lished).

[30]M. A. Virasoro, Phys. Rev. D 3, 2834 (1971).

[31]Bateman Manuscript Project, *Higher Transcendental Functions*, edited by A. Erdélyi (McGraw-Hill, New York, 1953), Vol. 1, pp. 255 and 257.

[32]This behavior implies an analogous exponential vanishing of the imaginary part of the five-point function. It would be very interesting to have a physical explanation for this behavior.

[33]The same results are obtained by a direct calculation of the asymptotic behaviors.

[34]Explicitly,

$$\alpha_{a\bar{a}}+\alpha_{x\bar{x}}-\alpha_{x\bar{x}\bar{b}}-\alpha_{x\bar{x}b}-1$$
$$=\alpha_{b\bar{b}}+[-a_{\bar{b}\bar{b}}+a_{a\bar{a}}+a_{x\bar{x}}-\alpha_{x\bar{x}\bar{b}}(m_b^2)-\alpha_{x\bar{x}b}(m_b^2)-1],$$
$$-\alpha_{\bar{x}\bar{b}}-\alpha_{b\bar{x}}+\alpha_{b\bar{b}\bar{x}}-1$$
$$=\alpha_{b\bar{b}}+[-a_{bb}-a_{\bar{x}b}-a_{b\bar{x}}-2m_x^2-2m_b^2+\alpha_{\bar{b}\bar{b}\bar{x}}(m_x^2)-1].$$

For reasonable masses and intercepts both brackets are negative so that the diagram in (A8) has a higher power. However, for example, for sufficiently negative intercepts the second bracket is positive.

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Failure of the Eikonal Approximation for the Vertex Function in a Boson Field Theory*

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By an explicit calculation of the high-energy behavior of the three-point function in the $\lambda\phi^3$ theory, through terms of order λ^5, we demonstrate that the eikonal approximation for generalized ladder exchange fails in that model, for this vertex function. In a given order, it underestimates the exact asymptote.

The high-energy behavior of Feynman graphs and of field-theoretic amplitudes is currently studied with the help of the eikonal approximation.[1,2] This technique has been extended from its original domain of applicability, fermion–vector-boson models, to other field theories, in particular the $\lambda\phi^3$ model.[3] The latter model has two important features which make it an attractive theoretical laboratory: (1) The (hopefully) inessential complications of spin are absent and the Feynman rules are correspondingly simple; (2) Regge-type behavior has been established in this theory. Unfortunately, there is no proof that the amplitudes of this model, when evaluated in the eikonal approximation, bear any relation to the high-energy asymptote of the *exact* expressions, even in perturbation theory. Indeed, a recent investigation by Tiktopoulos and Treiman[4] of certain scattering graphs, involving crossed ladder exchanges, indicates that there may exist groups of Feynman diagrams whose high-energy behavior is not necessarily given by the eikonal approximation.

The purpose of this paper is to publicize the fact that the generalized ladder exchange graphs, contributing to the three-point function (vertex function) in the $\lambda\phi^3$ theory, when calculated to fifth-order perturbation theory, possess "leading logarithms" in the high-energy region which are not properly given by the eikonal approximation. This result makes use of an old calculation, performed some years ago by one of us (R.J.) in connection with the evaluation of the high-energy behavior of the vertex function in a spinor–vector-meson theory: quantum electrodynamics.[5] For the latter theory, the eikonal approximation correctly describes the high-energy asymptote, at least order by order in perturbation theory.[6]

The use of the eikonal techniques to calculate the vertex function is just as plausible as the analogous computation of the scattering amplitude since the eikonal approximation is correct in this context for spinor–vector-meson theories.[6] Because our calculation is in a (relatively) low order of perturbation theory, all the contributing ladder exchange graphs can be analyzed completely, and it is evi-

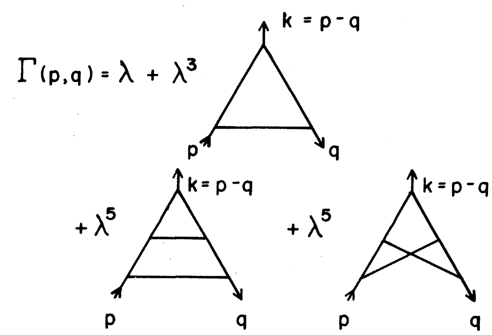

FIG. 1. Generalized ladder graphs, through λ^5, which contribute to $\Gamma(p,q)$.

