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Implications of a Charge Asymmetry in the ω^0 Dalitz Plot*

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We analyze the possible theoretical implications of the recent evidence for a charge asymmetry in the ω Dalitz plot. In particular, we discuss the asymmetry in terms of theoretical models based on (a) the Okubo-Yuta background-interference effect, (b) an interfering mass-degenerate exotic resonance $\tilde{\rho}^0$ ($I^G=1^-$, $J^{PC}=1^{-+}$), and (c) C violation in $\rho^0\to\pi^+\pi^-\pi^0$ decay with coherence of the ρ^0 and ω^0 production amplitudes.

I. INTRODUCTION

Recently, Abrams *et al.*[1] observed a significant charge asymmetry on the ω^0 Dalitz plot for the channel $\pi^+p\to\omega^0\Delta^{++}$ at 3.7 GeV/c. The pertinent features of the data are

(i) The effect is large and is localized in t', the momentum transfer squared,[2] being most prominent in the t' interval $0.08\le t'\le 0.20$ (GeV/c)^2. The charge asymmetry is consistent with zero outside of this concentrated kinematic region.

(ii) The observed charge asymmetry is indicative of an interfering coherent $I=1$, $J^{PC}=1^{-+}$ amplitude in the region of the ω^0 mass.

(iii) The asymmetry parameter α_I ($I=1$) has a value $\alpha_{I=1}=0.18\pm0.05$ for t' in the interval 0.08–0.2 (GeV/c)^2. The asymmetry is symmetric with respect to $\epsilon_\omega=2(m_\omega-m)/\Gamma_\omega$ for $\epsilon_\omega>0$ and $\epsilon_\omega<0$.

(iv) Coherence between ρ^0 and ω^0 production, which has been observed via the ω^0-ρ^0 interference effect in the $\pi^+\pi^-$ mass spectrum, for the t' interval 0–0.14 (GeV/c)^2 *cannot* be established in the data of Abrams *et al.*[1] for the *full* t' interval in which the asymmetry is observed.

The present work analyzes the theoretical implications of such a charge asymmetry for the ω decay. Owing to the limited experimental information currently available, unambiguous conclusions cannot be drawn at this stage. However, several suggestive models have been proposed[1,3] as possible interpretations of this asymmetry effect. We study these models here to extract the pertinent theoretical information and (where possible) testable consequences. In Sec. II we discuss the Okubo-Yuta[4] coherent-background-interference mechanism; in Sec. III an interfering-resonance model based on the postulated existence of a mass-degenerate C-exotic resonance $\tilde{\rho}$ ($I^G=1^-$, $J^{PC}=1^{-+}$) is described; finally, in Sec. IV we speculate on the relevance of C violation in $\rho^0\to\pi^+\pi^-\pi^0$

decay and the coherence of the ρ^0 and ω^0 production amplitudes[3] as a possible source of explanation for this charge asymmetry.

II. THE OKUBO-YUTA BACKGROUND-INTERFERENCE MODEL

On the basis of the data available in this one experiment, the possibility that the observed asymmetry results from interference with some unknown but coherent background amplitude cannot be excluded.

The maximum possible asymmetry resulting from the estimated 10% background under the ω signal can be found using the formalism of Okubo and Yuta.[4] If we assume that the background amplitude B is entirely coherent with the ω, is entirely in the $J^{PC}=1^{-+}$, $I=1$ state, and is imaginary relative to the ω, we have

$$\alpha^{\max} \leqslant \left(\frac{2\pi\Gamma_\omega}{\Delta m}\,\frac{\sigma_B}{\sigma_\omega}\right)^{1/2}, \tag{1}$$

where

$$\sigma_B = \frac{|B|^2}{\mu^4}\,2m_\omega\,\Delta m$$

is the cross section for background production, Γ_ω is the ω^0 width, and Δm is the experimental mass interval. We find

$$\alpha^{\max} \lesssim 20\%. \tag{2}$$

Although this estimate shows that the asymmetry may result from the Okubo-Yuta effect, there are several reasons for exploring the problem further.

One reason is that in the order-of-magnitude estimate above, it has been assumed that the entire observed background (10%) is involved in the interference. Actually, there are numerous three-pion states that can be produced in $\pi^+ p \to (3\pi)\Delta^{++}$. Among these states, with various isospin and spin-parity values, the ω interference projects out only that coherent component with $J^{PC}=1^{-+}$, $I=1$. It remains to be explained why this particular component dominates the background.

