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## Electromagnetic Interactions of High-Energy Cosmic-Ray Muons

O. C. Allkofer, C. Grupen,\* and W. Stamm

*Institut für Reine und Angewandte Kernphysik, Universität Kiel, Kiel, Germany*

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The electromagnetic interactions of cosmic-ray muons in the energy range up to 1 TeV were investigated with a spark-chamber calorimeter in combination with the Kiel spectrograph. The purpose was to investigate the different kinds of electromagnetic processes (knock-on, direct-pair, bremsstrahlung, and multiple pion production) in the range of energy transfer from 0.2 to 100 GeV. The production of pions is in reasonable agreement with the theoretical prediction of Daiyasu *et al.* In the region of high energy transfer (>2 GeV) the experimental results agree with Bhabha's theory of the knock-on process as well as with Murota's theory of direct pair production. However, in the region of energy transfer around 1 GeV there are minor deviations from these theories, in contradiction with accelerator data.

### I. INTRODUCTION

In this paper we present results of a spark-chamber experiment designed to determine the cross sections for the different electromagnetic interactions of muons in order to extend the knowledge of these cross sections beyond accelerator energies. A large number of such measurements have been performed using cosmic-ray muons. However, the interpretation of the cosmic-ray data has been difficult, since the spectrum of incident muons has to be folded into the differential probabilities for the various kinds of interactions. The lower energy limit and the shape of the muon spectrum influence the theoretical predictions calculated in this way. The interpretation of underground experiments requires an energy-range relation for muons; an additional source of error is thereby introduced into the theoretical prediction. Moreover, it is rather difficult to distinguish the different kinds of interactions, especially in the region of high energy transfer, since the electron shower which is initiated by a secondary particle obliterated

the initial signature of the event. These difficulties were overcome in this experiment by measuring the muon energy for each individual event in the spark-chamber calorimeter by means of the Kiel spectrograph. The knowledge of the muon energy permits, in addition, a discrimination between the various processes by the use of theoretical predictions.

Some authors<sup>1-8</sup> find agreement with the theories of Bhabha<sup>9,10</sup> for the knock-on process and of Murota *et al.*<sup>11</sup> for the process of direct pair production, while other experiments<sup>4,12-15</sup> cannot be explained by these theories. The bremsstrahlung cross section<sup>16</sup> is very small for muons with energies less than a few hundred GeV. For this reason it is difficult to measure this cross section with high accuracy, even though our experiment involves muons of energies up to 1 TeV. The investigations on multiple pion production are considerably complicated by the variety of theoretical predictions<sup>17-20</sup> about the virtual-photon spectrum of the muon, as well as by the uncertainty about the photonuclear cross section for pion production.