dent that the noneikonal contributions do not disappear from the sum. Moreover, the noneikonal part is of the same form as the eikonal result; thus, there is no easy way to separate one from the other. Therefore, we conclude, in agreement with the conjecture by Tiktopoulos and Treiman, that the eikonal approximation cannot be used in the $\lambda\phi^3$ theory. The three-point function is the first amplitude for which the eikonal method fails, and this is, in our opinion the simplest example of that phenomenon.

The details of our calculation are the following. The leading logarithms, arising from generalized ladder exchange and contributing to the vertex function $\Gamma(p, q)$ on the mass shell ($p^2=m^2=q^2$) with large momentum transfer $[-k^2=-(p-q)^2 \gg m^2]$,

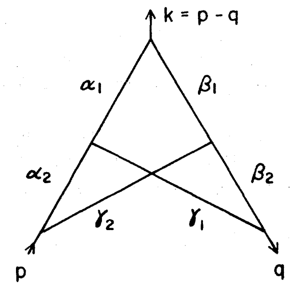

FIG. 2. Parameter labeling of $\Gamma_{5b}(k^2)$.

are represented through fifth order by the graphs of Fig. 1. These graphs have the following integral representation[7]:

$$\Gamma(p, q)=\lambda+\lambda^3\Gamma_3(k^2)+\lambda^5\Gamma_{5a}(k^2)+\lambda^5\Gamma_{5b}(k^2), \tag{1}$$

$$\Gamma_3(k^2)=i\int\frac{d^4r}{(2\pi)^4}\,[(r^2+2p\cdot r)(r^2+2q\cdot r)(r^2-m^2)]^{-1}, \tag{2}$$

$$\Gamma_{5a}(k^2)=-\int\frac{d^4rd^4s}{(2\pi)^8}\,\{(r^2+2p\cdot r)[(r+s)^2+2p\cdot(r+s)][(r+s)^2+2q\cdot(r+s)](r^2+2q\cdot r)(r^2-m^2)(s^2-m^2)\}^{-1}, \tag{3}$$

$$\Gamma_{5b}(k^2)=-\int\frac{d^4rd^4s}{(2\pi)^8}\,\{(r^2+2p\cdot r)[(r+s)^2+2p\cdot(r+s)][(r+s)^2+2q\cdot(r+s)](s^2+2q\cdot s)(r^2-m^2)(s^2-m^2)\}^{-1}. \tag{4}$$

These formulas are written in analogy with quantum electrodynamics. They are not the complete vertex function in this theory for two reasons: (1) Self-energy and selected vertex correction graphs have not been included; we have not analyzed the behavior of these. (2) In a scalar theory, the crossed-ladder exchange graph enters as $\frac{1}{2}\lambda^5\Gamma_{5b}(k^2)$ in Eq. (1) because of symmetry effects; i.e., the direct ladder is enhanced by a factor of 2 relative to the crossed ladder. However, this numerology *does not lead to an exponentiation* of the vertex function [see Eqs. (6) and (7) below], and we shall ignore it. The asymptotic formulas for (2) and (3) may be readily found by straightforward computation or from the literature.[5-7] These expressions, in the limit $-k^2 \gg m^2$, are

$$\Gamma_3(k^2)\to(1/32\pi^2k^2)\ln|k^2/m^2|, \tag{5}$$

$$\Gamma_{5a}(k^2)\to\tfrac{1}{6}[\Gamma_3(k^2)]^2. \tag{6}$$

The calculation of $\Gamma_{5b}(k^2)$, which is outlined below, yields

$$\Gamma_{5b}(k^2)\to\tfrac{4}{3}[\Gamma_3(k^2)]^2. \tag{7}$$

Therefore, we find the following asymptotic form for the fifth-order leading logarithms arising from generalized ladder exchange graphs:

$$\Gamma_5(k^2)=\Gamma_{5a}(k^2)+\Gamma_{5b}(k^2)\to\tfrac{3}{2}[\Gamma_3(k^2)]^2. \tag{8}$$

