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Behavior of Space Components of Currents Under $SU(3) \otimes SU(3)$ Transformations*

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We suggest that the equal-time commutators between the axial charges and the space components of the currents should be modified. We then present a model which provides a natural explanation for the value $\xi = -1$ for the ratio of form factors in K_{l3} decays and, in addition, leads to the usual good results of current algebra.

I. INTRODUCTION

Quite successful calculations dealing with strong-interaction processes have been carried out over the past few years.[1] The two main assumptions employed in these calculations have been:

(1) the hypothesis that equal-time commutators (ETC) between current operators may be abstracted from simple models and postulated to be exact, and

(2) the hypothesis of the partially conserved axial-vector current (PCAC), i.e., the assumption that the relevant matrix elements are slowly varying between $q^2=0$ and $q^2=-m_\pi^2$ when $(\sqrt{2}/f_\pi m_\pi^2)\times\partial_\mu A_\mu^\alpha(x)$ is used as an interpolating pion field. Here $A_\mu^\alpha(x)$ denotes the axial-vector current and q_μ is the pion momentum.

On the other hand, the application of PCAC and current algebra to processes involving weak and electromagnetic interactions has only been partially successful, and in particular, has led to predictions for the K_{l3} decays and the decays $\pi^0\to 2\gamma$ and $\eta\to 3\pi$ which are in striking disagreement with experiment. Notice, however, that there is an essential difference between current-algebra calculations for strong interactions and similar calculations for weak and electromagnetic processes, in that, while the former make use of only the ETC between the time components of currents, the latter require in addition the specification of ETC between their time and space components. It is therefore plausible that the failure of the calculations for these weak and electromagnetic decays may be due to a deviation of the ETC between the time and space components of currents from their usually assumed values.[2,3]

Motivated by the preceding discussion, we allow for additional terms in the time-space current commutators. We then present a model in which the usual good results of current algebra are maintained, and which in addition leads to a successful description of the K_{l3} decays. Unfortunately, the usual results of current algebra for $\pi^0\to 2\gamma$ and $\eta\to 3\pi$ also remain unaffected.[3a] We now proceed to state our main assumptions.

(1) Following Gell-Mann, we shall assume that the usual ETC between the time components of the 16 vector and axial-vector currents $J_\mu^\beta(x)$ hold, i.e.,

$$\begin{aligned}
[Q^\alpha(x_0), V_0^\beta(x)] &= if^{\alpha\beta\gamma}V_0^\gamma(x),\\
[Q^\alpha(x_0), A_0^\beta(x)] &= if^{\alpha\beta\gamma}A_0^\gamma(x),\\
[Q_5^\alpha(x_0), V_0^\beta(x)] &= if^{\alpha\beta\gamma}A_0^\gamma(x),\\
[Q_5^\alpha(x_0), A_0^\beta(x)] &= if^{\alpha\beta\gamma}V_0^\gamma(x),
\end{aligned} \tag{1.1}$$

where $V_0^\gamma(x)$ and $A_0^\gamma(x)$ are the time components of the vector and axial-vector currents, while $Q^\alpha(x_0)$ and $Q_5^\alpha(x_0)$ are the corresponding charges.

(2) We assume, as is customary, that the vector and axial-vector currents $J_\mu^\alpha(x)$ transform as octets under $SU(3)$, i.e.,

$$\begin{aligned}
[Q^\alpha(x_0), V_\mu^\beta(x)] &= if^{\alpha\beta\gamma}V_\mu^\gamma(x),\\
[Q^\alpha(x_0), A_\mu^\beta(x)] &= if^{\alpha\beta\gamma}A_\mu^\gamma(x).
\end{aligned} \tag{1.2}$$

Actually it may be shown[4,5] that Eq. (1.2) follows from Eq. (1.1) and covariance if the vector current is conserved.

(3) In dealing with ETC between axial charges and space components of currents, we make use of the most general form for these ETC which is compatible with Lorentz covariance and Eq. (1.1).[4,5] Therefore, we write

$$\begin{aligned}
[Q_5^\alpha(x_0), V_k^\beta(x)] &= if^{\alpha\beta\gamma}A_k^\gamma(x) - S_k^{\beta\alpha}(x),\\
[Q_5^\alpha(x_0), A_k^\beta(x)] &= if^{\alpha\beta\gamma}V_k^\gamma(x) - T_k^{\beta\alpha}(x),
\end{aligned} \tag{1.3}$$

with

$$\begin{aligned}
S_k^{\beta\alpha}(0) &= +\int d^3x\, x_k[V_0^\beta(0), \partial_\mu A_\mu^\alpha(x)],\\
T_k^{\beta\alpha}(0) &= +\int d^3x\, x_k[A_0^\beta(0), \partial_\mu A_\mu^\alpha(x)].
\end{aligned} \tag{1.4}$$

(4) We shall make use of a slight extension of the PCAC hypothesis. Before discussing this generalization, let us first note that the time-ordered products $T(\partial_\rho A_\rho^\alpha(x)J_\mu^\beta(y))$ are not covariant[3] due to the presence of the extra terms $S_k^{\beta\alpha}$ and $T_k^{\beta\alpha}$ in Eq. (1.4). Therefore, a seagull term must be added to covariantize the off-mass-shell amplitudes. [The seagull contribution vanishes on the mass shell

since it appears multiplied by the kinematical factor $(q^2+m_\pi{}^2)$.] Since a seagull is required by covariance we are naturally led to interpret PCAC as the assumption that, if $(\sqrt{2}/f_\pi m_\pi{}^2)\partial_\mu A_\mu^\alpha(x)$ is used as an interpolating pion field, the scattering amplitude is a slowly varying function between $q^2=0$ and $q^2=-m_\pi{}^2$ in the presence of the "minimal" seagull addition which is dictated by covariance.

There are two possible ways of making such minimal additions of the seagull terms.[3] For example, consider the case of $S_k^{\beta\alpha}$. Denoting the seagulls by $P_\mu^{\beta\alpha}(x)\delta^4(x-y)$, these are

(a) $P_k^{\beta\alpha}(x)=0$, while $P_0^{\beta\alpha}$ and $S_k^{\beta\alpha}$ are the components of a four-vector, or

(b) $P_0^{\beta\alpha}(x)=0$, while $P_k^{\beta\alpha}(x)=-S_k^{\beta\alpha}(x)$.

The choice (a) corresponds to the case in which the extra term $S_k^{\beta\alpha}$ contributes to the soft-pion limit, whereas the choice (b) trivially reproduces earlier calculations. Let us stress, however, that in either case the presence of the terms $S_k^{\beta\alpha}$ will lead to restrictions on the scaling properties of current divergences.[6]

In the following sections we limit ourselves to the choice (a) for the addition of seagull terms. In Secs. II and III we present a model for the terms $S_k^{\beta\alpha}$ and $T_k^{\beta\alpha}$, and show that the introduction of these terms leads to an understanding of the K_{l3} decays. We then show in detail in Secs. IV and V that the model also yields the usual good results of current algebra.

II. THE DECAYS $K\to\pi l\nu$ AND $\pi^\pm\to\pi^0 l\nu$

The relevant matrix element for K_{l3} decays is given by

$$\langle\pi^0(p)|V_\mu^{(4-i5)}(0)|K^+(q)\rangle = -(1/\sqrt{2})[f_+(p^2,t)(q+p)_\mu+f_-(p^2,t)(q-p)_\mu], \tag{2.1}$$

with $t=-(p-q)^2$.

