Nonlocal Symmetry Operations, the Cabibbo Angle, and the Leptonic Decays of Vector Mesons*

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(Received 27 July 1970)

Symmetries which are not exact but which leave the vacuum invariant cannot be implemented locally. In this note we consider simple nonlocal symmetries. For the in- and out-fields, the nonlocality involved can be simply some trivial scale transformations that accompany the internal-symmetry index transformations. In particular, with proper relative normalizations, a set of in-fields $\hat{\phi}_\alpha^{\rm in}(0)$ will transform irreducibly under the nonlocal group. If one assumes that the vector currents $V_\alpha^\mu(0)$ behave like vector-meson fields $\Box\hat{\phi}^\mu_{\alpha,\,\rm in}(0)$, as far as the vacuum to one-vector-meson matrix elements are concerned, one finds $\tan\theta_Y = \tan\theta_B(m_\phi/m_\omega)^2$ and $\frac{1}{3}m_\rho\Gamma(\rho\to ee) = m_\omega\Gamma(\omega\to ee) + m_\phi\Gamma(\phi\to ee)$, both previously derived from Weinberg's first sum rule. If one assumes that the weak axial-vector currents behave like the pseudoscalar meson fields $\partial_\mu\hat{\phi}_\alpha^{\rm in}(0)$, as far as the vacuum to one-pseudoscalar-meson matrix elements are concerned, one finds $m_\pi/m_K = (f_K/f_\pi)\tan\theta_A$.

I. INTRODUCTION

Internal symmetries of the isospin-invariance type are usually assumed to act locally; that is, if G is an internal-symmetry transformation, and if A and B are any two fields that are relatively local, then

$$[GA(x)G^{-1}, B(y)] = 0 \quad \text{for } (x-y)^2 < 0, \tag{1}$$

where the bracket denotes a commutator or an anticommutator as appropriate for the type of fields. Often it is also assumed that G leaves the vacuum invariant. It has been shown recently[1] that each such G must commute with the Poincaré group, and hence with the Hamiltonian. There can be no mass splitting within a multiplet for a local internal-symmetry group, if the vacuum is unique and invariant under the symmetry transformations.

The strong-interaction symmetries such as $SU(3)$ certainly cannot be exact, although often one likes to treat the vacuum as being invariant under an $SU(3)$ transformation. Then by the above result the symmetry cannot be implemented locally. Furthermore, even if one allows the vacuum to be noninvariant, there is another inconvenience when the symmetry is not exact, associated with the nonexistence of space integrals over the fourth component of nonconserved local currents.[2] Ordinarily these integrals are thought to define the symmetry generators. Since, in practice, we often use the symmetry operators to derive not only relations in the limit of perfect symmetry, but also deviations from perfect-symmetry results, such as the Gell-Mann–Okubo mass formula, to say that some of the symmetry operators are ill defined except in the limit of perfect symmetry is not very useful. This can be gotten around,[2] but is inconvenient.

For these reasons we would like to consider nonlocal symmetry operations that do not obey Eq. (1).

Since we have no *a priori* notions as to what kind of complicated nonlocal effects need be considered, we will start by looking for simple nonlocal symmetry transformations on the asymptotic fields.[3] Once these transformations are defined on the in-fields, then in principle the transformations of interacting fields are determined by the expansion of the interacting fields in the in-fields.

One type of nonlocal transformations that one may consider are those generated by the lightlike charges.[4] In the free-quark model with mass splitting, these charges still transform some components of the fields locally, but transform the other components nonlocally. The lightlike charges do not commute with some of the homogeneous Lorentz transformations. We will not study the nonlocal transformations generated by lightlike charges in this paper. Instead we will study a more trivial type of nonlocal symmetry, which acts simply on the in-fields, and which commutes with the homogeneous Lorentz group.

II. SYMMETRY OF THE ASYMPTOTIC FIELDS

To be specific, let us first consider just a set of pion in-fields, satisfying the equations of motion

$$\begin{aligned}(\Box + m_0{}^2)\phi_0(x) &= 0,\\ (\Box + m_1{}^2)\phi_\pm(x) &= 0,\end{aligned} \tag{2}$$

with the charged-pion mass m_1 not equal to the neutral-pion mass m_0. We omit the subscript "in" on the in-fields throughout this section, since all the fields are in-fields.

