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Binding Energy of the Deuteron*

M. G. Hare† and G. Papini

Department of Physics, University of Saskatchewan, Regina Campus, Regina, Saskatchewan, Canada

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The binding energy of the deuteron is written in terms of once-subtracted, sidewise dispersion relations for the mass-energy form factors of the neutron, proton, and deuteron. The threshold region is assumed to dominate, and accordingly a cutoff is introduced into the dispersion integrals. This cutoff is, however, related to the proton-neutron mass difference δM, so that no free parameter exists in the work. The binding energy B is reproduced for a cutoff value corresponding to $\delta M \sim 1.7$ MeV, this being the δM calculated by Pagels. Both electromagnetic effects and the deuteron breakup into an S-wave, triplet-state nucleon pair are considered for contributions to B. No realistic estimate has been made of the effect on the result of π mesons in the intermediate-state expansion. The two-nucleon-state contribution dominates the results, accounting for approximately 90% of B at the experimental value of 2.2 MeV.

I. INTRODUCTION

In this paper we are concerned with the calculation of the binding energy of the deuteron, using dispersion relations involving mass-energy form factors. The method used has been devised and successfully applied by Pagels[1] to the determination of the neutron-proton mass difference. It is essentially based on the close analogy existing between electric charge and gravitational mass as quantities indicating the source strength of the electromagnetic and gravitational fields, respectively. This may also be expressed by saying that as the electric and magnetic structures of a particle are contained in the particle-photon vertex, so its mechanical properties are contained in the particle-graviton vertex. Dispersion relations may then be obtained for the form factors appearing in both vertices.

In conventional theories, the deuteron's binding energy B can be calculated from a knowledge of the scattering length and effective range of the two-nucleon forces.[2] These parameters may in turn be calculated from a knowledge of the two-nucleon potential, or determined from scattering data, for instance. A dispersive method would differ from phenomenological theories in that it should relate B to fundamental constants, such as coupling constants and masses, that would occur in a Lagrangian description of the particles involved in the scattering. A way of doing this was outlined by Blankenbecler and Cook[3] (starting from the n-d-p vertex instead of the d-graviton vertex we are considering) but, to our knowledge, has never been used in any actual calculation of the deuteron's binding energy. In this work we are forced to use the triplet effective range for the deuteron as an input parameter, essentially because an exact theory for the deuteron is not available.

In practice, however, calculations from "first principles" are never completely possible in dispersion theory, where usefulness lies in relating unknown quantities to experimentally measurable ones. The objective of our calculation is to relate B to known scattering processes. Not all of these are measurable in our case. In fact those involving a gravitation cannot be measured experimentally because of the extreme weakness of the gravitational interaction, but use can still be made of their analytic properties.

The advantage of this method is that it allows a direct interpretation of the final result in terms of the mass contributions of the single processes involved.

The deuteron is treated as a spin-one elementary particle or quasiparticle capable of breaking up into an S-wave, triplet-state nucleon pair, somewhat along the lines discussed by Amado[4] and Weinberg.[5] This assumption simplifies the calculations to some extent, and some of its implications will be discussed later.

The paper is divided into seven sections. In Sec. II we study the general particle-graviton vertices to obtain the mass-energy form factors, which are then related to the binding energy in Sec. III, by means of suitable dispersion relations. The form factors appearing in the dispersion integrals are explicitly found by expanding the vertices in terms of intermediate states in Sec. IV, and calculating the resulting scattering amplitudes in Sec. V. The final results are contained in Sec. VI, and are followed in Sec. VII by a summary and conclusion. For the sake of completeness, dispersion relations for the nucleon and deuteron mass-energy form factors are proved in the Appendix.

II. GENERAL VERTEX EXPRESSIONS

It is instructive to obtain the threshold values of the pertinent form factors before the expression for the binding energy is derived (Sec. III). To do this we consider Fig. 1, where W represents the off-shell particle and l is the massless ($l^2=0$) graviton. Actually, for the purposes of this paper, the invariant trace of the energy-momentum tensor is sufficient to represent the graviton,[6] so the full tensor, corresponding to gravitons of spin 2, will not be considered here.

For nucleons, the most general expression for the vertex can be composed[7] of I, γ_5, γ_μ, $\sigma_{\mu\nu}$, $\gamma_5\gamma_\mu$, l_μ, W_μ, and N_μ, where N is the nucleon (p or n) momentum. By use of TCP invariance, momentum conservation, and the Dirac equation, the only possibility for the vertex is

$$\bar{u}(N)\Sigma(W^2)=\bar{u}(N)[G_N(W^2)+G_N'(W^2)\not{l}]\,, \tag{1}$$

where Σ represents the vertex and $\not{l}=\gamma_\mu l_\mu$. The functions G and G' are just the mass-energy form factors.

At the threshold $l=0$ and $W^2=N^2=M_N^2$, and Eq. (1) gives us

$$\bar{u}(N)\Sigma(M_N^2)u(N)=M_N\bar{u}(N)u(N)=M_N$$
$$=\bar{u}(N)G_N(M_N^2)u(N)=G_N(M_N^2)\,, \tag{2}$$

so that we can conclude that

$$G_N(M_N^2)=M_N, \tag{3}$$

which is just the coupling constant for the nucleon-graviton interaction at threshold.[8]

Thus a projection operator is needed to extract $G_N(W^2)$ from the vertex (1), so that this form factor can be used in the dispersion relations. The operator will be

$$P(W^2)=\frac{M_N\not{l}u(N)}{W^2-M_N^2}\,, \tag{4}$$

as one can verify by substitution back into Eq. (1).

For the deuteron, which we take as a spin-1 or vector system, the most general vertex function can consist of the polarization four-vectors, ϵ_μ and ϵ_μ^W, for the on-shell and off-shell deuterons, respectively, as well as d_μ, l_μ, and W_μ. The Lorentz condition, $d\cdot\epsilon=0$ (which has the property of eliminating the spin-0 or scalar representation), and TCP invariance restrict the choice to the vector

$$\epsilon_\mu\Sigma_{\mu\nu}(W^2)=F_d(W^2)\epsilon_\nu+F_d'(W^2)\,\epsilon\cdot l\; l_\nu\,, \tag{5}$$

where the functions F and F' are the form factors.