Making use of the soft-pion limit and the ETC (1.1) and (1.3), we obtain

$$q_\mu[f_+(m_K{}^2,0)+f_-(m_K{}^2,0)] = q_\mu\frac{f_K}{f_\pi}-\frac{2i}{f_\pi}\langle\Omega|S_\mu^{(3,4-i5)}(0)|K^+(q)\rangle. \tag{2.2}$$

Assuming that $\xi\equiv(f_-/f_+)\cong-1$, as seems to be the case experimentally, we then obtain

$$\langle\Omega|S_\mu^{(3,4-i5)}(0)|K^+(q)\rangle\approx-iq_\mu(f_K/2). \tag{2.3}$$

A similar calculation for the π_{l3} decays yields

$$\langle\Omega|S_\mu^{(3,1-i2)}(0)|\pi^+(q)\rangle\approx0. \tag{2.4}$$

III. TRANSFORMATION PROPERTIES OF SPACE COMPONENTS OF CURRENTS

In Eqs. (2.3) and (2.4) we have obtained some of the matrix elements of the terms $S_k^{\beta\alpha}$. In this section we consider specific forms for $S_k^{\beta\alpha}$ and $T_k^{\beta\alpha}$.

A. An Unacceptable Ansatz

Let us assume as our first ansatz for these terms the following equal-time commutation relations:

$$[Q_5^\alpha(x_0),V_k^\beta(x)]=if^{\alpha\beta\gamma}(1+\lambda_1)A_k^\gamma(x) \tag{3.1}$$

and

$$[Q_5^\alpha(x_0),A_k^\beta(x)]=if^{\alpha\beta\gamma}(1+\lambda_1')V_k^\gamma(x). \tag{3.2}$$

Combining Eqs. (2.3) and (3.1) we obtain $\lambda_1\approx-1$, whereas Eqs. (2.4) and (3.1) lead to $\lambda_1\approx0$. In other words, λ_1 and λ_1' must be dependent on $SU(3)$ indices in order that Eqs. (3.1) and (3.2) be compatible with Eqs. (2.3) and (2.4). Moreover, commuting Eqs. (3.1) and (3.2) with $Q_5(x_0)$ and making use of the Jacobi identity, it can be shown that the choice $\lambda_1\approx-1$ leads to a value for λ_1' which is given by

$$\lambda_1'=-\lambda_1/(1+\lambda_1)\simeq\infty. \tag{3.3}$$

Thus, the simple modification of only altering the scale of the usual commutation relations by the introduction of the parameters λ_1 and λ_1' is physically unacceptable.

B. The Model

We now add more terms to the right-hand side of Eqs. (3.1) and (3.3) and assume, as the next simplest possibility,

$$[Q_5^\alpha(x_0),V_k^\beta(x)]=if^{\alpha\beta\gamma}(1+\lambda_1)A_k^\gamma(x)+i\lambda_3f^{\alpha\beta\gamma}\partial_k v^\gamma(x) \tag{3.4}$$

and

$$[Q_5^\alpha(x_0),A_k^\beta(x)]=if^{\alpha\beta\gamma}(1+\lambda_1')V_k^\gamma(x)+i\lambda_3'd^{\alpha\beta\gamma}\partial_k u^\gamma(x), \tag{3.5}$$

where u^γ and v^γ are members of a $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$. Thus we have

$$[Q^\alpha(x_0),u^\beta(x)]=if_{\alpha\beta\gamma}u^\gamma(x), \tag{3.6}$$

$$[Q^\alpha(x_0),v^\beta(x)]=if_{\alpha\beta\gamma}v^\gamma(x), \tag{3.7}$$

$$[Q_5^\alpha(x_0),u^\beta(x)]=-id_{\alpha\beta\gamma}v^\gamma(x), \tag{3.8}$$

and

$$[Q_5^\alpha(x_0),v^\beta(x)]=id_{\alpha\beta\gamma}u^\gamma(x). \tag{3.9}$$

In the above equations the indices β and γ in $d_{\alpha\beta\gamma}$ run from 0 to 8. It should be noted that Eqs. (3.4) and (3.5) are the most general expressions involving V_k^α, A_k^α, $\partial_k u^\alpha$, and $\partial_k v^\alpha$ which are consistent with P, C, and T invariance.

We can now commute Eqs. (3.4) and (3.5) with $Q^{\delta}(x_0)$ and $Q_5^{\delta}(x_0)$, respectively, and use the Jacobi identity to derive possible relations between (λ_1, λ_3) and (λ_1', λ_3'). On commuting with $Q^{\delta}(x_0)$ we obtain only the usual relations between the structure constants $f_{\alpha\beta\gamma}$ and the constants $d_{\alpha\beta\gamma}$. These are

$$f_{\delta\beta\gamma}f_{\alpha\gamma\rho}+f_{\delta\alpha\gamma}f_{\gamma\beta\rho}=f_{\delta\gamma\rho}f_{\alpha\beta\gamma},$$
$$d_{\delta\beta\gamma}f_{\alpha\gamma\rho}+f_{\delta\alpha\gamma}d_{\gamma\beta\rho}=d_{\alpha\beta\gamma}f_{\delta\gamma\rho}. \tag{3.10}$$

On commuting Eqs. (3.4) and (3.5) with $Q_5^{\delta}(x_0)$ we derive

$$(1+\lambda_1)(1+\lambda_1')(f_{\delta\beta\gamma}f_{\alpha\gamma\rho}-f_{\delta\gamma\rho}f_{\alpha\beta\gamma})=-f_{\delta\alpha\gamma}f_{\gamma\beta\rho} \tag{3.11}$$

and

$$\lambda_3(1+\lambda_1')(f_{\alpha\beta\gamma}f_{\delta\beta\rho}-f_{\delta\beta\gamma}f_{\alpha\gamma\rho})=\lambda_3'(d_{\delta\beta\gamma}d_{\alpha\gamma\rho}-d_{\alpha\beta\gamma}d_{\delta\gamma\rho}). \tag{3.12}$$

Application of the well-known identities satisfied by the coefficients $d_{\alpha\beta\gamma}$ and $f_{\alpha\beta\gamma}$ reduces relations (3.11) and (3.12) to

$$(1+\lambda_1)(1+\lambda_1')=1 \tag{3.13}$$

and

$$\lambda_3'(1+\lambda_1)+\lambda_3=0. \tag{3.14}$$

In the next section we present estimates for the λ's in some simple models.

IV. EVALUATION OF THE λ PARAMETERS

We expect the parameters λ_1 and λ_1' to be rather small. However, we do not expect the terms involving λ_3 and λ_3' to be negligible, because their contribution could lead to a large, negative value (≈ -1) for the ξ parameter in K_{l3} decays. In order to obtain numerical estimates, we study the commutators of Eqs. (3.4) and (3.5) in the Gell-Mann–Oakes–Renner (GOR) model[7] and in the quark model.[8]

We begin by writing Eqs. (2.3) and (2.4) in the form

$$f_K(1+\lambda_1)+\lambda_3\langle\Omega|v^{4-i5}|K^+(q)\rangle\cong 0 \tag{4.1}$$

and

$$f_\pi\lambda_1+\lambda_3\langle\Omega|v^{1-i2}|\pi^+(q)\rangle\cong 1. \tag{4.2}$$

Thus the matrix elements of the pseudoscalar densities are

$$\langle\Omega|v^{1-i2}|\pi^+(q)\rangle=-(\lambda_1/\lambda_3)f_\pi \tag{4.3}$$

and

$$\langle\Omega|v^{4-i5}|K^+(q)\rangle=-f_K(1+\lambda_1)/\lambda_3. \tag{4.4}$$

Incidentally, one may note that from Eqs. (4.3) and (4.4) it follows that the vacuum state is not $SU(3)$-invariant.