It is easy to write down the nonlocal and local conserved currents:

$$\frac{1}{\sqrt{2}}\hat{J}^{\pm}_{\mu}(x) \equiv \pm\left[\frac{m_0}{m_1}\phi_{\pm}\left(\frac{m_0}{m_1}x\right)\frac{\partial}{\partial x^{\mu}}\phi_0(x) - \phi_0(x)\frac{\partial}{\partial x^{\mu}}\frac{m_0}{m_1}\phi_{\pm}\left(\frac{m_0}{m_1}x\right)\right],$$

$$\hat{J}^{3}_{\mu}(x) \equiv \phi_+(x)\frac{\partial}{\partial x^{\mu}}\phi_-(x) - \phi_-(x)\frac{\partial}{\partial x^{\mu}}\phi_+(x).$$

Although the usual proof that the space integral of the fourth component of a conserved current determines a charge operator[5] does not hold for nonlocal currents, in this specific case one can verify that the integrals $\int d^3x\,\hat{J}^{\pm}_0(x)$ do define operators. We will denote them by $\hat{I}^{\pm}$ and $\hat{I}^3$, to distinguish them from the usual isospin operators. The point is that in $\hat{I}^{\pm}$ only the combination of creation and destruction operators $a^{\dagger}a$ appears, and not[6] the combinations aa and $a^{\dagger}a^{\dagger}$. This circumstance is analogous to the case of lightlike charges.

The charge operators $\hat{I}_i$ satisfy the commutation relations

$$[\hat{I}_i, \hat{I}_j] = i\epsilon_{ijk}\hat{I}_k,$$

and generate the group $SU(2)_{\text{in}}$. The transformations under $SU(2)_{\text{in}}$ of the in-fields, normalized by the usual convention $\langle 0|\phi_i(0)|\pi^i, \vec{p}\rangle (2\pi)^{3/2} = 1$, with $\langle \pi^i, \vec{p}|\pi^i, \vec{p}\,'\rangle = 2E_p\,\delta^3(\vec{p}-\vec{p}\,')$, are as follows:

$$\begin{aligned}
&[\hat{I}_3, \phi_0(x)] = 0, \quad [\hat{I}_3, \phi_{\pm}(x)] = \pm\phi_{\pm}(x),\\
&[\hat{I}_{\pm}, \phi_0(x)] = \pm\sqrt{2}\,\frac{m_0}{m_1}\phi_{\pm}\left(\frac{m_0}{m_1}x\right), \quad [\hat{I}_+, \phi_+(x)] = 0,\\
&[\hat{I}_+, \phi_-(x)] = -\sqrt{2}\,\frac{m_1}{m_0}\phi_0\left(\frac{m_1}{m_0}x\right), \quad [\hat{I}_-, \phi_-(x)] = 0,\\
&[\hat{I}_-, \phi_+(x)] = \sqrt{2}\,\frac{m_1}{m_0}\phi_0\left(\frac{m_1}{m_0}x\right).
\end{aligned} \tag{3}$$

We see that (a) the triplet of fields $\hat{\phi}_0(0) = \phi_0(0)$ and $\hat{\phi}_{\pm}(0) \equiv (m_0/m_1)\phi_{\pm}(0)$ transforms like an $\hat{I}=1$ multiplet under $SU(2)_{\text{in}}$, and (b) an $SU(2)_{\text{in}}$ rotation can be accompanied by a simultaneous scale transformation in the space-time coordinates. The corresponding transformations of the plane-wave states, whose normalization we have specified above, are

$$\begin{aligned}
&\hat{I}_+|\pi^-, \vec{p}\rangle = -\sqrt{2}\,(m_0/m_1)|\pi^0, (m_0/m_1)\vec{p}\rangle,\\
&\hat{I}_+|\pi^0, \vec{p}\rangle = \sqrt{2}\,(m_1/m_0)|\pi^+, (m_1/m_0)\vec{p}\rangle,\\
&\hat{I}_3|\pi^+, \vec{p}\rangle = |\pi^+, \vec{p}\rangle,
\end{aligned} \tag{4}$$

and similarly for other combinations.[7] In short, the states $|\hat{\pi}^0, \vec{p}\rangle \equiv |\pi^0, \vec{p}\rangle$, $|\hat{\pi}^+, (m_1/m_0)\vec{p}\rangle \equiv (m_1/m_0)|\pi^+, (m_1/m_0)\vec{p}\rangle$, and $|\hat{\pi}^-, (m_1/m_0)\vec{p}\rangle \equiv (m_1/m_0)|\pi^-, (m_1/m_0)\vec{p}\rangle$ transform like members of a triplet under $SU(2)_{\text{in}}$, and we have

