Comments and Addenda

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in **Physical Review Letters** *and are not appropriate for regular Articles. It includes only the following types of communications: (1) comments on papers previously published in* **The Physical Review** *or* **Physical Review Letters**; *(2) addenda to papers previously published in* **The Physical Review** *or* **Physical Review Letters**, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section should be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.*

Comment on a Proof of the $Z=0$ Condition for Compositeness

E. Tirapegui

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
and Department of Mathematics, Rutgers – The State University, New Brunswick, New Jersey 08903

(Received 5 August 1971)

A proof of the $Z=0$ condition for compositeness is examined. In a previous criticism of this proof a paradoxical result was pointed out. It is proved that this result can be understood by precisely explaining the meaning of the equality of the elementary field and the Haag-Nishijima-Zimmermann field for the composite bound state.

It has been claimed by Fried and Jin[1] that Jouvet's $Z=0$ condition for compositeness[2] can be obtained by a rigorous argument when the Haag-Nishijima-Zimmermann (HNZ) field[3] coincides with the elementary field. Before discussing their argument let us briefly recall the HNZ construction of a local field. If in a field theory we have a one-particle state $|\phi_{\vec{p}}\rangle$ and if some product of the fields, for instance $\phi_a(x)\phi_b(y)$, is such that $\langle 0|\phi_a(x)\phi_b(y)|\phi_{\vec{p}}\rangle \neq 0$, then we can construct an HNZ field

$$\hat{\phi}(x)=\lim_{\xi\to 0;\ \xi\ \text{spacelike}} \frac{T(\phi_a(x-\frac{1}{2}\xi)\phi_b(x+\frac{1}{2}\xi))}{\langle 0|\phi_a(-\frac{1}{2}\xi)\phi_b(\frac{1}{2}\xi)|\phi_{\vec{p}}\rangle} \qquad (1)$$

for the particle represented by $|\phi_{\vec{p}}\rangle$. (See W. Zimmermann, Ref. 3, for details.) However, this construction is by no means unique; any other product of fields with a nonvanishing matrix element between the physical vacuum $|0\rangle$ and $|\phi_{\vec{p}}\rangle$ gives another HNZ field – this multiplicity being just a consequence of Borchers's theorem[4] on the nonuniqueness of the interpolating fields in field theory. One usually calls "the" HNZ field the simplest one, namely (1).

Now, the argument of Fried and Jin consists in calculating the vacuum expectation value $\langle 0|[\hat{\phi}(\vec{x},t), \partial_t\hat{\phi}(\vec{x}\,',t)]|0\rangle = iB\delta(\vec{x}-\vec{y})$, and in showing that $B=\infty$. Since it is known that for an elementary field, B is the inverse of the wave-function renormalization constant of the field, they conclude from $B=\infty$ that the renormalization constant of the field $\hat{\phi}(x)$ vanishes. There are two ways of interpreting this result. The first one is to consider that there is no elementary field corresponding to the state $|\phi_{\vec{p}}\rangle$ in the theory. Then (1) is just a definition of a local "composite" field for that state and the Fried and Jin calculation would be correct with their definition of Z, for a composite particle, as B^{-1}. This conclusion is valid if expression (3) of their work is divergent. We note here that it can be explicitly verified in solvable models that this need not be the case. (See Ref. 5, p. 199, remark a.) Let us also remark that the commutator $[\hat{\phi}(\vec{x},t), \partial_t\hat{\phi}(\vec{x}\,',t)]$ is not in general a c number in this case, as can be seen by direct computation (for instance in the Lee model), and that B^{-1} does not correspond to the renormalization constant one usually considers in the $Z=0$ theory. Indeed the content of the $Z=0$ condition has to do with the fact that it relates a theory in which there is no elementary field corresponding to a particle state $|\phi_{\vec{p}}\rangle$, and another theory in which there is an elementary field corresponding to $|\phi_{\vec{p}}\rangle$. It is the renormalization constant of the elementary field in the second theory which vanishes in the composite limit as we have discussed in detail in a recent paper.[6] This distinction is partially made by Osborn[7] in a discussion of the Fried and Jin paper but we do not agree completely with his results. We will return to this particular point elsewhere.

