ning efficiencies. After these restrictions 21.6% of the muon candidates remain. The expected fraction detected is calculated in a manner similar to that for $\Sigma \to ne\nu$, except that the muon events must have a muon momentum between 35 and 70 MeV/c in the Σ center-of-mass system. See Fig. 4.

There remain 13 events after the constraints; the scanning efficiency for these events is 70%. The branching ratio becomes

$$R(\Sigma^- \to n\mu\nu) = 13\,\frac{1}{1.82\times 10^6}\times\frac{1}{0.70}\times\frac{1}{0.216} \times\frac{1}{\text{fraction where } e^- \text{ turns more than } 180^\circ}\,.$$

Figure 5 illustrates the dependence of the branching ratio on g_A/g_V for selected values of g_W. Using currently accepted values for the coupling constants,[6] $g_A/g_V = -0.30$ and $g_W = 0.27$, the branching ratio becomes

$$R(\Sigma^- \to n\mu\nu) = (0.38 \pm 0.11)\times 10^{-3}.$$

These results are in good agreement with recent results at Maryland,[7] Princeton,[8] and Heidelberg.[9]

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Calculated Cosmic-Ray Muon Spectra at High Energies (>20 GeV)

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Calculations of sea-level cosmic-ray muon spectra have been made at 75°, 80°, 85°, and 88.75° between 20 and 1000 GeV, and compared with measurements made at Argonne National Laboratory. Although the experimental results are a consistent 60% of the calculated values, leading to too few muons being found at high zenith angles, it is felt that this does not support the Utah anomaly, as the discrepancy is energy- and angle-independent. Similarly, no exotic processes, such as the failure of special relativity, seem to be operating.

I. INTRODUCTION

Cosmic rays are the only source of information on nucleon-nucleus collisions at energies above those available from accelerators. Pion partial inelasticities, cross-section energy dependence, possible production processes, and the primary nucleon flux incident on the earth's atmosphere have, in the past, been estimated on a approximate basis from the dependence of muon fluxes on energy and angle.[1-3]

For these reasons, a calculation of muon spectra at a variety of angles, based on transport theory, depending on a reasonable model of nucleon-nucleus collisions and taking into account all relevant physical processes would be of interest. Substantial agreement with measured spectra would be evidence for the essential correctness of the physical picture incorporated in the calculations. Such calculations have been performed here, and indicate the validity of the "conventional" assumption that nearly all observed sea-level muon fluxes at energies up to 1000 GeV and high zenith angles arise from pion decay. No exotic effects are needed to account for the data.

II. THEORY

This paper presents a calculation of sea-level muon spectra based on a phenomenological model of nucleon-nucleus collisions which is incorporated into a solution to the Boltzmann equation for hadron and muon transport. The solution is for a sort of Green's function, that is, for the spectrum produced by an incident nucleon flux of unit intensity per (cm^2 sec sr) which is integrated over a primary nucleon spectrum to yield secondary muon fluxes.[4]

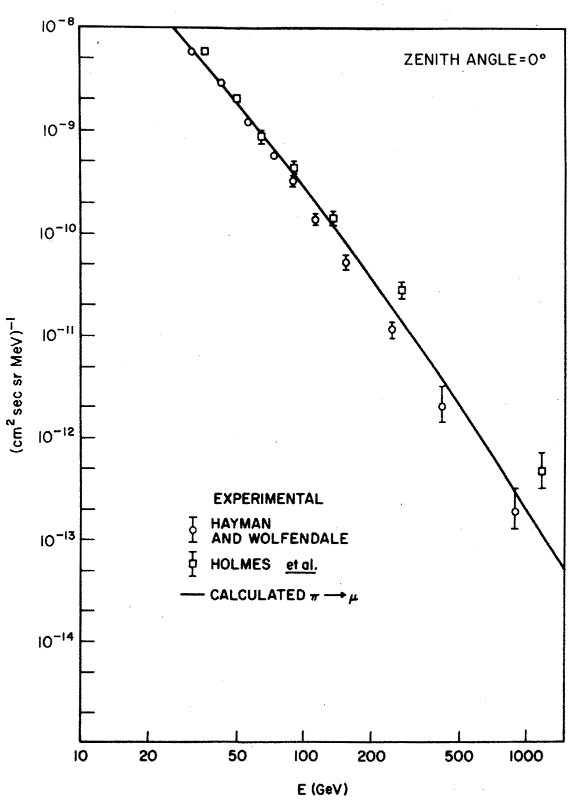

FIG. 1. Calculated and measured muon spectra at a zenith angle of 0°.