$$\begin{aligned}
&\langle 0|\hat{\phi}_{\pi^0}(0)|\hat{\pi}^0, \vec{p}\rangle = \langle 0|\hat{\phi}_{\pi^{\pm}}(0)|\hat{\pi}^{\pm}, (m_1/m_0)\vec{p}\rangle,\\
&\langle \hat{\pi}^0, \vec{p}|\hat{\pi}^0, \vec{p}\,'\rangle = \langle \hat{\pi}^{\pm}, (m_1/m_0)\vec{p}|\hat{\pi}^{\pm}, (m_1/m_0)\vec{p}\,'\rangle.
\end{aligned} \tag{5}$$

The operators $\hat{I}_i$ act additively on multiparticle in-states. They commute with the Lorentz transformations, but, of course, not with translations. It is easy to check that the equations of motion (2), as well as the action $A = \int d^4x\,\mathcal{L}_0(x)$, where $\mathcal{L}_0$ is the free Lagrangian density with mass splitting, are invariant under $SU(2)_{\text{in}}$.

These considerations allow a straightforward extension to include fermions, such as the proton-neutron system. One finds, for example,

$$[\hat{I}_+, \psi_n(x)] = \left(\frac{M_n}{M_p}\right)^{3/2}\psi_p\left(\frac{M_n}{M_p}x\right).$$

The extension to $SU(3)_{\text{in}}$ is also straightforward. Thus the usual statement that the mass operator transforms like certain tensors in the derivation of the Gell-Mann–Okubo formula is, from our viewpoint, better formulated as the assumption that it transforms like these tensors under $SU(3)_{\text{in}}$. The $SU(3)_{\text{in}}$ generators exist in the presence of mass differences, so that there are no ambiguities, and the noncommutivity of these generators with translations, and hence with the (mass)^2 operator $P_{\mu}P^{\mu}$, is explicitly allowed for.

If we treat the vector mesons as particles also, and consider the space of in-states for the vector mesons, then, since they do not obey the Gell-Mann–Okubo mass formula very well, we must allow for the noncommutivity of the mass operator with the Casimir operators of $SU(3)_{\text{in}}$ as well. Thus the physical vector mesons will be a mixture of an $SU(3)_{\text{in}}$ singlet state $|\hat{s}\rangle$ with an $SU(3)_{\text{in}}$ octet state $|\hat{8}\rangle$. Since $SU(3)_{\text{in}}$ is to be well defined regardless of the noncommutivity with the mass operator, we must define $|\hat{8}\rangle$ and $|\hat{s}\rangle$ to be orthogonal; hence [cf. Eq. (5)]

$$\begin{aligned}
&|\hat{s}, \vec{p}\rangle \equiv \cos\theta|\hat{\omega}, \vec{p}\rangle + \sin\theta|\hat{\phi}, (m_{\phi}/m_{\omega})\vec{p}\rangle,\\
&|\hat{8}, \vec{p}\rangle \equiv -\sin\theta|\hat{\omega}, \vec{p}\rangle + \cos\theta|\hat{\phi}, (m_{\phi}/m_{\omega})\vec{p}\rangle.
\end{aligned}$$

In terms of an octet and a singlet vector fields, we have

$$\begin{aligned}
&\hat{\phi}^s_{\mu}(0) \equiv \cos\theta\,\hat{\omega}_{\mu}(0) + \sin\theta\,\hat{\phi}_{\mu}(0),\\
&\hat{\phi}^8_{\mu}(0) \equiv -\sin\theta\,\hat{\omega}_{\mu}(0) + \cos\theta\,\hat{\phi}_{\mu}(0),
\end{aligned} \tag{6}$$

which satisfy $\langle 0|\hat{\phi}^s_{\mu}(0)|\hat{8}, \vec{p}\rangle = \langle 0|\hat{\phi}^8_{\mu}(0)|\hat{s}, \vec{p}\rangle = 0$.