At threshold Eq. (5) gives us

$$(2d_0)^{-1/2}\epsilon_\mu\Sigma_{\mu\nu}(M_d^2)\epsilon_\nu(2d_0)^{-1/2}$$
$$=M_d=(2M_d)^{-1/2}\epsilon_\mu F_d(M_d^2)(2M_d)^{-1/2}\epsilon_\mu$$
$$=F_d(M_d^2)/2M_d\,, \tag{6}$$

since $\epsilon_\mu\epsilon_\mu=1$. Thus we can conclude that

$$F_d(M_d^2)=2M_d^2\,. \tag{7}$$

We need a projection operator to extract $F_d(W^2)$ as this is the form factor that contributes at threshold (the coupling constant). It is actually more convenient to extract $F_d(W^2)/2M_d\equiv G(W^2)$, and the following operator does this for us:

$$P_\nu(W^2)=\frac{M_d}{4(M_d^2+d\cdot l)}\epsilon_\nu^W\left[\epsilon\cdot\epsilon^W+\frac{M_d-d\cdot l}{(d\cdot l)^2}\epsilon\cdot l\,\epsilon^W\cdot l\right], \tag{8}$$

which one can verify by operating on Eq. (5) with it. Note also that $\epsilon^W\cdot W=0$ at threshold.

In constructing Eq. (8), use has been made of the completeness relation

$$\sum_{\text{pol}}\epsilon_\alpha\epsilon_\beta=-g_{\alpha\beta}+k_\alpha k_\beta/M_d^2\,, \tag{9}$$

where k is the deuteron momentum. The second term in (9) is a direct consequence of the finite rest mass of the deuteron.

III. DISPERSION RELATIONS

If the form factors are analytic in W^2, then dispersion relations may be written for them. If these relations are assumed to be once subtracted, then we may write

$$G(W^2)=M+\frac{W^2-M^2}{\pi}\int_{M^2}^\infty\frac{\text{Im}G(W'^2)dW'^2}{(W'^2-M^2)(W'^2-W^2-i\epsilon)}\,, \tag{10}$$

where the pole term M is the threshold value of $G(W^2)$. The integration starts at M^2 because, as is shown in Sec. IV, ImG vanishes below this point. The analyticity of ImG is shown in the Appendix and closely follows the method of Bincer.[9]

The binding energy of the deuteron,

$$B=M_n+M_p-M_d\,, \tag{11}$$

can be obtained from Eq. (10) as

$$B=G_p(W^2)+G_n(W^2)-G_d(W^2)$$
$$-\frac{W^2-M_p^2}{\pi}\int_{M_p^2}^\infty\frac{\text{Im}G_p(W'^2)dW'^2}{(W'^2-M_p^2)(W'^2-W^2-i\epsilon)}$$
$$-\frac{W^2-M_n^2}{\pi}\int_{M_n^2}^\infty\frac{\text{Im}G_n(W'^2)dW'^2}{(W'^2-M_n^2)(W'^2-W^2-i\epsilon)}$$
$$+\frac{W^2-M_d^2}{\pi}\int_{M_d^2}^\infty\frac{\text{Im}G_d(W'^2)dW'^2}{(W'^2-M_d^2)(W'^2-W^2-i\epsilon)}\,. \tag{12}$$

We will now assume that as W^2 approaches infinity

$$\lim_{W^2\to\infty}[G_p(W^2)+G_n(W^2)-G_d(W^2)]\to 0\,, \tag{13}$$

as the similar type of assumption being made by Pagels.[1] With this important assumption, Eq. (12) yields

$$B=\frac{1}{\pi}\int_{M_n{}^2}^{\infty}\frac{\mathrm{Im}G_n(W^2)dW^2}{W^2-M_n{}^2}+\frac{1}{\pi}\int_{M_p{}^2}^{\infty}\frac{\mathrm{Im}G_p(W^2)dW^2}{W^2-M_p{}^2}-\frac{1}{\pi}\int_{M_d{}^2}^{\infty}\frac{\mathrm{Im}G_d(W^2)dW^2}{W^2-M_d{}^2}\,, \tag{14}$$

where a change of variable from W' to W has been made.

It can be seen that the assumption of once-subtracted relations is essential to this work, since it is the subtraction terms that give B directly.

In this theory we are assuming low-energy dominance and so, since the high-energy contribution will not be calculated, the integrals appearing in Eq. (14) must be cut off. As it is the integration range that controls the nature and number of the intermediate states, we require that it be the same for the three integrals of Eq. (14). In other words, if $\lambda^2 M_d{}^2$ represents the upper limit of the deuteron's (last) integral and $\lambda'^2 M_N{}^2$ the upper limit of the nucleons' (first two) integrals, then λ and λ' will be related by

$$\lambda^2 M_d{}^2-M_d{}^2=\lambda'^2 M_N{}^2-M_N{}^2\,. \tag{15}$$

Now λ' is determined by the neutron-proton mass difference,[1] so λ is also determined[10] within the limits of reasonable agreement with this mass difference δM.

IV. INTERMEDIATE-STATE EXPANSIONS

The vertex diagram, Fig. 1, can be expanded in terms of intermediate states as shown in Figs. 2, 3, and 4 below. We write the nucleon vertex as

$$\bar{u}(N)\Sigma(W^2)=(N_0/M_N)^{1/2}\langle N,\, l|\bar{\eta}(0)|0\rangle\,, \tag{16}$$

where $\bar{\eta}(x)$ is the nucleon current $(\not{N}-M_N)\bar{\Psi}(x)$, $\bar{\Psi}(x)$ being a nucleon creation operator. Contracting out the graviton[11] gives us

$$\begin{aligned}\bar{u}(N)\Sigma(W^2)&=i\left(\frac{N_0}{M_N}\right)^{1/2}\int d^4x\, e^{-il\cdot x}\langle N|T[\Theta(x)\bar{\eta}(0)]|0\rangle\\&=i\left(\frac{N_0}{M_N}\right)^{1/2}\int d^4x\, e^{-il\cdot x}\hat{\theta}(x_0)\langle N|[\Theta(x),\bar{\eta}(0)]|0\rangle+i\left(\frac{N_0}{M_N}\right)^{1/2}\int d^4x\, e^{-il\cdot x}\langle N|\bar{\eta}(0)\Theta(x)|0\rangle\,,\end{aligned} \tag{17}$$

where T is the time-ordering operator, $\hat{\theta}(x_0)$ is the time step function, and $\Theta(x)$ is the graviton current (the trace of the energy-momentum tensor). The last term is the equal-time term and contributes only at the pole, so we neglect it from here on.[9]

