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# Deuteron Electromagnetic Form Factor*

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A vector-mesonic correction to the deuteron form factors is considered which is analogous to the Glauber double-scattering process. This correction dominates the form factors at large momentum transfer. A fit to the static magnetic moment of the deuteron yields a satisfactory fit to the magnetic scattering at large momentum transfer.

## I. INTRODUCTION

The study of the electromagnetic form factor of the deuteron has been a sensitive probe of the neutron-proton interaction. Most treatments have been limited by their consideration of only the single-impulse contributions to the form factor. In this note we wish to present a calculation of a correction to the deuteron form factor based on the ideas of vector-meson dominance[1] and of double scattering, such as occurs in the Glauber treatment[2] of high-energy scattering off the deuteron. However, both of these concepts will have to be considerably extended for the present application.

As in the Glauber double scattering, we will find that this additional contribution can become dominant at sufficiently large momentum transfers $t$, thus preventing a simple interpretation of the large-$t$ data in terms of the short-distance behavior of the deuteron wave function.[3] However, even at $t=0$ the correction is still present, and in fact allows us to obtain agreement with the experimentally observed magnetic moment of the deuteron for a wave function with a 7% $d$-state probability, as preferred by scattering and quadrupole-moment measurements. In the intermediate-$t$ region, we find that it is possible to understand the inability of even the best existing deuteron wave functions to describe the experimentally observed[4] magnetic scattering simultaneously with the somewhat smaller electric scattering of the deuteron, when these are calculated in the single-impulse approximation.

An exact relativistic description of the deuteron would be very desirable but very complex. There are many relativistic and field-theoretic corrections to the Schrödinger description, and some of these have been examined in Ref. 3. Since a full treatment of these effects is not available, we have chosen to keep our model as simple and as physical as possible. It is to be expected that the above considerations will modify the numerical predictions of our simple model, but the qualitative features should persist.

The model is based on the observation that the deuteron form factor has a behavior not unlike that of the scattering amplitude for a projectile off the deuteron (see Fig. 1). Since double scattering is an important correction to the latter process, one would certainly expect the analogous contributions to be important in the former. In the scattering process, the $t=0.4$ (GeV/$c$)$^2$ break resulting from interplay between spherical and quadrupole contributions is substantially modified by the double-scattering process, which becomes important in this region because the projectile can transfer momentum to both nucleons and leave them with a low relative momentum and hence a large

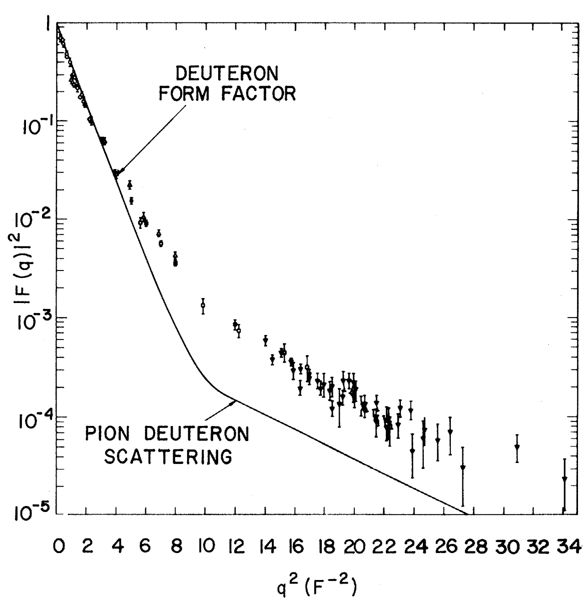


FIG. 1. The experimental points for the deuteron form factor (see Ref. 10, where the sources of the data are identified). The solid curve outlines the data for $\pi d$ scattering at 9.0 GeV [F. Bradamante *et al.*, Phys. Letters 31B, 87 (1970)].