## II. THEORIES

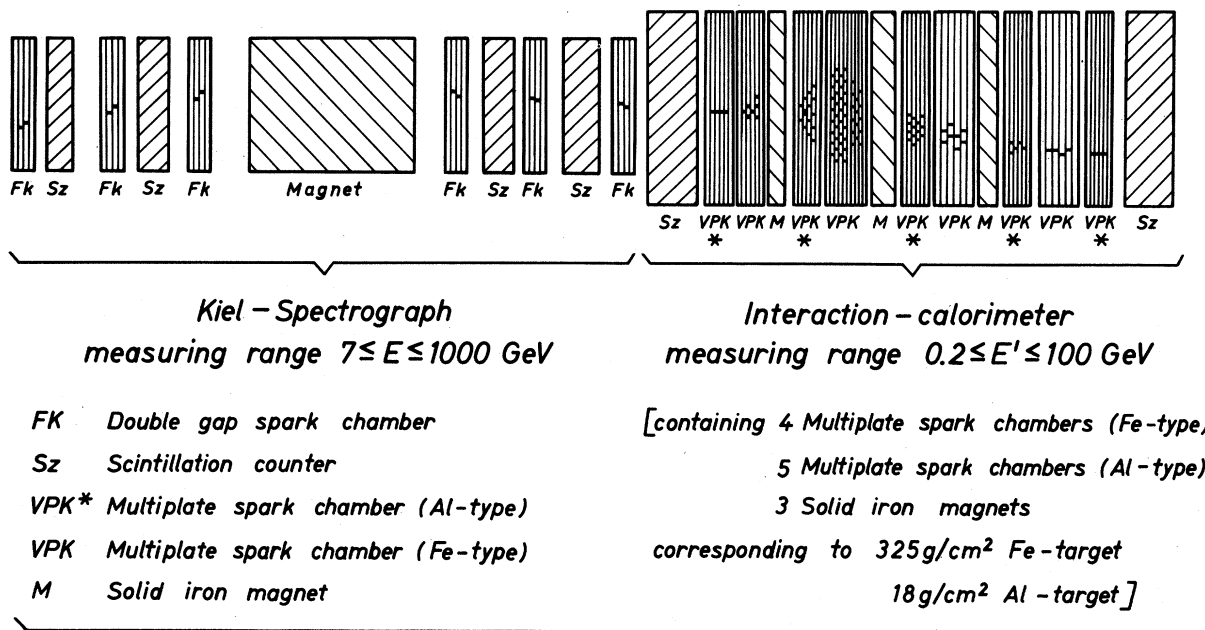
The differential probabilities for muon interactions depend on the muon's energy  $E$  and the energy of the secondary particles  $E'$ . The knock-on process is described theoretically by Bhabha,<sup>9,10</sup> Higher-order corrections<sup>21,22</sup> have little influence on the results. This cross section has a weak dependence on the muon energy  $E$  and varies as  $1/E'^2$  as far as the energy transfer is concerned. The probability for the production of bremsstrahlung is given by Christy and Kusaka.<sup>16</sup> The cross section varies as  $\log E$  and  $1/E'$ , respectively. Recent calculations<sup>23</sup> on this process yield agreement with the results of Christy and Kusaka to within 20%. The differential probability for direct pair production was calculated after Murota *et al.*<sup>11</sup> Unfortunately, the Murota theory contains indeterminate constants to which arbitrary values can be assigned, making the cross sections uncertain to about a factor of 2.<sup>2,4,13,14,24,25</sup> Our calculations are based on the approximate formulas (31) and (34) of Murota's theory. The value of  $\alpha$  was chosen to be 2. The integration over the energy partition of the electron pair was performed according to the original Murota integration limits. Recent calculations<sup>26-29</sup> on the process of direct pair production do not suffer from the disadvantage of indeterminate constants and should be preferred to the Murota theory. The cross section for direct pair production depends on the muon energy, the depen-

dence varying from  $\log E$  to  $E^2$ , and also on the energy transfer, the dependence varying from  $1/E'$  to  $1/E'^2$  and then to  $1/E'^3$  with increasing energy transfers.

The cross section for multiple pion production was calculated on the basis of different models.<sup>17-20</sup> The photonuclear cross section for this process was taken from accelerator experiments and calculations.<sup>30-32</sup> A comparison of recent calculations on the virtual-photon spectrum with those of Williams and Weizsäcker,<sup>19,20</sup> Kessler and Kessler,<sup>18</sup> and Daiyasu *et al.*<sup>17</sup> can be taken from Cassidy.<sup>33</sup>

## III. EXPERIMENTAL ARRANGEMENTS

This experiment is performed in combination with the Kiel cosmic-ray muon spectrograph.<sup>34-37</sup> Before the muons pass the interaction calorimeter their momentum is measured to be within the range of 7 GeV/c to 1 TeV/c. The spectrograph is adjusted to a zenith angle of  $83^\circ$  in order to increase the number of high-energy muons. The large zenith angle permits a horizontal arrangement of the calorimeter; 40% of the muons coming from the spectrometer pass the calorimeter. The calorimeter has an effective volume of about  $1 \times 1 \times 2.2$  m<sup>3</sup>. It is built up in a sandwich technique. The interaction stack is composed of four spark chambers of Fe type and five spark chambers of Al type. The Fe-type spark chambers consist of iron tar-



Detector arrangement in the interaction experiment

FIG. 1. Experimental arrangement of the calorimeter and spectrograph.

gets and two double-gap spark chambers; their electrodes are made of aluminum plates of thickness 2 mm. The Al-type is constructed of aluminum plates, only. Three solid iron magnets, which are situated between the spark chambers, increase the lateral spread of the shower particles, thus improving the multitrack efficiency of the spark chambers. The experimental arrangement of the calorimeter is shown in Fig. 1. An iron target which is equivalent to two radiation lengths is followed by a double-gap spark chamber. The total target amounts to 325 g/cm<sup>2</sup> of iron and 34 g/cm<sup>2</sup> of aluminum or Lucite. The spark chambers are filled with Neogal. In order to prevent any impurity in the filling, an excess pressure of 1 Torr C<sub>2</sub>H<sub>5</sub>OH is maintained by an automatic gas-control circuit. Each double-gap spark chamber is triggered by a separate spark-gap unit.<sup>38</sup> The passage of a muon is photographed stereoscopically by using two cameras.

