

## Proposed Method for a Search for $C$ Violation via $\rho^0$ Interference in $\omega$ Decay\*

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A method is suggested to search for the existence of a  $C$ -violating transition in  $\rho^0$  decay. The method requires a simultaneous analysis of both the usual  $\rho^0$  decay channel  $\pi^+\pi^-$ , and the  $G$ -violating channel  $\pi^+\pi^-\pi^0$ . The  $\pi^+\pi^-$  mass distribution is used to determine the kinematic regions of coherence of the  $\rho^0$  and  $\omega$  production amplitudes and the relative  $\rho$ - $\omega$  production phase. It is shown that one may then use the  $\omega \rightarrow \pi^+\pi^-\pi^0$  Dalitz plot as an analyzer of the isospin of the  $3\pi$  final state from  $\rho^0$  decay via a generalized asymmetry analysis of the  $\omega$  Dalitz plot for data samples in which  $\rho$ - $\omega$  coherence is observed. A general discussion of the effects of the  $\rho \rightarrow 3\pi$  decay mode upon the  $\omega$  is presented, with emphasis on the search for the isospin-conserving,  $C$ -violating decay mode. The relation of the  $\rho$ - $\omega$  interference method to other methods to search for  $C$  violation is also discussed.

### I. INTRODUCTION

We want to point out that the  $\omega$ - $\rho$  interference effect, which is now experimentally well established,<sup>1</sup> may be useful in a search for  $C$  violation in electromagnetic or substrong  $\Delta I=0$  transitions. The proposed method consists of first identifying a set of two processes which lead to a coherent sample of  $\rho$  and  $\omega$  events. The coherence of the reactions is ascertained from an examination of the  $\pi^+\pi^-$  mass spectrum. If interference effects with the  $\omega \rightarrow 2\pi$  decay mode are present, this is an indicator for coherence and allows one to measure the relative  $\omega$ - $\rho$  production phase (which we shall call  $\beta$  throughout this paper). If such coherence between  $\omega$  and  $\rho$  production is established, the  $\rho \rightarrow 3\pi$  decay mode will also interfere with the  $\omega \rightarrow 3\pi$  decay. This interference can occur between the  $\omega \rightarrow \pi^+\pi^-\pi^0$  decay and any  $\pi^+\pi^-\pi^0$  (and hence  $G=-1$ ) final state to which the  $\rho$  decays. We may thus distinguish between four possible transitions of the  $\rho^0$  to  $\pi^+\pi^-\pi^0$  states of definite isospin:

- (a) The  $\Delta I=1$  electromagnetic transition to the state  $I=0$ ,  $J^{PC}=1^{--}$ . This amplitude can be estimated from  $\omega$ - $\rho$  mass-mixing theories which relate it to the  $\omega \rightarrow 2\pi$  decay mode.<sup>2</sup>
- (b) The  $\Delta I=0$   $C$ -violating transition to the state  $I=1$ ,  $J^{PC}=1^{-+}$ . This amplitude is the main object of the proposed search.
- (c) The  $\Delta I=1$  electromagnetic transition to the state  $I=2$ ,  $J^{PC}=1^{--}$ .
- (d) The  $\Delta I=2$   $C$ -violating transition to the state  $I=3$ ,  $J^{PC}=1^{-+}$ .

A study of the  $\pi^+\pi^-\pi^0$  Dalitz plot in the region of the  $\omega$  mass can be used as a test for the presence and the nature of any coherent  $J^P=1^-$   $\pi^+\pi^-\pi^0$  amplitude. In the region of the  $\omega$  mass, the  $\pi^+\pi^-\pi^0$  mass spectrum is dominated by the large  $I=0$   $\omega$  amplitude. Thus if all decay angles of the normal

to the  $3\pi$  system are summed over, the  $\pi^+\pi^-\pi^0$  Dalitz plot becomes a sensitive detector for the presence of small contributions from any *other*  $I \neq 0$ ,  $J^P=1^-$  coherent amplitude.