The eikonal approximation, applied to the third-order contribution $\Gamma_3(k^2)$, is correct[8]:

$$\Gamma_3^E(k^2)\equiv i\int\frac{d^4r}{(2\pi)^4}[(2p\cdot r)(2q\cdot r)(r^2-m^2)]^{-1}\to\Gamma_3(k^2). \tag{9}$$

On the other hand, when the fifth-order ladder exchange graphs are evaluated in the eikonal approximation, the following *incorrect* result is obtained:

$$\begin{aligned}\Gamma_5^E(k^2)&\equiv-\int\frac{d^4rd^4s}{(2\pi)^8}\{(2p\cdot r)[2p\cdot(r+s)][2q\cdot(r+s)](r^2-m^2)(s^2-m^2)\}^{-1}\left(\frac{1}{2q\cdot r}+\frac{1}{2q\cdot s}\right)\\&=-\frac{1}{2}\int\frac{d^4rd^4s}{(2\pi)^8}\left(\frac{1}{2p\cdot r}+\frac{1}{2p\cdot s}\right)\{[2p\cdot(r+s)](2q\cdot r)(2q\cdot s)(r^2-m^2)(s^2-m^2)\}^{-1}=\frac{1}{2}[\Gamma_3^E(k^2)]^2.\end{aligned} \tag{10}$$

In passing from the first to the second lines in the right-hand side of (10), the integrand was symmetrized in s and r. Thus, in the usual way, the eikonal approximation leads to an exponentiation,

$$\Gamma^E(p,q)=\lambda\exp[\lambda^2\Gamma_3^E(k^2)]\,. \tag{11}$$

Unfortunately, as a comparison of (8) with (10) indicates, this result is in error.

We emphasize that the analogous eikonal computation in quantum electrodynamics appears to be correct. In that theory one finds, as $-k^2\to\infty$,[5,6]

$$\Gamma^\mu(p,q)\to\gamma^\mu\Gamma(k^2)=\gamma^\mu\Gamma^E(k^2)=e\gamma^\mu\exp\left(\frac{-e^2}{16\pi^2}\ln^2\frac{|k^2|}{\mu^2}\right), \tag{12}$$

where μ is the photon mass. The validity of the eikonal approximation in this theory has been checked through seventh order,[5,6] and as we shall argue below, this should be true to all finite orders.

We now indicate the derivation of (7). Two independent methods are presented. The first makes use of a parametric representation for $\Gamma_{5b}(k^2)$. The second employs the modified Feynman rules which are useful for studying high-energy behavior. These rules were developed by K. Wilson and one of us (R.J.)[5] and are analogous to Weinberg's infinite-momentum rules.[9] The parametric representation for $\Gamma_{5b}(k^2)$ is

$$\Gamma_{5b}(k^2)=\left(\frac{1}{16\pi^2}\right)^2\int_0^1 d\alpha_1 d\alpha_2 d\beta_1 d\beta_2 d\gamma_1 d\gamma_2\,\delta(1-\alpha_1-\alpha_2-\beta_1-\beta_2-\gamma_1-\gamma_2)D^{-2}, \tag{13a}$$

$$D=-k^2f+m^2(g+2f), \tag{13b}$$

$$f=\beta_1\gamma_1(\alpha_1+\alpha_2)+\alpha_1\gamma_2(\beta_1+\beta_2)+\alpha_1\beta_1(\alpha_2+\beta_2), \tag{13c}$$

$$\begin{aligned} g=&[(\alpha_2+\gamma_2)(\beta_2+\gamma_1)+(\alpha_1+\beta_1)(\beta_2+\gamma_1)+(\alpha_1+\beta_1)(\alpha_2+\gamma_2)](\gamma_1+\gamma_2)\\ &+[\alpha_1{}^2+(\beta_1+\beta_2)^2](\alpha_2+\gamma_2)+[\beta_1{}^2+(\alpha_1+\alpha_2)^2](\beta_2+\gamma_1)+(\alpha_2{}^2+\beta_2{}^2)(\alpha_1+\beta_1). \end{aligned} \tag{13d}$$

Here the parameters are associated with the propagators as indicated in Fig. 2.