The partial inelasticities obtained from the model are shown in Table I. The vertical muon spectrum per (cm^2 sec sr) is shown in Fig. 1. The experimental data are from Hayman and Wolfendale[5] and Holmes *et al.*[6] The calculation is in essential agreement with the generally accepted results of the Durham group[5]: quite close below 100 GeV, within about 30% at higher energies. All reaction cross sections are constant and geometric, and muon stopping and decay are properly accounted for. The atmosphere was assumed to be a flat slab 1033 g/cm^2 thick. No attempt was made to account for the curvature of the atmosphere.

The incident nucleon spectrum assumed for this calculation is given in Table II.

III. RESULTS AND DISCUSSION

The calculations are compared with the recent measurements performed at Argonne National Laboratory by Asbury *et al.*[7] for zenith angles of 75° to 88.75°.

The calculations and experiments are shown in Figs. 2 and 3. Both muons from pion decay and muons from kaon decay were calculated. The kaon contribution is, as would be expected, quite small. The experimental data are a systematic 60% of the calculated results at all energies and angles except for 88.75°. The comparison at this highest angle is obscured in any event by the neglect of the earth's curvature, which will significantly affect the muon fluxes.

The error flags were calculated by this author from the data given by Asbury *et al.*[7] on the assumption of Poisson statistics and 68% confidence limits (which differ from one standard deviation when small numbers of events are con-

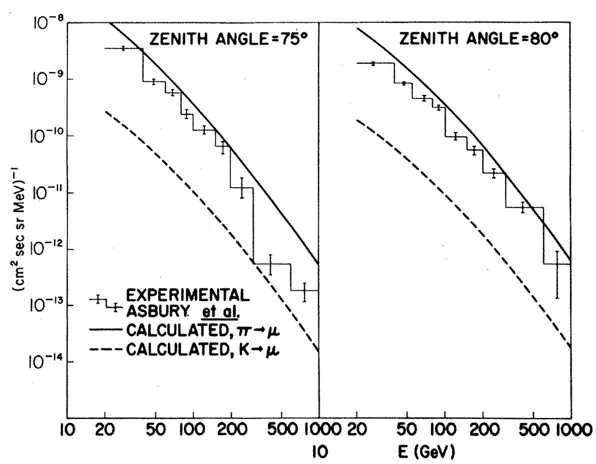

FIG. 2. Calculated and measured muon spectra at zenith angles of 75° and 80°.

TABLE I. Partial inelasticities for proton-air and neutron-air collisions.

q	K_{qp} (protons)	K_{qn} (neutrons)
p	0.211	0.211
n	0.211	0.211
π^+	0.180	0.112
π^-	0.112	0.180
π^0	0.180	0.180
K^+	0.034	0.022
K^-	0.022	0.034
K^0	0.034	0.034

sidered).

The most probable reason for the discrepancy between calculation and measurement observed at 75°, 80°, and 85° is a systematic error in the determination of the muon detection efficiency, a possibility mentioned by Asbury *et al.*[7] If the detection efficiency used by the Argonne National Laboratory group was a factor 1.7 too high, then excellent agreement would be found for these angles.

This method of calculation has yielded good agreement with vertical sea-level nucleon, muon, and pion spectra.[4] In addition, good agreement with sea-level ionization from the total and hard component has been reported.[8] Furthermore, it is noted that this calculation yields a value of 2.60×10^{-6} per (cm^2 sec sr MeV/c) for the vertical muon flux at 1 GeV/c, which agrees well with Rossi's "standard" value of 2.45×10^{-6}.[9] Hence, a 60% discrepancy between calculation and measurement at all angles and energies is not expected.

It might be argued that the agreement at zero

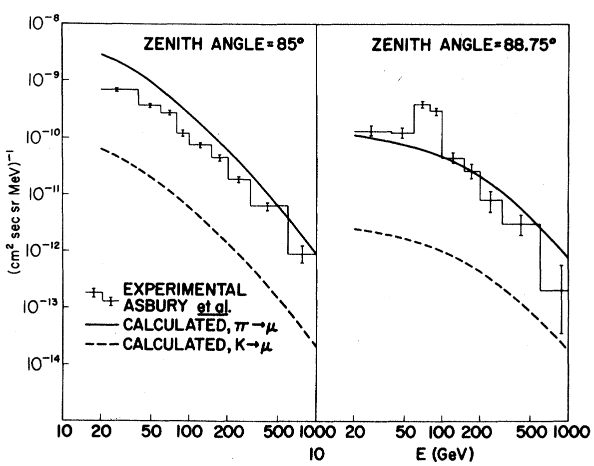

FIG. 3. Calculated and measured muon spectra at zenith angles of 85° and 88.75°.