The observation of a charge asymmetry on the  $\omega$  Dalitz plot provides unambiguous evidence for the presence, in the region of the  $\omega$  mass, of a coherent amplitude with  $I=1$  and  $J^{PC}=1^{-+}$  (see Sec. III B). However, a unique interpretation of the dynamical origin of this interfering amplitude is not possible for a single experimental data sample. (In fact, as we will discuss, a unique interpretation can only be obtained if results from several experiments on different reactions become available.) We consider here four possible sources for this amplitude:

- (1) The  $\Delta I=1$   $C$ -violating decay of the  $\omega$ .<sup>3</sup>
- (2) The  $\Delta I=0$  decay from a coherent  $\rho^0$  amplitude, which would result from  $C$  violation in electromagnetic or substrong interactions as conjectured by Bernstein, Feinberg, and Lee.<sup>4</sup>
- (3) An unknown background of the type postulated by Yuta and Okubo<sup>5</sup> in connection with the  $\eta$ -decay asymmetry experiments.<sup>6</sup> Such a background will in general consist of a number of different isospin and spin-parity nonresonant components: Charge asymmetry in  $\omega$  decay projects out the particular component  $I=1$ ,  $J^{PC}=1^{-+}$ .
- (4) The presence of a small amplitude due to an  $I=1$ ,  $J^{PC}=1^{-+}$  state (i.e., a resonance  $\tilde{\rho}$  with opposite  $C$  to the  $\rho$ ) produced coherently with the  $\omega$ .<sup>7</sup>

As far as case (1) is concerned, since the  $I=1$  state is an  $\omega$ -decay product, the  $\omega$  Dalitz-plot effect should be independent of the  $\omega$  production process. A charge asymmetry would then be expected for *all*  $\omega$  events; such an asymmetry has not been observed in two experiments,<sup>8,9</sup> each with  $\sim 4000$   $\omega$  events. Further discussion of this decay mode is given in Ref. 8. A study of the  $\omega$  Dalitz

plot for data samples with different initial and final states, as well as for different kinematical selection criteria, may be able to distinguish among cases (2) to (4). In case (2) we can relate the  $\omega$  Dalitz-plot asymmetry expected in a particular set of reactions to the value of  $\beta$  measured from the interference observed in the  $\pi^+\pi^-$  mass distribution. We cannot predict the behavior for different experiments of the asymmetry in case (4) due to a lack of knowledge of the production mechanism of such a state. If the behavior turns out different from the predictions for case (2), then at any rate  $C$  violation in  $\rho^0$  decay is not the correct interpretation. It is, however, more difficult to distinguish between cases (3) and (4), for the interference is observed only within a narrow mass region in the vicinity of the  $\omega$  mass. Experimentally untangling this problem would require evidence bearing on the purity of the quantum assignments for the interfering amplitude and on the resonant nature of the amplitude. We do not discuss this problem further in this paper; evidence for such an amplitude and a discussion of its properties are given in the following paper.<sup>8</sup>

Finally we outline the organization of this paper. In Sec. II we briefly discuss the  $\omega$ - $\rho$  interference effect in the  $\pi^+\pi^-$  spectrum to introduce our notation. A phenomenological description is given in Sec. III of possible  $\rho^0$  interference effects observable in  $\pi^+\pi^-\pi^0$  final states, considering separately  $\pi^+\pi^-\pi^0$  states with  $I=0$  and with  $I\neq 0$ . In Sec. IV we present a brief experimental review of the observed  $\rho$ - $\omega$  relative phase  $\beta$  for several reactions, and predictions for the  $\omega$  Dalitz-plot asymmetry as  $\beta$  is varied. Section V concludes with a consideration of the relationship of our analysis to the  $\eta \rightarrow \pi^+\pi^-\pi^0$  charge asymmetry analysis, and to  $\eta$  (and  $\eta'$ )  $\rightarrow \pi^+\pi^-\gamma$  analyses.

## II. THE $\omega$ - $\rho$ INTERFERENCE EFFECT IN THE $\pi^+\pi^-$ MASS SPECTRUM

The  $\omega$ - $\rho$  interference effect as observed in the  $\pi^+\pi^-$  mass spectrum has been discussed in detail in the literature<sup>1,10-12</sup> and we will be very brief here, introducing terms merely to make our notation clear.