In the GOR model the matrix elements of the divergences of the axial-vector currents between the vacuum and single-meson states are given by

$$\langle\Omega|\partial_\mu A_\mu^{1,2,3}(0)|\pi(q)\rangle=[(\sqrt{2}+c)/\sqrt{3}]\langle\Omega|v^{1,2,3}(0)|\pi(q)\rangle=f_\pi m_\pi^{\,2} \tag{4.5}$$

and

$$\langle\Omega|\partial_\mu A_\mu^{4,5,6,7}(0)|K(q)\rangle=[(\sqrt{2}-\tfrac{1}{2}c)/\sqrt{3}]\langle\Omega|v^{4,5,6,7}(0)|K(q)\rangle=f_K m_K^{\,2}, \tag{4.6}$$

where c is a measure of the ratio of $SU(3)$ breaking to that of $SU(2)\otimes SU(2)$ breaking. We shall assume that $c\neq-\sqrt{2}$ and $c\neq 2\sqrt{2}$, because either of these possibilities would lead to a conserved axial-vector current and hence to $\lambda_1=\lambda_3=\lambda_1'=\lambda_3'=0$. We emphasize that we need only make use of Eqs. (4.5) and (4.6) to obtain the λ's in the GOR model, and hence do not require in this determination the full set of assumptions which are usually made in that model.

From Eqs. (4.3)–(4.6) we obtain

$$\lambda_1=\frac{m_\pi^{\,2}}{m_K^{\,2}}\,\frac{(\sqrt{2}-\frac{1}{2}c)}{[(c+\sqrt{2})-(m_\pi^{\,2}/m_K^{\,2})(\sqrt{2}-\frac{1}{2}c)]}, \tag{4.7}$$

$$\lambda_3=-\frac{c+\sqrt{2}}{\sqrt{3}}\,\frac{(\sqrt{2}-\frac{1}{2}c)}{[m_K^{\,2}(c+\sqrt{2})-m_\pi^{\,2}(\sqrt{2}-\frac{1}{2}c)]}, \tag{4.8}$$

$$\lambda_1'=-\frac{m_\pi^{\,2}}{m_K^{\,2}}\,\frac{(\sqrt{2}-\frac{1}{2}c)}{(c+\sqrt{2})}, \tag{4.9}$$

and

$$\lambda_3'=\frac{1}{m_K^{\,2}}\left(\frac{\sqrt{2}-\frac{1}{2}c}{\sqrt{3}}\right). \tag{4.10}$$

We note that λ_1 and λ_1' are of order $(m_\pi/m_K)^2$ and hence terms proportional to λ_1 and λ_1' may be neglected.

Let us next consider the quark model, and assume that the pseudoscalar densities v^α satisfy the equal-time commutation relations of the quark model. Using a saturation scheme for the matrix elements of these commutators, we may solve for the matrix elements of the v^α given in Eqs. (4.3) and (4.4) and thereby obtain another estimate for the λ parameters.

The equal-time commutators for the scalar densities u^α in the quark model are

$$\left[\int u^\alpha(x)d^3x,\int u^\beta(y)d^3y\right]=-\sqrt{3}\begin{pmatrix}8&8&8_a\\ \alpha&\beta&\gamma\end{pmatrix}\int V_0^\gamma(x)d^3x. \tag{4.11}$$

Moreover, the u^α transform as members of a $(3,\bar{3})\oplus(\bar{3},3)$ representation of $SU(3)\otimes SU(3)$. [See Eqs. (3.6)–(3.9).] We therefore have

$$[u^8(x), Q_5^{1+i2}] = -(i/\sqrt{3})v^{1+i2} \tag{4.12}$$

and

$$[u^8(x), Q_5^{4+i5}] = (i/2\sqrt{3})v^{4+i5}. \tag{4.13}$$

On evaluating matrix elements of the commutators (4.12) and (4.13) between the vacuum and a pseudoscalar meson state, we obtain

$$\langle\Omega|v^{1+i2}|\pi(q)\rangle = (\sqrt{3}/2)f_\pi\langle\pi(q)|u^8|\pi(q)\rangle \tag{4.14}$$

and

$$\langle\Omega|v^{4+i5}|K^-(q)\rangle = (-\sqrt{3})f_K\langle K^-(q)|u^8|K^-(q)\rangle. \tag{4.15}$$

We now evaluate the matrix elements of Eq. (4.11) between single-particle states and saturate the commutator with single-particle intermediate states. Let us define

$$\langle\mu,\gamma|u^\beta|\underline{8},\alpha\rangle = (4m_\alpha m_\gamma)^{1/2}\sum_\xi F_{\mu\xi}\begin{pmatrix} 8 & 8 & \mu_\xi \\ \alpha & \beta & \gamma\end{pmatrix}, \tag{4.16}$$

where $F_{\mu\xi}$ are the reduced matrix elements. Following Lee's procedure,[9] we derive relations between the $F_{\mu\xi}$ by saturating the commutator with octet and singlet states. [We assume the absence of exotic mesons belonging to the $\underline{10}$ and the $\underline{27}$ representations of $SU(3)$.] We obtain

$$F_{8_1}{}^2 + F_{8_2}{}^2 + \tfrac{1}{4}F_1{}^2 = 3, \tag{4.17}$$

$$-\tfrac{4}{5}F_{8_1}{}^2 + \tfrac{1}{4}F_1{}^2 = 0, \tag{4.18}$$

and

$$4F_{8_1}F_{8_2} = 0. \tag{4.19}$$

The solutions for F_{8_1}, F_{8_2}, and F_1 obtained from Eqs. (4.17), (4.18), and (4.19) are used to completely define the matrix elements (4.16). Then, using Eqs. (4.14) and (4.15), we solve for the λ parameters in Eqs. (4.3) and (4.4) and obtain the following sets of solutions.

Solution 1:

$$\lambda_1 = -5m_\pi/(7m_K + 5m_\pi), \quad \lambda_3 = \pm 1/(7m_K + 5m_\pi),$$
$$\lambda_1' = m_\pi/m_K, \quad \lambda_3' = \mp 1/(7m_K). \tag{4.20}$$

Solution 2:

$$\lambda_1 = -5m_\pi/(7m_K + 5m_\pi), \quad \lambda_3 = \pm 5/3(7m_K + 5m_\pi),$$
$$\lambda_1' = m_\pi/m_K, \quad \lambda_3' = \mp 5/(21m_K). \tag{4.21}$$

Solution 3:

$$\lambda_1 = -m_\pi/(3m_K + m_\pi), \quad \lambda_3 = \pm 1/5(3m_K + m_\pi),$$
$$\lambda_1' = m_\pi/3m_K, \quad \lambda_3' = \mp 1/(15m_K). \tag{4.22}$$

Solution 4:

$$\lambda_1 = -m_\pi/(3m_K + m_\pi), \quad \lambda_3 = \pm 1/3(3m_K + m_\pi),$$
$$\lambda_1' = m_\pi/9m_K, \quad \lambda_3' = \mp 1/(9m_K). \tag{4.23}$$

The content of this section may be summarized as follows: We have shown that both in the GOR model and in the quark model λ_1 and λ_1' depend on the ratio m_π/m_K and hence may be neglected. On the other hand, the constants λ_3 and λ_3' are dimensional constants which are proportional to $(1/m_K)^\alpha$, where $\alpha = 2$ in the GOR model and $\alpha = 1$ in the quark model. The terms proportional to λ_3 and λ_3' are not negligible and their presence leads to a natural explanation for the large value of the ξ parameter in the K_{l3} decays.