Intermediate states are now inserted into the commutators and the projection operator is applied to extract the required form factor. Also $i\hat{\theta}(x_0)$ is taken to $\frac{1}{2}$, which takes G to $\mathrm{Im}G$.[12] Thus Eq. (17) becomes

$$\begin{aligned}\mathrm{Im}G_N(W^2)=\tfrac{1}{2}\sum_{\text{spin}}(N_0/M_N)^{1/2}\int d^4x\, e^{-il\cdot x}\Big[&\int d^3n(2\pi)^{-3}\sum_n\langle N|\Theta(0)|n\rangle\langle n|\bar{\eta}(0)|0\rangle P(W^2)e^{ix\cdot(n-N)}\\&-\int d^3n'(2\pi)^{-3}\sum_{n'}\langle N|\bar{\eta}(0)|n'\rangle\langle n'|\Theta(0)|0\rangle P(W^2)\,e^{-ix\cdot n'}\Big]\,,\end{aligned} \tag{18}$$

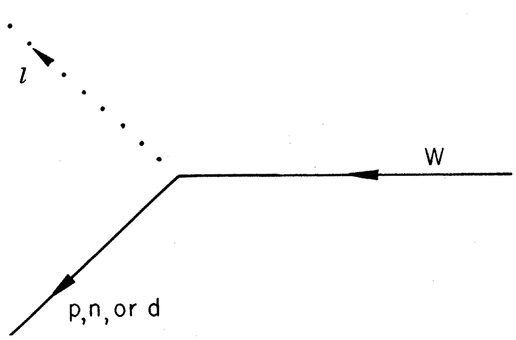

FIG. 1. The general graviton-particle vertex.

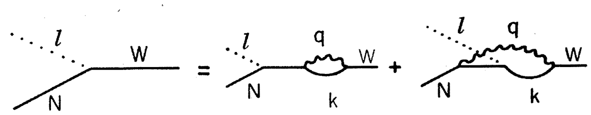

FIG. 2. The nucleon-photon intermediate state.

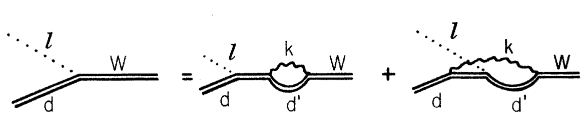

FIG. 3. The deuteron-photon intermediate state.

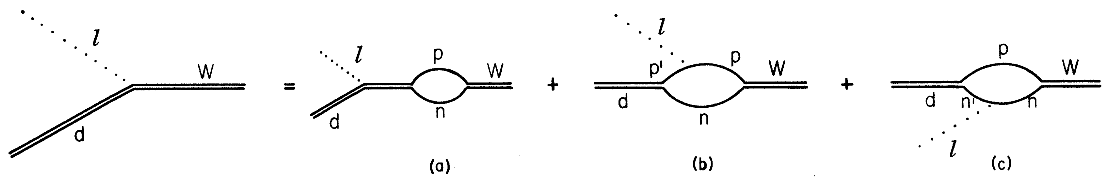

FIG. 4. The two-nucleon intermediate state.

where the sums are understood to be over the spins of the indicated intermediate-state particles.

The x integration is then carried out yielding momentum-conserving δ functions, which in turn allow the intermediate-state integrations to be performed. The result is

$$\begin{aligned}\mathrm{Im}G_N(W^2) = &\sum_{\text{spin}} (N_0/M_N)^{1/2}\pi \sum_n 2(n_0+l_0)\delta((n+l)^2 - M_n{}^2)\hat{\theta}(n_0+l_0)\langle N|\Theta|n+l\rangle\langle n+l|\bar{\eta}|0\rangle P \\ &+ \sum_{\text{spin}} (N_0/M_N)^{1/2}\pi \sum_{n'} 2l_0\delta(l^2 - M_{n'}{}^2)\hat{\theta}(-l_0)\langle N|\bar{\eta}|-l\rangle\langle -l|\Theta|0\rangle P\,. \end{aligned} \tag{19}$$

Now the second term exist only when $l^2 = M_{n'}{}^2$, but $l^2 = 0$, so this intermediate state is just the vacuum. This in turn results in the matrix element $\langle N|\bar{\eta}|0\rangle$ which vanishes,[11] so the second term is dropped. In the first term the intermediate state cannot be just a nucleon, because this also results in the vanishing term $\langle N|\bar{\eta}|0\rangle$. The intermediate state must consist of more than a nucleon, and it will be taken to be a nucleon plus a photon. It is thus seen that the smallest mass for the intermediate state is M_N, which explains the lower limit on the dispersion integrals for the nucleons, Eq. (10).

For the deuteron we can follow an analogous procedure. The starting point is the deuteron vertex

$$\epsilon_\mu \Sigma_{\mu\nu}(W^2) = (2d_0)^{1/2}\langle d, l|j_\nu^\dagger(0)|0\rangle\,, \tag{20}$$

where $j_\nu^\dagger(x)$ is the deuteron current $(\Box - M_d{}^2)\Phi_\nu^\dagger(x)$, $\Phi_\nu^\dagger(x)$ being the deuteron creation operator. The steps now followed are: the graviton is contracted out, the equal-time term is neglected, intermediate states are inserted into the commutator, G is taken to ImG, and the x integration and the intermediate-state integrations are performed, giving us

$$\begin{aligned}\mathrm{Im}G_d(W^2) = &\sum_{\text{pol}} (2d_0)^{1/2}\pi \sum_n 2(d_0+l_0)\delta((d+l)^2 - M_n{}^2)\hat{\theta}(d_0+l_0)\langle d|\Theta|d+l\rangle\langle d+l|j_\nu^\dagger|0\rangle P_\nu \\ &+ \sum_{\text{pol}} (2d_0)^{1/2}\pi \sum_{n'} 2l_0\delta(l^2 - M_{n'}{}^2)\hat{\theta}(-l_0)\langle d|j_\nu^\dagger|-l\rangle\langle -l|\Theta|0\rangle P_\nu\,. \end{aligned} \tag{21}$$

As before, the second term vanishes, and the first term will also vanish if the intermediate state is just a deuteron because $\langle d|j_\nu^\dagger|0\rangle = 0$.

The intermediate states are taken to be a deuteron plus a photon for one possibility, and a two-nucleon state for another possibility. The lowest allowable mass is M_d, explaining the lower limit on the deuteron's dispersion integral.