In a first-order theory<sup>11</sup> one can express the  $\pi^+\pi^-$  mass distribution including the  $\omega$ - $\rho$  interference effect as

$$\left. \frac{dn}{dm} \right|_{2\pi} = |\sqrt{N_\rho} b_\rho(m) + \sqrt{N_\omega} \delta(2\pi/\Gamma_\omega)^{1/2} e^{i\beta} b_\rho(m) b_\omega(m)|^2, \quad (1)$$

where

$$b_\lambda(m) = (\Gamma_\lambda/2\pi)^{1/2} / (m_\lambda - m - \frac{1}{2}i\Gamma_\lambda);$$

the ratio of the  $\omega$  and  $\rho$  production amplitudes is expressed phenomenologically as

$$a(\omega)/a(\rho) = (N_\omega/N_\rho)^{1/2} e^{i\beta},$$

with  $N_\lambda$  the number of events of particle  $\lambda$  observed in the  $t$  interval under study. In Eq. (1)  $\delta$  is the off-diagonal element of the  $\omega$ - $\rho$  mass mixing matrix. In a more general discussion the interference term must be multiplied by a coherence factor  $\xi$ , where  $0 \leq \xi \leq 1$ .

## III. THE EFFECTS OF $\rho^0$ INTERFERENCE IN $\omega \rightarrow \pi^+\pi^-\pi^0$ DECAY

The coherence of  $\rho^0$  and  $\omega$  production amplitudes can lead to observable effects in both the  $\pi^+\pi^-\pi^0$  differential cross section and in the  $3\pi$  Dalitz plot. We define the Dalitz variables

$$X = (T_+ - T_-) / Q\sqrt{3} \quad (2a)$$

and

$$Y = T_0 / Q, \quad (2b)$$

where  $T_n$  and  $\vec{p}_n$  are the kinetic energy and momentum of the pion with charge  $n$  in the  $\pi^+\pi^-\pi^0$  center-of-mass system and

$$Q = m_\omega - m_{\pi^+} - m_{\pi^-} - m_{\pi^0}.$$

The lowest-order decay amplitudes  $D_I$  for  $\pi^+\pi^-\pi^0$  states with  $J^P = 1^-$  and isospin  $I$  may be expressed in terms of the Dalitz variables as<sup>13</sup>

$$\vec{D}_0 = c_0 \vec{q}, \quad (3a)$$

$$\vec{D}_1 = c_1 \vec{q} X, \quad (3b)$$

$$\vec{D}_2 = c_2 \vec{q} (1 - 3Y), \quad (3c)$$

$$\vec{D}_3 = c_3 \vec{q} X [(1 - 3Y)^2 - 3X^2] \quad (3d)$$

(where  $\vec{q} = \vec{p}_+ \times \vec{p}_-$ ). The  $c_I$  are positive real numbers defined so that

$$\int_{\text{all } X, Y} |\vec{D}_I|^2 dX dY = 1.$$

The Dalitz-plot intensity as a function of  $m(\pi^+\pi^-\pi^0)$  for  $\omega$  and  $\rho^0$  production (with  $\rho^0$  decay into  $\pi^+\pi^-\pi^0$ ) is then

$$\left. \frac{d^3n}{dm dX dY} \right|_{3\pi} = \left| \sqrt{N_\omega} b_\omega(m) \vec{D}_0 + \sqrt{N_\rho} b_\rho(m) e^{-i\beta} \left[ b_\omega(m) \vec{D}_0 \delta(2\pi/\Gamma_\rho)^{1/2} + \sum_{I=1}^3 (\Gamma_I/\Gamma_\rho)^{1/2} \vec{D}_I e^{i\beta_I} \right] \right|^2. \quad (4)$$

Here  $\beta_I$  for  $I=1, 2, 3$  is the  $\rho^0$  decay phase (relative to the  $\rho^0 \rightarrow \pi^+\pi^-$  decay amplitude), and  $\Gamma_I$  the partial width for  $\rho^0$  decay into a  $\pi^+\pi^-\pi^0$  state with isospin  $I$  (the "direct"  $I=0$  term is neglected since  $\omega$ - $\rho$  mixing is the dominant effect for  $I=0$ ). Note that  $\beta$  as used in Eq. (4) is the same quantity as measured from  $\omega$ - $\rho$  interference in the  $\pi^+\pi^-$  mass spectrum using Eq. (1).