Another open question is the t dependence. The observed asymmetry peaks near a momentum transfer $t' \simeq 0.2$ (GeV/c)^2 [feature (i) of the Introduction]. If it is assumed, for example, that the background is produced by the same exchanges as the ω, then a forward peaking of B, and therefore of the asymmetry, is suggested.

We conclude that, although the Okubo-Yuta effect is a possible explanation, there remains significant doubt about it and other explanations should be considered.

III. THE INTERFERING-RESONANCE MODEL

The fact [as stated in (i) and (ii) of the Introduction] that the observed charge asymmetry in the ω^0 Dalitz plot is noted in only a limited kinematic region argues for a t dependence of the coherent ($I^G=1^-$, $J^{PC}=1^{-+}$) production amplitude more akin to that of *resonance production* rather than to a miscellaneous-background amplitude.[1] We shall assume in this section that the interfering amplitude is that associated with a resonant particle $\tilde{\rho}^0$ with quantum numbers $I^G=1^-$, $J^{PC}=1^{-+}$ and *nearly mass degenerate to the known* ω^0 ($I^G=0^-$, $J^{PC}=1^{--}$). We discuss below the magnitude of the charge asymmetry in this model, the possible experimental searches for such a $\tilde{\rho}^0$, and the relevance of $\tilde{\rho}^0$ to contending theories of exotic states in meson spectra.

Magnitude of Asymmetry

The distribution in mass m of the (3π) final state is given by

$$\frac{dn}{dm} = \left|\frac{F_{\tilde{\rho}}(b_{\tilde{\rho}}\Gamma_{\tilde{\rho}})^{1/2}}{m-m_{\tilde{\rho}}+\frac{1}{2}i\Gamma_{\tilde{\rho}}} + \frac{F_\omega(b_\omega\Gamma_\omega)^{1/2}}{m-m_\omega+\frac{1}{2}i\Gamma_\omega}\right|^2, \tag{3}$$

where $b_{\tilde{\rho}}$, b_ω denote the branching ratio for $\tilde{\rho}\to 3\pi$ and $\omega^0\to 3\pi$; $F_{\tilde{\rho}}$, F_ω are the production amplitudes for ($\tilde{\rho}$, ω^0) in the reaction under consideration. Let us call

$$P_{\tilde{\rho}} = F_{\tilde{\rho}}\sqrt{b_{\tilde{\rho}}},\quad P_\omega = F_\omega\sqrt{b_\omega}\,, \qquad P_{\tilde{\rho}}^*/P_\omega = |P_{\tilde{\rho}}/P_\omega|\,e^{i\phi}. \tag{4}$$

Assume that $\tilde{\rho}$ and ω^0 are essentially mass degenerate, and then let

$$E = m - m_{\tilde{\rho}} = m - m_\omega,\quad dE = dm\,; \tag{5}$$

hence,

$$\frac{dn}{dE} = \left|\frac{P_{\tilde{\rho}}\sqrt{\Gamma_{\tilde{\rho}}}}{E+\frac{1}{2}i\Gamma_{\tilde{\rho}}} + \frac{P_\omega\sqrt{\Gamma_\omega}}{E+\frac{1}{2}i\Gamma_\omega}\right|^2. \tag{6}$$

Note that up to a relative phase ϕ,

$$P_{\tilde{\rho}} = -\sqrt{N_{\tilde{\rho}}}/\sqrt{2\pi}\,,\quad P_{\omega^0} = -\sqrt{N_\omega}/\sqrt{2\pi}$$

in the notation of Abrams *et al.*[3] The asymmetry will depend upon a factor

$$\mathrm{Re}\left(\frac{1}{E-\frac{1}{2}i\Gamma_{\tilde{\rho}}}\;\frac{1}{E+\frac{1}{2}i\Gamma_\omega}\,e^{i\phi}\right). \tag{7}$$

Hence, in general, the imaginary part of one resonance may interfere with either the real or imaginary part of the other giving rise, respectively, to a charge asymmetry which is symmetric or antisymmetric in energy about the central ω mass. In the cases $\phi \simeq 0$, π, the sign of the asymmetry is independent of m over the entire ω^0 mass region. This type of asymmetry has been denoted by α in Ref. 1 and corresponds to an interference with the imaginary part of the ω^0 amplitude. When $\phi \simeq \frac{1}{2}\pi$ the sign of the asymmetry is opposite for $m-m_\omega>0$ and $m-m_\omega<0$ and the asymmetry (denoted by α') corresponds to interference with the real part of the ω^0 amplitude. Experimentally, it is the α type of asymmetry[1] which has been observed in $\pi^+p\to\omega^0\Delta^{++}$.[5]