The second way of interpreting Ref. 1 is to suppose that there is in the theory an elementary field $\phi(x)$ corresponding to the state $|\phi_{\vec{p}}\rangle$, and that $\phi(x)=\hat{\phi}(x)$. It seems to us that this is the point of view of Fried and Jin since on page 1153 of their work they state that the divergence of their formula (3) has been used by Nishijima to prove relation (1) of their paper. With the first interpretation, relation (1) is just a definition of a local field for the state $|\phi_{\vec{p}}\rangle$ and there is nothing to prove. This second point of view has been taken by Cordero[8] and by Brandt *et al.*[9] in their analysis of Ref. 1. In Ref. 9 the study is done in the $N\theta$ sector of the Lee model, which is sufficient for clearly understanding the situation. We consider the Lee model with a stable V particle and we recall that the self-mass $\delta\nu=Z(m_{V_0}-m_V)$ and the renormalization constant Z of the V field are given by (for calculations and notation see Refs. 5 and 10)

$$\delta\nu=\frac{g^2}{4\pi^2}\int_\mu^\infty d\omega\,\frac{|f(\omega)|^2(\omega^2-\mu^2)^{1/2}}{\omega+m_N-m_V}, \tag{2}$$

$$Z=1-\frac{g^2}{4\pi^2}\int_\mu^\infty d\omega\,\frac{|f(\omega)|^2(\omega^2-\mu^2)^{1/2}}{(\omega+m_N-m_V)^2}. \tag{3}$$

Let us denote by $\psi_V(x)$, $\psi_N(x)$, and $A(x)$ the fields of the V, N, and θ particles, respectively. One has $\langle 0|\psi_N A|V_{\vec{p}}\rangle\neq 0$, where $|V_{\vec{p}}\rangle$ is the one-particle state of particle V, so we can construct an HNZ field $\hat{\psi}_V(x)$ for this state which in momentum space will be given by [see also formula (6) of Ref. 9]

$$\hat{b}_V(\vec{p},t)=\frac{g}{4\pi^{3/2}\delta\nu}\int d^3q\,d^3k\,\delta(\vec{p}-\vec{q}-\vec{k})\frac{f(\omega_k)}{\sqrt{\omega_{\vec{k}}}}\times b_N(\vec{q},t)a(\vec{k},t). \tag{4}$$

Now, Brandt *et al.* remark that the proof of Fried and Jin is erroneous due to dubious interchange of limits. The fact that the Fried and Jin proof is not correct is clearly exhibited in their paper, but the reason why this is so is not given. What they have done is to reproduce the proof in the Lee model. They first remark that when $\delta\nu=\infty$ and $Z\neq 0$ it follows from the equation of motion of the V particle,

$$Z\left(i\frac{\partial}{\partial t}-m_V\right)b_V(\vec{p},t)=\delta\nu\, b_V(\vec{p},t)-\frac{g}{4\pi^{3/2}}\int d^3q\,d^3k\,\delta(\vec{p}-\vec{q}-\vec{k})\frac{f(\omega_{\vec{k}})}{\sqrt{\omega_{\vec{k}}}}\,b_N(\vec{q},t)a(\vec{k},t), \tag{5}$$

that $b_V(\vec{p},t)=\hat{b}_V(\vec{p},t)$. The next step is to calculate

$$\langle 0|\{\hat{b}_V(\vec{p},t),\hat{b}_V^\dagger(\vec{p}\,',t)\}|0\rangle=B\delta(\vec{p}-\vec{p}\,'),$$

to put $B=Z^{-1}$ because of the relation $b_V=\hat{b}_V$, and to conclude that if $B=\infty$, then $Z=0$. This last step can be done exactly in this model using (4) and one obtains

$$\begin{aligned}\langle 0|\{\hat{b}_V(\vec{p},t),\hat{b}_V^\dagger(\vec{p},t)\}|0\rangle &=\delta(\vec{p}-\vec{p}\,')\left(\frac{g}{2\pi\delta\nu}\right)^2\int_0^\infty dk\,\frac{k^2|f(k)|^2}{\omega_k}\\ &=B\delta(\vec{p}-\vec{p}\,').\end{aligned} \tag{6}$$

One can then verify that if the form factor $f(k)$ is such that $|f(k)|\to k^{\alpha/2}$ as $k\to\infty$, with $-1\leqslant\alpha<0$, then from (3), $Z\neq 0$; from (2), $\delta\nu=\infty$; and from (6), $B=\infty$. So we arrive at a paradox that clearly invalidates the Fried and Jin proof. Let us see now how this situation arises. Defining

$$\rho(p)=(2\pi)^3\sum_Z|\langle 0|\psi_V(0)|Z\rangle^2\delta(p-p_Z),$$

and similarly defining $\hat{\rho}(p)$ (replacing ψ_V by $\hat{\psi}_V$), one easily shows that

$$\langle 0|\{\hat{b}_V(\vec{p},t),\hat{b}_V^\dagger(\vec{p}\,',t)\}|0\rangle=\delta(\vec{p}-\vec{p}\,')\int\hat{\rho}(a)da$$

and $Z^{-1}=\int\rho(a)da$ when Z as defined by (3) does not vanish. [This is the case discussed in Ref. 9; see also Eq. (5).] One also has that propagator $S_R'(s)$ in momentum space, i.e., the Fourier transform of $\langle 0|T(\psi_V(x)\psi_V^\dagger(y))|0\rangle$, is given by