#### A. $\rho^0$ Decay into $\pi^+\pi^-\pi^0$ in the $I=0$ State

The events arising from  $\rho^0$  decay into  $\pi^+\pi^-\pi^0$  in the  $I=0$  state and from  $\rho$ - $\omega$  interference will have the same distribution on the Dalitz plot as pure  $\omega$  events. The only observable effect due to  $\rho$ - $\omega$  interference in this case will thus be a modulation<sup>14</sup> of the  $d\sigma/dt|_{3\pi}$  distribution (an additional modulation of the  $3\pi$  mass distribution<sup>15</sup> is not observable at present levels of resolution and statistics). Should the  $\rho$ - $\omega$  interference occur predominantly in one polarization state (say the  $m=0$  state as expected for models with  $\pi$ -exchange dominance for  $\rho^0$  production and  $B$ -exchange dominance for  $\omega$  production), the  $\pi^+\pi^-\pi^0$  density-matrix elements could also show a measurable effect.<sup>16</sup> Since the unmodulated  $d\sigma/dt$  is experimentally inaccessible, observation of such effects at present is model dependent.<sup>17</sup> However, for favored kinematic regions as much as (10-20)% of the  $\omega$  signal may be due to  $\rho$ - $\omega$  interference,<sup>14</sup> permitting a model-independent analysis.

To obtain a model-independent effect, we note that  $\omega$ - $\rho$  mixing and  $\omega$ - $\rho$  interference will induce a breakdown of charge symmetry in each of the two sets of reactions

$$\pi^+n \rightarrow \rho^0 p, \quad (5)$$

$$\pi^-p \rightarrow \rho^0 n \quad (6)$$

and

$$\pi^+n \rightarrow \omega p, \quad (7)$$

$$\pi^-p \rightarrow \omega n. \quad (8)$$

As was pointed out,<sup>11</sup> this follows from the dominance of isovector exchange in the  $t$  channel: A change in sign of the  $t$ -channel amplitude at the  $\rho^0$  production vertex is expected between the  $\pi^+$  and  $\pi^-$  reactions (5) and (6), while no sign change should occur at the  $\omega$  vertex for reactions (7) and (8). Thus independent of a possible sign change at the nucleon vertex,  $\beta(\pi^+)$  is expected to differ from  $\beta(\pi^-)$  by  $180^\circ$ . A charge-symmetry violation may then be observed by measuring a nonzero difference between the differential cross sections for reactions (7) and (8).

A somewhat cleaner method to display a possible breakdown of charge symmetry due to the existence of a  $\rho^0 \rightarrow \pi^+\pi^-\pi^0$  decay mode is to measure the

difference between the differential cross sections for the reactions

$$\pi^+d \rightarrow \omega p p$$

and

$$\pi^-d \rightarrow \omega n n.$$

The virtue of this method is that if the  $\omega$  is detected via its decay products by the same apparatus independent of the sign of the charge of the incident pion, both reactions may be studied with the same experimental setup. The equivalent reactions (7) and (8) do not lend themselves to a similar method, since the deuterium reaction (7) is probably most easily performed in a bubble chamber, while reaction (8) has two neutral particles in the final state, and therefore requires additional detection apparatus.