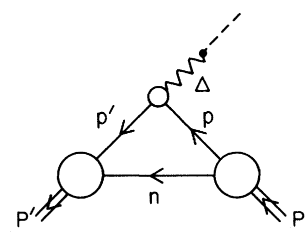


FIG. 2. Single-impulse contributions.

probability of binding. A similar mechanism will prove to be important for the deuteron magnetic form factor. One important difference arises in that the double-scattering term acts in a subtractive manner due to the nearly pure imaginary nature of the scattering amplitudes involved, whereas the electromagnetic form factors must be real, and in fact the preferred correction turns out to be positive.

## II. THEORY

Since the deuteron is an isoscalar particle, it is assumed that the virtual photon changes to an $\omega$ meson (the $\phi$ cannot couple if one takes the usual quark-mixing angle for which the $\phi$ is composed purely of strange quarks which cannot be absorbed by nucleons). The $\omega$ either scatters from one nucleon, giving up roughly half of its momentum, and is then absorbed on the other nucleon, or else is transformed into a $\rho$ by the scattering. The $\rho$ is then absorbed on the second nucleon. The Adler-Drell calculation involved the $\gamma$-$\rho$-$\pi$ vertex and is contained in the latter class of diagrams. Our correction will thus involve two independent scattering amplitudes, which can be chosen to fit both the electric form factor and the magnetic form factor. Experiments which separate these two terms at larger momentum transfers will provide a test of the model discussed here.

To introduce the model and the notation to be used, we consider the evaluation of the diagram of Fig. 2, neglecting spin effects for the moment. The photon-$\omega$ coupling constant is denoted by $g_\omega$, and the $\omega$-nucleon coupling is described by $G_\omega(t)$. The contribution of this diagram to the deuteron electric form factor is

$$g_\omega(\omega^2+\Delta^2)^{-1}G_\omega(\Delta)F(\Delta)=F^S(\Delta)F(\Delta), \qquad (1)$$

where $F^S$ is the isoscalar nucleon form factor, and $F(t)$ is the Fourier transform of the deuteron nucleon distribution.

The presence of the second nucleon in the vicinity of the vector meson will certainly modify the above impulse contribution, as shown in Fig. 3. The evaluation of this diagram is complicated but straightforward. One can proceed by using the eikonal Green's function approach, or by working directly in momentum space. Since the deuteron is loosely bound, the nucleon momenta in the diagram must be

$$p\sim n\sim \tfrac{1}{2}P, \quad p'\sim n'\sim \tfrac{1}{2}P',$$

and hence $l\sim\frac{1}{2}\Delta$. Using these approximations for $n$ and $p'$ in the energy denominator and defining $l=\frac{1}{2}\Delta-\delta$, the contribution of Fig. 2 can be written in the form

$$4\pi(\omega^2+\Delta^2)^{-1}\int\frac{d^3\delta}{(2\pi)^3}F^S(\tfrac{1}{2}\Delta-\delta)f_{\omega\omega}(\tfrac{1}{2}\Delta+\delta)F(2\delta), \qquad (2)$$

where $f_{\omega\omega}$ is an effective $\omega$-nucleon scattering amplitude which is assumed to be independent of the energy. The factors $F^S$ and $f_{\omega\omega}$ vary slowly compared to $F(2\delta)$. It is convenient to introduce $I(\Delta)$ by

$$F^S(\tfrac{1}{2}\Delta)f_{\omega\omega}(\tfrac{1}{2}\Delta)I(\Delta) \equiv\int\frac{d^3\delta}{(2\pi)^3}F^S(\tfrac{1}{2}\Delta-\delta)f_{\omega\omega}(\tfrac{1}{2}\Delta+\delta)F(2\delta), \qquad (3)$$

where $I$ is a slowly varying function of $\frac{1}{2}\Delta$ and will be taken to be a constant. Were we considering the case of a non-hard-core potential, it would be possible to neglect the dependence of $F^S$ and $f_{\omega\omega}$ on $\delta$. Then $I$ would become