Since an α-type asymmetry is involved, we take $\phi\simeq 0$ and upon integration of E, we find for the asymmetry

$$\alpha=\frac{4|P_{\tilde\rho}||P_\omega|}{|P_{\tilde\rho}|^2+|P_\omega|^2}\int\frac{dx(x^2+1)}{(x^2+1)^2+\lambda x^2}\Big/\int\frac{dx}{x^2+\frac14}$$

$$=\frac{4|P_{\tilde\rho}||P_\omega|I}{|P_{\tilde\rho}|^2+|P_\omega|^2}\,, \tag{8}$$

where

$$\lambda=\frac{(\Gamma_{\tilde\rho}-\Gamma_\omega)^2}{\Gamma_{\tilde\rho}\Gamma_\omega}\,. \tag{9}$$

The limits of integration can be set from $-\infty$ to $+\infty$ if the range of E is much greater than the widths of $\Gamma_{\tilde\rho}$ and Γ_ω. Using contour integration, we find

$$I=\frac{i}{2}\,\frac{X_1X_2-1}{X_1X_2(X_1+X_2)}\,,$$

$$X_1=i\{\tfrac12(\lambda+2)-\tfrac12[\lambda(\lambda+2)]^{1/2}\}^{1/2}\,, \tag{10}$$

$$X_2=i\{\tfrac12(\lambda+2)+\tfrac12[\lambda(\lambda+2)]^{1/2}\}^{1/2}\,.$$

It is evident from Eq. (8) that $\lambda=0$ (and hence, $\Gamma_{\tilde\rho}=\Gamma_\omega$) will yield maximal asymmetry

$$\alpha_{\max}=\frac{2|P_{\tilde\rho}||P_\omega|}{|P_{\tilde\rho}|^2+|P_\omega|^2}\,, \tag{11}$$

independent of the range of integration of E or x in (8). For $|P_{\tilde\rho}|\ll|P_\omega|$ we have

$$\alpha_{\max}=2|P_{\tilde\rho}|/|P_\omega|\,. \tag{12}$$

If we take the extreme case where one resonance is ten times as wide as the other, $\lambda=10$, $I=0.175$. From (8) we have

$$\alpha=\frac{0.7|P_{\tilde\rho}P_\omega|}{|P_{\tilde\rho}|^2+|P_\omega|^2}\,. \tag{13}$$

In order to obtain a rough upper limit on the relative production amplitudes of $\tilde\rho$ and ω, we have examined several experiments[6] in which the charged three-pion decay mode of $\tilde\rho^\pm$ might have been detected. The effective mass plots for charged three-pion states are not inconsistent with the presence of a resonance at 784 MeV with a production amplitude smaller by a factor of three or four than that of the ω.

We shall not attempt here to obtain a quantitative relationship between the production amplitude and the width of $\tilde\rho$. It is anticipated that $\tilde\rho$ will be narrower than the ω^0 because of the barrier factors.[7] The relativistic barrier factor (in the wave function) for the $\tilde\rho$ has an exponent 3 vs an exponent 2 for the ω. (The nonrelativistic factor gives an even stronger suppression.) We consider the ratio $\Gamma_{\tilde\rho}/\Gamma_\omega=1/10$ to be a pessimistic lower limit. As mentioned above, this gives a suppression by a factor of 3.5 in the asymmetry.

Using the values $P_{\tilde\rho}/P_\omega=\frac14$ and $\Gamma_{\tilde\rho}/\Gamma_\omega=\frac{1}{10}$, we find from Eq. (8), $\alpha\simeq 15\%$. That is, the asymmetry is a very sensitive measure of the presence of even a narrow and suppressed resonance degenerate with the ω.