#### B. $\omega$ Dalitz-Plot Analysis for Interference due to $I \neq 0$ States

As a consequence of Eq. (4), the  $\omega$  Dalitz plot should reveal the presence of coherent  $I \neq 0$ ,  $J^P = 1^-$  amplitudes via interference with the  $\omega$ . Considering explicitly the amplitude for  $I=1$ , and including the coherence factor, Eq. (4) may be written as

$$\begin{aligned} \frac{d^3n}{dm dXdY} = & N_\omega |\vec{D}_0 b_\omega(m)|^2 + N_\rho \frac{\Gamma_1}{\Gamma_\rho} |\vec{D}_1 b_\rho(m)|^2 \\ & + 2\xi [N_\omega N_\rho (\Gamma_1/\Gamma_\rho)]^{1/2} \vec{D}_0 \cdot \vec{D}_1 \\ & \times \text{Re}[b_\omega^*(m) b_\rho(m) e^{i(\beta_1 - \beta)}]. \end{aligned} \quad (9)$$

Thus  $\omega$  interference with an  $I=1$  amplitude has a Dalitz-plot dependence from the  $\vec{D}_0 \cdot \vec{D}_1$  term which changes sign as  $X$  changes sign, and hence, gives rise to an asymmetry in  $X$ , i.e., charge asymmetry. Furthermore, integration of Eq. (9) over the entire  $\omega$  Dalitz plot causes the interference contribution to vanish.

The mass dependence of the interference term in Eq. (9) may be used to determine the  $\Delta I=0$   $C$ -violating decay phase  $\beta_1$ . Here we assume that  $\beta$  is already measured separately via interference in the  $\pi^+\pi^-$  mass distribution.

To illustrate the method, we use the approximation  $m_\rho \cong m_\omega$ , neglect  $\Gamma_\omega$  compared to  $\Gamma_\rho$ , and neglect the quadratic contribution of the  $I=1$  amplitude. Then Eq. (9) simplifies to

$$\begin{aligned} \frac{d^3n}{dm dXdY} = & N_\omega |D_0 b_\omega(m)|^2 \\ & + \frac{4\xi}{\pi} \left( \frac{N_\omega N_\rho \Gamma_1}{\Gamma_\omega \Gamma_\rho^2} \right)^{1/2} \vec{D}_0 \cdot \vec{D}_1 \\ & \times \text{Re} \left( \frac{\epsilon_\omega - i}{\epsilon_\omega^2 + 1} e^{i(\beta' + \beta_1 - \beta)} \right). \end{aligned} \quad (10)$$

Here we have set  $\epsilon_\omega = 2(m_\omega - m)/\Gamma_\omega$ ; the  $\rho^0$  decay phase  $\beta'$  is determined by the phase of the  $\rho$  Breit-Wigner amplitude at  $m_\omega$ , i.e.,

$$\beta' = \tan^{-1}[(\Gamma_\rho/2)/(m_\rho - m_\omega)]$$

which in the present approximation equals  $\pi/2$ . The interference term has contributions which are both even and odd with respect to  $\epsilon_\omega$ , while the  $\omega$  intensity term is even in  $\epsilon_\omega$ . Depending on the value of  $\beta_1 - \beta$ , we may distinguish between two general types of charge asymmetries. For  $\beta_1 - \beta \approx 0$  (or  $\pi$ ) the asymmetry has a sign which is independent of  $m$  over the entire  $\omega$  mass region. This corresponds to interference with the imaginary part of the  $\omega$  amplitude. For  $\beta_1 - \beta \approx \pi/2$  the asymmetry is positive below the  $\omega$  mass (i.e., for  $\epsilon_\omega > 0$ ) and negative above (i.e., for  $\epsilon_\omega < 0$ ), etc. This corresponds to interference with the real part of the  $\omega$  amplitude. In practice this latter type of asymmetry will be diluted due to the experimental resolution which moves events across the  $m = m_\omega$  boundary.