V. APPLICATIONS

The applications discussed in this paper will include an evaluation of K_{l4} decays, the nonleptonic hyperon decays, the CP-conserving nonleptonic decays of K mesons, and pion photoproduction. In each instance we show explicitly that the modifications of the ETC between time and space components of currents expressed by Eqs. (3.4) and (3.5) do not significantly alter the results based on an application of the usual current-algebra commutators and the soft-pion limit.

A. The K_{l4} Form Factors

The matrix element of the axial-vector current in the K_{l4} decays is defined by

$$\langle\pi^\alpha(q)\pi^\beta(p)|A_\lambda^\gamma(0)|K^\delta(k)\rangle = i(2\pi)^{-9/2}(8q_0p_0k_0)^{-1/2}(m_K)^{-1}[F_1(q+p)_\lambda + F_2(q-p)_\lambda + F_3(k-q-p)_\lambda], \tag{5.1}$$

where the form factors F_1, F_2, and F_3 are functions of the variables

$$s = -(q+p)^2, \quad t = -(k-q-p)^2, \quad \text{and} \quad \eta = k\cdot(q-p).$$

Following Weinberg,[10] we reduce out both the pions and use the PCAC relation

$$\partial_\mu A_\mu^\alpha(x) = (f_\pi/\sqrt{2})m_\pi{}^2\varphi^\alpha(x)$$

to obtain

$$-\tfrac{1}{2}f_\pi{}^2 m_\pi{}^4 \frac{(4q_0p_0)^{1/2}(2\pi)^3}{(q^2+m_\pi{}^2)(p^2+m_\pi{}^2)}\langle\pi^\alpha(q);\pi^\beta(p)|A^\gamma_\lambda(0)|K^\delta(k)\rangle$$
$$=\int\int d^4x\,d^4y\,e^{-i(qx+py)}\langle\Omega|T(\partial_\mu A^\alpha_\mu(x);\partial_\nu A^\beta_\nu(y);A^\gamma_\lambda(0))|K^\delta(k)\rangle. \tag{5.2}$$

The time-ordered product in (5.2) can be expanded as follows:

$$\begin{aligned}T(\partial_\mu A^\alpha_\mu(x);\partial_\nu A^\beta_\nu(y);A^\gamma_\lambda(0)) &= \partial_\mu\partial_\nu T(A^\alpha_\mu(x),A^\beta_\nu(y),A^\gamma_\lambda(0)) - \delta(x_0-y_0)T([A^\beta_0(y),\partial_\mu A^\alpha_\mu(x)]A^\gamma_\lambda(0))\\ &-\tfrac{1}{2}\delta(x_0-y_0)T\left(\left(\frac{\partial}{\partial y_\nu}+\frac{\partial}{\partial x_\nu}\right)[A^\alpha_0(x),A^\beta_\nu(y)]A^\gamma_\lambda(0)\right)\\ &-\tfrac{1}{2}\left(\frac{\partial}{\partial y_\nu}-\frac{\partial}{\partial x_\nu}\right)\delta(x_0-y_0)T([A^\alpha_0(x),A^\beta_\nu(y)]A^\gamma_\lambda(0))\\ &-\tfrac{1}{2}\delta(x_0)\delta(y_0)\{[A^\beta_0(y),[A^\alpha_0(x),A^\gamma_\lambda(0)]]+[A^\alpha_0(x),[A^\beta_0(y),A^\gamma_\lambda(0)]]\}\\ &-\delta(x_0)T([A^\alpha_0(x),A^\gamma_\lambda(0)]\,\partial_\nu A^\beta_\nu(y))-\delta(y_0)T([A^\beta_0(y),A^\gamma_\lambda(0)]\,\partial_\mu A^\alpha_\mu(x)).\end{aligned} \tag{5.3}$$

We utilize the commutators of Eqs. (1.1), (1.2), (3.4), and (3.5) to simplify the time-ordered product (5.3). As is usually done,[10] we neglect the first three terms. The first term is of order $(q\cdot p)$ and can be neglected; the second term is the σ term and its contribution is expected to be small; for the third term we write

$$-\tfrac{1}{2}\delta(x_0-y_0)T\left(\left(\frac{\partial}{\partial y_\nu}+\frac{\partial}{\partial x_\nu}\right)[if_{\alpha\beta\epsilon}(1+\lambda_1'\delta_{\nu i})V^\epsilon_\nu+id_{\alpha\beta\epsilon}\lambda_3'\delta_{\nu i}\partial_\nu u^\epsilon]\,\delta^3(x-y)A^\gamma_\lambda(0)\right),$$

where the $\delta_{\nu i}$ $(i=1,2,3)$ terms arise from the commutators of Eqs. (3.4) and (3.5). The divergence of the conserved vector current is zero, and the above expression is again of order $\vec{q}^{\,2}$ and hence can be neglected. The fourth term includes a K-meson pole and cannot be dropped. The fifth term has two double commutators and it is straightforward to show, using Eqs. (3.14) and (3.15), that its contribution to the physical K_{l4} decays is unaltered by the presence of the extra terms in the commutators (3.4) and (3.5). The last two terms do have extra terms proportional to derivatives of the scalar densities u^α, which originate in our modification of the usual time-space commutation relations.

On retaining only terms of order p and q, we obtain

$$\begin{aligned}-\tfrac{1}{2}f_\pi{}^2 m_\pi{}^4 &\frac{(4q_0p_0)^{1/2}(2\pi)^3}{(q^2+m_\pi{}^2)(p^2+m_\pi{}^2)}\langle\pi^\alpha(q);\pi^\beta(p)|A^\gamma_\lambda(0)|K^\delta(k)\rangle\\ &=\tfrac{1}{2}(f_{\alpha\gamma\epsilon}f_{\beta\epsilon\rho}+f_{\beta\gamma\epsilon}f_{\alpha\epsilon\rho})\langle\Omega|A^\rho_\lambda(0)|K^\delta(k)\rangle\\ &-\frac{f_\pi m_\pi{}^2(2\pi)^{3/2}(2q_0)^{1/2}}{\sqrt{2}\,(q^2+m_\pi{}^2)}\langle\pi^\beta(p)|[f_{\alpha\gamma\epsilon}(1+\lambda_1'\delta_{\nu i})V^\epsilon_\lambda+\lambda_3'\delta_{\nu i}d_{\alpha\gamma\epsilon}\partial_\lambda u^\epsilon]|K^\delta(k)\rangle\\ &-\frac{f_\pi m_\pi{}^2(2\pi)^{3/2}(2q_0)^{1/2}}{\sqrt{2}\,(p^2+m_\pi{}^2)}\langle\pi^\alpha(q)|[f_{\beta\gamma\epsilon}(1+\lambda_1'\delta_{\nu i})V^\epsilon_\lambda+\lambda_3'\delta_{\nu i}d_{\beta\gamma\epsilon}\partial_\lambda u^\epsilon]|K^\delta(k)\rangle\\ &+\tfrac{1}{2}(p-q)_\nu\int d^4x\,e^{-i(q+p)x}\langle\Omega|T([f_{\alpha\beta\epsilon}(1+\lambda_1'\delta_{\nu i})V^\epsilon_\nu(x)+d_{\alpha\beta\epsilon}\lambda_3'\delta_{\nu i}\partial_\nu u(x)]A^\gamma_\lambda(0))|K^\delta(k)\rangle.\end{aligned} \tag{5.4}$$