$$I\sim|\psi(0)|^2. \qquad (4)$$

If the intermediate vector meson is a $\rho$ meson, with isospin index $\alpha$, instead of an $\omega$, it is necessary to introduce the transition amplitude $f_{\rho\omega}\tau_\alpha$, and the $\rho$-nucleon coupling $G_\rho\tau_\alpha$. Since the deuteron is an isoscalar, $\langle\vec\tau^{(1)}\cdot\vec\tau^{(2)}\rangle=-3$. Neglecting the $\rho$-$\omega$ mass difference, the resulting contribution to the deuteron form factor can be written as

$$-3(\omega^2+\Delta^2)^{-1}4\pi F^V(\tfrac{1}{2}\Delta)(g_\omega/g_\rho)f_{\rho\omega}(\tfrac{1}{2}\Delta)I, \qquad (5)$$

where $F^V$ is the isovector nucleon form factor.

The introduction of spin is now straightforward. The contribution of the single-impulse diagrams was evaluated by Jankus.[5] The vector-meson contributions of Fig. 3 will be evaluated by neglecting any spin dependence of the amplitudes $f_{\omega\omega}$ and $f_{\rho\omega}$. Since the intermediate vector meson has momen-

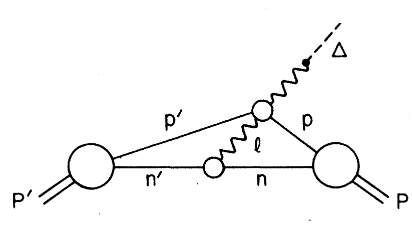


FIG. 3. Double-scattering correction.

tum $\frac{1}{2}\Delta$, its contribution to the magnetic moment is one-half its contribution to the electric moment. Also, since both contributions depend on $I$, which is determined by the behavior of the deuteron wave function near the origin, the $d$ state does not contribute significantly.

The final result for the deuteron form factors can now be written down. It is customary to divide out the isoscalar nucleon form factor in order to separate the nucleon form-factor effects from those of the basic deuteron distribution. With this convention and using the scaling of the nucleon form factors, the final results for the electric, magnetic, and quadrupole form factors are

$$\begin{aligned} \mathfrak{F}_E &= N^2[F_E + D(f_1 - f_2)H(t)], \\ \mathfrak{F}_M &= N^2[F_M + D(\mu^S f_1 - \mu^V f_2)H(t)], \\ \mathfrak{F}_Q &= N^2 F_Q, \end{aligned} \tag{6}$$

where

$$\begin{aligned} N^2 &= [1 + D(f_1 - f_2)]^{-1}, \\ H(t) &= \frac{F^S(\frac{1}{2}\Delta) f_{\omega\omega}(\frac{1}{2}\Delta)}{F^S(\Delta) f_{\omega\omega}(0)(1+\Delta^2/\omega^2)}, \\ D &= 4\pi I/\omega^2, \\ \mu^S &= \tfrac{1}{2}(\mu^p + \mu^n), \\ f_1 &= f_{\omega\omega}(0), \\ f_2 &= (3g_\omega/g_\rho) f_{\rho\omega}(0). \end{aligned} \tag{7}$$

It has been assumed that $f_{\omega\omega}$ and $f_{\rho\omega}$ have the same dependence on $t$. The terms denoted by $F_E$, $F_M$, and $F_Q$ are the standard impulse contributions. Since the vector-meson terms can affect the total charge in the deuteron, the electric form factor must be renormalized to unity at $t=0$. There is some ambiguity as to the correct procedure for doing this. We have chosen the most common procedure (see Ref. 3), that of modifying the normalization of the deuteron wave function by a factor $N$, in analogy to the normalization procedure for a covariant Bethe-Salpeter wave function.