The usual asymmetry analysis considers the number of events with  $\vec{D}_0 \cdot \vec{D}_1 > 0$  ( $N_+$ ) and the number with  $\vec{D}_0 \cdot \vec{D}_1 < 0$  ( $N_-$ ):

$$\alpha = (N_+ - N_-)/(N_+ + N_-). \quad (11)$$

To allow for the second type of asymmetry, we refine this analysis by including a second subscript which bears the sign of  $\epsilon_\omega$ ; for example,

$$N_{-+} = \int_{X < 0} dX \int_0^\infty d\epsilon_\omega \int_{\text{all } Y} dY \frac{d^3 n}{d\epsilon_\omega dX dY},$$

$$N_{--} = \int_{X < 0} dX \int_{-\infty}^0 d\epsilon_\omega \int_{\text{all } Y} dY \frac{d^3 n}{d\epsilon_\omega dX dY}, \text{ etc.}$$

We then define the generalized asymmetries<sup>18</sup>

$$\alpha_1 = [N_{++} + N_{+-}] - (N_{-+} + N_{--})/N, \quad (12a)$$

$$\alpha'_1 = [(N_{++} + N_{--}) - (N_{+-} + N_{-+})]/N, \quad (12b)$$

where  $N = N_{++} + N_{+-} + N_{-+} + N_{--}$ . Equation (12a) is seen to be identical to Eq. (11). The utility of the pair  $\{\alpha_1, \alpha'_1\}$  lies in their projection properties:

$$\alpha_1 \propto \xi \sqrt{\Gamma_1} \sin(\beta' + \beta_1 - \beta),$$

$$\alpha'_1 \propto \xi \sqrt{\Gamma_1} \cos(\beta' + \beta_1 - \beta).$$

Hence (in the above stated approximation)  $\alpha_1$  measures the interference of the  $I=1$ ,  $\rho-3\pi$  amplitude with the imaginary part of the  $\omega$  Breit-Wigner amplitude, and  $\alpha'_1$  interference with the real part.

For the general case we define the number of events  $N'_{ij}$ , where

$$i \equiv + \text{ for } \vec{D}_0 \cdot \vec{D}_I > 0,$$

$$i \equiv - \text{ for } \vec{D}_0 \cdot \vec{D}_I < 0,$$

$$j \equiv + \text{ for } \epsilon_\omega > 0, \text{ i.e., below } m_\omega,$$

$$j \equiv - \text{ for } \epsilon_\omega < 0, \text{ i.e., above } m_\omega,$$

and evaluate using Eq. (4) the generalized asymmetries

$$\alpha_I = [(N'_{++} + N'_{+-}) - (N'_{-+} + N'_{--})]/N, \quad (13a)$$

$$\alpha'_I = [(N'_{++} + N'_{--}) - (N'_{+-} + N'_{-+})]/N. \quad (13b)$$

Thus measuring  $\{\alpha_I, \alpha'_I\}$  for  $I=1, 2$ , and 3 allows a determination of the decay phases  $\beta_I$ , and if  $\xi$  is known, the partial widths  $\Gamma_I$ .

As a final remark on our Dalitz-plot analysis, we comment briefly on an alternative notation for the Dalitz coordinates using polar variables  $(r, \theta)$ , where<sup>3</sup>

$$T_0 = \frac{1}{3}Q(1 + r \cos \theta),$$

$$T_\pm = \frac{1}{3}Q[1 + r \cos(\frac{2}{3}\pi \mp \theta)],$$

so

$$Y = \frac{1}{3}(1 + r \cos \theta)$$

and

$$X = \frac{1}{3}r \sin \theta.$$

In terms of these variables, Eq. (3) may be rewritten

$$\vec{D}_0 = C_0 \vec{q}, \quad (14a)$$

$$\vec{D}_1 = C_1 \vec{q} r \sin \theta, \quad (14b)$$

$$\vec{D}_2 = C_2 \vec{q} r \cos \theta, \quad (14c)$$

$$\vec{D}_3 = C_3 \vec{q} r^3 \sin 3\theta. \quad (14d)$$

Equations (14) then provide a pictorial representation of the asymmetries for interference of the different  $3\pi$  isospin states with the  $\omega$  amplitude:

$I=1$  Charge (right-left) asymmetry ( $\sin \theta > 0, < 0$ ),

$I=2$  Up-down asymmetry ( $\cos \theta > 0, < 0$ ),

$I=3$  Alternating (sextant) asymmetry ( $\sin 3\theta > 0, < 0$ ).