Production and Decay of $\tilde\rho$

The most tangible proof that the interfering-resonance model is the correct interpretation of the data would be the experimental establishment of the $\tilde\rho$ state with $I^G=1^-$, $J^{PC}=1^{-+}$ and mass degenerate with the known ω^0 state. The $\tilde\rho$ state is, however, "*C* exotic" (or "type-II exotic") in the classification of Rosner.[8] It is not coupled to $(q\bar q)$, nor to $N\bar N$, $\Sigma\bar\Sigma$, $\Xi\bar\Xi$, and, to the extent that $SU(3)$ is good, not to $\Lambda\bar\Sigma$ as well. Hence, on general grounds, its production cross section *is expected to be small* and the present lack of evidence for the state is not necessarily detrimental to the interpretation[9] as already discussed above. For instance, in

$$p+p\to d+\tilde\rho^+\,, \tag{14}$$

the $n(940)$ and $\Delta(1238)$ exchanges are forbidden (the latter by isospin considerations).

General analysis based on available phase space and barrier factors would suggest that for a $J^{PC}=1^{-+}$ state in the 800-MeV mass range, the dominant strong-decay mode is going to be the $\eta\pi$ mode. Typically, one might expect (roughly) that normalized to the ρ width, $\tilde\rho\to\eta\pi\lesssim 30$ MeV. The strong decay $\tilde\rho^0\to 3\pi$ (which might participate in the charge-asymmetry experiment[1]) may be a secondary mode; hence $\tilde\rho^0\to 3\pi$ can be much smaller than 30 MeV. On the other hand, it has been shown by Lipkin and Meshkov[10] that members of a meson octet with negative *CP* cannot decay into two pseudoscalar mesons due to an $SU(3)$ selection rule. In this case the $\tilde\rho\to 3\pi$ mode would be dominant. However, since the $\tilde\rho$ is not a (usual) quark-model state, it may be premature at the present time to insist upon a specific $SU(3)$ assignment, and a search for a narrow peak in $\tilde\rho^-$ in $\pi^-p\to pX^-$ using missing-mass techniques[11] may be appropriate. Despite the dominant $\eta\pi$ decay mode of $\tilde\rho$, it is unlikely that the η^0-exchange diagram plays a significant role in $\pi^-+p\to p+\tilde\rho^-$ because of the "attenuated" $\eta^0\bar pp$ coupling from $SU(3)$ considerations. Abrams *et al.*[1] suggest that ρ exchange may be dominant in the production of a possible $\tilde\rho_0$ state in $\pi^+p\to\tilde\rho_0\Delta^{++}$. Production of $\tilde\rho^-$ due to ρ exchange from $\pi^-+p\to p+\tilde\rho^-$ at large momentum transfer may allow some separation of this phenomenon from the dominant production of ρ^- at small momentum transfer (via pion exchange)–though there may be significant contribution to ρ^- production at large momentum transfer due to ω^0 exchange. Study of the $\eta\pi$ and 3π final-state decay in $K^- + n \to \Lambda^0 + \tilde{\rho}^-$ (K^* exchange) should be feasible. It is evident that techniques which trigger specifically on the ($\eta\pi$) final-state mass spectra are likely to be promising in the search for $\tilde{\rho}$. The conjectured narrow width for $\tilde{\rho}$ may lend itself more readily to discovery by cascade methods analogous to those used by Baud *et al.*[12] in the CBS search for a *narrow* $\delta(962)$.

Finally, the possible existence of "C-exotic" mesons such as[13,14] $\tilde{\rho}(784)$ and $\delta(962)$ need not be in contradiction to the rather stringent upper limits set on *exchange* of exotic particles, since the latter most severely test exchanges involving particles satisfying $\Delta S = -\Delta Q$, $\Delta Q = 2$, and $\Delta S = 2$.[15] As emphasized already, *production* of $\tilde{\rho}(784)$ can be achieved through normal ρ exchange.