### III. NUMERICAL RESULTS AND CONCLUSIONS

Let us first review the experimental and previous theoretical situation with respect to the deuteron. In Fig. 4 we show the predictions for the deuteron electric form factor of a variety of hard-core models, as compared to the experimental points. Out to the largest $t$ for which a separation $\mathfrak{F}_E$ and $\mathfrak{F}_M$ has been obtained, the Partovi[6] and other hard-core potentials adequately describe the electric data. The situation for $\mathfrak{F}_M$ is quite different. It is well known that for a hard-core potential to adequately describe the scattering data and the measured quadrupole moment of the deuteron, a $d$-state probability of $P_D \cong 7\%$ is required. This $d$-state probability leads, however, to too low a value for the deuteron magnetic moment. As illustrated in Fig. 5, this situation persists and indeed worsens as $t$ increases; the experimentally measured magnetic scattering [defined by $\mathfrak{M} = 2(\frac{2}{3})^{1/2}\mathfrak{F}_M$] slowly diverges from the predictions of the Partovi and other hard-core models. With the inclusion of our correction it is, however, possible to remedy this situation. Since the agreement with the electric-form-factor data of $F_E$ (the unmodified Partovi prediction) is satisfactory, we will take $f_1 \approx f_2$, for which $N=1$. We would then have

$$\mathfrak{F}_M = F_M + D(\mu^S - \mu^V) f_1 H(t), \tag{8}$$

which at $t=0$ becomes

$$\mu_D = 2\mu^S - 3(\mu^S - \tfrac{1}{4})P_D + D(\mu^S - \mu^V) f_1. \tag{9}$$

The presence of the last term in Eq. (9) allows us to obtain the observed value of $\mu_D$ [0.857 (Ref. 7)] provided that[8]

$$D(\mu^S - \mu^V) f_1 = 0.017. \tag{10}$$

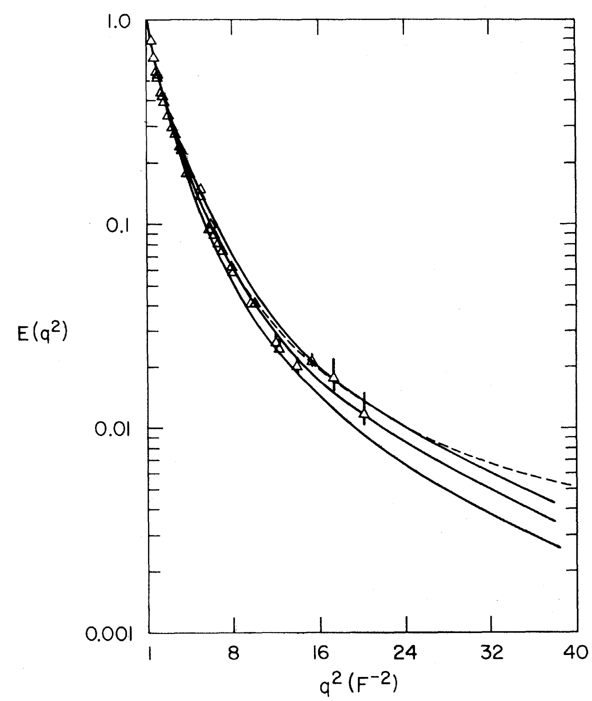


FIG. 4. Electric form factor of the deuteron. The experimental points and the curves are from Buchanan, Ref. 4. The outer solid lines represent the reasonable extremes of hard-core models. The inner line is the Partovi prediction. The dashed line is our correction term added to the bottom solid line for the case $f_1 \neq f_2$, as explained in the text.