#### IV. DATA ANALYSIS

The different kinds of interactions could not be distinguished from one another as far as the processes are concerned which initiate electromagnetic cascades. At energy transfers exceeding 0.2

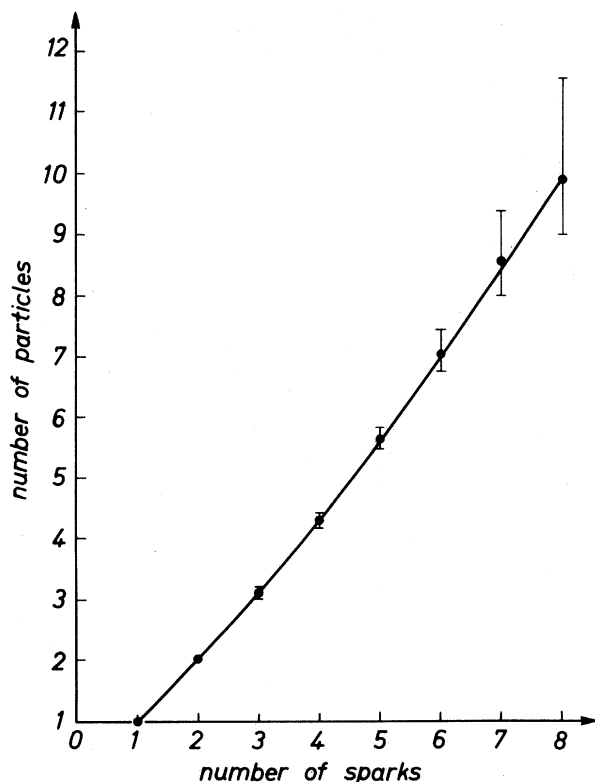


FIG. 2. Relation between the number of sparks and corresponding particles.

GeV, the secondary particles produce showers, thus obliterating the initial signature of the interaction. However, a knowledge of the muon energy for each individual event permits a separation of the processes from the theoretical point of view, because of their different dependence on the muon energy and energy transfer. For example, the knock-on process can be investigated at relatively low muon energies, the direct pair production at high muon energies, and the bremsstrahlung process at high muon energies and high energy transfers.

It was possible to discriminate the process of multiple pion production by pattern recognition of the events. The behavior of the secondary pions is characterized by the interaction length in contrast to the radiation length which describes the multiplication of electrons and photons.

The estimation of energy transfer involves uncertainties due to the conversion from the total spark number to the total number of electron track segments and due to the error in the minimum energy of the electrons in the shower, which can be detected in a spark chamber.<sup>39-46</sup> The energy estimation was carried out as follows: Firstly, the multitrack efficiency of the spark chambers was determined. Multitrack efficiency is defined as the ratio of sparks to the number of particles which passed through the chamber volume. A difficulty arises from the fact that the actual number of particles which passed through the spark chamber is not known. However, there exists a reference spark which originates from the muon. It is obvious that the muon has passed through the chamber in each event. The efficiency of the muon spark in the presence of other sparks is a measure of the value of multitrack efficiency. This method for the determination of multitrack efficiency is described in detail in Ref. 47. The relation between the number of sparks per gap and the corresponding number of particles can be taken from

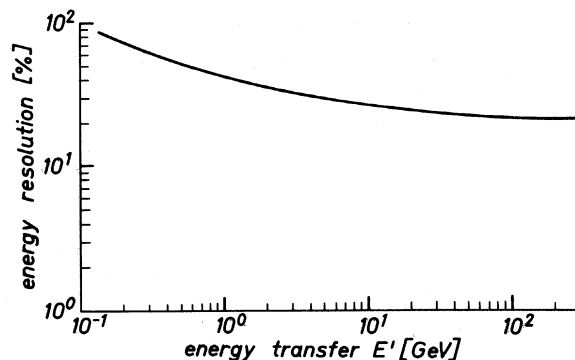


FIG. 3. Energy resolution as a function of the shower energy.

Fig. 2. This relation is linear at low particle numbers; but the corrections are stronger than linear at high particle densities. Multitrack efficiency decreases by about 2.5% per additional particle.