Theoretical Implication

The most striking feature about the possible existence of an $\tilde{\rho}$ state with $I^G = 1^-$, $J^{PC} = 1^{-+}$ is its expected near mass degeneracy with the known $I^G = 0^-$, $J^{PC} = 1^{--}$ $\omega^0(784)$. As emphasized recently,[16] "accidental" degeneracies (especially if not an isolated case) involving two states of the same spin-parity but one of which is not of the $q\bar{q}$ type [e.g., $\tilde{\rho}(784)$] generally suggest a different underlying symmetry at work. Related to this remark is the idea[13,17] that we may have besides the $SU(3)$ multiplets (of $q\bar{q}$ and qqq types), multiplets of another group G, say $SU(2)_{I_1} \otimes SU(2)_{I_2}$, $I_1 + I_2 = I$ = isospin [G not contained in $SU(3)$], analogous to the use of two different overlapping schemes in atomic physics: jj coupling and LS coupling. For physically meaningful assignments to $SU(2)_{I_1} \otimes SU(2)_{I_2}$ multiplets, we must require that the *G parity of members of such a multiplet be the same.*[13,18] *The pair $\tilde{\rho}(784)$ and $\omega^0(784)$ thus lends itself naturally to assignment to the $(\frac{1}{2}, \frac{1}{2})$ representation of this clashing $SU(2) \times SU(2)$ symmetry.* Other $(\frac{1}{2}, \frac{1}{2})$ representations have been suggested in connection with $\eta'(959)$-$\delta(962)$ degeneracy[13] where the $\delta(962)$ may be related to recent observations of a narrow peak[12,14] in the 955-MeV mass region.

The existence of C-type exotic mesons has also been proposed in connection with C violation in electromagnetism. The gist here is really that field-current-identity theory suggests that to every current we can associate at least one particle. Thus, for instance, the usual isovector (J_μ^V) and isoscalar (J_μ^S) electromagnetic currents are closely identified with the (ρ^0, ω^0, ϕ^0) system, so a C-violating K_μ current can be associated with the h meson[19] (for K_μ^S) or the $\tilde{\rho}$ meson[20] (for K_μ^V). Of course, it must be clearly understood that K_μ currents *may* imply $\tilde{\rho}$ or h mesons but *not* vice versa. Indeed, even if C violation were electromagnetic in origin, construction of K_μ^V and K_μ^S currents (see Sec. IV, below) can be made on the basis of the anomalous-magnetic-moment model for the (ρ^0, ω^0, ϕ^0) system without necessarily invoking the additional assumption of field-current identity for the K_μ currents.

Mandula *et al.*[21] have proposed economical solutions to the particle spectrum (richer than the naive quark model) on the basis of duality considerations which include, among other C-exotic states, a state with the quantum numbers of the $\tilde{\rho}$. Neither this duality model nor the C-violating field-current model would appear to explain the near degeneracy between $\tilde{\rho}$ and ω, though.

IV. C VIOLATION VIA ρ^0 INTERFERENCE IN ω^0 DECAY

Features (iii) and (iv) of Sec. I do not speak well for a C-violation (via ρ^0 interference in ω^0 decay) interpretation of the data.[1] The magnitude of the asymmetry (~18%) and the inability to establish coherence of ρ^0- and ω^0-production amplitudes in the kinematic t' range, where the asymmetry is most prominent, are definite problems for this interpretation. Nevertheless, as a proposed *method* to search for the existence of a C-violating transition in ρ^0 decay, the ρ^0-ω^0 interference model has interesting possibilities, as stressed especially by Abrams *et al.*[3]

Setting aside the magnitude of the asymmetry for the moment, we can still attempt to understand features (i)–(iv) in terms of a C-violating model based on ρ^0-ω^0 interference.

General Notions

We consider only the $I = 0$ and $I = 1$ amplitudes. For ρ^0 (ω^0) $\to 3\pi$, the $I = 2$ final state is C conserving while the 3π $I = 3$ state (though C violating) is suppressed by barrier factors.[7] There are then two distinct mechanisms specifically associated with the ρ-ω complex that can give rise to an asymmetry via C violation.

(a) The ω decay itself has a C-violating $I = 1$ amplitude. Asymmetry in this case should not be t'-dependent, since a single-particle ω^0 decaying into two opposite C channels will generally have no memory so far as asymmetry on the t' dependence. This is in contradiction to feature (i) of data. Furthermore, the magnitude of the asymmetry in this case is estimated to be[22] ~1%.

(b) An $I = 1$ C-violating amplitude due to $\Delta I = 0$ $\rho^0 \to 3\pi$ decay. We shall henceforth concentrate only on this case.