$$\int\rho(a)(s+a-i\epsilon)^{-1}da,$$

and the corresponding formula holds for $\hat{S}_R'(s)$ with $\hat{\psi}_V$ and $\hat{\rho}$ replacing ψ_V and ρ. We can then compute $\sigma(a)=\rho(a)-\delta(a-m_V)$ and $\hat{\sigma}(a)=\hat{\rho}(a)-\delta(a-m_V)$ by calculating the imaginary parts of the propagators $S_R'(s)$ and $\hat{S}_R'(s)$ for $-s>m_N+\mu$. One obtains

$$\hat{\sigma}(-s)=\sigma(-s)\,\frac{(s+m_{V_0})^2}{(m_{V_0}-m_V)^2}. \tag{7}$$

For the purpose of our discussion let us everywhere replace the form factor $f(\omega)$ by $\tilde{f}(\omega)=f(\omega)\theta(\Lambda-\omega)$, and let us denote (when necessary to avoid confusion) the new quantities so defined by $\delta\nu_\Lambda$, Z_Λ, etc. It is clear that we will recover the original theory when $\Lambda\to\infty$. Moreover we suppose that $|f(k)|\to k^{\alpha/2}$ as $k\to\infty$, $-1\leqslant\alpha<0$, so that when $\Lambda\to\infty$ we will have $Z\neq 0$ and $\delta\nu=\infty$. In this limiting procedure we keep the observable mass m_V fixed and finite, and we see then from $\delta\nu=Z(m_{V_0}-m_V)$ that $m_{V_0}(\Lambda)\to\infty$ when $\Lambda\to\infty$.

Now, from (7) we deduce that when $\Lambda\to\infty$ we have $\lim\hat{\sigma}(a)=\sigma(a)$; then $Z^{-1}=\int\lim\hat{\rho}(a)da$, which is finite, and we do not obtain for the left-hand side of (6) the result $B\delta(\vec{p}-\vec{p}\,')$ [where B is defined by the right-hand side of (6) and is divergent in the limit]. But if we integrate first, i.e., we compute $\int\hat{\rho}(a)da$ before taking the limit $\Lambda\to\infty$, we see from (7) that the integral will diverge. More precisely the divergent contribution to the integral will come from the term $(m_{V_0}{}^2)^{-1}\int\rho(a)a^2da$ which will behave in the limit as $\Lambda^{-\alpha}$ for $-1<\alpha<0$, and as $\Lambda(\ln\Lambda)^{-2}$ for $\alpha=-1$, which is precisely the behavior of B computed from (6) in the limit $\Lambda\to\infty$. [We note that we have used the facts that $\rho(a)\sim a^{\alpha-1}$ as $a\to\infty$ and $m_{V_0}\sim\Lambda^{\alpha+1}$ for $-1<\alpha<0$, $m_{V_0}\sim\ln\Lambda$ for $\alpha=-1$, as can be seen from (2).] This then shows that the computation of the vacuum expectation value of the anticommutator is ambiguous, and that one does not necessarily obtain Z^{-1} from (6) [see Ref. 9, formula (7)]. Let us now explain the origin of this difficulty.

The reason for expecting that (6) should give $Z^{-1}\delta(\vec{p}-\vec{p}\,')$ is the equality $b_V=\hat{b}_V$ that Brandt *et al.* deduce from the equation of motion (5) when $\delta\nu=\infty$. Indeed this relation can be shown to be valid in perturbation theory to all orders for the perturbation expansions of the vacuum expectation values of T products of the fields.[11] But one must be careful to interpret $b_V=\hat{b}_V$ in other cases; indeed, as has already been remarked by Zimmermann,[12] this equality can only hold for matrix elements between vectors $|\alpha\rangle$ and $|\beta\rangle$ such that the left-hand side of the equation of motion (5) divided by $\delta\nu_\Lambda$ tends to zero when $\Lambda\to\infty$. Calling $|N_{\vec{q}},\theta_{\vec{k}}\rangle^{(\pm)}$ the scattering states of the $N\theta$ sector (see Ref. 10) and $F(t)$ the left-hand side of (5), one easily verifies that the matrix elements

$$\langle 0|F(t)|V_{\vec{p}}\rangle,\quad \langle 0|F(t)|N_{\vec{q}},\theta_{\vec{k}}\rangle,\quad \langle V_{\vec{p}'}|F(t)|V_{\vec{p}}\rangle,$$

$$\langle N_{\vec{q}'},\theta_{\vec{k}'}|F(t)|N_{\vec{q}},\theta_{\vec{k}}\rangle,\quad\text{and } \langle V_{\vec{p}}|F(t)|N_{\vec{q}},\theta_{\vec{k}}\rangle$$