Clearly the leading contribution for large $|k^2|$ arises from the region of parameter space where f vanishes. In order to expose that region, we first make the following transformation:

$$\alpha_1\to\sigma_1\alpha_1',\quad \beta_1\to\sigma_1\beta_1',\quad \beta_2\to\sigma_1\beta_2',\quad \gamma_1\to\sigma_1\gamma_1',\quad d\alpha_1d\beta_1d\beta_2d\gamma_1\to d\alpha_1'd\beta_1'd\beta_2'd\gamma_1'\sigma_1{}^3d\sigma_1\delta(1-\alpha_1'-\beta_1'-\beta_2'-\gamma_1');$$

$$\alpha_1'\to\sigma_2\alpha_1'',\quad \alpha_2\to\sigma_2\alpha_2',\quad \beta_1'\to\sigma_2\beta_1'',\quad \gamma_2\to\sigma_2\gamma_2',\quad d\alpha_1'd\alpha_2d\beta_1'd\gamma_2\to d\alpha_1''d\alpha_2'\,d\beta_1''d\gamma_2'\sigma_2{}^3d\sigma_2\delta(1-\alpha_1''-\beta_1''-\alpha_2'-\gamma_2').$$

Next, we scale all remaining sets of parameters which can cause f to vanish when they themselves vanish (in the terminology of Tiktopoulos,[10] these are the "minimal sets"):

$$\alpha_1''\to\rho_1\alpha_1''',\quad \alpha_2'\to\rho_1\alpha_2'',\quad d\alpha_1''d\alpha_2'\to d\alpha_1'''d\alpha_2''\rho_1d\rho_1\delta(1-\alpha_1'''-\alpha_2'');$$

$$\beta_1''\to\rho_2\beta_1''',\quad \gamma_2'\to\rho_2\gamma_2'',\quad d\beta_1''d\gamma_2'\to d\beta_1'''d\gamma_2''\rho_2d\rho_2\delta(1-\beta_1'''-\gamma_2'');$$

$$\alpha_1'''\to\rho_3\alpha_1^{\mathrm{IV}},\quad \gamma_1'\to\rho_3\gamma_1'',\quad d\alpha_1'''d\gamma_1'\to d\alpha_1^{\mathrm{IV}}d\gamma_1''\rho_3d\rho_3\delta(1-\alpha_1^{\mathrm{IV}}-\gamma_1'');$$

$$\beta_1'''\to\rho_4\beta_1^{\mathrm{IV}},\quad \beta_2'\to\rho_4\beta_2'',\quad d\beta_1'''d\beta_2'\to d\beta_1^{\mathrm{IV}}d\beta_2''\rho_4d\rho_4\delta(1-\beta_1^{\mathrm{IV}}-\beta_2'');$$

$$\alpha_1^{\mathrm{IV}}\to\rho_5\alpha_1^{\mathrm{V}},\quad \beta_1^{\mathrm{IV}}\to\rho_5\beta_1^{\mathrm{V}},\quad d\alpha_1^{\mathrm{IV}}d\beta_1^{\mathrm{IV}}\to d\alpha_1^{\mathrm{V}}d\beta_1^{\mathrm{V}}\rho_5d\rho_5\delta(1-\alpha_1^{\mathrm{V}}-\beta_1^{\mathrm{V}}).$$