III. LEPTONIC DECAYS OF PSEUDOSCALAR AND VECTOR MESONS

It is clear that $SU(3)_{\text{in}}$ cannot commute with the physical S matrix having nontrivial interactions, because of the requirement of momentum conservation in reaction processes. Thus the generators of $SU(3)_{\text{in}}$ are in general different from those of $SU(3)_{\text{out}}$. It is an interesting but difficult dynamical question as to whether the generators of these two groups together satisfy any simple algebraic relations, or whether there exists another group which interpolates between $SU(3)_{\text{in}}$ and $SU(3)_{\text{out}}$. We do not know the answer. A more modest question is how the interacting fields transform under $SU(3)_{\text{in}}$, as far as the *one-particle* matrix elements $\langle 0|\phi(0)|1\rangle$ are concerned. Since a one-particle in-state is the same as a one-particle out-state, the action of $SU(3)_{\text{in}}$ on it is the same as that of $SU(3)_{\text{out}}$. We may ask whether the linear terms in the in-field expansion of a set of interacting fields, such as the weak and electromagnetic currents, are more $SU(3)$-symmetric in terms of $\hat{\phi}_{\text{in}}$ or in terms of the usual ϕ_{in}. Admittedly the difference is just in trivial relative normalizations, but numerically these differences can be quite large.

The vacuum to one-hadron matrix elements of the currrents that are measurable correspond to those that occur in K_{l2} and π_{l2}, and in the decay of vector mesons into lepton pairs (in the approximation where the vector resonances are treated as particles). With field-current identities[8] the vector currents J^μ_α are proportional to $m_\alpha{}^2\phi^\mu_\alpha$ or $[P^2, \phi^\mu_\alpha]$, where ϕ^μ_α denotes the vector-meson field of particle type α. In any case, we are only going to look at the one-particle matrix elements, and dimensionally the currents are of dimension $\Box\phi^\mu_\alpha$. We will therefore consider the terms linear in $\hat{\phi}_{\alpha,\text{in}}$ in a normal ordered in-field expansion of the usual vector currents V^μ_α, in the form

$$V^\mu_3(0) = c_8[P^2, \hat{\rho}^\mu_{\text{in}}(0)] + \cdots, \tag{7a}$$

$$V^\mu_8(0) = c_8[P^2, \hat{\phi}^\mu_{8,\text{in}}(0)] + \cdots, \tag{7b}$$

$$V^\mu_0(0) = c_0[P^2, \hat{\phi}^\mu_{s,\text{in}}(0)] + \cdots. \tag{7c}$$

We wish to see whether it is possible to have the same constant c_8 for the whole octet. Similarly, for the weak axial-vector currents which appear in the effective weak Hamiltonian in the form

$$H_{\text{wk}} = \frac{G}{\sqrt{2}}\,[(\mathcal{V}^{\pi^+} + \mathcal{A}^{\pi^+}) + (\mathcal{V}^{K^+} + \mathcal{A}^{K^+}) + l^+]\cdot[\text{H.c.}],$$

we consider the symmetric form for the one-pseudoscalar-meson terms, with the same constant f:

$$\mathcal{A}^{\pi^+}_\mu(0) = f\partial_\mu \hat{\pi}^+_{\text{in}}(0) + \cdots,$$
$$\mathcal{A}^{K^+}_\mu(0) = f\partial_\mu \hat{K}^+_{\text{in}}(0) + \cdots, \tag{8}$$

where $\partial_\mu\hat{\phi}(0) \equiv i[P_\mu, \hat{\phi}(0)]$. Equation (7) is to be compared with the usual definition of the five parameters f_ρ, f_Y, f_B, and θ_Y, θ_B through

$$V^\mu_3(0) = f_\rho{}^{-1} m_\rho{}^2 \rho^\mu_{\text{in}}(0) + \cdots, \tag{9a}$$

$$V^\mu_8(0) = \tfrac{1}{2}\sqrt{3}\,Y^\mu(0) = \tfrac{1}{2}\sqrt{3}\,f_Y{}^{-1}[\cos\theta_Y\, m_\phi{}^2\phi^\mu_{\text{in}}(0) - \sin\theta_Y\, m_\omega{}^2\omega^\mu_{\text{in}}(0)] + \cdots, \tag{9b}$$

$$V^\mu_0(0) = \tfrac{3}{2}B^\mu(0) = \tfrac{3}{2}f_B{}^{-1}[\sin\theta_B\, m_\phi{}^2\phi^\mu_{\text{in}}(0) + \cos\theta_B\, m_\omega{}^2\omega^\mu_{\text{in}}(0)] + \cdots. \tag{9c}$$