#### IV. POSSIBLE APPLICATIONS OF THE $\rho$ - $\omega$ INTERFERENCE METHOD

There is presently available ample evidence<sup>1</sup> that with a proper choice of reactions, the relative phase  $\beta$  between  $\rho^0$  and  $\omega$  production can be stepped through the entire range of  $360^\circ$ . A summary of some of the relevant experiments is given in Table I, where  $\beta$  is chosen at  $90^\circ$  intervals. (A more complete compilation is given in Ref. 1.)

A stringent test, then, of the identification of an interfering  $I=1$ ,  $\pi^+\pi^-\pi^0$ ,  $J^P=1^-$  amplitude with the  $\Delta I=0$ ,  $\pi^+\pi^-\pi^0$  decay mode of the  $\rho^0$  is that the decay phase  $\beta_1$  be the same for the complete set of reactions in Table I. Thus if  $\beta$  is changed by  $180^\circ$ ,

TABLE I. A partial list of  $\rho$ - $\omega$  production processes as a function of the (approximate)  $\rho$ - $\omega$  relative production phase  $\beta$ .

$\beta$ (degrees)	Process	Reference
0	$\gamma + C \rightarrow (\rho, \omega) + C$	19 <sup>a</sup>
90	$\pi^+ p \rightarrow (\rho, \omega) \Delta^{++}$	10
	$\pi^+ d \rightarrow (\rho, \omega) p p$	20
180	$\pi^- p \rightarrow (\rho, \omega) \pi^- p$	21
270	$\pi^- p \rightarrow (\rho, \omega) n$	22

<sup>a</sup>The value  $\beta=0^\circ$  is expected theoretically for diffractive production of  $\rho^0$  and  $\omega$ ; however, we caution that the experimental situation is not clear as yet. Values of  $\beta$  which could be as large as  $40^\circ$ - $100^\circ$  are not excluded as yet.

$$\{\alpha_1, \alpha'_1\} \rightarrow \{-\alpha_1, -\alpha'_1\},$$

and if  $\beta$  is increased by  $90^\circ$ ,

$$\{\alpha_1, \alpha'_1\} \rightarrow \{\alpha'_1, -\alpha_1\}, \text{ etc.}$$

Besides these phase restrictions, one can also determine, in principle,  $\xi\sqrt{\Gamma_1}$  for each reaction, so that if  $\xi$  is measured from the  $\pi^+\pi^-$  mass spectrum, the  $\omega$  Dalitz-plot asymmetries are also uniquely prescribed in magnitude.

Since the observed magnitude of the asymmetry will be a function of the ratio of the  $\rho^0$  and  $\omega$  production amplitudes  $A_\rho/A_\omega$ , as well as the degree of coherence  $\xi$ , a definite prediction for the magnitude of the asymmetry as a function of  $\Gamma_1/\Gamma_\rho$  cannot be made. As an indication of the order of magnitude to be expected, however, we note that for the experimental conditions observed in Ref. 8 a value of  $\Gamma_1/\Gamma_\rho=0.01$  yields an asymmetry of  $\alpha_1 \cong 0.1$ . Since  $\alpha_1 \propto \sqrt{\Gamma_1}$ , we thus expect that the more realistic estimate<sup>3</sup> for  $\Gamma_1/\Gamma_\rho$  of  $10^{-4}$  should give a 1% asymmetry on the  $\omega$  Dalitz plot. Further, the more detailed discussion of Ref. 8 suggests that poor mass resolution mainly affects the determination of  $\alpha'_i$ ; it is also shown there that  $\alpha'_i$  is as effective an analyzer as  $\alpha_i$  in the limit of perfect resolution.