Secondly, the relation between the total track-segment number and the energy transfer has to be determined. This was carried out using the calculation of Crawford and Messel.<sup>41-43</sup> There exists a linear relation between the number of electrons

( $N$ ) and the shower energy ( $E'$ ):

$$E' = kN,$$

The constant  $k$  depends on the critical energy of the material, the angle of incidence, the absorber thickness, and the minimum energy of electrons in the shower which can be detected by a spark chamber. We used a minimum energy of 10 MeV. The reason is as follows: It is true that if one takes 10 MeV as the low-energy cutoff of the electrons,

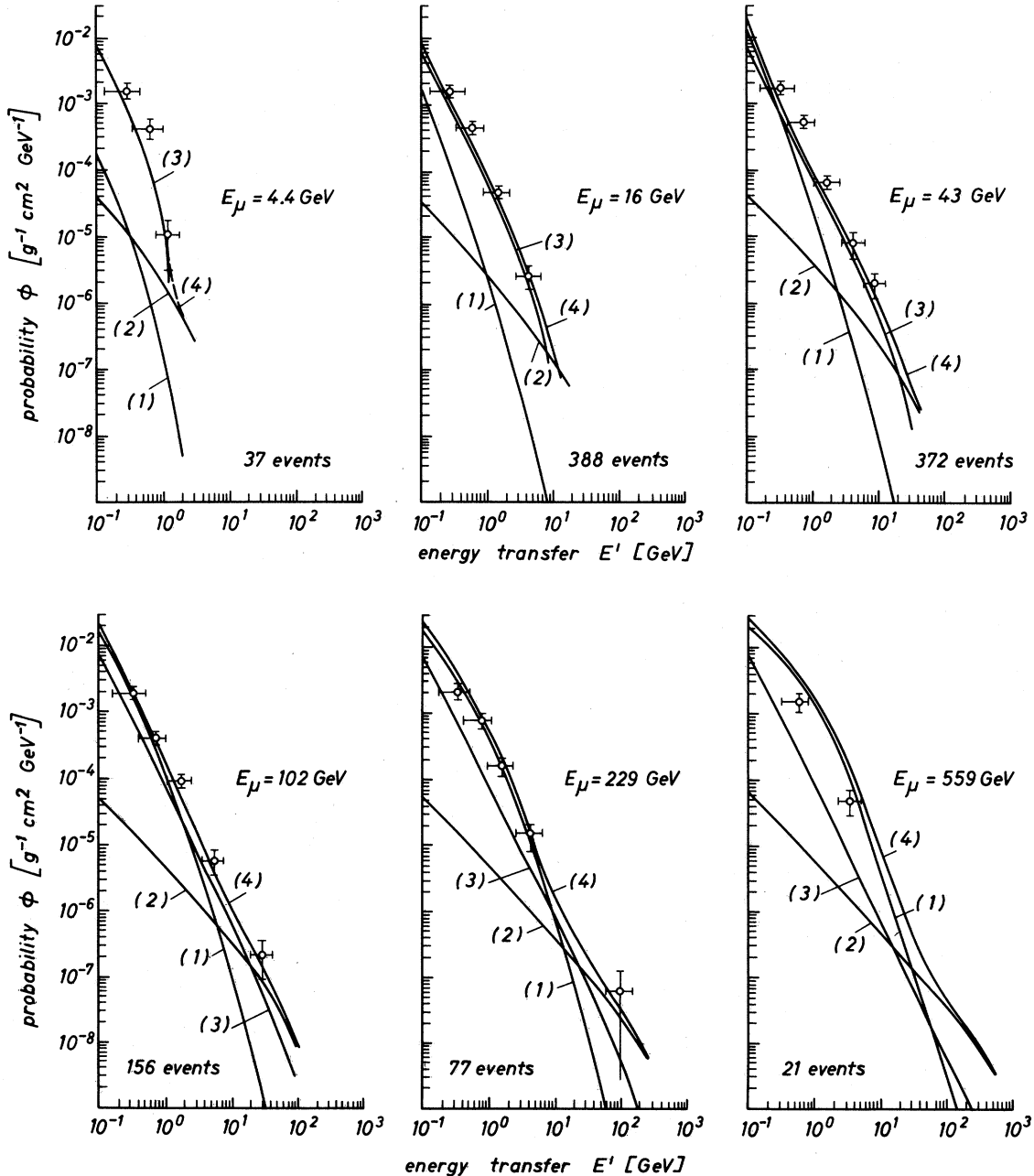


FIG. 4. Experimental results of muon interactions at different muon energies. The curves represent the theoretical predictions of the yield by the pair-production process (1), bremsstrahlung (2), knock-on (3), and the sum of these processes (4).

one still has to take electrons of low energy into account, since the spark chamber is capable of detecting particles of zero energy if these particles have entered the sensitive volume of the chamber. However, the electrons in a shower are spread over all angles with respect to the shower axis. (The shower axis lies in the direction of the electrical field in our experiment.) The multitrack efficiency of spark chambers tends to zero with increasing angle of incidence of the shower particles. The critical angle is about  $30^\circ$ , i.e., electrons with angles of incidence exceeding  $30^\circ$ , with respect to the electrical field, cannot be detected.