Thus, in detail, we desire to calculate

$$\begin{aligned}\mathrm{Im}G_N^\gamma(W^2) = \left(\frac{N_0}{M_N}\right)^{1/2} \frac{\pi}{(2\pi)^3}\int d^4q\, d^4k\, 2q_0 2k_0 \delta(k^2 - M_N{}^2)\delta(q^2)\hat{\theta}(q_0)\hat{\theta}(k_0)\delta(q+k-N-l) \\ \times \sum_{\text{spin}} \langle N|\Theta|q,k\rangle\langle q,k|\bar{\eta}|0\rangle P(W^2)\,, \end{aligned} \tag{22a}$$

$$\begin{aligned}\mathrm{Im}G_d^\gamma(W^2) = (2d_0)^{1/2}\frac{\pi}{(2\pi)^3}\int d^4d'\, d^4k\, 2d_0' 2k_0 \delta(d'^2 - M_d{}^2)\delta(k^2)\hat{\theta}(d_0')\hat{\theta}(k_0)\delta(d'+k-d-l) \\ \times \sum_{\text{pol}} \langle d|\Theta|d',k\rangle\langle d',k|j_\nu^\dagger|0\rangle P_\nu(W^2)\,, \end{aligned} \tag{22b}$$

$$\begin{aligned}\mathrm{Im}G_d^{p,n}(W^2) = (2d_0)^{1/2}\frac{\pi}{(2\pi)^3}\int d^4p\, d^4n\, 2p_0 2n_0 \delta(p^2 - M_p{}^2)\delta(n^2 - M_n{}^2)\hat{\theta}(p_0)\hat{\theta}(n_0)\delta(p+n-d-l) \\ \times \sum_{\text{pol, spin}} \langle d|\Theta|p,n\rangle\langle p,n|j_\nu^\dagger|0\rangle P_\nu(W^2)\,, \end{aligned} \tag{22c}$$

where the explicit symbols used for the particles are defined by Figs. 2, 3, and 4, respectively, and the superscript on the left-hand side indicates the intermediate state.

V. INTERMEDIATE-STATE CALCULATIONS

The detailed calculations of Eqs. (22) are very lengthy, and so we give only the essential steps here.

For the nucleons there are two situations, one for the proton and one for the neutron. Considering the proton and Fig. 2, the spin sum is[1]

$$\sum_{\text{spin}} \langle p|\Theta|k,q\rangle\langle k,q|\bar{\eta}|0\rangle P=\sum_{\text{spin}}\left[\left(\frac{M_p}{p_0}\right)^{1/2}\bar{u}(p)iM_p\left(i\frac{\not{k}+\not{q}+M_p}{(k+q)^2-M_p^2}\right)ie\left(\gamma_\nu+\frac{ik}{2M_p}\sigma_{\nu\mu}q_\mu\right)\frac{\epsilon_\nu}{(2q_0)^{1/2}}\right.$$
$$\times\left(\frac{M_p}{k_0}\right)^{1/2}u(k)\left(\frac{M_p}{k_0}\right)^{1/2}\bar{u}(k)\frac{\epsilon_\alpha}{(2q_0)^{1/2}}ie\left(\gamma_\alpha-\frac{ik}{2M_p}\sigma_{\alpha\beta}q_\beta\right)u(p)\frac{M_p\not{l}}{W^2-M_p^2}$$
$$+\left(\frac{M_p}{p_0}\right)^{1/2}\bar{u}(p)ie\left(\gamma_\nu+\frac{ik}{2M_p}\sigma_{\nu\mu}q_\mu\right)\left(i\frac{\not{p}-\not{q}+M_p}{(p-q)^2-M_p^2}\right)iM_p\frac{\epsilon_\nu}{(2q_0)^{1/2}}$$
$$\left.\times\left(\frac{M_p}{k_0}\right)^{1/2}u(k)\left(\frac{M_p}{k_0}\right)^{1/2}\bar{u}(k)\frac{\epsilon_\alpha}{(2q_0)^{1/2}}ie\left(\gamma_\alpha-\frac{ik}{2M_p}\sigma_{\alpha\beta}q_\beta\right)u(p)\frac{M_p\not{l}}{W^2-M_p^2}\right]. \tag{23}$$

Here $k=+1.79$ is the anomalous part of the proton's magnetic dipole moment. Because a threshold calculation is being performed, the anomalous moment terms do not have to be corrected by off-shell factors or by negative-energy contributions.[9] The photon's polarization is ϵ. The scattering matrix element $\langle p|\Theta|k,q\rangle$ is given by all the factors up to $u(k)$ in Eq. (23), the vertex matrix element $\langle k,q|\bar{\eta}|0\rangle$ is given by the factors up to $u(p)$, and the projection operator, Eq. (4), is the last term, $M_p\not{l}/(W^2-M_p^2)$.

The spin sums are performed and the resulting traces calculated. Then the intermediate states are integrated out, and after much simplification the result is

$$\operatorname{Im}G_p^\gamma(W^2)=-\frac{\alpha M_p^3}{4W^2}\left\{8+\frac{W^2}{M_p^2}-\frac{M_p^2}{W^2}-\frac{8W^2}{W^2-M_p^2}\ln\frac{W^2}{M_p^2}+\tfrac{1}{2}k\left[8+\frac{11W^2}{2M_p^2}-\frac{3M_p^2}{2W^2}-\frac{12W^2}{W^2-M_p^2}\ln\frac{W^2}{M_p^2}\right]\right.$$
$$\left.+\tfrac{1}{4}k^2\left[\frac{(W^2-M_p^2)(-21W^6-4W^4M_p^2-19W^2M_p^2+8M_p^6)}{12W^4M_p^2}+\frac{3W^2}{M_p^2}\ln\frac{W^2}{M_p^2}\right]\right\}. \tag{24}$$

The expression for the neutron has only the k^2 term appearing (as the neutron has no net electric charge) where $k=+1.91$, and has M_p replaced by M_n. The electromagnetic coupling constant α is 1/137.