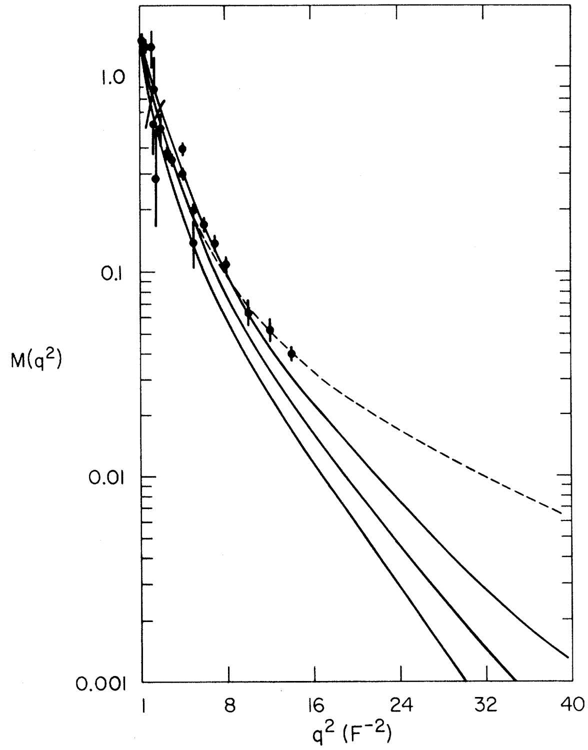


FIG. 5. Magnetic form factor. The notation is the same as in Fig. 4. The dashed line is the prediction including our correction for the case $f_1 = f_2$ and the Partovi form factors.

Thus

$$\mathfrak{M} = M + 0.028\, H(t). \tag{11}$$

$H(t)$ is determined except for the $t$ dependence of the $\omega$-$\omega$ and $\rho$-$\omega$ amplitudes, which we take from experiment to behave like $e^{+at}$ with $a = 4.3$.[9] The prediction of Eq. (11) is plotted also in Fig. 5. One should note that at large values of $t$, this particular choice of $f_1 = f_2$ would lead to a complete dominance of the scattering by our correction to the magnetic form factor, a term which in the forward direction is only a 2% effect. Indeed at $t = 30$, $\mathfrak{M}(30) = 0.0112$ whereas $\mathfrak{F}_E(30) = 0.0057$. It should be emphasized that one can fit the magnetic data equally well by using a different deuteron wave function and $f_1 \neq f_2$.

One can also fit the present electric data by

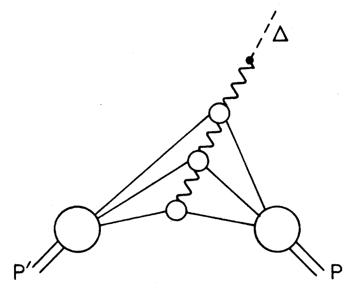


FIG. 6. Multiple-scattering contribution to the $He^3$ form factor.

choosing $f_1 \neq f_2$ and a different potential model. Data at higher momentum-transfer values will distinguish these alternatives, as is seen in Fig. 4 where we plot also the case for which $D(f_1 - f_2) = 0.01$. At large values of $t$, our correction will then dominate both $\mathfrak{F}_E$ and $\mathfrak{M}$, which in this limit should fall off in the same way, as given by $H(t)$. In any case it is now easy to obtain agreement with the deuteron data of Fig. 1,[10] for large $t$. The contribution which we have studied is of just the right size to provide the flattening observed experimentally.

We would like to stress that just as the double-scattering contributions of the Glauber theory can be expected to dominate the large-$t$ behavior for projectile scattering from deuterium, one expects that analogous contributions dominate the large-$t$ behavior of the deuteron electromagnetic form factor. It is also true that both provide small but important corrections in the low- to intermediate-$t$ region.

This type of vector-meson contribution could also play an important role at large momentum transfer in the form factors of $H^3$, $He^3$, and $He^4$. It has been demonstrated[11] that the present $He^3$ wave functions, which are derived from nucleon-nucleon potentials that fit the two-body scattering data, are unable to describe the structure at $t = 0.4$ $(\mathrm{GeV}/c)^2$. The type of correction which should dominate for large $t$ is given diagrammatically in Fig. 6. It falls off more slowly in $t$ than $H(t)$ for the deuteron, and may well be important for $t \geqslant 0.4$. It could in fact be responsible for the dip at $t = 0.4$. Experimental data for larger $t$ are again needed to decide if vector-meson corrections do in fact dominate.

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