The matrix element of the vector current between a K-meson state and a pion state is given in terms of the K_{l3} form factors [see Eq. (2.1)]. For the present calculation we shall use the value $\xi=f_-/f_+\cong-1$ for the ratio of form factors. However, it may be shown that our results hold for all values of ξ. (See the discussion at the end of this subsection.) We also know the matrix elements of the scalar densities u^α between meson states, since by an application of the soft-pion technique they may be related to the matrix elements of the pseudoscalar densities v^α between the vacuum and one-meson states.[7] Then, using Eqs. (4.3) and (4.4), we may express these matrix elements in terms of the λ parameters. We now evaluate the contribution of the last term in Eq. (5.4).

Consider the matrix elements

$$M^f_{\nu\lambda}(k,p)=\int d^4x\, e^{-ipx}\langle\Omega|T((1+\lambda_1'\,\delta_{\nu i})V^\epsilon_\nu(x)A^\gamma_\lambda(0))|K^\delta(k)\rangle \tag{5.5}$$

and

$$\delta_{\nu i}M^d_{\nu\lambda}(k,p)=\int d^4x\, e^{-ipx}\langle\Omega|T(\lambda_3'\,\delta_{\nu i}\,\partial_\nu u^\epsilon(x)A^\gamma_\lambda(0))|K^\delta(k)\rangle . \tag{5.6}$$

Again following Weinberg,[10] we apply Low's procedure for evaluating these matrix elements to $O(p)$ and $O(q)$. The matrix element $M^f_{\nu\lambda}$ is essentially the one evaluated by Weinberg.[10] The matrix element $M^d_{\nu\lambda}\delta_{\nu i}$ can be expanded in a power series in k and p and we have

$$\delta_{\nu i}M^d_{\nu\lambda}(k,p)=(2\pi)^{-3/2}(2k_0)^{-1/2}\left[-\frac{f_K(k-p)_\lambda p_\nu(ig)\delta_{\nu i}}{2k\cdot p-p^2}+\delta_{\nu i}(a_1+k_\lambda k_\nu a_2+p_\lambda k_\nu a_3+k_\lambda p_\nu a_4+p_\lambda p_\nu a_5+\cdots)\right], \tag{5.7}$$

where the constant g is defined by the matrix element

$$\langle P^\sigma(p)|\partial_\mu u^\epsilon|P^\delta(k)\rangle=ig_{\delta\epsilon\sigma}(k-p)_\mu . \tag{5.8}$$

On contracting with p_ν, the left side of Eq. (5.7) contains only terms $O(p^2)$ and hence is negligible within our approximations. (The $\delta_{\nu i}$ factor requires that there be no ETC terms in $p_\nu M^d_{\nu\lambda}$.) On the right side of Eq. (5.7) the pole term is $O(p^2)$ and hence it is also negligible. Thus all the coefficients a_i in Eq. (5.7) are zero and the term $p_\nu M^d_{\nu\lambda}\delta_{\nu i}$ does not contribute to the K_{l4} decays. So all the terms in Eq. (5.4) are known up to $O(p)$ and $O(q)$. It is now straightforward to extract the form factors in the various K_{l4} decays after covariantizing the expressions using the prescription discussed in Sec. I.

The final expressions for the form factors are given below, with $A=m_K/f_\pi$, and $B=m_Kf_K/f_\pi{}^2$.

(1) $\bar K^0\to\pi^+\pi^0 l\bar\nu_l$:

$$F_1^{+0}=0,\quad F_2^{+0}=\sqrt2[-(1+\lambda_1')A+\lambda_1'B/2],\quad F_3^{+0}=(1+\lambda_1')(B/\sqrt2)[k\cdot(p-q)/k\cdot(p+q)]. \tag{5.9}$$

(2) $K^-\to\pi^0\pi^0 l\bar\nu_l$:

$$F_1^{00}=-A,\quad F_2^{00}=0,\quad\text{and}\quad F_3^{00}=-B/2. \tag{5.10}$$

(3) $K^-\to\pi^+\pi^- l\bar\nu_l$:

$$F_1^{+-}=-(1+\lambda_1')A,\quad F_2^{+-}=(1+\lambda_1')A+\lambda_1'B/2,\quad F_3^{+-}=-\tfrac12 B[1+(1+\lambda_1')k\cdot(p-q)/k\cdot(p+q)]. \tag{5.11}$$

We have already shown in Sec. IV that the parameter λ_1' is negligibly small. We are thus able to reproduce the results obtained by Weinberg[10] even though we use $\xi=-1$. In particular, we obtain $|F_1^{+-}|=|F_2^{+-}|$, and the decay rate for $K^+\to\pi^+\pi^-e^+\nu_e$ is given by[11]

$$\Gamma(K^+\to\pi^+\pi^-e^+\nu_e)=2.53\times10^3\ \text{sec}^{-1}. \tag{5.12}$$

This agrees with the experimental result of Ely *et al.*[12] based on approximately 300 events, which is $\Gamma_{+-}=(2.60\pm0.30)\times10^3\ \text{sec}^{-1}$ with four possible solutions for F_2^{+-}/F_1^{+-}, namely, -1.61 ± 0.15, 1.33 ± 0.17, -1.66 ± 0.15, or 1.12 ± 0.13.

It may appear surprising at first glance that Weinberg's results are reproduced in our model even though we make use of Eqs. (3.4) and (3.5) rather than of the usual ETC between time and space components of currents. However, this may be easily understood upon noticing that he makes use of the Callan-Treiman-Mathur-Okubo-Pandit (CTMOP) relation[13] while we, on the other hand, also allow for a large negative value of the ξ parameter, as suggested by recent experiments. In fact, if one were to assume that $\xi=-1$ in Weinberg's derivation, one would *not* obtain results which agree with the experimental ones. The effect of the extra terms in Eqs. (3.4) and (3.5) is to compensate for the deviations from the CTMOP relation so that Weinberg's results hold. Let us note that although we have presented our derivation only for the value $\xi=-1$, the same results may be obtained for all ξ. To show this it suffices to note that Eqs. (4.3) and (4.4) are then correspondingly modified, and the compensation referred to above still occurs.