Comparison of Eqs. (7b) and (7c) with Eqs. (9b) and (9c) shows that

$$\begin{aligned} \tan\theta &= \tan\theta_Y(m_\omega/m_\phi), \\ \tan\theta &= \tan\theta_B(m_\phi/m_\omega), \\ \tan\theta_Y &= \tan\theta_B(m_\phi/m_\omega)^2. \end{aligned} \tag{10}$$

Equation (10) has been derived from Weinberg's first sum rule,[9] and is also consistent with the current-mixing model.[8] Oakes and Sakurai have emphasized[9] that Eq. (10) means that the orthogonality of the transformation between singlet and octet states, and ω and ϕ states, can no longer be maintained. We see that, from our viewpoint, on the contrary Eq. (10) is a consequence of this orthogonality as expressed under Eq. (6). The transformation between $(\hat{\phi}_s, \hat{\phi}_8)$ and $(\hat{\omega}, \hat{\phi})$ is orthogonal, although that between (V_0, V_8) and $(\hat{\omega}, \hat{\phi})$ is not because of the P^2 factor in Eqs. (7). Equations (7a) and (7b) also imply a relation between f_ρ and f_Y, yielding a relation between decay rates,

$$\tfrac{1}{3}m_\rho\Gamma(\rho\to ee) = m_\omega\Gamma(\omega\to ee) + m_\phi\Gamma(\phi\to ee). \tag{11}$$

This relation has also been derived from Weinberg's first sum rule.[10] Both Eq. (10) and Eq. (11) are in reasonable agreement with the experiments.[11] We see that both follow naturally if the relations between $V^\mu_\alpha(0)$ and $\hat{\phi}^\mu_{\alpha,\text{in}}(0)$ are of the symmetric form in Eqs. (7).

Equations (8) are to be compared with the usual relations in terms of Cabibbo's angle θ_A, and the PCAC (partially conserved axial-vector current) constants f_π and f_K,

$$\mathcal{A}^{\pi^+}_\mu(0) = \cos\theta_A\, f_\pi\partial_\mu\pi^+_{\text{in}}(0) + \cdots,$$
$$\mathcal{A}^{K^+}_\mu(0) = \sin\theta_A\, f_K\partial_\mu K^+_\mu(0) + \cdots.$$

Comparison of the relative ratios gives

$$\frac{m_\pi}{m_K} = (\tan\theta_A)\frac{f_K}{f_\pi}. \tag{12}$$

Equation (12) is also reasonably accurate numerically.[12]

We now proceed to evaluate critically the significance of the numerical agreements between Eqs. (7) and (8) and the experimental results.

(a) The first question is how arbitrary are Eqs. (7) and (8). These equations are based on the criterion that no symmetry breaking is to be introduced through multiplicative constants, c_8 and f being the same within each octet. Deviations from symmetry enters through the P^2 and P_μ factors only, that is, through the noncommutivity of $SU(3)_{in}$ with translations. Although any arbitrariness in powers of P^2 will correspond to multiplicative factors of mass ratios, covariance and locality require that the arbitrariness can at most correspond to integral powers of P^2, and hence to even powers of mass ratios.

In the case of Eqs. (7), not only is the power of P^2 appearing there (i.e., first power) rather "natural" as argued before, but also, since the difference between $\hat{\phi}_{in}$ and ϕ_{in} is proportional to one power of mass ratios, *it is not possible* to write an equation corresponding to Eqs. (7) using ϕ_{in} instead of $\hat{\phi}_{in}$, and a universal c_8', that will lead to Eq. (10) and Eq. (11). Thus, as far as the one-vector-meson matrix elements of the vector currents are concerned, $SU(3)_{in}$ does appear to be a better symmetry than the ordinary $SU(3)$, and this is independent of any possible arbitrariness in the powers of P^2 in Eq. (7). At the very least, the formulation in terms of Eq. (7) is simple and requires fewer distinct parameters to adequately describe the experimental results.

