\epsilon^0\sigma'$ mixing angles

$$b'\chi_1+a'\chi_3=\begin{cases}-2.8\times10^{-2}\\ +4.6\times10^{-2}\end{cases}, \tag{4.11}$$

where the upper value corresponds to the plus sign in (4.10) and the lower value corresponds to the minus sign. Numerically, (4.11) is about three to four times the magnitude of the analogous combinations of the $\pi^0\eta$ and $\pi^0\eta'$ mixing angles. This may not be unreasonable since Table I shows[24] that the "tadpole" part of the κ-squared mass shift is about three times the kaon-squared mass shift for large $\epsilon_+{}^2$.

Finally, we remark that if one adopts *scale invariance* as a good symmetry for V_0 in (2.1), relations between χ_1 and χ_3 and $f_{\sigma\pi\eta}$ and $f_{\sigma'\pi\eta}$ may be obtained. Using formulas (6.3) and (6.7) of paper III we find

$$(\epsilon^2-\sigma^2)(\sqrt{2}\,b'-Wa')\chi_1+(\epsilon^2-\sigma'^2)(\sqrt{2}\,a'+Wb')\chi_3=\frac{\alpha_1-\alpha_2}{\alpha}\epsilon^2, \tag{4.12}$$

$$f_{\sigma\pi\eta}(\sqrt{2}\,b'-Wa')+f_{\sigma'\pi\eta}(\sqrt{2}\,a'+Wb')+g_{\epsilon\pi\eta}\left[\frac{\alpha_1-\alpha_2}{2\alpha}+(\sqrt{2}\,b'-Wa')\chi_1+(\sqrt{2}\,a'+Wb')\chi_3\right]=0. \tag{4.13}$$

V. $\eta'\to3\pi$ DECAY

The main decay mode of the $\eta'(960)$ meson is the strong process $\eta'\to\eta2\pi$. Because the Q value of this decay is very low the width is quite small and competing electromagnetic modes may be observable.

The computation of $\eta'\to\pi^+\pi^-\pi^0$ in the present model is entirely analogous to the work in Sec. IV. Thus we just state the final result:

$$T(\eta' \to \pi^+\pi^-\pi^0) = \frac{b'}{\alpha} f_{\sigma\pi\eta'}\left(1 - \frac{\sigma^2 - \pi^2}{\sigma^2 - \pi^2 - \eta'^2 + 2\eta'\omega_0}\right) + \frac{a'}{\alpha} f_{\sigma'\pi\eta'}\left(1 - \frac{\sigma'^2 - \pi^2}{\sigma'^2 - \pi^2 - \eta'^2 + 2\eta'\omega_0}\right)$$
$$+\frac{a}{\alpha^2}(\chi_1 b' + \chi_3 a')(\epsilon^2 - \eta'^2)\left(1 - \frac{\epsilon^2 - \pi^2}{\epsilon^2 - \eta'^2 - \pi^2 + 2\eta'\omega_0}\right)$$
$$+\frac{a}{\alpha^2}(\psi_1 b + \psi_3 a)\left[(2\epsilon^2 - \eta'^2 - \pi^2) - (\epsilon^2 - \pi^2)(\epsilon^2 - \eta'^2)\left(\frac{1}{\epsilon^2 - \eta'^2 - \pi^2 + 2\eta'\omega_+} + \frac{1}{\epsilon^2 - \eta'^2 - \pi^2 + 2\eta'\omega_-}\right)\right]. \tag{5.1}$$

In the limit where σ^2, σ'^2, and ϵ^2 are large compared with π^2 and η'^2, each term in (5.1) by itself yields the characteristic spectrum shape

$$1 - 2\omega_0/\eta'. \tag{5.2}$$

However, since η'^2 is large, the approximation of neglecting it may not be very good for finite values of σ^2, σ'^2, and ϵ^2. Then it is more accurate to use the formula (5.1).

It may be of some interest to make a *very crude estimate* of the $\eta' \to 3\pi$ rate. In the approximation where the scalar masses are large and only the $(\psi_1 b + \psi_3 a)$ and $(\chi_1 b' + \chi_3 a')$ terms are retained, Eqs. (4.7) and (5.1) yield

$$\frac{T(\eta' \to \pi^+\pi^-\pi^0)}{T(\eta \to \pi^+\pi^-\pi^0)} \simeq \frac{a}{b}\left(\frac{\eta'}{\eta}\right)^2\left(1 - \frac{2\omega_0}{\eta'}\right)\left(1 - \frac{2\omega_0}{\eta}\right)^{-1}. \tag{5.3}$$

Denoting the Q values of these decays by $Q(\eta')$ and $Q(\eta)$ and using the nonrelativistic phase-space formula[23] gives, finally,

$$\Gamma(\eta' \to \pi^+\pi^-\pi^0) \simeq \left(\frac{a}{b}\right)^2\left(\frac{\eta'}{\eta}\right)^3\left[\frac{Q(\eta')}{Q(\eta)}\right]^2 \Gamma(\eta \to \pi^+\pi^-\pi^0)$$
$$\simeq 0.1 \text{ MeV}. \tag{5.4}$$

In (5.4) terms of order ω_0 have been neglected for simplicity.