B. The Nonleptonic Hyperon Decays

We shall discuss the nonleptonic hyperon decays within the framework of the current × current model. The weak-interaction Hamiltonian in this model is given by

$$H_w=(G/\sqrt2)\{J_\mu,J_\mu^\dagger\}, \tag{5.13}$$

where

$$J_\mu = J_\mu\,(\text{leptonic}) + \cos\theta\,(V_\mu + A_\mu)^{(1+i2)} + \sin\theta\,(V_\mu + A_\mu)^{(4+i5)}. \tag{5.14}$$

Following Sugawara and Suzuki,[14] we evaluate the decays $B^\alpha(p) \to B^\beta(p') + \pi^\gamma(q)$ in the soft-pion limit from the following identity:

$$\lim_{q_\mu \to 0} [-i(2q_0)^{1/2}(2\pi)^{3/2}\langle B^\beta(p');\,\pi^\gamma(q)|H_w(0)|B^\alpha(p)\rangle]$$
$$= -(\sqrt{2}/f_\pi)\langle B^\beta|[Q_5^\gamma, H_w(0)]|B^\alpha\rangle + \lim_{q_\mu \to 0}[\sqrt{2}\,(q^2+m_\pi{}^2)/f_\pi m_\pi{}^2]\int d^4x\, e^{-iqx}\langle B(p')|\theta(x_0)[iq_\mu A_\mu^\gamma(x), H_w(0)]|B^\alpha(p)\rangle. \tag{5.15}$$

The Hamiltonian (5.13) is CP-invariant and the terms responsible for the strangeness-changing nonleptonic decays are symmetric under the T-L transformations[15] [i.e., symmetric under the interchange of the indices $2 \leftrightarrow 3$ in the $SU(3)$ tensor notation]. From this it follows that only the ETC term on the right side of Eq. (5.15) contributes to the s-wave parity-violating (p.v.) decay amplitudes. The retarded commutator terms in Eq. (5.15) contain baryon-pole terms which contribute only to the p-wave parity-conserving (p.c.) decay amplitudes.[16]

With the modifications of the usual current commutation relations [see Eqs. (1.1), (1.2), (3.4), and (3.5)] the usual chiral properties of the Hamiltonian no longer hold:

$$[Q_5^\alpha, H_w(0)] \neq [Q^\alpha, H_w(0)]. \tag{5.16}$$

Despite this, the sum rules for the p.v. amplitudes derived by Sugawara and Suzuki follow in our model. In order to demonstrate this, let us consider for simplicity the decay modes in which a neutral pion is emitted. The ETC of Eq. (5.15) is given by

$$(-\sqrt{2}/f_\pi)[Q_5^3, H_w(\text{p.v.})]$$
$$= -(iG\sin\theta\cos\theta/2f_\pi)\{[(S^{2,4}+P^{2,4}) - (S^{1,5}+P^{1,5})] + \delta_{\mu k}[\lambda_1(P^{2,4}-P^{1,5}) + \lambda_1'(S^{2,4}-S^{1,5})]$$
$$+ \delta_{\mu k}\lambda_3[2\{\partial_\mu v^2, A_\mu^4\} - \{\partial_\mu v^4, A_\mu^2\} - 2\{\partial_\mu v^1, A_\mu^5\} + \{\partial_\mu v^5, A_\mu^1\}]$$
$$+ \delta_{\mu k}\lambda_3'[\{\partial_\mu u^4, V_\mu^1\} + \{\partial_\mu u^5, V_\mu^2\}]\}, \tag{5.17}$$

where

$$S^{\alpha,\beta} = \{V_\mu^\alpha, V_\mu^\beta\}, \quad P^{\alpha,\beta} = \{A_\mu^\alpha, A_\mu^\beta\}. \tag{5.18}$$

The matrix elements of the symmetrized products of currents occurring in Eq. (5.17), evaluated between single baryon states, yield the p.v. decay amplitudes. (Before evaluating the matrix elements, these symmetrized products are made covariant by the "minimal" additions of seagull terms, as discussed in Sec. I.) We note that the terms proportional to λ_3' do not contribute to the matrix elements because, by a partial integration, they can be expressed in the form $\{u^\alpha, \partial_\mu V_\mu^\beta\}$, and such terms are zero by the conserved-vector-current (CVC) hypothesis. All the other symmetrized products $S^{\alpha,\beta}$, $P^{\alpha,\beta}$, and $\{\partial_\mu v^\alpha, A_\mu^\beta\}$ can be expressed as $\underline{27}$ and $\underline{8}_s$ $SU(3)$ tensors. We have therefore reproduced the conditions under which Sugawara and Suzuki[14] derived their sum rules, viz.,

$$A(\Lambda_-^0) = -\sqrt{2}A(\Lambda_0^0), \tag{5.19a}$$

$$A(\Xi_-^-) = \sqrt{2}A(\Xi_0^0), \tag{5.19b}$$

$$\sqrt{2}A(\Sigma_0^+) + A(\Sigma_-^-) = -A(\Sigma_+^+), \tag{5.19c}$$

and

$$2A(\Xi_-^-) - A(\Lambda_-^0) = (\tfrac{3}{2})^{1/2}A(\Sigma_-^-) = -\sqrt{3}A(\Sigma_0^+) - (\tfrac{3}{2})^{1/2}A(\Sigma_+^+). \tag{5.19d}$$

The amplitudes in Eqs. (5.19) have been expressed in the standard notation. The amplitude $A(\Sigma_+^+)$ obtains contributions only from the 27-plet tensors contained in Eq. (5.17). In the limit of octet dominance $A(\Sigma_+^+) = 0$, and the Lee-Sugawara relation[17]

$$2A(\Xi_-^-) - A(\Lambda_-^0) = -\sqrt{3}A(\Sigma_0^+) \tag{5.20}$$

now follows from Eq. (5.19d).

All the decay amplitudes can be expressed in terms of three parameters which correspond to the $\underline{27}$, $\underline{8}_s$,

$\underline{8}_a$ matrix elements of Eq. (5.17) between single baryon states. These parameters can actually be evaluated if the saturation scheme of Chiu, Schechter, and Ueda[18] is adopted. The procedure is to evaluate the symmetrized products of currents by inserting a complete set of intermediate states and assuming that the octet and decuplet baryon states alone saturate the sum over intermediate states. Further assumptions have to be made regarding the behavior of various form factors of the currents at large momentum transfer, before numerical estimates can be obtained. It has been shown in Sec. IV that the coefficients λ_1 and λ_1' are small and hence terms proportional to these coefficients can be dropped forthwith. The matrix elements of the terms $\{\partial_\mu v^\alpha, A_\mu^\beta\} = -\{v^\alpha, \partial_\mu A_\mu^\beta\}$ are model-dependent to the extent that the matrix elements of v^α between baryon states are not known. However, if we assume that the f/d ratios in these matrix elements and in the matrix elements of the axial-vector currents are the same, and are close to $(\frac{1}{3})^{1/2}$, then these terms will show octet dominance irrespective of the nature of the form factors.[19] The contribution of these terms must now be added on to the contribution from the usual $S^{\alpha,\beta}$ and $P^{\alpha,\beta}$ terms.

So far we have shown that the modifications introduced in the current commutation relations do not alter the results of Sugawara and Suzuki for the s-wave decay amplitudes. Since the ETC terms do not contribute to the p-wave amplitudes, these modifications also do not affect the parity-conserving amplitudes.