The method of determining the value of multitrack efficiency was based on particle tracks which are nearly parallel to the muon track. Thus we have to consider the number of particles with energies larger than zero which lie within the cone of  $\pm 30^\circ$  with respect to the direction of the electrical field. This number of particles, which is equal to the number of electrons with energies larger than 10 MeV which lie within a cone  $\pm 90^\circ$ , is relevant for the energy estimation of our events. For this reason we can use the Monte Carlo calculation of Crawford and Messel with a low-energy cutoff of 10 MeV.

The error of the energy estimation depends on the actual number of sparks in the event. An additional error is introduced by the uncertainty in the cone of efficiency of the spark chamber. The energy error as a function of the shower energy is plotted in Fig. 3. A typical energy resolution of (30–50)% could be obtained.

The energy estimation of the events involving multiple pion production was performed using the energy-range relations for charged pions. These events were corrected for neutral pions.

## V. RESULTS

The experimental results are based on about 10 000 muon traversals and about 2700 events with energy transfers in excess of 0.2 GeV. The muons were divided into six groups of mean energies 4.4, 16, 43, 102, 229, and 559 GeV. The experimental results at these muon energies can be taken from Fig. 4. The figures with muon energies below 43 GeV yield information about the knock-on process, while figures with energies in excess of 229 GeV are used to investigate direct pair production. At large energy transfers the bremsstrahlung production can be determined.

There is reasonable agreement between the results and the theoretical predictions except in the range of energy transfers near 1 GeV for the knock-on process as well as the direct pair production.

Because the muon spectrograph determines the

charge of the incident muons it is possible to give results about a possible positive-negative asymmetry in the interaction of cosmic-ray muons. This asymmetry is indicated by some cosmic-ray experiments,<sup>12,48-50</sup> though it has not been found in experiments with accelerator muons.<sup>7</sup> The results of our investigation are shown in Fig. 5 in their dependence on the muon energy as well as the energy transfer. The integral value is based on about 4000 muon traversals and about 1000 interactions in excess of 0.2 GeV energy transfer. This result indicates that the cross section for positive muons exceeds the cross section for negative muons by about 20%. This tendency in the behavior of the positive-negative asymmetry is in agreement with the results of Neddermeyer *et al.*<sup>12,48-50</sup>

Out of the set of about 2700 events, 30 muon-induced nuclear interactions could be identified. These events of multiple pion production by muons are corrected for neutral pions according to the relation

$$E' = 1.5nE_\pi,$$

where  $n$  represents the multiplicity of charged pions, and the factor 1.5 takes neutral pions into ac-

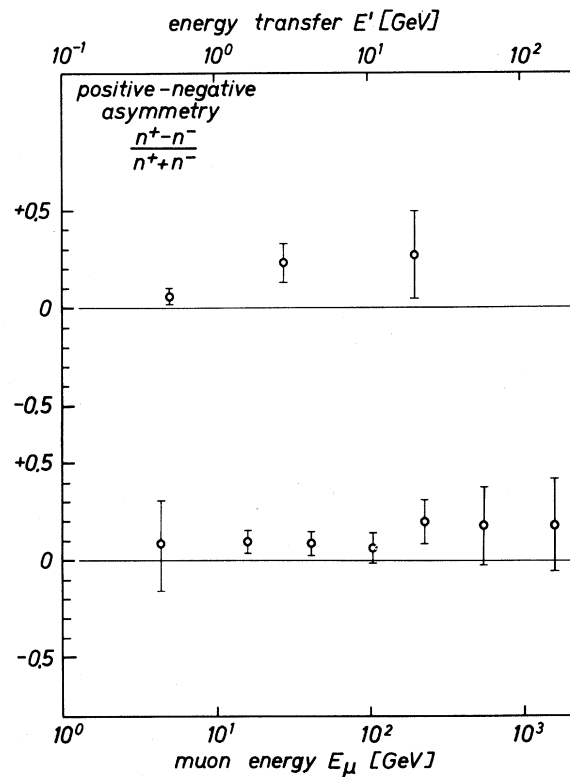


FIG. 5. Positive-negative asymmetry of muon interactions as a function of the muon energy and energy transfer.  $n^+$  is the number of events initiated by positive muons;  $n^-$ , the corresponding quantity for negative muons.