TABLE II. Postulated primary nucleon spectrum used in calculations.

Energy (GeV)	Nucleon flux (cm^2 sec sr MeV)$^{-1}$
20	9.52×10^{-7}
37	2.52×10^{-7}
69	6.34×10^{-8}
129	1.52×10^{-8}
240	3.56×10^{-9}
447	8.09×10^{-10}
832	1.80×10^{-10}
1550	3.94×10^{-11}
2885	8.46×10^{-12}
5371	1.78×10^{-12}
10 000	3.70×10^{-13}

degrees zenith angle alluded to above is fortuitous. The experimental data might include directly produced muons. It would then follow that calculations at high zenith angles would overestimate the muon flux in a manner similar to that observed by the Utah group.[10]

There are two arguments against this. The first is that if the vertical spectrum includes a significant number of directly produced muons at high energies, its spectral index will be close to −2.5, the index of the primary spectrum in this energy range. However, at 1000 GeV the index has been shown to be −3.55 by Hayman *et al.*,[5] which, as shown by Barrett *et al.*,[11] would be expected if muons result from pion precursors. Hence the energy dependence of the vertical spectrum is inconsistent with the Utah hypothesis near or below 1000 GeV.

The second argument is that the discrepancy extends all the way down to 20 GeV, and is remarkably constant. The Utah effect is not expected to set in below 300 GeV.[2,12]

It is seen that the discrepancy between theory and measurement is consistent with a systematic overestimate of the detector sensitivity by a factor 1.7. On this assumption, the essentially constant proportion between theory and experiment between 75° and 85° and from 20 to 1000 GeV indicates that at these energies muons are produced by pion decay in accordance with the nucleon model described earlier.[4] Energy dependent phenomena such as the Utah effect or other exotic processes such as the failure of special relativity which might affect meson and muon lifetimes and hence the population of muons at large depths,[13] and which were discussed by Asbury *et al.*,[7] do not appear to be operating.

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Electromagnetic Interactions of High-Energy Cosmic-Ray Muons

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The electromagnetic interactions of cosmic-ray muons in the energy range up to 1 TeV were investigated with a spark-chamber calorimeter in combination with the Kiel spectrograph. The purpose was to investigate the different kinds of electromagnetic processes (knock-on, direct-pair, bremsstrahlung, and multiple pion production) in the range of energy transfer from 0.2 to 100 GeV. The production of pions is in reasonable agreement with the theoretical prediction of Daiyasu *et al*. In the region of high energy transfer (>2 GeV) the experimental results agree with Bhabha's theory of the knock-on process as well as with Murota's theory of direct pair production. However, in the region of energy transfer around 1 GeV there are minor deviations from these theories, in contradiction with accelerator data.

I. INTRODUCTION

In this paper we present results of a spark-chamber experiment designed to determine the cross sections for the different electromagnetic interactions of muons in order to extend the knowledge of these cross sections beyond accelerator energies. A large number of such measurements have been performed using cosmic-ray muons. However, the interpretation of the cosmic-ray data has been difficult, since the spectrum of incident muons has to be folded into the differential probabilities for the various kinds of interactions. The lower energy limit and the shape of the muon spectrum influence the theoretical predictions calculated in this way. The interpretation of underground experiments requires an energy-range relation for muons; an additional source of error is thereby introduced into the theoretical prediction. Moreover, it is rather difficult to distinguish the different kinds of interactions, especially in the region of high energy transfer, since the electron shower which is initiated by a secondary particle obliterates the initial signature of the event. These difficulties were overcome in this experiment by measuring the muon energy for each individual event in the spark-chamber calorimeter by means of the Kiel spectrograph. The knowledge of the muon energy permits, in addition, a discrimination between the various processes by the use of theoretical predictions.

Some authors[1-8] find agreement with the theories of Bhabha[9,10] for the knock-on process and of Murota *et al.*[11] for the process of direct pair production, while other experiments[4,12-15] cannot be explained by these theories. The bremsstrahlung cross section[16] is very small for muons with energies less than a few hundred GeV. For this reason it is difficult to measure this cross section with high accuracy, even though our experiment involves muons of energies up to 1 TeV. The investigations on multiple pion production are considerably complicated by the variety of theoretical predictions[17-20] about the virtual-photon spectrum of the muon, as well as by the uncertainty about the photonuclear cross section for pion production.