The three-pion |amplitude|^2 in this case is approximately[3]

$$N_\omega\left|\frac{D_0(\Gamma_\omega/2\pi)^{1/2}}{\epsilon_\omega - i}\right|^2\left(\frac{2}{\Gamma_\omega}\right)^2 + \frac{2\xi}{\pi}\left(\frac{N_\omega N_\rho \Gamma_1}{\Gamma_\rho}\right)^{1/2}\vec{D}_0\cdot\vec{D}_1 \times \mathrm{Re}\left[\left(\frac{\Gamma_\rho}{\Gamma_\omega}\right)^{1/2}\left(\frac{\epsilon_\omega - i}{\epsilon_\omega^2+1}\right)e^{i(\pi/2+\beta_1-\beta)}\right], \tag{15}$$

where $\epsilon_\lambda = 2(m_\lambda - m)/\Gamma_\lambda$ ($\epsilon_\omega \gg \epsilon_\rho$), N_λ is the number of events of particle λ observed in the t' interval under study, β is the relative production phase measured from $\omega\rho$ interference in the $\pi^+\pi^-$ mass spectrum, β_1 is the $I=1$ ρ^0 decay phase (relative to the $\rho^0 \to \pi^+\pi^-$ decay amplitude) and Γ_1 is the $I=1$ partial width for ρ^0 decay into $\pi^+\pi^-\pi^0$.

Using the |amplitude|^2 above, the energy-symmetric (α) and energy-antisymmetric (α') asymmetries can be written in the form[3]

$$\alpha = \alpha_0 \sin(\tfrac{1}{2}\pi + \beta_1 - \beta), \qquad \alpha' = \alpha_0' \cos(\tfrac{1}{2}\pi + \beta_1 - \beta), \tag{16}$$

where α_0 and α_0' are unknown constants. It is to be expected that $\alpha_0' < \alpha_0$, due to the problem of experimental sensitivity. An experimental selection of events according to $\epsilon > 0$ or $\epsilon < 0$ is subject to uncertainty due to the fact that the experimental mass resolution is larger than the known ω width. Thus the uncertainty in the value of ϵ for each event tends to wash out the α'-type asymmetry but not the α type.[3]

A Difficult Point

The asymmetry formulas (16) indicate that no matter what the phase $\beta_1 - \beta$ is, the asymmetry is *always* nonvanishing for one of α, α'. However, for $t' \leq 0.14$ *the experimental asymmetry vanishes*.[1] Also note that it is precisely here that the asymmetry should be largest. The reason is as follows. The $\omega^0 \to 3\pi$ signal is the dominant one. The $\rho^0 \to 3\pi$ is smaller (and induces an asymmetry by interference). The ρ amplitude is largest at small t' (via one-pion exchange); therefore, it should produce the greatest interference at *small* t'.

Possible Resolution of Dilemma

Assuming the value $\beta = 86° \pm 17°$ which was established from the 2π interference effect,[2] recall that this value is known only for $t' \leq 0.14$. Above $t' = 0.14$ the (2π) interference disappears. This is consistent with two hypotheses: (1) ρ and ω are incoherent for $t' > 0.14$, or (2) the value of β is near zero here.[23]

If we adopt (2), then the average value of β for $t' \leq 0.14$ is $86° \pm 17°$, but above $t' = 0.14$ it shifts by 90° to a value near zero.

Examining the asymmetries given by Eq. (16), it can be seen that a change of 90° in β results in a shift from an α'-type asymmetry to an α type, if we choose $\beta_1 \simeq 0°$. So according to this model, *the asymmetry is of the α' type (energy antisymmetric) for $t' \leq 0.14$ and of the α type (energy symmetric) for $t' > 0.14$.* The reason that the asymmetry is not seen in the small t' interval is lack of experimental resolution for an asymmetry of the α' type.

Both types of asymmetry are expected to vanish above $t' = 0.22$ due to weakness of the $\rho^0 \to 3\pi$ signal.

The simplest prediction of this model is that for the (2π) reaction induced by a π^-, e.g., $\pi^- p \to \pi^+\pi^- n$, the behavior of β implies that the interference peak here also disappears above $t' = 0.14$.

The model also predicts that the asymmetry seen in the (3π) reaction induced by a π^- [e.g., $\pi^- p \to (3\pi) n$] should be of the α type for small t' and of the α' type for large t'.

Magnitude of the Asymmetry

This remains a formidable problem for the C-violation interpretation. It has been pointed out to us[24] that the effect of ρ^0 upon ω^0 should generally be expected to be small because only a small number of ρ^0 decays occur in the ω energy band. Goldhaber[23] estimates that, for the C-conserving electromagnetic transition $\rho^0 \to (3\pi)_{I=0}$,

$$\left|\frac{\Gamma(\rho \to (3\pi)_{I=0})}{\Gamma(\omega^0 \to (3\pi)_{I=0})}\right| \sim 10\%, \tag{17}$$

where the dominant contribution comes from mixing between $J^{PC} = 1^{--}$ ρ and ω states. However, such a mixing enhancement is irrelevant to the discussion of the C-violating $\Delta I = 0$ ρ decay to $(3\pi)_{I=1}$.