It should be noticed that since $G(M^2)=M$, where M is real, then $\operatorname{Im}G(M^2)=0$. This can be expressed in Eq. (24) as

$$\lim_{W^2\to M_p^2}\operatorname{Im}G_p^\gamma(W^2)\to\lim_{W^2\to M_p^2}\frac{\alpha(W^2-M_p^2)}{2M_p}\left(+1-\tfrac{1}{4}k+\tfrac{3}{8}k^2\right)\to 0. \tag{25}$$

For the deuteron there are two situations, one for each possible intermediate state. For the first consider Fig. 3 where the polarization sum is

$$\sum_{\text{pol}}\langle d|\Theta|d',k\rangle\langle d',k|j_\nu^\dagger|0\rangle P_\nu=\sum_{\text{pol}}\left\{\frac{\epsilon_\mu}{(2d_0)^{1/2}}\left[i2M_d^2g_{\mu\alpha}\left(i\frac{-g_{\alpha\beta}+(d+l)_\alpha(d+l)_\beta/M_d^2}{(d+l)^2-M_d^2}\right)\right.\right.$$
$$\times ie[-(d+l+d')_\sigma g_{\beta\rho}+d'_\beta g_{\sigma\rho}+(d+l)_\rho g_{\sigma\beta}]$$
$$+ie[-(2d-k)_\sigma g_{\alpha\mu}+(d-k)_\mu g_{\alpha\sigma}+d_\alpha g_{\mu\sigma}]$$
$$\left.\times\left(i\frac{-g_{\alpha\beta}+(d-k)_\alpha(d-k)_\beta/M_d^2}{(d-k)^2-M_d^2}\right)i2M_d^2g_{\beta\rho}\right]$$
$$\times\frac{\epsilon_\sigma^k}{(2k_0)^{1/2}}\frac{\epsilon'_\rho}{(2d'_0)^{1/2}}\frac{\epsilon_\gamma^k}{(2k_0)^{1/2}}\frac{\epsilon'_\lambda}{(2d'_0)^{1/2}}ie[-(d'+W)_\gamma g_{\lambda\nu}+W_\lambda g_{\gamma\nu}+d'_\nu g_{\gamma\lambda}]$$
$$\left.\times\frac{M_d}{4(M_d^2+d\cdot l)}\epsilon_\nu^W\left(\epsilon_\delta\epsilon_\delta^W+\frac{M_d^2-d\cdot l}{(d\cdot l)^2}\epsilon_\delta l_\delta\epsilon_\omega^W l_\omega\right)\right\}. \tag{26}$$

Here ϵ^k is the photon's polarization and ϵ' is the intermediate-state deuteron's polarization. In Eq. (26) the factors $\epsilon'_\rho\epsilon_\gamma^k$ separate the scattering matrix, $\langle d|\Theta|d',k\rangle$, from the vertex matrix, $\langle d',k|j_\nu^\dagger|0\rangle$. The projection operator is given by Eq. (8).

The polarization sums are performed and the intermediate states are integrated out, yielding

$$\mathrm{Im}G_d^\gamma(W^2)=\frac{\alpha M_d^3(W^2-M_d^2)}{4W^2(W^2+M_d^2)}\left[\frac{17M_d^4W^2+18M_d^2W^4+W^6}{2M_d^2(W^2-M_d^2)}\ln\frac{W^2}{M_d^2}+\frac{-2M_d^{10}+10M_d^8W^2+9M_d^6W^4-87M_d^4W^6+12M_d^2W^8+4W^{10}}{3M_d^4W^4(W^2-M_d^2)}\right], \qquad (27)$$

which has the limit

$$\lim_{W^2\to M_d^2}\mathrm{Im}G_d^\gamma(W^2)=\lim_{W^2\to M_d^2}\frac{\alpha(W^2-M_d^2)^9}{8M_d}\to 0\,, \qquad (28)$$

this being consistent with the real nature of $G_d(M_d^2)$.

The final diagram to be considered is Fig. 4, in which case the polarization and spin sums give us

$$\sum_{\text{pol, spin}}\langle d|\Theta|p,n\rangle\langle p,n|j_\nu^\dagger|0\rangle P_\nu=\sum_{\text{pol, spin}}\left\{\frac{-\epsilon_\mu}{(2d_0)^{1/2}}\left[i2M_d^2g_{\mu\alpha}i\frac{\epsilon'_\alpha\epsilon'_\beta}{W^2-M_d^2}\bar\nu(\bar n)\Gamma'_\beta u(p)\bar u(p)\Gamma'_\lambda\nu(\bar n)+iM_p\bar\nu(\bar n)\Gamma_\mu i\frac{p'+M_p}{p'^2-M_p^2}u(p)\bar u(p)\Gamma_\lambda\nu(\bar n)iM_n\bar\nu(\bar p)\Gamma_\mu i\frac{n'+M_n}{n'^2-M_n^2}u(n)\bar u(n)\Gamma_\lambda\nu(\bar p)\right]\right.$$
$$\left.\times\frac{M_d}{4(M_d^2+d\cdot l)}\epsilon_\lambda^W\left[\epsilon_\delta\epsilon_\delta^W+\frac{M_d^2-d\cdot l}{(d\cdot l)^2}\epsilon_\delta l_\delta\epsilon_\omega^W l_\omega\right]\left(\frac{M_n}{n_0}\frac{M_p}{p_0}\frac{M_n}{\bar n_0}\frac{M_p}{\bar p_0}\right)^{1/2}\right\}, \qquad (29)$$

where the spinors $\bar\nu$ and ν indicate antiparticles of momentum $\bar n=-n$ and $\bar p=-p$, and p' and n' are the off-shell nucleons. The spinors $u(p)$ and $\bar u(p)$ [or $u(n)$ and $\bar u(n)$] separate the deuteron plus graviton to two-nucleon scattering matrix element, $\langle d|\Theta|p,n\rangle$, from the two-nucleon–deuteron vertex matrix element, $\langle p,n|j_\nu^\dagger|0\rangle$.

The quantities

$$\Gamma'_\alpha(p^2)=iR(p^2)\gamma_\alpha\,,\qquad \Gamma_\alpha(p^2)=\Gamma'_\alpha(p^2)+iV(p^2)n_\alpha\,, \qquad (30)$$

represent the n-d-p vertices, respectively, with the deuteron off shell and one nucleon off shell. It is through Γ' and Γ that information about the deuteron is introduced into the calculation. In principle, the functions R and V can be calculated and expressed in terms of deuteron, nucleon, and meson masses, and strong-coupling constants.[3] This would, however, complicate the calculations enormously. In what follows we set R and V equal to their on-shell values and express them in terms of the low-energy parameters of the deuteron.[13]

Summing and integrating Eq. (29) gives us

$$\mathrm{Im}G_d^{p,n}(W^2)=\frac{M^3(W^2-4M^2)^{1/2}}{4\pi W(W^2+4M^2)}\left\{R^2\left(\frac{2W^4-4W^4M^2+16M^4}{3W^2M^2}\right)-2R^2\left[\frac{16M^2}{W^2-4M^2}-\frac{W^2}{M^2}\frac{8WM^2}{(W^2-4M^2)^{1/2}}\ln\left(\frac{W+(W^2-4M^2)^{1/2}}{W-(W^2-4M^2)^{1/2}}\right)\right]\right.$$
$$\left.-2MVR\left[\frac{4W^2+8M^2}{3W^2}+\frac{(W^2-4M^2)(-W^4+14W^2M^2+8M^4)}{24W^2M^4}-\frac{W}{(W^2-4M^2)^{1/2}}\ln\left(\frac{W+(W^2-4M^2)^{1/2}}{W-(W^2-4M^2)^{1/2}}\right)\right]\right\}, \qquad (31)$$

which has the limit

$$\lim_{W^2\to 4M^2}\mathrm{Im}G_d^{p,n}(W^2)\to\lim_{W^2\to 4M^2}\frac{R^2(W^2-4M^2)^{1/2}}{6}\to 0\,. \qquad (32)$$

Note that there are no terms in V^2 in (31) due to Eqs. (30) where Γ' does not contain V.