With these substitutions, we have

$$\begin{aligned}\Gamma_{5b}(k^2)=\left(\frac{1}{16\pi^2}\right)^2\int_0^1 & d\alpha_1^{\mathrm{V}}d\alpha_2''d\beta_1^{\mathrm{V}}d\beta_2''d\gamma_1''d\gamma_2''d\sigma_1d\sigma_2d\rho_1d\rho_2d\rho_3d\rho_4d\rho_5(\sigma_1\sigma_2)^3\rho_1\rho_2\rho_3\rho_4\rho_5\\ &\times\delta(1-\sigma_1\sigma_2\rho_1\rho_3\rho_5\alpha_1^{\mathrm{V}}-\sigma_2\rho_1\alpha_2''-\sigma_1\sigma_2\rho_2\rho_4\rho_5\beta_1^{\mathrm{V}}-\sigma_1\rho_4\beta_2''-\sigma_1\rho_3\gamma_1''-\sigma_2\rho_2\gamma_2'')\\ &\times\delta(1-\sigma_2\rho_1\rho_3\rho_5\alpha_1^{\mathrm{V}}-\sigma_2\rho_2\rho_4\rho_5\beta_1^{\mathrm{V}}-\rho_4\beta_2''-\rho_3\gamma_1'')\delta(1-\rho_1\rho_3\rho_5\alpha_1^{\mathrm{V}}-\rho_2\rho_4\rho_5\beta_1^{\mathrm{V}}-\rho_1\alpha_2''-\rho_2\gamma_2'')\\ &\times\delta(1-\rho_3\rho_5\alpha_1^{\mathrm{V}}-\alpha_2'')\delta(1-\rho_4\rho_5\beta_1^{\mathrm{V}}-\gamma_2'')\delta(1-\gamma_1''-\rho_5\alpha_1^{\mathrm{V}})\delta(1-\beta_2''-\rho_5\beta_1^{\mathrm{V}})\delta(1-\alpha_1^{\mathrm{V}}-\beta_1^{\mathrm{V}})D^{-2}.\end{aligned} \tag{14}$$

In the above, D is to be expressed in terms of the new variables. When this is done, it may be observed that the leading asymptote comes from that region of the ρ_5 integration where ρ_5 is near zero. Hence we set ρ_5 to zero in all the δ functions, as well as in g. In f, ρ_5 is set to zero whenever it is compared to a nonvanishing variable. Equation (14) now simplifies to the following, when the remaining δ functions are used, and the superscripts are suppressed:

$$\Gamma_{5b}(k^2) \to \left(\frac{1}{16\pi^2}\right)^2 \int_0^1 d\sigma_1 d\sigma_2 d\rho_1 d\rho_2 d\rho_3 d\rho_4 d\rho_5 \sigma_1\sigma_2\rho_1\rho_2\rho_3\rho_4\rho_5 \delta(1-\sigma_1-\sigma_2) \times \delta(1-\rho_1-\rho_2)\delta(1-\rho_3-\rho_4)(|k^2|\sigma_1\sigma_2\rho_1\rho_2\rho_3\rho_4\rho_5+m^2)^{-2}. \tag{15}$$

There are eight disjoint regions of integration that contribute to the leading behavior of $\Gamma_{5b}(k^2)$. These arise whenever ρ_5 as well as any *one* member of each of the following pairs are near zero: (σ_1, σ_2), (ρ_1, ρ_2), (ρ_3, ρ_4). (There are eight ways to select one member out of each of the three pairs.) Each region gives

$$(1/16\pi^2)^2(1/4!\,|k^2|^2)\ln^4(|k^2|/m^2).$$

Hence

$$\Gamma_{5b}(k^2) = \tfrac{4}{3}(1/32\pi^2)^2(1/|k^2|^2)\ln^4(|k^2|/m^2).$$

This verifies (7).

We now discuss the second evaluation of (7). The method involves decomposing each propagator into two pieces,

$$(r^2 - m^2 + i\epsilon)^{-1} = (2\omega_r)^{-1}[(r_0 - \omega_r + i\epsilon)^{-1} - (r_0 + \omega_r - i\epsilon)^{-1}], \quad \omega_r \equiv (\vec{r}^{\,2} + m^2)^{1/2},$$

and performing all integrations over the zeroth components by closing the contour. Evidently this decomposition results in 64 "secondary" diagrams which describe $\Gamma_{5b}(k^2)$. From this set, only those are selected which have the property, at large momentum transfer, that a denominator can get very small during the integration. The dominant contribution can come only from these. One is then left with two secondary diagrams. These have the integral representation[11]

$$\Gamma_{5b}(k^2) = I_1 + I_2, \tag{16a}$$

$$I_1 = (1/2^{12}\pi^6)\int d^3r\, d^3s\; I(\vec{r}, \vec{s}; \vec{p}), \tag{16b}$$