The situation is somewhat different with respect to Eqs. (8). The powers of P^2 and P_μ appearing in Eqs. (8) also seem reasonable enough. However, while $\mathfrak{A}_\mu^\pi$ and $\mathfrak{A}_\mu^K$ are the currents which appear naturally in the weak-interaction Hamiltonian, the success of the Cabibbo theory and current algebra imply that it is on the other hand $(\cos\theta_A)^{-1}\mathfrak{A}_\mu^\pi$ and $(\sin\theta_A)^{-1}\mathfrak{A}_\mu^K$ which belong to the same octet with proper relative normalizations under the *local* $SU(3)$. As long as f_K/f_π is approximately unity, then not only the equations

$$\begin{aligned} \mathfrak{A}_\mu^\pi &= f\partial_\mu\hat{\pi}_{in} + \cdots, \\ \mathfrak{A}_\mu^K &= f\partial_\mu\hat{K}_{in} + \cdots, \end{aligned} \tag{8}$$

with the same f, are in good agreement with the experiments, but also the equations

$$\begin{aligned} (\cos\theta_A)^{-1}\mathfrak{A}_\mu^\pi &= c\partial_\mu\pi_{in} + \cdots, \\ (\sin\theta_A)^{-1}\mathfrak{A}_\mu^K &= c\partial_\mu K_{in} + \cdots, \end{aligned} \tag{8$'$}$$

with the same c, are in approximate agreement with the experiments. For this reason, although Eqs. (8) do give better results, we do not argue that the agreement between Eqs. (8) and the K_{l2}/π_{l2} decay-rate ratio is a strong indication that $SU(3)_{in}$ is a better symmetry for the one-pseudoscalar-meson component of the axial currents. Rather, we regard Eqs. (8) and (8$'$) as suggesting an interesting viewpoint that the need for the Cabibbo angle arises from mediating between the nonlocal $SU(3)_{in}$ and the usual local $SU(3)$. From this viewpoint the significance of the relation (12) is not to suggest an alternative to the Cabibbo theory (as attempted in one of the papers in Ref. 12), but rather to establish a connection between the magnitude of the Cabibbo angle and the mass ratio which appear naturally in the relations between $\hat{\phi}_{in}$ and ϕ_{in}.

(b) The main difficulty at the present time of further testing the usefulness of the $SU(3)_{in}$ symmetry is to go beyond the vacuum to one-particle matrix elements. Some higher matrix elements, such as those of the weak vector currents between pseudoscalar mesons relevant for K_{l3} and π_{l3} decays, can still be reduced to vacuum to one-particle matrix elements at the soft-pion point, if one assumes some current commutation relations. Equations (8) can then lead to results for K_{l3} and π_{l3} essentially equivalent to those in the one-angle Cabibbo theory. However, to deal with symmetry relations among genuine three-point and higher-point functions, one must be able to either write down the nonlinear terms in the in-field expansion of interacting fields, or have some algebraic relations between the generators of $SU(3)_{in}$, $SU(3)_{out}$, and the Poincaré group. So far we have been unable to arrive at a satisfactory formulation.

(c) Equations (7) and (8) have been written for $x=0$. More generally, if one defines $\hat{\phi}_{in}(x) = e^{iPx}\phi_{in}(0)e^{-iPx}$, one will have, for example,

$$\begin{aligned} \mathfrak{A}_\mu^\pi(x) &= f\partial_\mu\hat{\pi}_{in}(x) + \cdots, \\ \mathfrak{A}_\mu^K(x) &= f\partial_\mu\hat{K}_{in}(x) + \cdots. \end{aligned}$$