The above result should only be interpreted as an order-of-magnitude estimate. The experimental[16] *upper limit* is

$$\frac{\Gamma(\eta' \to \pi^+\pi^-\pi^0)}{\Gamma(\eta' \to \text{all})} < 5\%.$$

Taking the (poorly determined) total width to be 10 MeV would give an upper limit of about 0.5 MeV which is consistent with our estimate. Further experimental investigation of this mode would be highly desirable.

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[14]Essentially we are multiplying (3.13) of paper III by the general infinitesimal rotation

$$\begin{pmatrix} 1 & \psi_1 & \psi_3 \\ -\psi_1 & 1 & \psi_2 \\ -\psi_3 & -\psi_2 & 1 \end{pmatrix}$$

and absorbing ψ_2 in the angle θ_p without loss of generality.

[15]The value $W=1.7$ corresponds to fitting the η' mass exactly with our formulas. This is not very different from the value $2F_K/F_\pi - 1 \simeq 1.56$ predicted from weak interactions. The value of $\alpha \simeq \frac{1}{2}\pi^0$ corresponds to $\alpha = \frac{1}{2}F_\pi$. See paper I.

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[20]It should be stressed that our "tadpole" model is actually quite different in detail from the one of Coleman and Glashow. Thus our treatment is different from earlier approaches using the "tadpole" idea. See, for example, S. K. Yun, Phys. Rev. 173, 1764 (1968); L. Clavelli, Nuovo Cimento 52A, 73 (1967).

[21]Experimentalists parametrize the decay amplitude as being proportional to

$$1+\beta y\,,$$

where $y=3T_0/Q-1$. T_0 is the kinetic energy $\omega_0-\pi^0$ of the π^0. The form $1-2\omega_0/\eta$ corresponds to $\beta=-0.48$ while the experimental value [Columbia-Berkeley-Purdue-Wisconsin-Yale collaboration, Phys. Rev. 149, 1049 (1966)] is $\beta=-0.478$.

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[23]If the T amplitude for $\eta\to\pi^+\pi^-\pi^0$ is expanded as $T=\tau_0+\tau_1(\omega_0-\pi^0)+\cdots$, the width in the nonrelativistic limit is given by

$$\Gamma(\eta\to\pi^+\pi^-\pi^0)=\frac{1}{64\eta(3.14)^3}\iint d\omega_+\,d\omega_0|T|^2$$

$$=\frac{\sqrt{3}Q^2}{1152(3.14)^2\eta}\left[\left(\tau_0+\frac{Q\tau_1}{3}\right)^2+\frac{Q^2\tau_1{}^2}{36}\right],$$

where Q is the decay Q value.

[24]For large $\epsilon_+{}^2$ we have, from (3.6) and (3.8), the result

$$\delta R^2=\frac{1+W}{1-W}\delta\tilde{K}^2\simeq-3.86\delta\tilde{K}^2\,.$$

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Nonleading Regge Behavior in νW_2 and the Possibility of Fixed Poles with Polynomial Residues*

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We discuss some of the theoretical arguments for the existence in Compton scattering of right-signatured fixed poles with polynomial residues. We show that if one could "switch off" the strong interactions, then a fixed pole with residue linear in q^2 (the photon mass squared) would be necessary for the consistency of the fixed-q^2 dispersion relation for νT_2 (whose absorptive part νW_2 is measured in inelastic electroproduction). We show that if the above conjecture is correct, then there must be some energy dependence in νW_2 over and above the conventional leading Regge form (Pomeranchukon plus f-A_2). Evidence is presented for the presence of such "nonleading behavior" in a similar process. In addition we show why the on-shell $\sigma_{\text{tot}}(\gamma p)$ could be compatible with the neglect of such a nonleading term. We find that a fixed pole with polynomial residue and the correct $q^2\to 0$ Thomson limit can be accommodated by the present data on νW_2 at large q^2. With the above assumptions on the fixed-pole behavior, we predict the high-energy behavior of νW_2 and find that asymptotically it must fall to a value substantially less than its present maximum magnitude.

I. INTRODUCTION

The amplitude for forward scattering of off-mass-shell photons on spin-averaged nucleons can be written in terms of

$$T_{\mu\nu}=4\pi^2\alpha\left[\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)T_1+\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\cdot q}{q^2}q_\nu\right)\frac{T_2}{M^2}\right], \tag{1.1}$$