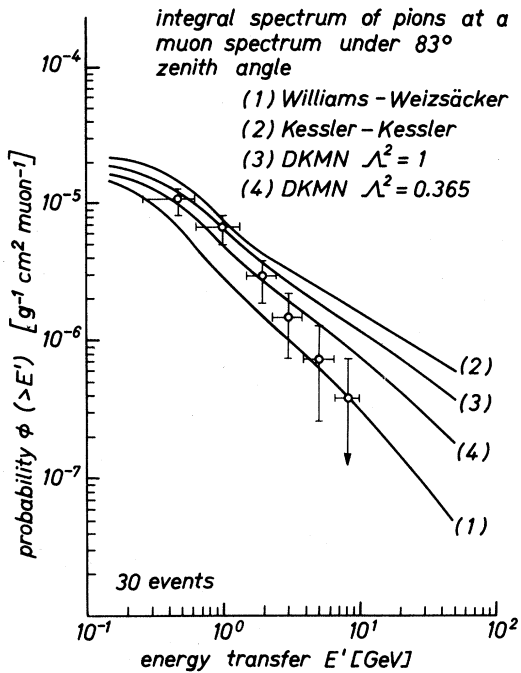


FIG. 6. Integral spectrum of muon-produced pions.

count.  $nE_{\pi}$  characterizes the total energy of the charged pions. Because of the poor statistics, the muon spectrum was folded into the cross section for multiple pion production. The integral spectrum of the pions, corrected for neutral pions, can be seen in Fig. 6. The results below 2.0 GeV energy transfer are in reasonable agreement with the theoretical prediction of Daiyasu *et al.* The experimental points above 3.0 GeV, which contain four events only, deviate from this theory. This

may be due to poor statistics in this energy region, or to the fact that pions escape from the stack. In this way the energy transfer would be underestimated. It should be mentioned, however, that the photonuclear cross section, assumed to be constant in excess of 2.0 GeV, might decrease with increasing energy of the virtual photon.

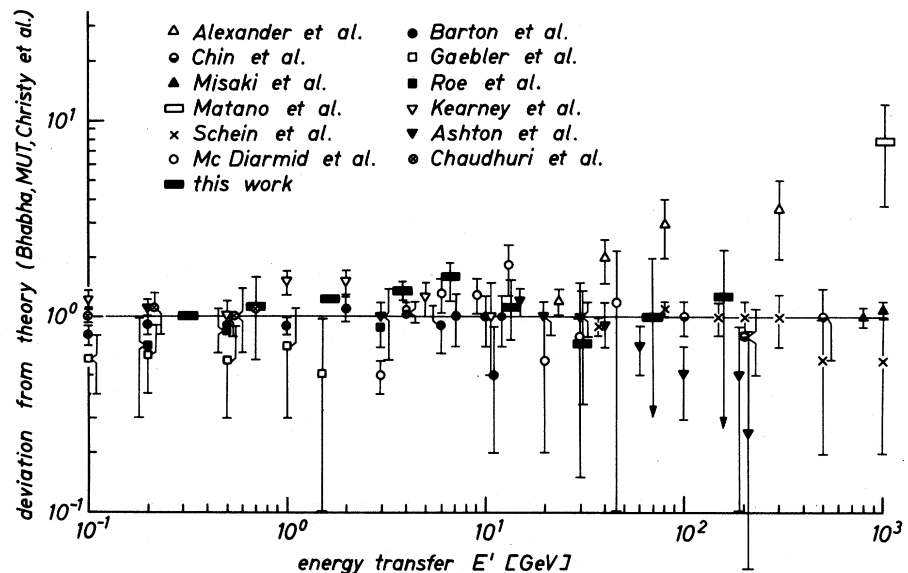
## VI. DISCUSSION

The experimental work on knock-on production, direct pair production, and bremsstrahlung production by muons was carried out under different experimental conditions. Few of the experiments performed have measured the incident muon energy, and the estimation of energy transfer often involves large systematic and random errors. The disadvantages of the unknown muon energy were overcome in this experiment by operating the calorimeter in coincidence with a muon spectrograph.