In the development of Eq. (31) we have made the simplification $M_n=M_p=\frac{1}{2}M_d=M$. This will affect the R^2 and MRV terms by no more than a few parts in a thousand and, as will be seen when the results are plotted (Fig. 5), will not affect the results significantly. The approximation greatly simplifies the calculation.

VI. RESULTS

The expressions for ImG, Eqs. (24), (27), and (31), are now inserted into the dispersion integrals, Eq. (14). The results are given in terms of individual mass shifts for the various contributions.

$$\delta M_p^{e^2} = \frac{\alpha M}{4\pi}\left[7\ln\lambda'^2 + \frac{1}{\lambda'^2} + 7 - \frac{8\lambda'^2}{\lambda'^2 - 1}\ln\lambda'^2\right], \tag{33a}$$

$$\delta M_p^{e^2k} = -\frac{\alpha M}{4\pi}\frac{k_p}{4}\left[-21 - \frac{3}{\lambda'^2} + 11\ln\lambda'^2 + \frac{24}{\lambda'^2 - 1}\ln\lambda'^2\right], \tag{33b}$$

$$\delta M_p^{e^2k^2} = -\frac{\alpha M}{4\pi}\frac{k_p^2}{48}\left[6 - 21\lambda'^2 - 4\ln\lambda'^2 + \frac{19}{\lambda'^2} - \frac{4}{\lambda'^4} + 36\varphi_1(\lambda'^2)\right], \tag{33c}$$

$$\delta M_n^{e^2k^2} = -\frac{\alpha M}{4\pi}\frac{k_n^2}{48}\left[6 - 21\lambda'^2 - 4\ln\lambda'^2 + \frac{19}{\lambda'^2} - \frac{4}{\lambda'^4} + 36\varphi_1(\lambda'^2)\right], \tag{33d}$$

$$\delta M_d^{\gamma} = +\frac{\alpha M}{4\pi}\frac{1}{3}\left[-\frac{2}{\lambda^4} + \frac{20}{\lambda^2} + 8\lambda^2 - 68\ln\lambda^2 + 92\ln(\lambda^2+1) + 28 - \frac{54}{\lambda^2 - 1}\ln\lambda^2 - 92\ln 2 + 3\varphi_1(\lambda^2)\right], \tag{33e}$$

$$\begin{aligned}\delta M_d^{p,n} = +\frac{M}{4\pi^2}\Bigg\{R^2\Bigg[&\frac{2}{t-1} - \frac{4}{3(t+1)} + \frac{8\sqrt{2}}{3}\ln\left(\frac{t+3+\sqrt{8}}{t+3-\sqrt{8}}\right) + \frac{8}{3}\ln t + \frac{8}{3} \\ &- \frac{2t}{(t-1)^2}\ln t - \tfrac{1}{2}\varphi_1(t) + \tfrac{1}{2}\varphi_2(t) + \tfrac{1}{2}\varphi_3(t) - \frac{8\sqrt{2}}{3}\ln\left(\frac{4+\sqrt{8}}{4-\sqrt{8}}\right)\Bigg] \\ +MRV\Bigg[&t - \frac{25}{t} - 10\sqrt{2}\ln\left(\frac{t+3+\sqrt{8}}{t+3-\sqrt{8}}\right) - 20\ln t + \frac{24}{t(t+1)} + 10\sqrt{2}\ln\left(\frac{4+\sqrt{8}}{4-\sqrt{8}}\right) + \tfrac{1}{2}\varphi_1(t) - \tfrac{1}{2}\varphi_2(t) - \tfrac{1}{2}\varphi_3(t) + 12\Bigg]\Bigg\}.\end{aligned} \tag{33f}$$

In Eqs. (33), M is taken to be 938 MeV; α, k_p, and k_n are given previously. The cutoff parameter and t are related by

$$t = \frac{\lambda + (\lambda^2 - 1)^{1/2}}{\lambda - (\lambda^2 - 1)^{1/2}}. \tag{34}$$

The three functions φ_1, φ_2, and φ_3 are given by

$$\begin{aligned}\varphi_1(a) &= \int_1^a \frac{\ln x}{x-1}\,dx, \\ \varphi_2(a) &= \int_1^a \frac{x\ln x}{x^2+6x+1}\,dx, \\ \varphi_3(a) &= \int_1^a \frac{\ln x}{x^2+6x+1}\,dx,\end{aligned} \tag{35}$$

and have to be evaluated numerically. φ_1 is sometimes called the Spence function. B is now defined as

$$\delta M_p^{e^2} + \delta M_p^{e^2k} + \delta M_p^{e^2k^2} + \delta M_n^{e^2k^2} - \delta M_d^{\gamma} - \delta M_d^{p,n}.$$

The values of R and V are determined from Ref. 13 and are

$$R^2 = \frac{8\pi}{M}N^2, \quad MRV = \frac{1}{2}\frac{8\pi}{M}N^2, \quad N^2 = \frac{2(MB)^{1/2}}{1-(MB)^{1/2}r}, \tag{36}$$

where r is the deuteron's triplet effective range of 1.73 F. From Eqs. (36) we have eliminated terms proportional to ρ, where ρ is the admixture of the d state. This is in agreement with Mathews and Deo[14] who greatly reduce the effects of this term, and also with Duck[15] who neglects it completely.

Equations (36) are inserted into Eqs. (33) and the results rearranged to find B. A computer was used for the final calculations, and the results are shown in Fig. 5, where the error bars are a measure of the error introduced by the approximation $M_d = 2M_n = 2M_p = 2M$. They assume an effect of five parts per thousand on the R^2 and MRV terms. As can be seen, they do not affect the results significantly.