#### V. THE $\eta$ -DECAY CHARGE-ASYMMETRY METHOD COMPARED TO THE $\rho$ - $\omega$ INTERFERENCE METHOD FOR POSSIBLE C-VIOLATION STUDIES

There is an important difference between the  $\eta \rightarrow \pi^+\pi^-\pi^0$  decay charge asymmetry study suggested by Friedberg, Lee, and Schwartz<sup>23</sup> and the  $\rho$ - $\omega$  interference method we are proposing here. The dominant  $\eta$ -decay amplitude connects the initial  $\eta$  state  $I=0, J^{PC}=0^{-+}$ , and hence,  $G=+1$ , via a  $\Delta I=1$  electromagnetic transition to the  $G=-1$

$\pi^+\pi^-\pi^0$  state with  $I=1, J^{PC}=0^{-+}$ . The alternate conjectured C-violating transitions which may produce charge asymmetries via interference with the dominant decay amplitude are

$$(a) \Delta I=0 \text{ to } I=0, J^{PC}=0^{-+}$$

or

$$(b) \Delta I=2 \text{ to } I=2, J^{PC}=0^{-+}.$$

For the conjectured C-violating decay of the  $\eta$  to the  $J^{PC}=0^{-+}$  final state, it is the  $\Delta I=0$  transition which leads to a Dalitz-plot distribution with zeros along the sextant lines, while the  $\Delta I=2$  transition leads to a Dalitz plot with a zero along the Y axis only. As Lee pointed out<sup>3</sup> on the basis of a dimensional analysis, by expansion in the polar variables defined above, the  $\Delta I=0$  C-violating transition is then expected to be considerably suppressed by angular-momentum-like barrier factors. Thus an  $\eta$ -decay asymmetry study is primarily a probe for the  $\Delta I=2$  C-violating transition yielding a charge asymmetry on the Dalitz plot. In fact, the current indication for such an effect is  $\alpha = (1.5 \pm 0.5)\%$  corresponding to  $\Delta I=2$ , while there is no indication of a sextant asymmetry corresponding to  $\Delta I=0$ .<sup>6</sup>

For the  $\rho$ - $\omega$  interference method proposed above (Sec. III), several distinctions may be made from the  $\eta$  case. Purely from a kinematic viewpoint, the  $\rho^0 \rightarrow \pi^+\pi^-\pi^0$  decay should be a more favorable analyzer of  $\Delta I=0$  transitions than  $\eta$  decay. Not only does the larger available energy at the  $\omega$  mass make barrier factors less restrictive, but, more importantly, for the  $J^{PC}=1^{-+}$  final state it is the  $\Delta I=2$  transition which is suppressed relative to the  $\Delta I=0$  transition. It is also possible that the dynamics enhances the C-violating decay mode at the larger  $\rho^0$  mass,<sup>24</sup> so that we conclude that the measured null result for  $\Delta I=0$  transitions in  $\eta$  decay does not negate the value of a careful experimental search for C violation in  $\rho^0$  decay. Furthermore, if the C-violating  $\Delta I=0$  transition is not due to C violation in the electromagnetic interaction, but rather in a substrong interaction, then other suggested tests of C noninvariance<sup>4</sup> such as charge asymmetry in  $\eta$  (and  $\eta'$ ) decay to  $\pi^+\pi^-\gamma$ , may not be as sensitive as the  $\rho$ - $\omega$  interference method proposed above.

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<sup>1</sup>For a recent review of the experimental evidence for  $\omega$ - $\rho$  interference, see G. Goldhaber, in *Experimental Meson Spectroscopy*, edited by C. Baltay and A. H. Rosenfeld (Columbia Univ. Press, New York, 1970), p. 59.

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<sup>5</sup>H. Yuta and S. Okubo, *Phys. Rev. Letters* **21**, 781 (1968).

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<sup>7</sup>The suggestion has been made by S. Barshay, *Phys. Rev. Letters* **24**, 490 (1970), that such a state, by analogy with a  $\pi^+\pi^0$  mass difference calculation, may exist near a mass of 1 GeV and be responsible for the violation of CP invariance in  $K_L^0$  decay.

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<sup>17</sup>Evidence for such an effect, though not model-independent, was presented earlier [G. S. Abrams, K. W. J. Barnham, W. R. Butler, D. G. Coyne, G. Goldhaber, B. H. Hall, and J. MacNaughton, *Phys. Rev. Letters* **25**, 617 (1970)].

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