A comparison of the obtained results with those of other experiments is rather difficult because of different experimental conditions. However, collecting the experimental data on muon interactions and plotting the results in one diagram was tried. The results are expressed in terms of a factor characterizing the deviation from the theory. The error quoted in this comparison refers only to the statistical fluctuations in the number of observed events. The influence of the error in energy determination is discussed later.

Figure 7 gives a survey of experimental results on the sum of all electromagnetic interactions of the muon.<sup>2,4,5,15,51-59</sup> This comparison involves a large uncertainty, since the muon spectrum was folded into the differential probabilities. However,

FIG. 7. Comparison between experimental results on the total cross section for muon interactions and the theories.



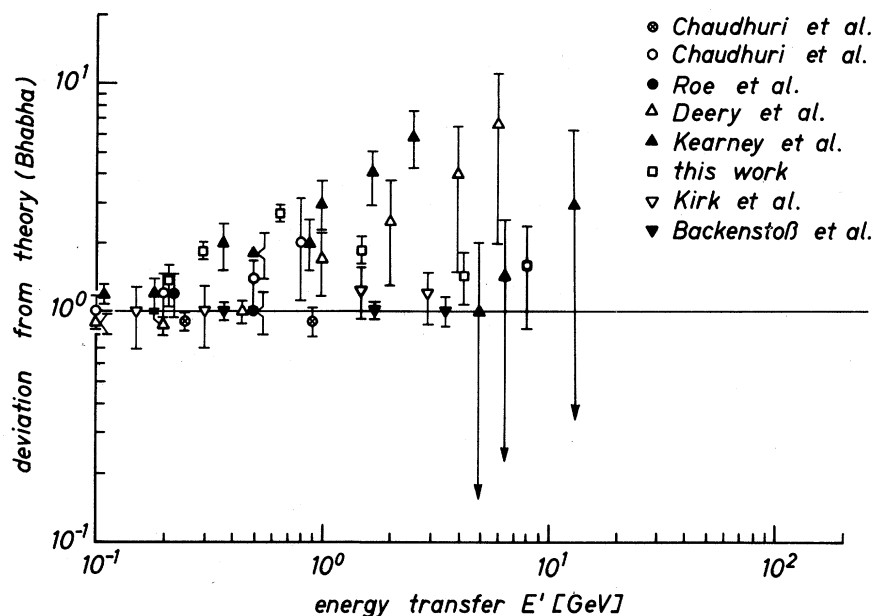


FIG. 8. Survey of experimental results on the knock-on process.

it can be seen that there is agreement between experiment and theory over a wide energy range.

Figure 8 summarizes the data on the knock-on process.<sup>1,2,4,6,7,12</sup> There is agreement between Bhabha's theory and the experimental results in the region of energy transfers beyond a few GeV. However, there are deviations in the region of medium energy transfers ( $\sim 1$  GeV).

This is in contradiction to the accelerator results obtained by Kirk and Neddermeyer,<sup>7</sup> as well as Backenstoss *et al.*<sup>6</sup> It is true that the estimation of energy transfer often involves large system-

atic and random errors, if the energy is determined by aid of spark-chamber calorimeters and multiplate cloud chambers. However, the estimation of the energy transfer in multilayer scintillation counters, which were used in the experiments of Kirk and Neddermeyer,<sup>7</sup> as well as Backenstoss *et al.*,<sup>6</sup> may be subject to systematic errors, because of photomultiplier gain drift.

From this point of view Kirk and Neddermeyer<sup>7</sup> interpreted their results as a confirmation of the validity of the applied corrections rather than a direct verification of Bhabha's theory.