The binding energy, 2.2 MeV, is roughly repro-

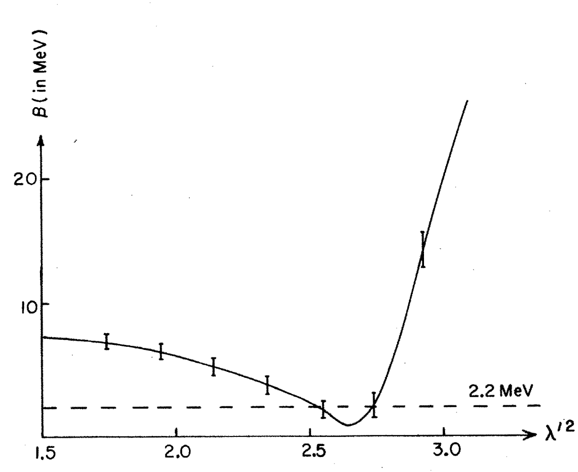

FIG. 5. The plot of B against λ'^2.

duced for a cutoff in the region $2.5 < \lambda'^2 < 2.8$, which, when inserted into Pagels's[1] work, gives a proton-neutron mass difference of $1.5 \rightarrow 1.7$ MeV. Pagels's work was not able to reproduce the proton-neutron mass difference exactly and 1.7 MeV was the value he obtained.

At $\lambda'^2 = 2.55$ (where $B \simeq 2.2$ MeV) the detailed results in MeV are

$$\delta M_p^{e^2} = 0.92, \quad \delta M_p^{e^2k} = -0.62, \quad \delta M_p^{e^2k^2} = 0.09,$$
$$\delta M_n^{e^2k^2} = 0.11, \quad \delta M_d^{\gamma} = 0.69, \quad M_d^{p,n} = -2.39,$$

and similar results hold at $\lambda'^2 = 2.75$. Most of the variation in the curve in Fig. 5 was due to the $\delta M_d^{p,n}$ term.

It must now be asked: "What effects will π mesons in the intermediate-state expansions have on the results?" Calculations were performed to determine the effects of the π meson on δM_n and δM_p. The necessary expressions are given by Pagels,[1] where it must be realized that we are finding $\delta M_p^{\pi} + \delta M_n^{\pi}$ rather than $\delta M_p^{\pi} - \delta M_n^{\pi}$. A significant contribution, in fact larger than δM_n^{γ} and δM_p^{γ}, was found.

No estimate of the effect of the π meson on δM_d was made. However, apart from a correction due to structure effects in the deuteron, such as Fermi motion, the π meson would be inserted into diagrams like those in Fig. 4, where it would interact with the nucleons. As in the case with photons in the intermediate state, the $\delta M_p^{\pi} + \delta M_n^{\pi}$ contribution would cancel this latter part of the δM_d^{π} contribution due to the minus sign in the definition of B [Eq. (11)]. The remaining π-meson correction, which is analogous to the calculated photon contribution of B, is unknown.

We thus concluded that π mesons in the intermediate-state expansions would not give significant results compared to the $\delta M_d^{p,n}$ contribution. Hence we did not calculate them.

VII. SUMMARY AND CONCLUSIONS

We have calculated the binding energy of the deuteron by using sidewise dispersion relations for the mass-energy form factors of the neutron, proton, and deuteron. The method involves a number of assumptions.

It is assumed that the form factors satisfy once-subtracted dispersion relations, and that they have the asymptotic behavior indicated by Eq. (13). We cannot add anything more to this except for stating with Pagels that these assumptions are essential to the method.[1] They are part of its heuristics.

The deuteron has been considered as an elementary particle capable of breaking up into two nucleons in an S-wave state; this condition of elementarity is also essential for the applicability of the method in its present form.

The binding energy may then be expressed in terms of the cutoff parameter λ. The parameter is not free, however, since threshold dominance relates it to the neutron-proton mass difference, which further corroborates the usefulness of this assumption already so successful in a number of problems.[1,6,16]

In particular, the value of B found agrees with the experimental value and corresponds to a proton-neutron mass difference of 1.5 to 1.7 MeV, which coincides with that obtained by Pagels[1] and compares favorably with the experimental one of 1.3 MeV. The value of B is made up of several contributions of which the electromagnetic ones are small and tend to cancel one another, while $\delta M_d^{p,n}$ is the dominant one. Its value is rather sensitive to the cutoff, since its R^2 and MRV contributions are large but of opposite sign. The contributions from Fig. 4(a) and from Figs. 4(b) and 4(c) also display this behavior.

Actually, in setting R and V equal to their on-shell value, we may not have been entirely correct in the region close to the shell value of W^2. This would certainly be true in the case with a composite deuteron where the form factors may be even a factor of 10^{-2} smaller than their on-shell values slightly off shell.[14] Moreover, off shell, the expression for the n-d-p vertex would contain more than two form factors. Since our deuteron is elementary, we cannot even be safely assured that the form factors behave in a way similar to that of Ref. 14, particularly because of the nonoccurrence of anomalous thresholds.

A refinement of the method would therefore require a more accurate calculation of the n-d-p vertex within our deuteron model, and would almost certainly increase the complexity of our work by several orders of magnitude. We are, however, considering the problem.

APPENDIX

Dispersion relations can be proved for the nucleons, as shown below (essentially following Bincer[9]). We let

$$G_N(W^2) = \sum_{\text{spin}} i\left(\frac{N_0}{M_N}\right)^{1/2} \int d^4x\, e^{-il\cdot x}\, \hat{\theta}(x_0) \times \langle N|[\Theta(x), \bar{\eta}(0)]|0\rangle P(W^2)$$
$$= \int d|\vec{x}| \int dx_0 f(|\vec{x}|, x_0; W^2) \exp[-ix_0(W_0 - M_N)]\,, \tag{A1}$$

where

$$f(|\vec{x}|, x_0; W^2) = \int d\varphi\, d(\cos\theta)\vec{x}^2 i \left(\frac{N_0}{M_N}\right)^{1/2} \hat{\theta}(x_0)\, e^{i\vec{W}\cdot\vec{x}} \sum_{\text{spin}} \langle N|[\Theta(x), \bar{\eta}(0)]|0\rangle P(W^2), \tag{A2}$$

which vanishes for $x_0 < 0$ because of $\hat{\theta}(x_0)$, and vanishes for $x_0 < |\vec{x}|$ because the commutator disappears for spacelike $|\vec{x}|$. The frame

$$N = (M_N, 0), \quad W = (W_0, \vec{W}), \quad l = (W_0 - M_N, \vec{W}), \tag{A3}$$

has been chosen. It leads us to

$$\vec{W}^2 = (W_0 - M_N)^2, \quad W_0 = (W^2 + M_N^2)/2M_N. \tag{A4}$$

Now, to show that there are no singularities in f in W^2, we perform the spin sum. We let

$$\langle N|[\Theta(x), \bar{\eta}(0)]|0\rangle P = \bar{u}(N)\langle 0|\text{function of } x|0\rangle P, \tag{A5}$$

as this is the only possibility that makes the matrix element a scalar, and is the only possibility that can be combined with the projection operator to form a scalar. Now we use Eq. (4) for P and obtain for the spin sum

$$\sum_{\text{spin}} \bar{u}(N) \frac{M_N \not{l}}{W^2 - M_N^2} u(N) = 1. \tag{A6}$$