$$I_2 = (1/2^{12}\pi^6)\int d^3r\, d^3s\; I(\vec{p} - \vec{r}, \vec{s}; \vec{p}), \tag{16c}$$

$$[I(\vec{r}, \vec{s}, \vec{p})]^{-1} = \omega_{p-r}\omega_r\omega_{p-r-s}\omega_{r+s}\omega_s^{\,2}(\omega_{p-r}+\omega_r-\omega_p)(\omega_{p-r-s}+\omega_{r+s}-\omega_p+m) \times(\omega_{p-r-s}+\omega_s+\omega_r-\omega_p+m)(\omega_{p-r-s}+\omega_s+\omega_r-\omega_p). \tag{16d}$$

We have presented the integrals in the rest frame of q. To evaluate them, one notices that the region of integration

$$R_1:\ \ p_{\parallel} >> r_{\parallel} >> s_{\parallel} >> m >> s_{\perp}^{\,2}/2s_{\parallel} >> r_{\perp}^{\,2}/2r_{\parallel}, \quad r_{\perp}, s_{\perp} >> m$$

gives the following contribution[12] to each integral I_i:

$$I_1^{(R_1)} = I_2^{(R_1)} = \tfrac{1}{3}(1/32\pi^2)^2(1/|k^2|^2)\ln^2(|k^2|/m^2). \tag{17}$$

In addition to the region R_1, there also exists the disjoint region R_2,[12]

$$R_2:\ \ p_{\parallel} \sim r_{\parallel} >> p_{\parallel} - r_{\parallel} >> s_{\parallel} >> m >> s_{\perp}^{\,2}/2s_{\parallel} >> r_{\perp}^{\,2}/2(p_{\parallel}-r_{\parallel});\quad r_{\perp}, s_{\perp} >> m,$$

which gives the same contribution as R_1 to each integral I_i:

$$I_i^{(R_2)} = I_i^{(R_1)}. \tag{18}$$

(This is evident from the fact that I_2 is related to I_1 by the transformation $\vec{r} \to \vec{p} - \vec{r}$.) Therefore we find, as before,[13]

$$\Gamma_{5b}(k^2) = \tfrac{4}{3}(1/32\pi^2)^2(1/|k^2|^2)\ln^4(|k^2|/m^2).$$

One may understand the failure of the eikonal approximation, in this model, in the following fashion. Working in the brick-wall frame, $p = (p^0, \vec{p})$, $q = (p^0, -\vec{p})$, $k = (0, 2\vec{p})$, we may route the large 3-momentum $\vec{p}$ through the crossed diagram in the four ways depicted in Fig. 3. The eikonal approximation correctly reproduces the sum of $\Gamma_{5b}(k^2)$ with *one* of the routings of Fig. 3(a) or 3(b). The presence of the additional routings indicates that the eikonal approximation *underestimates* the true answer, which is what we have found. Furthermore, it is clear that in a spinor–vector-meson theory, where the outside legs of Fig. 3 are *fermion* propagators with numerator factors, the direct routing of Fig. 3(a) dominates the alternate routings, Figs. 3(b), 3(c), and 3(d). The reason

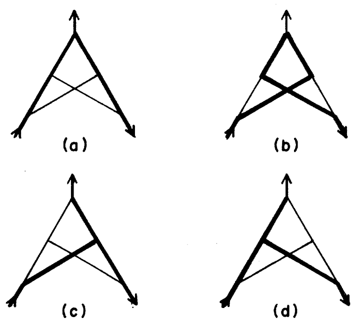

FIG. 3. Routing of large momentum through crossed graph. The heavy lines carry "hard" momentum; the light lines carry "soft" momentum.

for this is, of course, the presence of momentum factors in the numerator. Therefore, one would not expect the eikonal approximation to fail in that model, and explicit computation confirms this hope. Finally we remark that the phenomenon discussed here certainly is present in higher orders. For example, the graph pictured in Fig. 4(a) has the eikonal routing given in Fig. 4(b), and the noneikonal routing of Fig. 4(c). Note that the noneikonal routing is of length 4, in contrast to the eikonal, which is of length 6. Hence the former is $O(|k^2|^{-2})$ and dominates the latter, which is $O(|k^2|^{-3})$.