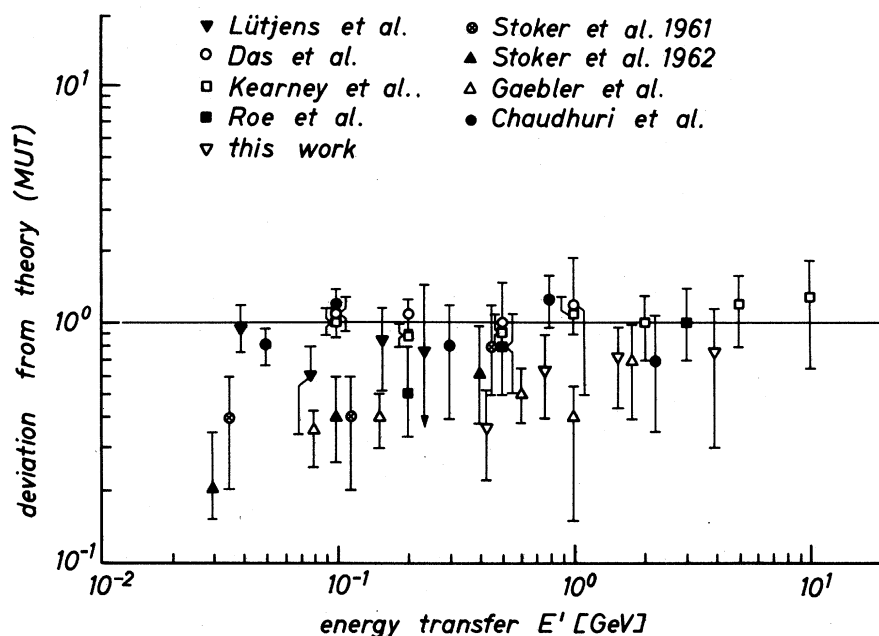


FIG. 9. Survey of experimental results on the direct pair production.

FIG. 10. Survey of experimental results on the bremsstrahlung production.

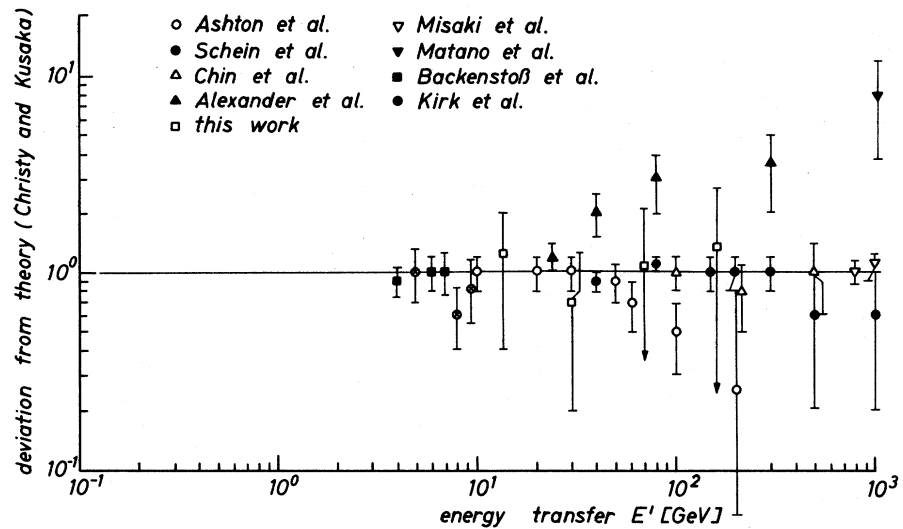


Figure 9 shows a survey about the results of the direct pair production.<sup>2-5,13-15,60</sup> The agreement is satisfactory except in the region of low energy transfer, where the spectrum of the secondaries seems to be flatter than predicted by Murota's theory. This lack of virtual soft photons was also found in a cosmic-ray experiment dealing with real photons (bremsstrahlung by electrons).<sup>61</sup>

The cross section for the bremsstrahlung production is rather small in comparison with that of the other processes. However, at large energy transfers this process becomes dominant. This is the reason why one can only compare data on this process at relatively large energy transfers (Fig. 10).<sup>6,7,51,53-57,59</sup> The agreement between theory and experimental results is satisfactory. However, statistics are very poor. The deviations from the theories quoted in the Figs. 7-10 refer to the number of observed events and the statistical error in this number. However, the comparison of the experimental results with the theories is seriously affected by the error in the energy determination of the event energy. Typical errors are about 30% if stacked assemblies of multiplate chambers or scintillation counters sandwiched with absorber plates are used. The main problem, which arises in all experiments on muon interactions, is to determine the energy of the secondary particles,

which initiate showers. If the influence of these energy errors is discussed (see Fig. 4), the accuracy on the quoted deviations is strongly modified. Within the large errors of energy estimations most of the mentioned deviations observed in experiments on muon interactions can be interpreted as rough agreement with the theories. The aim should be to perform energy determinations by aid of TASC<sup>62,63</sup> (total-absorption shower cascade) detectors to increase the energy resolution in comparison to stack assemblies of targets and detectors.

This experiment has shown that the behavior of the muon at energies considerably higher than those available at present accelerators can be understood in the framework of current theories about electromagnetic interactions of the muon, though there are still some deviations in the region of energy transfers around 1 GeV.

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\*Now at the University of Durham, United Kingdom.

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