This does not depend upon W^2, so f has no singularities in W^2. Thus

$$F(|\vec{x}|; W^2) = \int dx_0 f(|\vec{x}|, x_0; W^2) \exp[-ix_0(W_0 - M_N)] \tag{A7}$$

is analytic in W^2 because of its lack of singularities and because of its behavior in x_0. Then F can be written in a dispersion relation, assumed to be once subtracted:

$$F(|\vec{x}|; W^2) = F(|\vec{x}|; M_N^2) + \frac{W^2 - M_N^2}{\pi} \int \frac{dW'^2 \operatorname{Im} F(|\vec{x}|; W'^2)}{(W'^2 - M_N^2)(W'^2 - W^2 - i\epsilon)}, \tag{A8}$$

and since

$$G(W^2) = \int d|\vec{x}|\, F(|\vec{x}|; W^2), \tag{A9}$$

G may be written in a dispersion relation if the order of integration can be interchanged. It can, since $|\vec{W}|$ never becomes imaginary in the range of W_0, as seen by the relation $|\vec{W}| = W_0 - M_N$ [from Eq. (A4)]. This condition must be observed because the θ integration brings out a term $(\sin|\vec{W}||\vec{x}|)/(|\vec{W}||\vec{x}|)$ which could be singular if $|\vec{W}|$ were imaginary.

We show dispersion relations for the deuteron's form factor in an entirely analogous way, the only significant change being the polarization-sum step, which is

$$\langle d|[\Theta(x), j_\nu^\dagger(0)]|0\rangle P_\nu(W^2) = \epsilon_\nu \langle 0|\text{function of } x|0\rangle P_\nu(W^2). \tag{A10}$$

Only an ϵ_ν can come out to make the element a scalar, and be combined with P_ν to form a scalar. Using Eq. (8) for $P_\nu(W^2)$, the polarization sum becomes

$$\sum_{\text{pol}} \epsilon_\nu P_\nu(W^2) = \frac{W^4 - W^2 M_d^2 + 2M_d^4}{4M_d^3(W^2 + M_d^2)}. \tag{A11}$$

This certainly has no singularities in W^2 for the range of W^2 being considered in the dispersion integral, Eq. (14). Thus, proceeding as above, we can prove the dispersion relation for the form factor of the deuteron.

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Evidence on Duality and Exchange Degeneracy from Finite-Energy Sum Rules: $K^-n \to \pi^-\Lambda$ and $\pi^+n \to K^+\Lambda$†

R. D. Field, Jr. and J. D. Jackson

Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

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Using finite-energy sum rules for the reactions $K^-n \to \pi^-\Lambda$ and $\pi^+n \to K^+\Lambda$, we determine the effective "pole" parameters of the K^* and K^{**} Regge trajectories from a knowledge of the low-energy resonances and their couplings. The resonance parameters and the $D/(D+F)$ ratio for the $\frac{1}{2}^+$ baryon octet are varied somewhat to test the sensitivity of the high-energy predictions; $\frac{1}{2}^+$ octet couplings within the range of values found empirically in other reactions are preferred in our solution. We find that the s-channel resonances in $K^-n \to \pi^-\Lambda$ *do* add in such a way as to produce predominantly real amplitudes at high energies as predicted by duality diagrams. We find, however, that these predictions are not satisfied *exactly*. Although the phases of both A' and B are small and independent of t for $|t| < 0.5$ (GeV/c)^2, the residues of the even- and odd-signature Regge poles are closely exchange-degenerate only for the B amplitudes, and not for the A' amplitudes, thereby allowing an appreciable polarization for $K^-n \to \pi^-\Lambda$ as is observed experimentally. The Regge-pole parameters determined from the sum rules give a good fit to the reaction $K^-n \to \pi^-\Lambda$ over a wide range of energies, whereas they are unable to fit $\pi^+n \to K^+\Lambda$ at intermediate energies. Comparison of the resonance contributions to $K^-n \to \pi^-\Lambda$ and $\pi^+n \to K^+\Lambda$ shows that "peripheral" resonances dominate the sum rules in the first reaction, while "nonperipheral" states are important in the second. By supposing that "peripheral" resonances are dual to the leading Regge singularities in the t channel, while "nonperipheral" resonances are dual to lower-lying singularities, we are led to a rationale of why the simple model of two effective Regge poles is adequate for $K^-n \to \pi^-\Lambda$ even at intermediate energies, but inadequate there for $\pi^+n \to K^+\Lambda$.

I. INTRODUCTION

The duality diagrams introduced by Harari[1] and Rosner[2] conveniently illustrate the ramifications of duality and the absence of quark-model "exotic" states. Processes with planar duality diagrams supposedly have high-energy amplitudes with imaginary parts and t-dependent phases, whereas reactions with nonplanar diagrams have purely real amplitudes at high energy. Rosner explicitly states that his derivation of the duality diagrams from $SU(3)$ couplings applies only to the nonflip amplitude (A') of (0^-, $\frac{1}{2}^+$) scattering, and requires purely f coupling of the vector mesons, and purely d coupling of the tensor mesons to the pseudoscalar mesons. Harari, on the other hand, conjectures that whenever a diagram is nonplanar all the corresponding helicity amplitudes should be purely real at high energies. Thus Harari predicts that whenever the duality diagram for a reaction is nonplanar the polarization should vanish at high energy. One such process is $K^-n \to \pi^-\Lambda$, whose three duality diagrams are shown in Fig. 1. Although the quantum numbers allow resonances in all three channels, the s-t diagram, relevant for near-forward scattering at high s, is nonplanar. Following Harari's conjecture that both the nonflip (A') and flip (B) amplitudes are real, we should expect no high-energy polarization. However, experiments at 3.0 and 4.5 GeV/c show a