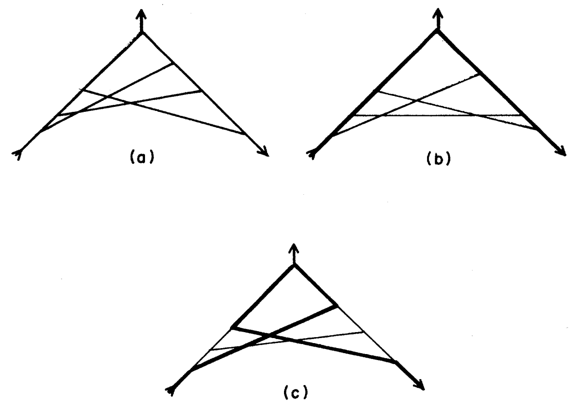

FIG. 4. Graph which possesses noneikonal routings: (a) full graph; (b) eikonal routing of length 6; (c) noneikonal routing of length 4.

The results of this investigation were developed during a time when we were learning about eikonal techniques from Professor K. Johnson. His instruction is gratefully acknowledged.

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[1]We use the expression "eikonal approximation" to describe the process of dropping quadratic terms in propagators which carry low momentum. This procedure, though not justified when a single diagram is considered, may accurately reproduce the high-energy asymptote when a symmetrization is performed over all possible soft-particle exchanges. Thus in a given order of perturbation theory one can hope that the eikonal approximation is valid. Frequently, when performed to all orders of perturbation theory, the eikonal approximation leads to the exponentiation of the lowest-order result.

[2]The contemporary interest is largely due to the many calculations of Cheng and Wu in quantum electrodynamics; see, e.g., H. Cheng and T. T. Wu, Phys. Rev. D 2, 2276 (1970). H. D. I. Abarbanel and C. Itzykson, Phys. Rev. Letters 23, 53 (1969), reminded us that these results make use of the eikonal approximation.

[3]See, for example, S. J. Chang and T. M. Yan, Phys. Rev. Letters 25, 1586 (1970); B. Hasslacher, D. K. Sinclair, G. M. Cicuta, and R. L. Sugar, *ibid.* 25, 1591 (1970).

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[5]R. Jackiw, Ph.D. thesis, Cornell University, 1966 (unpublished); Ann. Phys. (N. Y.) 48, 292 (1968).

[6]J. Primack, Ph.D. thesis, Stanford University, 1970 (unpublished); P. M. Fishbane and J. D. Sullivan, Phys. Rev. D 4, 458 (1971); E. Eichten, *ibid.* 4, 439 (1971); T. Appelquist and J. Primack (unpublished).

[7]M. Cassandro and M. Cini, Nuovo Cimento 34, 1719 (1964), also analyze some of the integrals under discussion here. The numerical factors occurring in their analysis have not been reproduced by the subsequent investigations, Refs. 5 and 6; nor do they agree with the eikonal approximation.

[8]The integration in Eq. (9) extends only over that region of r space consistent with the eikonal approximation: $2p\cdot r$, $2q\cdot r \gg r^2$. Therefore the integral converges.

[9]S. Weinberg, Phys. Rev. 150, 1313 (1966).

[10]G. Tiktopoulos, Phys. Rev. 131, 2373 (1963).

[11]For detailed analysis see Ref. 5. The same results, Eq. (16), may be obtained by use of the rules of Ref. 9.

[12]The explicit evaluation is given in Ref. 5. The second region, R_2, was erroneously omitted in Ref. 5. Note, however, that already R_1 gives a noneikonal result; the contribution from R_2 makes the discrepancy even larger. We emphasize here that the principal results of Ref. 5 are not affected by the omission of R_2, since that region *does not* contribute when one is discussing quantum electrodynamics.

[13]It is reassuring to note that if, through an inadvertence on our part, a region has been omitted which *does* contribute to asymptote, its inclusion can only *increase* our result which is already greater than the eikonal approximation. The reason for this is that the integrals (13) or (16) are positive, and we have analyzed contributions from disjoint regions.