C. The Nonleptonic Decays of K Mesons

Throughout the following discussion we neglect CP-violations in K-meson decays and we assume that the weak Hamiltonian responsible for the decays $K\to 2\pi$ and $K\to 3\pi$ is octet-dominant. A satisfactory theory of octet enhancement for the decays $K\to 3\pi$ does not exist; however, for $K\to 2\pi$ the saturation scheme of Chiu, Schechter, and Ueda[20] has at least yielded a qualitative understanding of this effect in this case. If we also assume, together with Hara and Nambu,[21] that the weak Hamiltonian is a sum of scalar and pseudoscalar densities which have the quark-model[8] commutation relations with the charges Q^α and Q_5^α, then the well-known results of Hara and Nambu, and of Abarbanel[21] for the decays $K\to 3\pi$ obviously remain unaffected. To see this, we note that our assumptions (3.4) and (3.5) regarding the ETC's of current components do not affect the commutator

$$[Q_5^\alpha, H_w] = [Q_5^\alpha, (\bar q\lambda_6 q) \pm i(\bar q\lambda_7\gamma_5 q)]. \tag{5.21}$$

In particular, the current-algebra predictions for the slopes S defined by[22]

$$|A(K^\delta \to \pi^\alpha\pi^\beta\pi^\gamma)|^2 = |A_{\text{av}}|^2(1+SQy/m_K) \tag{5.22}$$

remain unchanged. In Eq. (5.22) A_{av} is the amplitude averaged over the Dalitz plot, Q is the energy release in the reaction, and y is the Dalitz-plot variable corresponding to the energy of the "odd" pion.

Let us now study the effects of our modification [Eqs. (3.4) and (3.5)] of the usual current commutation relations on the calculations based on the current×current model. Since experiments have shown no significant violations of the $\Delta T = \frac{1}{2}$ rule,[23] we will adopt the point of view of Hara and Nambu,[21] and either (a) drop the 27-plet terms in the Hamiltonian of Eq. (5.13) constructed from charged Cabibbo currents, or (b) introduce neutral currents to express the octet Hamiltonian in the form

$$H_w^{(6)} = \mathcal{G}\, d_{6\alpha\beta}\{J_\mu^{(\alpha)}, J_\mu^{(\beta)}\}, \tag{5.23}$$

where $\mathcal{G}$ is an effective weak-interaction constant. We next recall that in the soft-pion limit the $K\to 3\pi$ decay matrix elements are given by

$$\lim_{q_{1\mu}\to 0} [(2q_0)^{1/2}(2\pi)^{3/2}\langle\pi^\alpha(q_1), \pi^\beta(q_2), \pi^\gamma(q_3)|H_w(\text{p.c.})|K^\delta(k)\rangle] = -i(\sqrt{2}/f_\pi)\langle\pi^\beta(q_2), \pi^\gamma(q_3)|[Q_5^\alpha, H_w(\text{p.c.})]|K^\delta(k)\rangle. \tag{5.24}$$

Using the ETC for the currents defined by Eqs. (1.1), (3.4), and (3.5) we can evaluate the commutator of Eq. (5.24). As mentioned before [see Eq. (5.16)] the usual chiral properties of the Hamiltonian no longer hold. Thus octet dominance of the current×current interaction no longer ensures that the matrix elements of the commutator in Eq. (5.24) will exhibit this property. We shall therefore have to assume that the 27-plet parts of the matrix elements of the ETC occurring in Eq. (5.24) vanish. We then obtain the Hara-Nambu[21] constraints on the decay amplitudes for the current×current Hamiltonian, whether or not we introduce neutral currents:

$$\begin{aligned} A(K^+\to\pi^-\pi^+\pi^+;\, q(\pi^-)=0) &= A(K^+\to\pi^+\pi_1^0\pi_2^0;\, q(\pi^0)=0) \\ &= A(K_2^0\to\pi^0\pi^+\pi^-;\, q(\pi^\pm)=0) = 0, \end{aligned} \tag{5.25}$$

$$A(K^+ \to \pi^-\pi^+\pi^+;\ q(\pi^+)=0) = A(K^+ \to \pi^+\pi^0\pi^0;\ q(\pi^+)=0)$$
$$= -i(1/f_\pi)\langle \pi^+\pi^- | [Q_5^{(1-i2)}, H_w(\text{p.c.})] | K^+\rangle = A(K^+), \tag{5.26}$$

and

$$A(K_2^0 \to \pi^0\pi^+\pi^-;\ q(\pi^0)=0) = A(K_2^0 \to 3\pi^0;\ q(\pi^0)=0)$$
$$= -i(\sqrt{2}/f_\pi)\langle \pi^+\pi^- | [Q_5^{(3)}, H_w(\text{p.c.})] | K_2^0\rangle = A(K_2^0). \tag{5.27}$$

We first note that if the amplitudes are expanded in terms of quadratic functions of the meson momenta, with the condition that in the physical limit the amplitudes are linear in the energy of the "odd" pion,

$$A(K^\delta(k) \to \pi^\alpha(q_1), \pi^\beta(q_2), \pi^\gamma(q_3)) = a + bk^2 + c(q_2{}^2 + q_3{}^2) + d(q_1{}^2) + e[(k-q_2)^2 + (k-q_3)^2] + f(k-q_1)^2, \tag{5.28}$$

then the constraints (5.25), (5.26), and (5.27) lead to the usual predictions for the slope parameters[21]

$$S(K_2^0 \to \pi^+\pi^-\pi^0) = S_{+-0} \cong -4Q/m_{K^0} \cong 0.30, \tag{5.29a}$$

$$S(K^+ \to \pi^+\pi^0\pi^0) = S_{+00} \cong -4Q/m_{K^+}, \tag{5.29b}$$

and

$$S(K^+ \to \pi^+\pi^-\pi^+) = S_{+-+} \cong 2Q/m_{K^+}. \tag{5.29c}$$

These results are in excellent agreement with the experimentally measured values. It should also be noted that the above results are independent of the magnitudes of the constants $A(K^+)$ and $A(K_2^0)$ of Eqs. (5.28) and (5.27).

We now evaluate the commutators of Eqs. (5.26) and (5.27). If the usual current commutation relations were used, we would be able to relate the amplitudes for $K \to 3\pi$ and $K \to 2\pi$. However, in the exact symmetry limit in the current×current model the CP-conserving $K \to 2\pi$ amplitudes are zero. Thus an application of current algebra leads to the unsatisfactory result that in the symmetry limit the $K \to 3\pi$ amplitudes are also zero. The usual way of avoiding this difficulty is to invoke symmetry-breaking effects to say that the $K \to 2\pi$ amplitudes do not actually vanish. On the other hand, the application of the commutation relations of Eqs. (3.4) and (3.5) leads to a different situation. Using the Hamiltonian of Eq. (5.23), we obtain

$$[Q_5^\alpha, H_w^{(6)}(\text{p.c.})] = i\mathfrak{g}\, d_{6\nu_1\nu_2}\{(f_{\alpha\nu_1\rho}\{V_\mu^{\nu_2}, [(1+\delta_{\mu k}\lambda_1)A_\mu^\rho + \lambda_3\delta_{\mu k}\partial_k v^\rho]\}$$
$$+ f_{\alpha\nu_1\rho}\{A_\mu^{\nu_2}, (1+\delta_{\mu k}\lambda_1')V_\mu^\rho\} + d_{\alpha\nu_1\rho}\{A_\mu^{\nu_2}, \lambda_3'\delta_{\mu k}\partial_k u^\rho\}) + (\nu_1 \leftrightarrow \nu_2)\}, \tag{5.30}$$

where $d_{\alpha\nu_1\rho}$ includes terms with $\rho=0$ and $d_{\alpha\nu_1 0} = (\frac{2}{3})^{1/2}\delta_{\alpha\nu_1}$. Just as in the case of hyperon decays, we shall ignore terms proportional to the parameters λ_1 and λ_1' which are negligible. Making use of a partial integration, we have $\{V_\mu, \partial_\mu v\} = -\{\partial_\mu V_\mu, v\}$ and we can therefore drop terms proportional to λ_3 on invoking the CVC hypothesis. Retaining only terms which transform as octet tensors under $SU(3)$, we have

$$[Q_5^\alpha, H_w^{(6)}(\text{p.c.})] = -2f_{6\alpha\lambda}H_w^{(\lambda)}(\text{p.v.}) + i\mathfrak{g}\, d_{6\nu_1\nu_2}\delta_{\mu k}\lambda_3'[(d_{\alpha\nu_1\rho}\{A_\mu^{\nu_2}, \partial_k u^\rho\} + (\tfrac{2}{3})^{1/2}\{A_\mu^{\nu_2}, \partial_k u^0\}) + (\nu_1 \leftrightarrow \nu_2)]$$
$$= -2f_{6\alpha\lambda}H_w^{(\lambda)}(\text{p.v.}) + i\lambda_3'[-\tfrac{3}{5}d_{6\alpha\lambda}\bar{H}_w^{(\lambda)} + 2(\tfrac{2}{3})^{1/2}d_{6\alpha\lambda}\bar{H}_w'^{(\lambda)}]. \tag{5.31}$$