Since $\hat{\pi}_{in}(x)$ and $\hat{K}_{in}((m_\pi/m_K)x)$ belong to the same octet, according to the discussion in Sec. II, to have the one-meson components in the same $SU(3)_{in}$ multiplet one must compare $\mathfrak{A}_\mu^\pi(x)$ not with $\mathfrak{A}_\mu^K(x)$, but with $\mathfrak{A}_\mu^K((m_\pi/m_K)x)$. This may appear to some as very unnatural. We wish to emphasize, however, that this circumstance is entirely reasonable from our viewpoint. After all, the one-meson in-field components in the axial currents are relevant for such matrix elements as $\langle 0|\mathfrak{A}_\mu^\pi|\pi\rangle$ and $\langle 0|\mathfrak{A}_\mu^K|K\rangle$. An internal symmetry is useful because it relates such matrix elements. However, there is no way to relate $\langle 0|\mathfrak{A}_\mu^\pi(x)|\pi,\vec{p}\rangle$ to $\langle 0|\mathfrak{A}_\mu^K(x)|K,\vec{p}\rangle$ for general x with only Clebsch-Gordan coefficients, since these must correspond to *different functions of* x no matter what $\vec{p}$ and $\vec{p}\,'$ are. (This is true as long as the three-momenta are not infinite. The use of an infinite-momentum frame is related to the lightlike charges, which also generate nonlocal transformations.) On the other hand, $\langle 0|\mathfrak{A}_\mu^\pi(x)|\pi,\vec{p}\rangle$ and $\langle 0|\mathfrak{A}_\mu^K((m_\pi/m_K)x)|K,(m_K/m_\pi)\vec{p}\rangle$ correspond to the same function of x, and therefore at least have a chance of being simply related for all x. Even when the symmetry is inexact, in practice one often deals with the first-order approximation in which one-particle states transform into one another. Presumably one would like the first-order approximation to be as good as possible. From this viewpoint we see that so far as the one-meson contributions are concerned, it is natural to compare $\mathfrak{A}_\mu^\pi(x)$ and $\mathfrak{A}_\mu^K((m_\pi/m_K)x)$. Once we do consider such nonlocal relations and the group of such transformations, we see that a definite multiplicative factor of mass ratio must appear concurrently with each nonlocal transformation, and this leads to some interesting consequences, as we have discussed in Sec. III.

ACKNOWLEDGMENTS

I wish to thank Alex Dragt and Claudio Orzalesi for a discussion. I am also grateful to Professor S. Oneda for explaining to me his work with Umezawa on asymptotic symmetries, and to Professor G. F. Dell'Antonio for useful comments.

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*Supported in part by the U. S. Air Force under grant AFOSR 68-1453A.

[1]L. J. Landau and E. H. Wichmann, J. Math. Phys. 11, 306 (1970). This is in some respects a global version of Coleman's theorem [S. Coleman, J. Math. Phys. 7, 787 (1966)], but stronger in the sense that it does not require the identification of symmetry generators with space integrals of currents.

[2]D. Kastler, D. W. Robinson, and A. Swieca, Commun. Math. Phys. 2, 108 (1966); B. Schroer and P. Stichel, *ibid.* 3, 258 (1966). See also the review article by C. Orzalesi, Rev. Mod. Phys. 42, 381 (1970).

[3]Although there might be nonlocal transformations on the interacting fields which are local on the in-fields, we do not consider this possibility; we will allow mass splittings within a multiplet of one-particle in-states.

[4]K. Bardakci and G. Segrè, Phys. Rev. 159, 1263 (1967); L. Susskind, *ibid.* 165, 1535 (1967); H. Leutwyler, *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer-Verlag, Berlin, 1968), Vol. 50, p. 29. Other references can be found from the last article.

[5]See Ref. 2.

[6]More precisely, terms of the form $a^\dagger a^\dagger$ in the space integral of $\hat{J}_0^\pm$ do not connect two-particle quasilocal states to the vacuum. One also notes that the integrals $\int d^4x f(x)\hat{J}_0^\pm(x)$ commute with fields $A(y)$, if y is spacelike not only to the support O of the test function f, but also to the region O' obtained from O by the scale transformation $x \rightarrow (m_0/m_1)x$. Consequently, the space integrals $\int d^3x \hat{J}_0^\pm(x)$ define bounded bilinear forms on a dense set, and can be extended to operators.

[7]Transformations of this type preserving the four-velocity seem to have first been considered by J. Formanek, Nuovo Cimento 43A, 741 (1966), who postulates algebraic relations between these transformations and the momentum operators.

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[9]R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters 19, 1266 (1967).

[10]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 19, 470 (1967); J. J. Sakurai, *ibid.* 19, 803 (1967).

[11]U. Becker *et al.*, Phys. Rev. Letters 21, 1504 (1968); J. E. Augustin *et al.*, Phys. Letters 28B, 503 (1968).

[12]There have been earlier attempts to justify this relation; see M. Gell-Mann, Phys. Rev. 125, 1067 (1962); R. Oehme, Ann. Phys. (N. Y.) 33, 108 (1965); T. Pradhan and M. Patnaik, Phys. Rev. D1, 200 (1970); A. Böhm and E. C. G. Sudarshan, Phys. Rev. 178, 2264 (1969). I wish to thank Professor Ivan Todorov for bringing this last reference to my attention, and for pointing out some similarities between their work and ours.