The Anomalous Magnetic-Moment Model of C Violation

One possible electromagnetic C-violation model (specific to the ω^0-ρ^0 system) which *might* give rise to an enhanced charge asymmetry in ω^0 decay, is the anomalous-magnetic-moment-type model, due to Lee and others.[25] For minimal-type electromagnetic currents, we can easily construct isoscalar K_μ^S and isovector K_μ^V currents out of the (ω^0, ϕ^0, ρ^0) vector-meson system,[26] e.g.,

$$K_\mu^V = \frac{\mu_V}{e}\left(\omega_\nu \frac{\partial \rho_\mu}{\partial x_\nu} - \rho_\nu \frac{\partial \omega_\mu}{\partial x_\nu}\right), \tag{18a}$$

$$K_\mu^S = \frac{\mu_S}{e}\left(\omega_\nu \frac{\partial \phi_\mu}{\partial x_\nu} - \phi_\nu \frac{\partial \omega_\mu}{\partial x_\nu}\right), \tag{18b}$$

where

$$\frac{\partial K_\mu^{(V,S)}}{\partial x_\mu} = 0$$

and

$$C_{st} K_\mu^{(v,s)} C_{st}^{-1} = K_\mu^{(v,s)} ,$$

with C_{st} the particle-antiparticle conjugation operator. The moment factor μ is unknown and *may* be large experimentally. A C-violating $\Delta I = 0$ transition $\rho^0 \to (3\pi)_{I=1}$ is of course consistent with a virtual second-order electromagnetic transition through $J_\mu^V K_\mu^V$ and/or $J_\mu^S K_\mu^S$, where J_μ^V and J_μ^S are the usual isovector and isoscalar electromagnetic currents. However, a substantial contribution from the K_μ^V component [for instance, from (18a)] is needed to explain the data of Gormley *et al.*[27] concerning the asymmetry in η^0 decay. Hence, at least the $J_\mu^V K_\mu^V$ combination is needed here also.

The ever-decreasing limits on the neutron electric dipole moment represent, of course, an increasing strain on electromagnetic C violation. Present experimental upper limit at 90% confidence level[28] is 5×10^{-23} cm. In a maximal electromagnetic C-violation model, the estimates have generally ranged from 10^{-21} to 10^{-22} e cm.[29] Pais and Treiman[30] have pointed out that if

$$K_\mu = \partial_\alpha \partial_\beta T_{\mu\alpha\beta}(x), \qquad T_{\mu\alpha\beta} = T_{\mu\beta\alpha}, \quad T_{\mu\alpha\beta} = -T_{\alpha\mu\beta}, \tag{19}$$

where the tensor $T_{\mu\alpha\beta}$ is sufficiently regular so that its matrix element exists in the zero-momentum-transfer limit, then the *neutron dipole moment vanishes* in the low-frequency limit for a full-strength K_μ current of form (19). A nonminimal K_μ^V current can be constructed from the spin-1 fields (ρ, ω) satisfying (19) as follows (in a free-field example):

$$K_\mu = \partial_\alpha \partial_\beta \partial_\mu (\rho_\alpha \omega_\beta + \omega_\alpha \rho_\beta) - \Box^2 \partial_\sigma (\rho_\mu \omega_\sigma + \rho_\sigma \omega_\mu) . \tag{20}$$

Little is known about the magnitude of the parameter μ experimentally.[31] Hopefully, it can lead to an enhanced contribution to $\rho^0 \to (3\pi)_{I=1}$ via $J_\mu^V K_\mu^V$ and, hence, to a large asymmetry in the ω^0 Dalitz plot[32] via interference. Indeed this type of model, where C violation occurs through the presence of a $\rho\omega\gamma$ vertex in the structure of the Feynman diagrams for the $\Delta I = 0$ transition $\rho^0 \to 3\pi$ and $\Delta I = 2$ transition $\eta^0 \to 3\pi$, can give some qualitative understanding of why asymmetry in the η^0 decay[27] is small (~1%) while being relatively large in the present case. Typical C-violation $\eta^0 \to 3\pi$ and $\rho^0 \to 3\pi$ decays can proceed via ρ-propagator tails $1/(k^2 + m_\rho^2)$, where k^2 is determined kinematically by the 2π invariant mass of the 3π final-state system at η^0 and ω^0 masses, respectively. To wit, if π_1, π_2, π_3 are the four-momenta of the three final-state pions, then $k^2 = (\pi_1 + \pi_2)^2$. The corresponding asymmetries α_ω and α_η can then be related (very roughly) to the ratio of the ρ propagator at these respective mass values as follows:

$$\frac{\alpha_\omega}{\alpha_\eta} = \frac{1}{k_\omega^2 + m_\rho^2} \bigg/ \frac{1}{k_\eta^2 + m_\rho^2} \sim 10 . \tag{21}$$

It is important to emphasize that a K_μ^V current is not yet ruled out by other tests of C violation in electromagnetism. For instance, the $(\eta^0 \to 3\pi^0)/(\eta^0 \to \pi^+ + \pi^0 + \pi^0)$ branching ratio is expected to remain close to 1.5, since $J_\mu^V K_\mu^V$ is small, ~1% to 2%, and $(K_\mu^V)^2$ is very small after integration over spectrum, so the decay branching ratio is dominated by the $\Delta I = 1$ $J_\mu^V J_\mu^S$ transition which leads to the 1.5 prediction. For the $\eta^0 \to \pi^0 + e^+ + e^-$ decay,[33]

$$\frac{\text{rate}(\eta^0 \to \pi^0 + e^+ + e^-)}{\text{rate}(\eta^0 \to 2\gamma)} \sim 0.04[\langle r^2 \rangle m_\eta^2]^2 \xi^2 , \tag{22}$$

where ξ is the $SU(3)$ suppression factor ($\sim\frac{1}{10}$). If we take $r \sim 1/2m_\eta$, then the rate ratio (22) $\sim 0.25 \times 10^{-4}$, which is compatible with the data. Experiments on $e^- + p \to e^- + \Delta^+$ test, in principle, for the presence of a K_μ^V through establishment of a T-violating phase δ; the present limits[34] are $|\delta| < 10°$, which is consistent with the maximum expected theoretically[35,36] of δ in the range 10° to 20°. The reason that sensitive tests of K_μ currents are difficult to achieve even in the $e^- + p$ inelastic collision, $\gamma N \rightleftarrows \pi N$ and $\gamma d \rightleftarrows np$ reciprocity relations, as well as in $e^- + d$ scattering, is that they generally involve vertices of type $\gamma p \to \Delta^+ \to N\pi$ as part or whole of the reaction diagram, where γ is either on the mass shell or nearly so. However, it is known that process $\gamma p \to \Delta^+ \to N\pi$ is largely understood already by Chew-Goldhaber-Low-Nambu dispersion theory, in which the J_μ *current* is known to play a dominant role.[36] Hence the K_μ contribution is masked. Finally, such accurately tested electromagnetic mass formulas as those due to Coleman and Glashow[37] will remain unchanged if we assume that the K_μ current belongs to the same $SU(3)$ octet as the usual J_μ current.

Remarks

If we extend the discussion to include also a K_μ^S current constructed analogous to Eq. (20) out of the (ω, ϕ) system, then the $\omega\phi\gamma$ C-violating vertex can play an important role (enhanced by mixing) in the A_2-splitting model proposed by one of us.[17] In particular, it would be reasonable to expect splitting in the neutral mode $A_2^0 \to K_1^0 K_1^0$ induced by $\pi^- p$ reaction for which there is now some empirical evidence.[38] Such a C-violating contribution might possibly also account for the conflicting results obtained on A_2 splitting from $\pi^+ p$ and $\pi^- p$ reactions.[39]

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[32]The small ω^0 asymmetry (~1%) expected from the C-violating decay $\omega^0 \rightarrow (3\pi)_{I=1}$ (cf. Ref. 22) via $J_\mu^S K_\mu^V$ has then to be attributed to possible dynamical suppression due to the $\Delta I = 1$ selection rule involved here. In *principle*, at least, the $\Delta I = 0$ decay $\rho^0 \rightarrow (3\pi)_{I=1}$ can be due to C violation in medium-strong interaction rather than electromagnetism.

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