In the above expression we have separated the contributions from $\partial_\mu u^\alpha$ and $\partial_\mu u^0$, and also made use of the coefficients $d_{\alpha\beta\gamma}$ to project the octet parts out of the symmetrized products of operators. Octet dominance of the matrix elements of the commutator of Eq. (5.31) then leads to the prediction that

$$A(K^+) = -A(K_2^0) \tag{5.32}$$

only for the Hamiltonian of Eq. (5.23). In the case of the Hamiltonian constructed out of only the charged Cabibbo currents the relation (5.32), which is a consequence of the $\Delta T = \frac{1}{2}$ rule, follows only if in addition we assume that

$$(\tfrac{2}{3})^{1/2}\langle\{\partial_\mu u^0, A_\mu^{(4+i5)}\}\rangle = \tfrac{1}{6}\langle\{\partial_\mu u^8, A_\mu^{(4+i5)}\}\rangle \tag{5.33}$$

for the matrix elements between a K state and the 2π final state.

As usual, in the exact symmetry limit the matrix elements of the first term in Eq. (5.31) vanish. However, since the matrix elements of the terms proportional to λ_3' are not related to the physical $K \to 2\pi$ amplitudes, the $K \to 3\pi$ amplitudes are nonvanishing. When symmetry breaking is introduced, the decay amplitudes are given in terms of the amplitudes for $K \to 2\pi$ and the above unknown matrix elements. An evaluation of these matrix elements is in progress at present.

D. The Kroll-Ruderman Theorem

The Kroll-Ruderman theorem[24] states that, up to the first order in photon momenta, the scattering amplitude for processes involving photons is given by the Born approximation. For pion photoproduction this result also follows by making use of usual current-algebra ETC and the soft-pion limit.[25] It is the purpose of this section to show that our modification of current algebra does not alter this fact. To see this it suffices to note that the ETC which is relevant for these processes is

$$[Q_5^\alpha, V_\mu^{\text{e.m.}}(0)] = [Q_5^\alpha, V_\mu^3(0) + (1/\sqrt{3})V_\mu^8(0)]. \quad (5.34)$$

We make use of Eq. (3.4) to evaluate the above commutator, and note that for these cases we should consider the matrix element of Eq. (5.34) only between states of equal momentum. In addition to the usual current-algebra term we may have possible contributions from the terms

$$\langle N|\{i\lambda_1[f_{\alpha 3\gamma} + (1/\sqrt{3})f_{\alpha 8\gamma}]A_\mu^\gamma + i\lambda_3[f_{\alpha 3\gamma} + (1/\sqrt{3})f_{\alpha 8\gamma}]\,\partial_\mu v^\gamma\}|N\rangle . \quad (5.35)$$

These terms naturally vanish since λ_1 is negligible and $\langle N|\partial_\mu v|N\rangle$ is zero for zero momentum transfer. The Kroll-Ruderman theorem thus follows in the usual manner.[25]

VI. CONCLUSIONS

Arguments based on Lorentz covariance indicate that the ETC between the time and the space components of currents may have terms, in addition to the usual terms, proportional to the space components of currents.[4,5] These additional terms would affect all earlier calculations involving equal-time current commutation relations, which arise in the soft-pion limit. We have presented a model for these terms [Eqs. (3.4) and (3.5)], and we have shown that the model leads to a simple explanation for the large negative values of the ξ parameter in K_{l3} decays. We have demonstrated that the presence of such terms is sufficient to ensure that Weinberg's results[10] for K_{l4} decays hold, whatever be the value of ξ. Furthermore, the Sugawara-Suzuki sum rules[14] for the nonleptonic decays of hyperons are maintained. The results for the nonleptonic decays of K mesons would suggest that the current×current model for the weak Hamiltonian which includes neutral currents is preferred over the Hamiltonian constructed out of charged Cabibbo currents, because one additional assumption is necessary in the latter model in order to maintain the results of the $\Delta T = \frac{1}{2}$ rule. This point is under study at present. The proof of the Kroll-Ruderman theorem based on the soft-pion limit and current algebra also remains unchanged.

Thus the modifications in Eqs. (3.4) and (3.5) of the usual current commutation relations do not alter the successful results of current algebra. A possible way of experimentally detecting the presence of the additional terms in these commutators would be to study the scaling properties of current divergences.[6] It can be shown[6] that if the dimensions of the current divergences are less than two, such modifications must be absent.

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Duality and Single-Particle Production*

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Following Mueller, we relate the single-particle production cross section for the process $a+b \to x+$ anything to a discontinuity of the six-line amplitude $a+b+\bar{x} \to a+b+\bar{x}$. Using the dual-resonance model for the six-line amplitude, we obtain an explicit form for the production cross section at high energies. The formula exhibits the expected features of limiting fragmentation, an invariant central region, and triple-Regge asymptotic behavior. In addition, it has a universal cutoff in transverse momentum of the form $e^{-4bp_\perp^2}$ in the central region, where b is the universal trajectory slope. We discuss for particle production some general consequences of duality in the missing mass. For example, we relate the behavior of two-body scattering amplitudes at wide angles to the transverse-momentum dependence of production cross sections. Finally, we discuss the possible experimental relevance of our results.

I. INTRODUCTION

Despite their phenomenological shortcomings, dual-resonance models[1] (DRM) have proven to be an extremely valuable theoretical laboratory for investigating the consequences for scattering amplitudes of the requirements of analyticity, crossing, and Regge asymptotic behavior in the absence of constraints imposed by unitarity. It is remarkable that two-body DRM scattering amplitudes possess such phenomenologically plausible high-energy features as narrow forward peaks and an exponential decrease at fixed wide angles, even though achieving thorough agreement with experiment seems to be impractical.[2]

Mueller[3] has discovered an ingenious method for describing single-particle production at high energies in general terms, using Regge-pole phenomenology. He has shown that single-pole dominance in Regge exchanges at high energies leads to a limiting distribution of produced particle momenta, i.e., the distribution has the property of limiting fragmentation,[4] an invariant central region, and triple-Regge behavior.[5] However, the Regge assumption alone does not provide an explicit description of the shape of the limiting distribution, and in particular does not explain the experimentally observed cutoff in transverse momenta. One must look to specific models to explore these questions.

The DRM has the Regge behavior required to produce a limiting distribution. Accordingly, we have applied the DRM to a study of the single-particle distribution.