all give zero in the limit $\Lambda\to\infty$ when divided by $\delta\nu_\Lambda$, so that $b_V=\hat{b}_V$ holds in the weak sense we have explained in these cases. Furthermore, in the definition of the spectral function $\rho(a)$ only the matrix elements between $|0\rangle$ and $|V_{\vec{p}}\rangle$ and $|N_{\vec{q}},\theta_{\vec{k}}\rangle$ appear, so that we must then obtain the same function when computing it with $\hat{b}_V$ instead of b_V, as we have directly verified [recall that $\rho(a)=\lim\hat{\rho}(a)$].

Let us consider now the case of the anticommutator. The question is whether we are allowed to replace b_V by $\hat{b}_V$ in the expression

$$\langle 0|\{b_V(\vec{p},t),b_V^\dagger(\vec{p}\,',t')\}|0\rangle|_{t=t'}.$$

We first remark that $\langle\{b,b^{\dagger\prime}\}\rangle=\langle bb^{\dagger\prime}\rangle$, where we have used a convenient short-hand notation. From the equation of motion (5) we see that $b=\hat{b}+F(t)/\delta\nu_\Lambda$, and using this relation we obtain

$$\langle bb^{\dagger\prime}\rangle=\langle\hat{b}\hat{b}^{\dagger\prime}\rangle+\left\langle\frac{F(t)}{\delta\nu_\Lambda}b^{\dagger\prime}\right\rangle+\left\langle b\frac{F(t')^\dagger}{\delta\nu_\Lambda}\right\rangle-\left\langle\frac{F(t)}{\delta\nu_\Lambda}\frac{F(t')^\dagger}{\delta\nu_\Lambda}\right\rangle. \tag{8}$$

For the relation $b=\hat{b}$ to hold in this case, the sum of the last three terms on the right-hand side of (8) should vanish when $\Lambda\to\infty$. But one easily verifies that the first two remain finite in the limit, while the last one diverges. In fact, this is necessary since we know by direct computation that $\langle\hat{b}\hat{b}^{\dagger\prime}\rangle=B=\infty$, so that another infinite quantity must arise to cancel B and give the finite results Z^{-1} for $\langle bb^{\dagger\prime}\rangle$. Let us show that the last term diverges in the same way as B (and in the same way as $\lim\int\hat{\rho}(a)da$, as expected). We get for the leading divergence

$$\left\langle\frac{F(t)}{\delta\nu_\Lambda}\frac{F(t')^\dagger}{\delta\nu_\Lambda}\right\rangle\sim\frac{1}{(\delta\nu_\Lambda)^2}\int da\,\rho(a)(a-m_V)^2\sim\frac{1}{(\delta\nu_\Lambda)^2}\int^\Lambda da\,a|f(a)|^2, \tag{9}$$

where we have used $\rho(a)\sim a^{-1}|f(a)|^2$ as $a\to\infty$. Comparison with B as given by (6) proves our assertion. We conclude then that we cannot replace b_V by $\hat{b}_V$ in the anticommutator, and that consequently one should expect the result $B\neq Z^{-1}$.

ACKNOWLEDGMENTS

The author is grateful to Professor B. Jouvet for many interesting suggestions and for crucial help. He is also indebted to Professor A. S. Wightman for an interesting discussion, and to Professor R. Hermann for his interest and continuous encouragement.

*Research sponsored in part by U. S. Air Force Office of Scientific Research under Contract No. F44620-71-C-0108.

[1] H. M. Fried and Y. S. Jin, Phys. Rev. Letters 17, 1152 (1966).

[2] B. Jouvet, Nuovo Cimento 5, 1 (1957).

[3] R. Haag, Phys. Rev. 112, 669 (1958); K. Nishijima, Phys. Rev. 111, 995 (1958); W. Zimmermann, Nuovo Cimento 10, 597 (1958).

[4] H. J. Borchers, Nuovo Cimento 15, 784 (1960).

[5] J. C. Houard, Nuovo Cimento 35, 194 (1965).

[6] E. Tirapegui, Nucl. Phys. (to be published).

[7] H. Osborn, Phys. Rev. Letters 19, 192 (1967).

[8] P. Cordero, Nuovo Cimento B60, 217 (1969).

[9] R. A. Brandt, J. Sucher, and C. H. Woo, Phys. Rev. Letters 19, 801 (1967).

[10] B. Jouvet and J. C. Houard, Nuovo Cimento 18, 466 (1960).

[11] K. Nishijima, Phys. Rev. B133, 204 (1964).

[12] W. Zimmermann, Commun. Math. Phys. 8, 66 (1968).